An Intermediation-Based Model of Exchange Rates

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Abstract

We develop a continuous time general equilibrium model with intermediaries at the heart of international financial markets. Global intermediaries bargain with households and extract rents from providing access to foreign claims. By tilting state prices, intermediaries' market power breaks monetary neutrality and makes international risk-sharing inefficient. Despite having zero net positions, markups charged by intermediaries significantly distort international asset prices, affecting exchange rate dynamics and their response to shocks. Our model can reproduce patterns consistent with several well-known exchange rate puzzles, such as deviations from Uncovered and Covered Interest Parity. All equilibrium quantities are derived in closed form, allowing us to pin down the underlying economic mechanisms explicitly.

Keywords: Financial Intermediation, Exchange Rates, Uncovered Interest Parity, Covered Interest Parity Deviations

JEL Classification Numbers: E44, E52, F31, F33, G13, G15, G23

1 Introduction

This paper develops a macroeconomic general equilibrium model in which international financial markets are subject to intermediation frictions. Intermediaries use their market power in a tiered market structure to charge markups for providing their clients with access to foreign financial instruments. These markups lead to demand imbalances and, by tilting state prices that investors in financial markets face, make international risk-sharing inefficient. We characterize the resulting endogenous non-linear dynamics of exchange rates in closed form for any number of shocks and any nature of shock dynamics. A calibration exercise shows that the model can help explain several well-known exchange rate puzzles.

Intermediaries are central to the functioning of international financial markets. Because of various frictions such as transaction costs, regulation, and costly information, asset trading exhibits increasing returns to scale, which makes it sub-optimal for most households to participate directly in financial markets. Instead, most individuals rely on intermediaries such as broker-dealers, commercial banks, pension funds, and mutual funds for borrowing and saving. The same economies of scale give intermediaries market power, allowing them to charge compensation in the form of markups, as reflected in various intermediation spreads.[1]

It is, therefore, natural to ask what this market power of intermediaries implies for

[1]In OTC markets, an identical asset is typically traded at different prices at a given point in time, depending on the identity of the trading counterparties. Trading in such markets is subject to frictions, whereby a handful of global intermediaries exert significant market power. For example, Hau et al. (2017) provide evidence for significant rent extraction in the FX derivatives markets. According to Hau et al. (2017), "A corporate client at the 75th percentile of average transaction costs pays a roughly 12 times larger spread than a corporate client at the 25th percentile." Wallen (2020) finds that markups are responsible for a significant component of spreads in FX markets, whereas Aldasoro et al. (2020) show a significant impact of market power and markups in dollar funding markets for foreign banks.

aggregate risk-sharing—a question this paper studies in the international context. Although much of the existing research has focused on intermediary balance sheet constraints, in our model, we purposely abstract from these important frictions and take the view of intermediaries as match-makers with significant bargaining power. In particular, our model captures the recent move away from a "principal-based model" of market-making (where dealers hold a non-trivial amount of open risk positions on their balance sheet) to an "agency model," in which they seek to economize on usage of their balance sheet by immediately offsetting trade with one client against opposing trading interest by another client (see, e.g., Adrian et al., 2017b; Fender and Lewrick, 2015).

To study the effect of intermediation markups on the macro economy and exchange rates, we introduce an imperfectly competitive intermediation sector into a classical, two-country, international cash-in-advance model similar to Lucas (1982). Each country is populated by a continuum of households that have direct access to trading domestic nominal risk-free bonds and a domestic Lucas tree (the claim on domestic output). However, the trading of all other securities happens over-the-counter (OTC) in the dealer-to-customer (D2C) market through global intermediary firms that are, in turn, owned by households. Upon contact, intermediaries take into account households' optimal demand for foreign financial asset exposures and use their bargaining power to extract rents and charge markups for catering to households' demand.[2] At the same time, global intermediation firms have access to a frictionless, centralized dealer-to-dealer (D2D) market to which they can turn to hedge

[2]Costinot et al. (2014) also emphasize the optimal markups (implemented via capital controls) on state-contingent transfers.

exposures or obtain funding. Pricing in the D2D market, in turn, defines the international pricing kernel at which dealers discount their cash flows.

We derive a closed-form solution to the bargaining problem in the D2C market and then embed this solution into the general equilibrium. Remarkably, despite the inherent complexity of the model with frictions, multiple shocks, and international financial markets, we can characterize equilibrium dynamics in closed form by leveraging the power of continuous-time methods. Based on our solution, we then explicitly show how the presence of intermediation markups affects equilibrium allocations and exchange rate behavior.

We show (both theoretically and numerically) that our model can help explain several well-known puzzles about the behavior of exchange rates. To this end, we study an economy with two symmetric countries in which interest rates are constant in the frictionless economy. As a result, absent intermediation frictions, the model cannot generate any joint dynamics between exchange rates and interest rates. By contrast, we show how intermediation markups help account for several known puzzles in exchange rate behavior, including the joint behavior of deviations from Covered and Uncovered Interest Parity. Consistent with the data, the model generates a large, positive coefficient in the Fama regression. This happens because, in equilibrium, price-discriminating intermediaries optimally influence the domestic households' demand for foreign risky and riskless assets, creating a negative association between the interest rate differential and the foreign exchange risk premium. Contrary to other existing models with financial frictions, our model achieves a significant R^2 in the Fama (1984) regression through a mechanism that is purely driven by trade and consumption risk sharing.

The model also generates a sizable Sharpe ratio for the carry trade, suggesting that the market power of intermediaries could drive a non-trivial fraction of this Sharpe ratio. The high R^2 in the Fama (1984) regression is possible because our model – in contrast to the frictionless benchmark – generates quantitatively realistic fluctuations in the interest rate differential. While such fluctuations could also be achieved through flexible preference specifications that produce enough volatility in the elasticity of intertemporal substitution, our model generates this volatility in a setting with logarithmic preferences. We also derive a closed-form expression for equilibrium CIP deviations and show how they are directly affected by the price pressure originating from households' desire to share fundamental shocks. The mechanism is stronger in the presence of a larger trade imbalance, highlighting how real demand forces lead to CIP deviations in our model. Our model can match the levels of UIP and CIP deviations quantitatively and generates realistic joint time-series dynamics of CIP and exchange rates, consistent with the recent findings of Avdjiev et al. (2019). In summary, this calibration exercise suggests that intermediation markups might be a quantitatively important channel behind some of the observed fluctuations in interest rates and exchange rates.

Roadmap. The remainder of the paper is structured as follows. Section 2 provides an overview of the relevant literature. Section 3 describes the model. Section 4 provides the equilibrium characterization. Section 5 investigates the link between intermediation frictions and various exchange rate anomalies. Section 6 concludes the paper.

2 Literature Review

The literature on general equilibrium models of exchange rates is vast. Most papers assume either complete markets[3] or an exogenously specified incompleteness in the form of portfolio constraints[4] or limits to market participation[5]. In contrast, in our model, market incompleteness and limits to international risk sharing are endogenous and determined by equilibrium intermediation markups.

Whereas much of the existing literature on intermediation frictions focuses on balance sheet constraints of intermediaries (see, e.g., Maggiori (2017) and Gabaix and Maggiori (2015), Itskhoki and Mukhin (2017, 2019), Fang and Liu (2021)),[6] our focus in this paper is different. To single out the effects of markups and market power, we assume that intermediaries are global risk-neutral firms that act as matchmakers to intermediate clients with different trading interests, maximizing firm value through rent extraction. Although dealer balance sheet constraints are among the key determinants of exchange rate dynamics (see, e.g., Du et al. (2019a)), recent empirical evidence (see, e.g., Aldasoro et al. (2020), Hau et al. (2017), and Wallen (2020)) suggests that markups are responsible for a significant component of spreads in FX derivatives markets. Our novel, tractable framework allows for

[3]See, e.g., Lucas (1982); Cole and Obstfeld (1991); Dumas (1992); Backus et al. (1992); Backus and Smith (1993), Obstfeld and Rogoff (1995); Pavlova and Rigobon (2007); Verdelhan (2010); Colacito and Croce (2011).

[4]See, e.g., Chari et al. (2002); Corsetti et al. (2008); Pavlova and Rigobon (2008), unspanned risk factors (Pavlova and Rigobon (2010, 2012), Farhi and Gabaix (2016), Brunnermeier and Sannikov (2017)).

[5]Alvarez et al. (2002, 2009), Bacchetta and Van Wincoop (2010) and Hassan (2013).

[6]Several papers (see, e.g., Jeanne and Rose (2002), Evans and Lyons (2002), Hau and Rey (2006), Bruno and Shin (2015), Camanho et al. (2017)) study the impact of frictions on exchange rates without modeling fundamentals such as exports and imports of multiple goods. Instead, they focus on how intermediaries' behavior and incentive structure shape market outcomes in FX.

arbitrary state-contingent contracts and can be easily adjusted to incorporate balance sheet constraints. We leave this possible extension for future research.

Most existing papers on intermediation frictions in international markets (see, e.g., Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017, 2019)) assume that households can only trade nominal domestic bonds, leading to an extreme form of market segmentation. In contrast, in our model, households could potentially share risks efficiently with each other, but intermediaries' market power affects state prices, and customers end up under- or over-insuring certain risks. Our paper's novel, tractable, continuous-time framework allows us to characterize these inefficiently insured risks and their equilibrium impact in closed form for any shock dynamics.

Another set of papers assumes *exogenous* shocks to Euler equations (Itskhoki and Mukhin (2017, 2019)) or an *exogenous* convenience yield (Jiang et al. (2018, 2019)). Our key theoretical innovation is a micro-foundation of convenience yields arising from macroeconomic demand forces. In our model, households' demand pressure in D2C markets creates a convenience yield (over-pricing relative to the D2D market) for securities with risk profiles that customers find attractive. Understanding the origins and micro-foundations of such convenience yields and linking them to risk (safety) characteristics of assets is crucial for deriving policy implications and predicting which securities will enjoy a convenience yield in the future.

Our model is also related to the prominent model of Alvarez et al. (2009) (see also Alvarez et al. (2002)), who were the first to study the impact of endogenous market segmentation on exchange rates. Alvarez et al. (2009) develop a general equilibrium monetary model with a continuum of households that differ in their fixed cost of participation in financial markets. This heterogeneity produces time variation in financial market participation, which in turn leads to a time variation in risk premia, even if the money supply follows a random walk. Despite exhibiting complex dynamics, the model of Alvarez et al. (2009) admits a closed-form solution, which the authors use to explain several stylized facts about Uncovered Interest Parity (UIP) deviations. Two key differences exist between our model and that of Alvarez et al. (2009). First, the cost of participation in our model is endogenous, determined by the household demand pressure. Second, the cost (the intermediation markup) is security-specific: Only securities with attractive risk profiles command a convenience yield.

Finally, our paper is also related to the recent literature on the breakdown of Covered Interest Parity (CIP). See, for example, Borio et al. (2016), Aldasoro et al. (2020), Rime et al. (2022), Avdjiev et al. (2019), and Du et al. (2019a). Several papers derive CIP deviations using models with different forms of limits to arbitrage. See, for example, Amador et al. (2020), Ivashina et al. (2015), Liao (2020), Hebert (2017), Andersen et al. (2017), Du et al. (2019b), Greenwood et al. (2019), Gourinchas et al. (2020), and Fang and Liu (2021). To the best of our knowledge, our model is the first macroeconomic general equilibrium model that generates a breakdown of CIP endogenously through segmentation effects in imperfect international financial markets without appealing to binding balance sheet constraints of intermediaries. Most importantly, CIP deviations in our model originate from trade in real goods that creates real imbalances and price pressure in the D2C markets. Securities offering insurance against desired states of the world enjoy an endogenous convenience yield that is reflected in observed CIP violations.

3 The Model

3.1 Agents, Preferences, and Consumption

We consider a continuous time pure-exchange economy monetary economy with intra-temporal cash-in-advance constraints. As in Lucas (1982), we assume that nominal interest rates in all countries are non-negative. There are two countries in the world economy: Home (H) and Foreign (F). Each country is endowed with a Lucas tree producing a strictly positive amount of country-specific perishable good, $X_{H,t}$ and $X_{F,t}$, respectively, for $t \geq 0$. Before the financial markets open, trees are owned by the respective country's households.

Each country is populated by a continuum of identical households whose preferences are represented by a time-additive log-linear utility over the consumption of both goods,

$$E\left[\int_0^\infty \Psi_{i,t} u_i(C_{i,t}, C^*_{i,t})dt\right], \tag{1}$$

with

$$u_i(C_i, C_i^*) \;=\; \beta_i \log(C_i) \;+\; (1-\beta_i)\log(C_i^*)\,,\; i \in \;\{H,\; F\}\,.$$

Here, $\beta_H,\ \beta_F$ are the weights on the Home goods in the utility of each country's households, accounting for the potential home bias in consumption. As is common in the literature, we use * to denote goods and quantities of the foreign country. The time-preference "demand shocks" $\Psi_{H,t},\ \Psi_{F,t}$ (henceforth, time discount factors) are arbitrary positive random variables, normalized so that $\Psi_{i,0} = 1$. Such demand shocks are commonly used in international economics. See, for example, Stockman and Tesar (1995), Pavlova and Rigobon (2007), and Itskhoki and Mukhin (2017).[7] Equilibrium prices are then pinned down by imposing market clearing for all goods (the economy-wide feasibility constraint):

$$\begin{aligned} C_{H,t} \ + \ C^*_{F,t} \ &= \ X_{H,t} \text{ (home goods market clearing)} \\ C^*_{H,t} \ + \ C_{F,t} \ &= \ X_{F,t} \text{ (foreign goods market clearing)} \end{aligned} \tag{2}$$

for $t \geq 0$, where $C_i, C_i^*, i = H, F$ are optimal consumption policies characterized below.

We denote by $P_{i,t},\ P^*_{i,t}$ the nominal prices of the two goods in the country $i = H, F$, in the units of country i currency. We also denote by $\mathcal{E}_t,\ t \geq 0$ the foreign currency price in home currency units; whenever $\mathcal{E}_t$ goes up, the foreign currency appreciates against the home currency. We assume a cash-in-advance constraint à la Lucas (1982) at the country level: All country k goods need to be purchased with country k currency, implying that total nominal expenditures for country k tradable goods' endowment $X_{k,t}$ always equals country k nominal output, $\mathcal{M}_{k,t}$:

$$P_{k,t}\, X_{k,t} \ = \ \mathcal{M}_{k,t}\ ,\ k = H,\ F\ ,\ t \geq 0\,. \tag{3}$$

[7]Some papers (such as Dornbusch et al. (1977), Pavlova and Rigobon (2008), and Pavlova and Rigobon (2012)) model demand shocks through random changes in $\beta_i,\ i = H, F$. It is straightforward to introduce such shocks into our model. The results are analogous to those for time-preference shocks.

In particular, in the absence of nominal rigidities, as in Lucas (1982), prices adjust immediately to monetary shocks so that inflation moves one-to-one with money supply: $P_{k,t} = \mathcal{M}_{k,t}/X_{k,t}$. Without loss of generality, we use the normalization $\mathcal{M}_{i,0} = 1$. It will be convenient for us to work with nominal consumption expenditures

$$\bar{C}_{i,t} \;=\; C_{i,t}P_{i,t} \;+\; C^*_{i,t}P^*_{i,t},\; t \geq 0\,, \tag{4}$$

the total domestic currency spending of households on their consumption bundles.

We assume country$-i$ households are endowed with country$-i$ output, so that, by (3), their nominal endowment is given by $\mathcal{M}_{i,t}$. In addition, they have access to a complete set of one-period state-contingent claims whose prices are summarized by the state price density (pricing kernel), $M_{i,t,t+dt}$. As a result, the joint dynamics of consumption expenditures, $\bar{C}_{i,t}$, and the value of assets (financial wealth), $W_{i,t}$, satisfy the standard inter-temporal budget constraint:

$$\underbrace{\bar{C}_{i,t}dt}_{Total\ Consumption} \quad + \quad \underbrace{E_t[M_{i,t,t+dt}\,W_{i,t+dt}]}_{Portfolio\ of\ state\ contingent\ securities} \quad = \quad \underbrace{W_{i,t}}_{beginning\ of\ period\ assets} \quad + \quad \underbrace{\mathcal{M}_{i,t}\,dt}_{Endowment}\;. \tag{5}$$

All of the above assumptions are completely standard and are used in most of the existing macroeconomic models. The only aspect that makes our model distinct is that the pricing kernels, $M_{i,t,t+dt}$, are determined through bargaining with intermediaries.

3.2 The D2C Bargaining Problem

Intermediaries in our model are represented by global intermediary firms. These firms can be viewed as agents within their respective countries, randomly assigned the role of intermediaries (financiers). We assume that financiers have access to a complete, frictionless dealer-to-dealer (D2D) market. This is the market dealers can use to offload any imbalances due to their client transactions to achieve a matched book with no directional exposure. As markets are complete, the prices of all financial securities traded in the inter-dealer market can be encoded in a single, international nominal pricing D2D kernel $M^I_{H,t}$ quoted in the units of home currency. We also use $M^I_{F,t}$ to denote the D2D kernel denominated in foreign currency. By no-arbitrage and D2D market completeness, we always have (see, e.g., Backus and Smith (1993))

$$\frac{M^I_{F,T}}{M^I_{F,t}} = \frac{M^I_{H,T}}{M^I_{H,t}} \frac{\mathcal{E}_T}{\mathcal{E}_t}. \tag{6}$$

Unlike intermediary firms, households do not have direct access to the inter-dealer market. We assume, however, that in each country H, F an all-to-all market exists where all local households and all global intermediation firms can trade two securities: a risky asset with nominal cash flows $\mathcal{M}_{i,t+dt}$ in local currency (henceforth, a Lucas tree) and a one-period country-specific nominal risk-free bond paying one unit of domestic currency at time $t + dt$. Households willing to trade any other financial instrument must contact an intermediation firm and bargain over-the-counter (OTC) in a D2C market. See Figure 1 in the Appendix for a graphical depiction of the market structure.

The objective of an intermediation firm is to maximize the firm value (i.e., the present discounted value of intermediation markups) under the D2D pricing kernel. We assume that intermediaries can observe households' country of origin and, hence, can charge country-specific markups. For example, intermediaries will charge higher markups to US households for insurance against a crash in the US stock market and, similarly, for Swiss households against a Swiss stock market crash.

As competitive intermediaries can freely trade in both the centralized local exchange and the global D2D market, they will equalize prices across these two markets in equilibrium. Hence, nominal domestic bonds and trees will trade at D2D prices on the domestic exchange. It is important to note that the bonds and stocks that are used in the intermediaries' problem are "redundant" securities, since households already have access to a complete set of state- and date-contingent securities from their interactions with the intermediaries. This makes the pricing of stocks and bonds straightforward: Formally, the D2C nominal pricing kernel $M_{i,t,t+dt}$ quoted by the intermediary must satisfy two constraints (fair pricing of domestic bonds and fair pricing of domestic trees) relating $M_{i,t,t+dt}$ to the D2D nominal pricing kernel (6) in the local currency:

$$E_t[M_{i,t,t+dt}] = E_t[M^I_{i,t,t+dt}] \qquad (fair\ pricing\ of\ bonds) \tag{7}$$

$$E_t[M_{i,t,t+dt}\mathcal{M}_{i,t+dt}] = E_t[M^I_{i,t,t+dt}\mathcal{M}_{i,t+dt}] \qquad (fair\ pricing\ of\ trees) \tag{8}$$

At the time t, a country i customer with nominal wealth $W_{i,t}$ gets matched with an intermediary who quotes him a one-period-ahead D2C pricing kernel $M_{i,t,t+dt}$ in the local currency.

Given these state prices quoted by the dealer, the customer then decides how to optimally finance their future wealth $W_{i,t+dt}$ through an OTC contract with the intermediary acquired in the D2C market, with a potentially complex state-contingent payoff.

Standard optimality conditions (see Lemma 1 below) imply that intermediaries are facing a downward-sloping demand curve from customers, state by state: $W_{i,t+dt}(M_{i,t,t+dt}) = W_{i,t} \frac{\Psi_{i,t+dt} D_{i,t+dt}}{\Psi_{i,t} D_{i,t}} (M_{i,t,t+dt})^{-1}$. Since intermediaries have access to complete D2D markets, their objective is to maximize the present value of cash flows in the D2C market under the D2D pricing kernel. Those cash flows are given by $E_t[M_{i,t,t+dt} W_{i,t+dt}]$ (the price paid by customers to intermediaries) at time t and by $-W_{i,t+dt}$ (the contractual payments of intermediaries to customers) at time $t + dt$. Thus, their present value under the D2D pricing kernel is given by the total intermediary rents,

$$\mathcal{I}_{i,t} \triangleq E_t[M_{i,t,t+dt} W_{i,t+dt}(M_{i,t,t+dt})] - E_t[M^I_{i,t,t+dt} W_{i,t+dt}(M_{i,t,t+dt})], \tag{9}$$

that is, the difference between the values of the claim $W_{i,t+dt}$ under the D2C and the D2D pricing kernels. The intermediary's goal is thus to maximize (9) under the no-arbitrage constraints (7)-(8).[8]

In order to derive households' downward sloping demand curves $W_{i,t+dt}(M_{i,t,t+dt})$ in (9),

[8]Importantly, since customers have iso-elastic preferences, their demand for Arrow securities is proportional to their wealth. Hence, intermediary quotes do not depend on customer wealth. Furthermore, the contract with each intermediary lasts for one period, and random matching implies that getting matched with the same customer again has zero probability. Hence, intermediaries maximize the markup from a given trade.

we note that they face the standard problem of maximizing (1) under (5). The solution to this problem is given in the following lemma.

Lemma 1 *Let $\bar{C}_{i,t}$ be the total nominal expenditure of country i households at time t (see (4)). Given the quoted D2C pricing kernel $M_{i,t}$ with $M_{i,0} = 1$, the optimal nominal expenditures satisfy*

$$\bar{C}_{i,t} \;=\; \Psi_{i,t}\, M_{i,t}^{-1}\, \bar{C}_{i,0}\,,$$

whereas the optimal wealth process is given by

$$W_{i,t} \;=\; E_t\Big[\int_t^\infty \frac{M_{i,s}}{M_{i,t}} \bar{C}_{i,s}\, ds\Big] \;=\; M_{i,t}^{-1} D_{i,t} \bar{C}_{i,0}\,, \quad i \in \{H, F\}\,,$$

where

$$D_{i,t} \;=\; E_t\left[\int_t^\infty \Psi_{i,s} ds\right]\,, \tag{10}$$

and the optimal consumption bundle $(C_{i,t}, C^*_{i,t})$ is given by

$$C_{i,t} \;=\; \beta_i \bar{C}_{i,t}/P_{i,t}\;,\;\; C^*_{i,t} \;=\; (1-\beta_i)\bar{C}_{i,t}/P^*_{i,t},\;\; t \geq 0\,. \tag{11}$$

4 Solving the Model in Continuous Time

To proceed further, we need to specify the dynamics of the fundamentals. As we now show, the continuous time setting allows us to solve the optimal contract in the D2C market and characterize equilibrium dynamics in closed form. Everywhere in the sequel, we use

$$\|\theta\|^2 \;=\; \sum_{k=1}^{N} \theta_k^2 \tag{12}$$

to denote the squared Euclidean norm of a vector $\theta \in \mathbb{R}^N$.

We assume that the fundamentals of the economy, $(\Psi_{H,t}, \Psi_{F,t}, \mathcal{M}_{H,t}, \mathcal{M}_{F,t})$, are driven by a N-dimensional standard Brownian motion B_t. The time preference shocks follow

$$\frac{d\Psi_{i,t}}{\Psi_{i,t}} \;=\; -\delta dt + (\theta_{i,t}^{\Psi})' dB_t\,,$$

while nominal output in the two countries follows

$$\frac{d\mathcal{M}_{i,t}}{\mathcal{M}_{i,t}} \;=\; \mu_i dt + \theta_i' dB_t\,, \quad i \in \{H, F\}\,,$$

with some shock exposure vectors $\theta_i,\ \theta_{i,t}^{\Psi} \in \mathbb{R}^N$. In particular, the volatility of the nominal output is given by

$$\mathrm{Var}_t[\frac{d\mathcal{M}_{i,t}}{\mathcal{M}_{i,t}}] \;=\; \|\theta_i\|^2 dt,$$

where $\|\theta_i\|^2$ is defined in (12). We purposely assume that the time discount rates, $\Psi_{i,t}$, have the same drift, ensuring that, on average, households in the two countries are equally patient. This assumption is necessary to ensure that a well-defined ergodic distribution of the model, in which both agents survive, exists. For simplicity, we assume that all parameters of the diffusion processes above are constant, except for the time preference shocks $\theta^{\Psi}_{i,t}$. The randomness of $\theta^{\Psi}_{i,t}$ will play an important role in our calibration exercise. However, all our closed-form solutions hold for arbitrary stochastic dynamics of all coefficients. Since $\Psi_{i,t}$ are geometric Brownian motions, formula (10) simplifies and we have $D_{i,t} \;=\; \Psi_{i,t}/\delta$.

The continuous time assumption makes the optimal contracting problem in the D2C market particularly tractable. Indeed, standard arguments imply that all stochastic discount factors in the diffusion setting admit a representation

$$-\frac{dM_{i,t}}{M_{i,t}} \;=\; r_{i,t}dt + (\eta_{i,t})'dB_t, \;\; -\frac{dM^I_{i,t}}{M^I_{i,t}} \;=\; r^I_{i,t}dt + (\eta^I_{i,t})'dB_t\,, \tag{13}$$

where $r_{i,t}$ and $r^I_{i,t}$ are the risk-free rates in the two market segments, and $\eta_{i,t},\ \eta^I_{i,t} \in \mathbb{R}^N$ are the vectors of *equilibrium risk premia* for each of the N sources of risk in our economy (the N-dimensional Brownian motions). Thus, there is a duality between the D2C bargaining problem and the problem of determining risk premia depending on the current state of the economy, where the fair pricing of bonds (condition (7)) implies that the interest rate $r_{i,t}$ offered in the D2C market has to coincide with that in the D2D market, and the same is true for the risk premia of the trees (condition (8)). Formally, conditions (7)-(8) can be rewritten as [9]

$$E_t\Big[\frac{dM_{i,t}}{M_{i,t}}\Big] \;=\; E_t\Big[\frac{dM^I_{i,t}}{M^I_{i,t}}\Big] \;\Leftrightarrow\; r_{i,t} \;=\; r^I_{i,t} \tag{14}$$

and

$$E_t\Big[\frac{dM_{i,t}}{M_{i,t}}\frac{d\mathcal{M}_{i,t}}{\mathcal{M}_{i,t}}\Big] \;=\; E_t\Big[\frac{dM^I_{i,t}}{M^I_{i,t}}\frac{d\mathcal{M}_{i,t}}{\mathcal{M}_{i,t}}\Big]\,. \tag{15}$$

Similarly, the objective (9) can be rewritten as

$$\max_{\eta_{i,t}}\left(E_t\Big[\frac{dM_{i,t}}{M_{i,t}}\frac{dW_{i,t}}{W_{i,t}}\Big]-E_t\Big[\frac{dM^I_{i,t}}{M^I_{i,t}}\frac{dW_{i,t}}{W_{i,t}}\Big]+E_t\Big[\frac{dM_{i,t}}{M_{i,t}}\Big]-E_t\Big[\frac{dM^I_{i,t}}{M^I_{i,t}}\Big]\right) \tag{16}$$

under the constraints (14)-(15). Applying Ito's lemma to $W_{i,t}=\delta^{-1}\Psi_{i,t}M^{-1}_{i,t}$, we obtain the following expression for household wealth dynamics

$$\frac{dW_{i,t}}{W_{i,t}} \;=\; \underbrace{\frac{d\Psi_{i,t}}{\Psi_{i,t}}}_{time\ discount} \;-\; \underbrace{\frac{dM_{i,t}}{M_{i,t}}}_{interest\ +\ premium} \;+\underbrace{\big(\frac{dM_{i,t}}{M_{i,t}}\big)^2}_{convexity}-\underbrace{\frac{d\Psi_{i,t}}{\Psi_{i,t}}\frac{dM_{i,t}}{M_{i,t}}}_{co-movement}\,. \tag{17}$$

[9] This can be seen easily as the $\Delta\to 0$ approximation to the following discrete-time optimization problem

$$E_t\Big[\frac{M_{i,t+\Delta}-M_{i,t}}{M_{i,t}}\frac{W_{i,t+\Delta}-W_{i,t}}{W_{i,t}}\Big]-E_t\Big[\frac{M^I_{i,t+\Delta}-M^I_{i,t}}{M^I_{i,t}}\frac{W_{i,t+\Delta}-W_{i,t}}{W_{i,t}}\Big].$$

Alternatively, note that for the log-utility agents, the markups maximization problem is essentially minimizing $E_t\Big[M^I_{i,t,t+\Delta}\Psi_{i,t,t+\Delta}/M_{i,t,t+\Delta}\Big]$, apply Ito's lemma to the process $M^I_{i,t}\Psi_{i,t}/M_{i,t}$, then note that

$$\frac{M^I_{i,t+\Delta}\Psi_{i,t+\Delta}}{M_{i,t+\Delta}}-\frac{M^I_{i,t}\Psi_{i,t}}{M_{i,t}} \;=\; \frac{M^I_{i,t}\Psi_{i,t}}{M_{i,t}}\Big(\frac{M^I_{i,t,t+\Delta}\Psi_{i,t,t+\Delta}}{M_{i,t,t+\Delta}}-1\Big)\,.$$

This gives the instantaneous risk premium maximization problem again.

The first term is the change in wealth due to shocks to time discount factors: Households' optimal consumption and investment choices allocate less wealth to states with lower $\Psi_{i,t}$. The second term is the contractual payment of the customer to the intermediary. The third term is a convexity adjustment because M^{-1} is convex in the state prices. Finally, the fourth term reflects the co-movement of state prices with time discounting. In the diffusion limit, however, the last two terms in (17) are negligible, and we can rewrite the extracted intermediary rents, $\mathcal{I}_{i,t}$, in (16) as

$$\begin{aligned} \mathcal{I}_{i,t} &= E_t\Big[\left(\frac{dM_{i,t}}{M_{i,t}} - \frac{dM^I_{i,t}}{M^I_{i,t}}\right)\frac{dW_{i,t}}{W_{i,t}}\Big] = E_t\Big[\left(\frac{dM_{i,t}}{M_{i,t}} - \frac{dM^I_{i,t}}{M^I_{i,t}}\right)\Big(\frac{d\Psi_{i,t}}{\Psi_{i,t}} - \frac{dM_{i,t}}{M_{i,t}}\Big)\Big] \\ &= (\eta^I_{i,t} - \eta_{i,t})'(\theta^\Psi_{i,t} + \eta_{i,t})\,dt\,. \end{aligned} \tag{18}$$

The intuition behind (18) is as follows. The intermediary has the incentive to charge the largest possible premium for exposure to shocks that the customer values the most. We refer to the difference $\eta^I_{i,t} - \eta_{i,t}$ as the *risk premium markup:* It reflects the additional price of risk that the intermediary charges to country-i customers for exposure to the B_t shocks (see (13)). The formula (18) for the *intermediary rents, $\mathcal{I}_{i,t}$,* implies the risk premium markup must be maximally aligned with the customers' wealth shock exposures, $\theta^\Psi_{i,t} + \eta_{i,t}$, given by the sum of two parts: time-discount shocks $\theta^\Psi_{i,t}$ and the risk premium $\eta_{i,t}$. At the same time, the same argument as above implies that the no-arbitrage constraint (15) for pricing the tree can be rewritten as

$$0 = E_t\Big[\left(\frac{dM_{i,t}}{M_{i,t}} - \frac{dM^I_{i,t}}{M^I_{i,t}}\right)\frac{d\mathcal{M}_{i,t}}{\mathcal{M}_{i,t}}\Big] = -(\eta_{i,t} - \eta^I_{i,t})'\,\theta_i\,dt\,. \tag{19}$$

The no-arbitrage condition (19) is very intuitive: It implies that the intermediary is constrained to choose the risk premium markup to be orthogonal to the risk premium vector of the domestic tree with cash flow $\mathcal{M}_{i,t}$.

We now introduce a modification of the base intermediation model that allows for a continuous transition between the models with and without intermediation frictions. To this end, we assume that the intermediaries are households that get randomly assigned to the job of financiers. These "special" households can offer the "regular" households that they get matched to the same SDF as their own without incurring any adjustment cost. However, if they deviate from this, they incur an adjustment cost $\|\eta_{i,t} - \eta_{i,t}^I\|^2$. We can think of this adjustment cost as a metaphor for having to run a balance sheet mismatch potentially or simply having to create new assets/liabilities for the households that they get matched with. Assume that the intermediaries penalize this adjustment cost at a rate of 0.5Γ. Then, the optimization problem (18)-(19) faced by the intermediaries takes the form

$$\begin{aligned} \max_{\eta_{i,t}} \quad & (\eta_{i,t}^I - \eta_{i,t})'(\theta_{i,t}^\Psi + \eta_{i,t}) - \frac{\Gamma}{2}(\eta_{i,t}^I - \eta_{i,t})^2 \\ \text{s.t.} \quad & (\eta_{i,t}^I - \eta_{i,t})'\theta_i \;=\; 0\,. \end{aligned} \tag{20}$$

That is, intermediaries maximize rents, $\mathcal{I}_{i,t}$, net of adjustment costs. Writing down the Lagrangian

$$(\eta_{i,t}^I - \eta_{i,t})'(\theta_{i,t}^\Psi + \eta_{i,t}) - \frac{\Gamma}{2}(\eta_{i,t}^I - \eta_{i,t})^2 \;-\; \lambda_{i,t}(\eta_{i,t}^I - \eta_{i,t})'\theta_i$$

and optimizing it with respect to $\eta_{i,t}$, we arrive at the following result.

Proposition 1 *The optimal, markup-maximizing vector of risk premia $\eta_{i,t}$ chosen by the intermediary for country-i customers is given by*

$$\eta_{i,t} \;=\; \frac{\Gamma}{2+\Gamma}\,\eta^I_{i,t} + \frac{2}{2+\Gamma}\left[\frac{1}{2}(\eta^I_{i,t} - \theta^{\Psi}_{i,t} + \lambda_{i,t}\theta_i)\right],\ \textit{with}\ \lambda_{i,t} \;=\; \frac{(\eta^I_{i,t} + \theta^{\Psi}_{i,t})'\theta_i}{\|\theta_i\|^2}\,. \tag{21}$$

Proposition 1 shows explicitly how the strength of the intermediation frictions depends on the adjustment cost parameter Γ. When Γ is large, the financier faces a high adjustment cost and, as a consequence, decides to equalize the quoted SDF to that in the D2D market: $\eta_{i,t} = \eta^I_{i,t}$. This corresponds to the equilibrium in a model without intermediation frictions. By contrast, when $\Gamma = 0$, the financier faces no adjustment cost, and the intermediary selects the pricing kernel that maximizes rents.

Effectively, $\eta^I_{i,t}$ represents the shadow costs of holding risk for intermediaries. Intermediaries pass this cost through to households, and (21) implies that the pass-through coefficient is given by

$$\underbrace{\frac{\partial \eta_{i,t}}{\partial \eta^I_{i,t}}}_{pass-through} \;=\; \frac{1+\Gamma}{2+\Gamma}\,. \tag{22}$$

The pass-through (22) is monotone increasing in Γ, and achieves its minimum of 0.5 in the zero-adjustment-cost case. That is, absent adjustment costs, intermediaries optimally pass on half of the shadow costs to customers. To understand why the pass-through is exactly

0.5, we note that the intermediary is maximizing the following tradeoff

$$-(\eta_{i,t}^I)'(-(\theta_{i,t}^{\Psi}+\eta_{i,t}))+\eta_{i,t}'(-(\theta_{i,t}^{\Psi}+\eta_{i,t}))\,.$$

The first component of this objective is the shadow cost intermediation (the true D2D market value of the OTC contract signed in the D2C market). The second term is the gain: The price of risk, $\eta_{i,t}$, that households pay for their optimal D2C contract, times the downward sloping demand, $-(\theta_{i,t}^{\Psi}+\eta_{i,t})$. At the optimum, marginal cost equals marginal gain, and the downward-sloping demand implies that $\eta_{i,t}^I\sim 2\eta_{i,t}$. The specific passthrough coefficient of 0.5 originates from the assumed logarithmic utility for the households. When the risk aversion is different from one, the passthrough coefficient is different from 0.5, but is always below one.

The formula (21) also shows how intermediaries maximize rent extraction by charging the highest prices for the states that the customer deems most valuable. This is achieved by aligning $-\eta_{i,t}$ with the diffusion vector $\theta_{i,t}^{\Psi}$ of time discount shocks. Finally, the constraint (19) always binds, limiting the intermediaries' ability to extract rents from risk premia that are aligned with the diffusion vector θ_i of $\mathcal{M}_{i,t}$. The Lagrange multiplier for this constraint, $\lambda_{i,t}$, is defined by how the vector $\eta_{i,t}^I+\theta_{i,t}^{\Psi}$ is aligned with θ_i. If both the D2D risk premia, $\eta_{i,t}^I$, and customer preference shocks, $\theta_{i,t}^{\Psi}$, are aligned with θ_i, charging markups for exposure to $\mathcal{M}_{i,t}$ becomes highly attractive; as a result, the constraint (19) binds, pushing up the value of $\lambda_{i,t}$.

4.1 Market Clearing, Consumption, and Exchange Rates

By the cash-in-advance constraint (3), we can equivalently formulate the economy-wide feasibility constraints (2) as the equality between total nominal consumption expenditures and the total nominal output. Namely, multiplying (2) by the respective prices and using (3), we get

$$\begin{aligned} P_{H,t}C_{H,t} \;+\; P_{H,t}C^{*}_{F,t} \;&=\; \underbrace{P_{H,t}X_{H,t}}_{\mathcal{M}_{H,t}} \\ P_{F,t}C^{*}_{H,t} \;+\; P_{F,t}C_{F,t} \;&=\; \underbrace{P_{F,t}X_{F,t}}_{\mathcal{M}_{F,t}}\,. \end{aligned} \tag{23}$$

Using the identity (11) for the optimal consumption bundles, we can rewrite (23) as

$$\begin{aligned} \underbrace{\beta_H\bar{C}_{H,t}}_{\text{home spending on home goods}} \;+\; \underbrace{(1-\beta_F)\bar{C}_{F,t}\mathcal{E}_t}_{\text{foreign spending on home goods}} \;&=\; \mathcal{M}_{H,t}\ \text{(home money market clearing)} \\ \underbrace{(1-\beta_H)\bar{C}_{H,t}\mathcal{E}_t^{-1}}_{\text{home spending on foreign goods}} \;+\; \underbrace{\beta_F\bar{C}_{F,t}}_{\text{foreign spending on foreign goods}} \;&=\; \mathcal{M}_{F,t}\ \text{(foreign money market clearing)} \end{aligned} \tag{24}$$

We now introduce two key state variables that will serve as the endogenous Markov state driving the equilibrium dynamics. We let $\pi_{i,t} = \frac{\beta_i\bar{C}_{i,t}}{\mathcal{M}_{i,t}}$, $i = H, F$ denote the nominal consumption share for the domestic goods, and arrive at the following result.

Lemma 2 (Equilibrium Relationship Between Consumption Shares and Exchange Rates) *Let $\mathcal{Q}_t = \frac{\mathcal{M}_{F,t}\mathcal{E}_t}{\mathcal{M}_{H,t}}$ be the value of one unit of home goods in terms of foreign goods, adjusted* *for money supply. Then,*

$$\pi_{H,t} = \Big(\frac{\beta_F}{1-\beta_F} - \mathcal{Q}_t\Big)\Big(\frac{\beta_F}{1-\beta_F} - \frac{1-\beta_H}{\beta_H}\Big)^{-1},$$
$$\pi_{F,t} = \Big(\frac{\beta_H}{1-\beta_H} - \frac{1}{\mathcal{Q}_t}\Big)\Big(\frac{\beta_H}{1-\beta_H} - \frac{1-\beta_F}{\beta_F}\Big)^{-1},$$

and the adjusted exchange rate satisfies

$$\mathcal{Q}_t \in \left(\frac{1-\beta_H}{\beta_H}, \frac{\beta_F}{1-\beta_F}\right). \tag{25}$$

Lemma 2 shows how fluctuations of the adjusted exchange rate inside the (25) interval lead to a redistribution of wealth between the two countries. When $\mathcal{Q}_t$ appreciates, F-households become richer, $\pi_{F,t}$ increases and $\pi_{H,t}$ decreases. As a result, exchange rate dynamics induce wealth transfers between the two countries.[10]

4.2 Equilibrium Without Intermediaries

To solve for the equilibrium, we need to characterize the dynamics of risk premia and interest rates. We start our analysis by deriving these dynamics for the frictionless model without intermediation markups. We use θ_{-i} to denote the risk premia for the respective other

[10] As we show below, all equilibrium quantities can be characterized explicitly in terms of the consumption share for the domestic goods $\pi_{i,t}$ and, hence, in terms of $\mathcal{Q}_t$: Exchange rates determine the wealth distribution and, as a result, the dynamics of risk premia. In turn, these risk premia determine exchange rates directly by the no-arbitrage forces that equalize state prices across two countries; see (6). A similar mechanism is also at play in general equilibrium models with recursive preferences (see, e.g., Colacito and Croce (2011)) and habit formation (see, e.g., Stathopoulos (2017)). In all these models, FX changes are linked to the ratio of SDFs, which in turn is associated with time-varying pseudo-Pareto weights. The time variation of these weights is closely linked to the dynamics of the wealth distribution.

country: $\theta_{-H} = \theta_F$, $\theta_{-F} = \theta_H$. We also use X^C to denote a quantity X in the case of Complete (Frictionless) markets.

Proposition 2 (Frictionless Equilibrium) *Without intermediation markups, the risk premia are given by*

$$\eta_{i,t}^C = \underbrace{\theta_i - \theta_{i,t}^{\Psi}}_{domestic\ risk} + \underbrace{(1 - \pi_{i,t})(\theta_{i,t}^{\Psi} - \theta_{-i,t}^{\Psi})}_{risk\ sharing}, \tag{26}$$

while the equilibrium interest rates are given by

$$r_{i,t}^C = \mu_i + \delta + \theta_i' \eta_{i,t}^C. \tag{27}$$

Absent intermediation frictions, money is neutral, and hence, adjustments in exchange rates fully undo any effect of the money supply. No sharing of risks arising from monetary policy shocks is possible (and neither is it necessary since money is neutral). Nominal risk premia in the country i only depend on domestic shocks plus the amount of trade (captured by the share of foreign goods, $1 - \pi_{i,t}$, times the risk sharing needs, as captured by the difference $\theta_{i,t}^{\Psi} - \theta_{-i,t}^{\Psi}$ between time discount shock exposures.

4.3 Equilibrium with Intermediaries

Having understood the dynamics of the frictionless model, we can now proceed with deriving the closed-form solution for the model with intermediaries. Intermediation frictions break money neutrality and make both the adjusted exchange rates, $\mathcal{Q}_t$, and the risk premia depend on monetary shocks in both countries. This significantly alters equilibrium dynamics, both quantitatively and qualitatively. In order to characterize these dynamics, we will need some preliminary results. Recall that $\pi_{i,t} = \frac{\beta_i \bar{C}_{i,t}}{\mathcal{M}_{i,t}}$, $i = H, F$. The following quantity will play an important role in our analysis:

$$\pi_t \triangleq 0.5 + \frac{1}{2}((1 - \pi_{H,t}) + (1 - \pi_{F,t}))\,.$$

Under autarky, when $\beta_H = \beta_F = 1$, we have $\beta_i \bar{C}_{i,t} = \mathcal{M}_{i,t}$, and hence $\pi_{H,t} = \pi_{F,t} = 1$, so that $\pi_t = 0.5$. Otherwise, π_t captures the average expenditures on non-domestic goods. Everywhere in the sequel, we will make the standard assumption of consumption home bias.

Assumption 1 (Consumption Home Bias) *We have $\beta_H > \frac{1}{2}$ and $\beta_F > \frac{1}{2}$.*

Under this assumption, it is possible to show that the following is true.

Lemma 3 *We have $\pi_{H,t} + \pi_{F,t} > 1$, i.e., $\pi_t \in (\frac{1}{2}, 1)$.*

We are now ready to characterize equilibrium dynamics in the presence of intermediation markups. To this end, we introduce some notation. We define

$$(\sigma_i)^2 = \|\theta_i\|^2 = d\langle \log \mathcal{M}_i \rangle_t / dt$$

to be the volatility of the centrally-traded tree in the country i and

$$\rho = \frac{(\theta_H)'\theta_F}{\sigma_H \sigma_F} = d\langle \log \mathcal{M}_H, \log \mathcal{M}_F \rangle_t / dt$$

to be the correlation between the trees across countries. Let also

$$\alpha_t \;=\; \frac{1+\Gamma}{2+\Gamma}\frac{2-2\pi_t}{2\pi_t-1}\,.$$

Proposition 3 (Equilibrium Risk Premia with Intermediation) *In equilibrium, risk premia are given by*

$$\begin{aligned}
\frac{1+\Gamma}{2+\Gamma}\eta^I_{i,t} \;&=\; \theta_i-\Big(\frac{1+\Gamma}{2+\Gamma}\theta^\Psi_{i,t}+\frac{1}{2+\Gamma}\lambda_{i,t}\theta_i\Big)\\
&-\;(\theta_{-i}-\theta_i)\left(-1+\frac{\pi_{-i,t}\alpha_t}{(1+\alpha_t)(2-2\pi_t)}+\frac{(1-\pi_{-i,t})}{(2\pi_t-1)(1+\alpha_t)}\right)\\
&-\;[\theta^\Psi_{i,t}-\theta^\Psi_{-i,t}]\left(\pi_{-i,t}\left(\frac{1}{1+\alpha_t}\frac{1}{2+\Gamma}-1\right)+\frac{\alpha_t}{1+\alpha_t}\right)\\
&-\;\big[\lambda_{-i,t}\theta_{-i}-\lambda_{i,t}\theta_i\big]\left(\pi_{-i,t}\frac{1}{1+\alpha_t}\frac{1}{2+\Gamma}-(1-\pi_{-i,t})\frac{1}{1+\Gamma}\frac{\alpha_t}{1+\alpha_t}\right).
\end{aligned}$$

In the case when $\Gamma=0$, this expression simplifies to

$$\eta^I_{i,t} = \underbrace{(2-\lambda_{i,t})\theta_i-\theta^\Psi_{i,t}}_{domestic\ risk} \;+\; \underbrace{\frac{1-\pi_{i,t}}{\pi_t}\big(\theta^\Psi_{i,t}-\theta^\Psi_{-i,t}+(\lambda_{i,t}-1)\theta_i-(\lambda_{-i,t}-1)\theta_{-i}\big)}_{risk\ sharing}\,, \tag{28}$$

where the dynamics of the Lagrange multipliers are given by

$$\lambda_{i,t} \;=\; 1+\frac{1}{\sigma_i^2}(\theta^\Psi_{i,t}-\theta^\Psi_{-i,t})'\Xi_{i,t} \;\; with\ \Xi_{i,t} \;=\; \frac{\theta_i+\frac{1-\pi_{-i,t}}{2-\pi_{i,t}}\rho\frac{\sigma_i}{\sigma_{-i}}\theta_{-i}}{\frac{2-\pi_{-i,t}}{1-\pi_{i,t}}-\frac{1-\pi_{-i,t}}{2-\pi_{i,t}}\rho^2}\,. \tag{29}$$

4.4 Economic Intuition

As we explain above, the adjustment costs parameter Γ controls the strength of intermediation frictions in our model. Here, we focus our discussion on the case of zero adjustment costs, corresponding to $\Gamma = 0$. The formula (28) shows explicitly how intermediation frictions affect risk sharing between the two countries through the two key quantities: the Lagrange multiplier, $\lambda_{i,t}$, and the degree of international trade, π_t. By Proposition 1, we have

$$\begin{aligned} \eta_{i,t} &= 0.5(\eta^I_{i,t} - \theta^\Psi_{i,t} + \lambda_{i,t}\theta_i) \\ &= \underbrace{\theta_i - \theta^\Psi_{i,t}}_{domestic\ risk} + \underbrace{0.5\frac{1-\pi_{i,t}}{\pi_t}\big(\theta^\Psi_{i,t} - \theta^\Psi_{-i,t} + (\lambda_{i,t}-1)\theta_i - (\lambda_{-i,t}-1)\theta_{-i}\big)}_{risk\ sharing}. \end{aligned}$$

Compared to the frictionless equilibrium of Proposition 2, we see that intermediation frictions introduce *time variations in international risk sharing*, whereby $(1-\pi_{i,t})\big(\theta^\Psi_{i,t} - \theta^\Psi_{-i,t}\big)$ gets multiplied by $\frac{0.5}{\pi_t}$. Since, by Lemma 3, $\pi_t \in (0.5, 1)$, intermediation serves as a barrier to the efficient allocation of risks. And given that π_t captures the average expenditures on non-domestic goods, this risk-sharing depends inversely on the degree of international trade, π_t. When there is more international trade (e.g., when the demand for domestic goods parameters, β_i, are smaller), households' desire to buy foreign goods pushes π_t up. However, in equilibrium, this translates into a corresponding desire for risk sharing via asset trading with intermediaries (households sell domestic securities to buy foreign goods). Intermediaries exploit this demand from households by charging higher markups. As a result, the amount of risk-sharing that households can actually achieve drops. In the extreme case when $\pi_t \approx 1$, households only achieve half of the risk sharing compared to the frictionless model.

The shadow cost of the constraint (15), $\lambda_{i,t}$, reflects the incentives of domestic households to adjust their exposure vis-a-vis domestic monetary shocks. When $\lambda_{i,t}$ is large, the domestic risk component of $\eta^I_{i,t}$ is dampened, while the risk sharing component increases. This is the mechanism behind monetary non-neutrality in our model: Intermediaries extract rents by exploiting households' desire to share monetary shocks. Thus, as in Itskhoki and Mukhin (2019), monetary non-neutrality arises due to intermediation frictions. However, while in Itskhoki and Mukhin (2019), this happens because intermediaries end up holding large amounts of nominal assets on their balance sheets, in our model, this happens because intermediaries strategically affect the exposure of households to monetary shocks. It is important to note that such monetary non-neutrality only arises when there are some real motives to trade. In our model, these real motives originate from differences in time discount shocks. When $\theta^{\Psi}_{i,t} - \theta^{\Psi}_{-i,t} \neq 0$, customers have incentives to share these shocks. This, in turn, triggers demand for risk sharing in the D2C market, whose sign and magnitude are determined by the alignment between the real motives for risk sharing, $\theta^{\Psi}_{i,t} - \theta^{\Psi}_{-i,t}$, and monetary shocks, as one can see from the explicit formula (29) for $\lambda_{i,t}$.

4.5 Equilibrium Dynamics of Interest Rates and Exchange Rates

We now discuss the equilibrium exchange rate dynamics. As above, we focus our analysis on the case $\Gamma = 0$. By no-arbitrage (equation (6)), exchange rates are pinned down as the quotient of intermediaries' pricing kernels. The dynamics of these kernels follow (13), and an application of the Ito formula leads to

$$d\log\mathcal{E}_t \;=\Big(\underbrace{(r^I_{H,t}-r^I_{F,t})}_{UIP}+\underbrace{\frac{1}{2}\big(\|\eta^I_{H,t}\|^2-\|\eta^I_{F,t}\|^2\big)}_{Currency\ risk\ premium}\Big)dt+(\eta^I_{H,t}-\eta^I_{F,t})'dB_t, \tag{30}$$

where the risk premium differentials are given by

$$\eta^I_{H,t}-\eta^I_{F,t}\;=\;\theta_H\;-\;\theta_F-\frac{1-\pi_t}{\pi_t}\big(\theta^\Psi_{H,t}-\theta^\Psi_{F,t}+(\lambda_{H,t}-1)\theta_H-(\lambda_{F,t}-1)\theta_F\big)\,.$$

When the cross-market (between D2D and D2C segments) arbitrage constraints in (20) bind, so that the Lagrange multiplier $\lambda_{i,t}-1\neq 0$, intermediation frictions create exposure of risk premia to monetary shocks, θ_i. When the Lagrange multipliers are low, households have no incentives to share monetary shocks. As a result, these shocks do not affect exchange rates. In equilibrium, the degree of monetary pass-through is determined by the degree of international trade. This stochastic pass-through of monetary shocks to exchange rates is a unique implication of our model.

As we show in the Appendix (see the proof of Proposition 2), equilibrium interest rates are given by

$$
\begin{aligned}
r_{i,t} = & \underbrace{\delta \; + \; \mu_i \; - \; \|\eta^I_{i,t}\|^2 \; - \; (\eta^I_{i,t})'(\pi_{i,t}\theta^\Psi_{i,t} + (1-\pi_{i,t})\theta^\Psi_{-i,t})}_{r^C_{i,t}} \\
& + \underbrace{\pi_{i,t}\mathcal{I}_{i,t} \; + \; (1-\pi_{i,t})\mathcal{I}_{-i,t}}_{global\ rents} \\
& + \underbrace{(\eta^I_{i,t})'\Big(\pi_{i,t}(\eta^I_{i,t} - \eta_{i,t}) + (1-\pi_{i,t})(\eta^I_{-i,t} - \eta_{-i,t})\Big)}_{risk\ premium\ markup\ alignment} .
\end{aligned}
\tag{31}
$$

The formula (31) shows explicitly how equilibrium interest rates deviate from their frictionless counterpart (27). The first term in (31) coincides with (27), with the complete market risk premia $\eta^C_{i,t}$ replaced by their D2D counterpart $\eta^I_{i,t}$. The second component, $\pi_{i,t}\mathcal{I}_{i,t} \; + \; (1-\pi_{i,t})\mathcal{I}_{-i,t} > 0$, is a weighted average of intermediation rents (18) extracted by intermediaries in the D2C market. In particular, this component is always positive. That is, *intermediaries' rent extraction pushes global interest rates up.*

The intuition behind this important result can be understood in a simple setting with a constant discount rate δ. In this case,

$$
C_{i,t} \;=\; e^{-\delta t} M^{-1}_{i,t} C_{i,0}
$$

and, hence,

$$
\underbrace{E_t[d\log C_{i,t}]}_{consumption\ growth} \;=\; \underbrace{-\delta dt}_{time\ discounting} \;+\; \underbrace{r_{i,t} dt}_{interest\ rates} \;+\; \underbrace{0.5\mathrm{Var}_t[d\log M_{i,t}]}_{risk}
$$

Intermediation frictions limit the households' ability to efficiently allocate wealth intertemporally, lowering the consumption growth rate. In equilibrium, global resource constraints (market clearing equations (24)) imply that $\log C_{i,t}$ has to grow at the same rate as the global output, and the interest rate needs to adjust upwards to ensure goods market clearing. This novel, surprising equilibrium channel is distantly related to the deposit channel introduced in Drechsler et al. (2017). Namely, in Drechsler et al. (2017), intermediaries (banks) affect interest rates through their market power in the risk-free (deposit) market. In our model, the market power in the deposit market is completely shut down by the no-arbitrage constraint (14). Yet, interest rates are impacted indirectly by the intermediary market power in the risky asset markets, thereby boosting intermediation rents. The fact that, by (31), interest rates positively co-move with intermediation rents is consistent with the empirical evidence (see, e.g., Borio et al. (2017)) on the positive relationship between bank profitability and interest rates. However, the mechanism suggested by our model is new, with the causality going in the opposite direction.

The last term in (31) depends on the alignment of risk premia, $\eta^I_{i,t}$, with risk premium markups, $\eta_{i,t} - \eta^I_{i,t}$ and $\eta_{-i,t} - \eta^I_{-i,t}$. By (20), intermediaries optimally align $\eta^I_{i,t} - \eta_{i,t}$ with $\eta_{i,t}$, while $\eta_{i,t}$ at the optimum is aligned with $\eta^I_{i,t}$. As a result, the last term in (31) also tends to be positive, pushing interest rates even higher.

To illustrate the capability of our model to explain real data, in our theoretical derivations below, we often make the following simplifying assumption.

Assumption 2 (Independent Shocks) *Output shocks are orthogonal to the demand shocks.*

That is, $\theta_i'\theta_{i,t}^{\Psi} = \theta_i'\theta_{-i,t}^{\Psi} = 0$, $i = H, F$. Furthermore, the demand shocks are orthogonal across countries, i.e., $(\theta_{i,t}^{\Psi})'\theta_{-i,t}^{\Psi} = 0$.

In the sequel, we focus our discussion on the zero-adjustment cost case ($\Gamma = 0$). Then, under Assumption 2, short-term rates follow

$$\begin{aligned} r_{i,t} \;=\; & \underbrace{\mu_i + \delta - \|\theta_i\|^2}_{r_i^C} \\ & + \underbrace{\frac{1-\pi_{i,t}}{2\pi_t}\frac{(1-\pi_{-i,t}) + (\pi_{i,t} - \pi_{-i,t})(2\pi_t - 2)}{2\pi_t}\|\theta_{i,t}^{\Psi} - \theta_{-i,t}^{\Psi}\|^2}_{intermediation\ frictions}, \end{aligned} \tag{32}$$

where r_i^C is the short rate in the frictionless model (27). By direct calculation, the correction to the short-term rate (the second line in (32)) is always positive, consistent with our discussion of the formula (31).

Since r_i^C is constant, intermediation frictions generate volatility in the short rates (32) through fluctuations in international trade, as captured by π_t and $\pi_{i,t}$, as well as demand shocks, as captured by $\theta_{i,t}^{\Psi}$. In both the data and realistic calibrations, π_t is persistent and slow-moving. As a result, generating realistic interest rate volatility requires an additional source of fluctuations in $\theta_{i,t}^{\Psi}$. We model this source as time-varying, persistent, and mean-reverting volatility of demand shocks (see, e.g., Dahlquist et al. (2023) for a similar specification).

Assumption 3 (Volatility of the demand Shocks) *We have*

$$\|\theta_{i,t}^{\Psi}\|^2 \;=\; (\bar{\sigma}^{\Psi})^2 \;+\; (\sigma_t^{\Psi})^2, \;\; i = H, F,$$

for some $\bar{\sigma}^{\Psi} > 0$, *where* $\sigma_t^{\Psi} = \exp(4\,x_t^2) - 1$, *with* x_t *being a standard Jacobi process* $x_t \in [-1, 1]$,

$$dx_t = -\kappa^x x_t\, dt + \sigma^x \sqrt{1 - x_t^2} dB_t^x \,.$$

B_t^x *is a Brownian motion that is independent of other shocks.*[11]

Under Assumption 3, equilibrium dynamics are driven by two state variables: The exogenous state x_t and the endogenous state $\mathcal{Q}_t$ driving the international trade (see, Lemma 2). The two components of expected exchange rate fluctuations, the interest rate differential (the UIP component in (30)) and the risk premium term in (30), always move in the opposite directions, driven by households substituting between risky and risk-free assets. The precise nature of this equilibrium behavior is highly nonlinear, as is illustrated by Figure 2. The next section shows that when shocks are sufficiently volatile, the risk premium component dominates the interest rate differential, leading to quantitatively realistic exchange rate dynamics.

4.6 The CIP Deviations

In our model, households willing to borrow in foreign currency need to do so through the intermediaries in the D2C market. We use $r^F_{H,t}$ to denote the rate that intermediaries charge to country-H households for borrowing in the country F currency; $r^H_{F,t}$ is defined similarly. Naturally, market segmentation, together with intermediary market power, implies

[11] See, Appendix C for more details.

a difference between the direct rate, $r_{F,t}$, in the D2D market, and the D2C rate. It is known that institutional short-term funding markets are highly segmented, with often a very large dispersion in the rates available to different market participants. See Rime et al. (2022) for a detailed analysis of these markets. The friction in the retail segment of funding (credit) markets is even larger, with banks often exercising very large market power over their retail customers. For example, Hungarian households willing to borrow in US dollars could do so through a local branch of an international bank, which would necessarily include a (large) markup into the quoted rate.

Another possibility for an H household willing to borrow in the F currency is to do so synthetically. Given the current rate $r_{H,t}$ and the forward rate $f_{t,t+dt}$ (quoted in the D2C market with a markup), the household can borrow x units of the H-currency and at the same time sign a forward contract to exchange $x(1+r_{H,t}dt)$ units of the H-currency at time $t+dt$ into $x(1+r_{H,t}dt)f_{H,t,t+dt}$ units of the F-currency. This is equivalent to borrowing $x/\mathcal{E}_t$ units of the F-currency at the *synthetic* rate $r^F_{H,t}$ such that $x(1+r^F_{H,t}dt) = x(1+r_{H,t}dt)f_{H,t,t+dt}/\mathcal{E}_t$. Due to the absence of arbitrage in the D2C market for H-households, $r^F_{H,t}$ has to coincide with the rate quoted D2C as part of the pricing kernel $M_{H,t,t+dt}$. This identity is commonly known as the *covered interest parity*; however, due to segmentation between D2C and D2D markets, the synthetic rate is generally different from the direct rate $r_{F,t}$. The differences between the observed direct rates and the synthetic rates,

$$CIP_{H,t} \;=\; r_{F,t} \;-\; r^F_{H,t}\,,$$

are called CIP deviations. Similarly, we can define these deviations for country F-households as

$$CIP_{F,t} \;=\; r_{H,t} \;-\; r_{F,t}^{H}\,.$$

As we show below, in our model, households always find it optimal to borrow in foreign currency. As a result, the quoted D2C rates $r_{i,t}^{-i}$ contain a positive markup that we can interpret as borrowing CIP-deviations documented in Rime et al. (2022). Note also that, in the existing empirical studies (see, e.g., Du et al. (2019a), Rime et al. (2022)), CIP deviations are always defined against the USD (i.e., $CIP_{H,t} = r_{\$,t} - r_{H,t}^{\$}$ in our notation). Investigating the joint behavior of $CIP_{F,t},\ CIP_{H,t}$ is an interesting direction for future research.

Recent work has pointed to the balance sheet constraints of intermediaries as the key mechanism responsible for CIP deviations. See, for example, Du et al. (2019a) and Du et al. (2023). Our model shows how sizable CIP deviations (e.g., up to hundreds of basis points; see Figure 2b in the Appendix) can arise in a general equilibrium macroeconomic model purely due to real demand forces coupled with intermediaries' market power with *risk-neutral intermediaries holding zero net positions.* The mechanism underlying CIP deviations in our model is as follows: Demand for real goods generates demand for foreign assets by households that intermediaries cater to. Intermediary market power endogenously creates market segmentation, and as a result, the demand for foreign assets pushes up their prices in the D2C market, leading to a widening of CIP deviations. Our continuous-time equilibrium allows us to derive a simple, closed-form expression for these deviations.

Proposition 4 (CIP deviations) *The short rate for borrowing in the country $(-i)$ currency quoted D2C to the country i household, $r_{i,t}^{-i}$, is given by*

$$
\begin{aligned}
r_{H,t}^{F} &= \underbrace{r_{F,t}}_{direct\ country-F\ rate} + \underbrace{(\eta_{H,t}^{I} - \eta_{F,t}^{I})'(\eta_{H,t} - \eta_{H,t}^{I})}_{markup} \\
r_{F,t}^{H} &= \underbrace{r_{H,t}}_{direct\ country-H\ rate} + \underbrace{(\eta_{F,t}^{I} - \eta_{H,t}^{I})'(\eta_{F,t} - \eta_{F,t}^{I})}_{markup} .
\end{aligned}
$$

Under Assumption 2, we have

$$
CIP_{i,t} = \frac{(1-\pi_{i,t})(\pi_t - 1)}{2(\pi_t)^2} \underbrace{\|\theta_{i,t}^{\Psi} - \theta_{-i,t}^{\Psi}\|^2}_{risk\ sharing} < 0 , \ i = H, F . \tag{33}
$$

H-households when seeking to borrow in country-F currency face the direct rate $r_{F,t}$ that prevails in the F currency for domestic residents plus "a spread / markup", given by $-CIP_{H,t} = r_{H,t}^{F} - r_{F,t}$. By Lemma 3, $\pi_t < 1$, while $\pi_{i,t} < 1$ for all i. Thus, $\text{CIP}_{i,t}$ is always negative.

This is surprising. It means that, in a symmetric setting with two ex-ante identical countries, demand pressure in the D2C market always makes borrowing in foreign currency more expensive. The size of the CIP deviations is proportional to the size of the potential real gains from risk sharing, as captured by the difference $\theta_{H,t}^{\Psi} - \theta_{F,t}^{\Psi}$ (see (26)). When the shock exposures $\theta_{H,t}^{\Psi}$, $\theta_{F,t}^{\Psi}$ are sufficiently different, households have strong motives to share the corresponding risks. One way to achieve this risk sharing is by borrowing in foreign currency and gaining exposure to foreign exchange rate fluctuations. This natural demand pressure in our setup with endogenous segmentation creates a wedge, $CIP_{i,t}$, between the two interest rates. By Assumption 3 (and assuming that $\mathrm{corr}(d\log\Psi_{i,t}, d\log\Psi_{-i,t})$ is constant, as in our calibration), the real motives to trade, $\|\theta^{\Psi}_{H,t} - \theta^{\Psi}_{F,t}\|^2 = const \cdot (\exp(4x_t^2) - 1)$ are monotone increasing in the common demand shock x_t and, hence, their size is controlled by the volatility of these shocks, $(\sigma^x)^2$: Large demand shock volatility implies large hedging demand pressures in the D2C market and, as a result, large intermediation markups for access to foreign risk-free assets. This mechanism is amplified in the presence of trade imbalance. In particular, if country H economy is closed (so that country i households only consume domestic goods), $1 - \pi_{H,t} \approx 0$ and $CIP_{H,t} \approx 0$: When households only consume domestic goods, then, in equilibrium, they end up not borrowing in the foreign currency, and there is no sense for intermediaries to charge markups for such borrowing.

5 Calibration

In this section, we perform a calibration exercise illustrating the ability of our model to quantitatively match several important empirically observed patterns in exchange rate dynamics.

We consider the following specifications for the dynamics of the output and demand shocks:

$$\begin{bmatrix} (\theta_H)' \\ (\theta_F)' \\ (\theta_H^\Psi)' \\ (\theta_F^\Psi)' \end{bmatrix} = \begin{bmatrix} \bar{\sigma}^{\mathcal{M}} & 0 & \bar{\sigma}^{\mathcal{M}} & 0 & 0 & 0 \\ 0 & \bar{\sigma}^{\mathcal{M}} & \rho^{\mathcal{M}}\bar{\sigma}^{\mathcal{M}} & \sqrt{1-(\rho^{\mathcal{M}})^2}\bar{\sigma}^{\mathcal{M}} & 0 & 0 \\ s^\Psi\bar{\sigma}^\Psi & 0 & 0 & 0 & \sqrt{1-(s^\Psi)^2}\sigma_t^\Psi & 0 \\ 0 & s^\Psi\bar{\sigma}^\Psi & 0 & 0 & \sqrt{1-(s^\Psi)^2}\rho^\Psi\sigma_t^\Psi & \sqrt{1-(s^\Psi)^2}\sqrt{1-(\rho^\Psi)^2}\sigma_t^\Psi \end{bmatrix} \tag{34}$$

Table 1 reports the calibrated coefficients in (34) (See Appendix C for details). σ_t^Ψ is defined in Assumption 3. Shock specification (34) ensures *symmetry*: Countries H and F are ex-ante identical and only differ through ex-post realizations of shocks. Such symmetry significantly limits the ability of the model to produce realistic dynamics of risk premia. In Appendix D, we investigate an extension of (34) (see (46)), allowing for asymmetric exposures of countries to shocks, and show how such an asymmetry allows us to achieve much better quantitative match for empirically observed risk premia.

We use our analytical solution from Proposition 3 with $\Gamma = 0$ to perform the simulations. As follows from Proposition 3, important quantities affecting the equilibrium dynamics are the cross-moments, $\theta_i'\theta_{i,t}^\Psi$ and $(\theta_{i,t}^\Psi)'\theta_{-i,t}^\Psi$. Here, $\theta_i'\theta_{i,t}^\Psi$ define the instantaneous covariances between output shocks, $\mathcal{M}_{i,t}$, and demand shocks, $\Psi_{i,t}$. Clearly, the sign and the magnitude of these covariances are key determinants of households' hedging behavior, defining whether the supply of the goods is abundant in the states that they value the most. By contrast, $(\theta_{i,t}^\Psi)'\theta_{-i,t}^\Psi$ is less important and only affects equilibrium through the size of risk sharing,

$\|\theta_{i,t}^{\Psi} - \theta_{-i,t}^{\Psi}\|^2$. A positive (respectively, negative) covariance, $(\theta_{i,t}^{\Psi})'\theta_{-i,t}^{\Psi}$ reduces (amplifies) the gains from risk sharing, acting as a scale parameter akin the volatility shock x_t. Apart from this scale effect, it has no impact on the signs and the nature of equilibrium moments.

Under Assumption 2, all these cross-moments are identically zero, implying that the frictionless interest rates are constant (see (27) and (32)), and making it impossible for the frictionless model to generate any joint dynamics for interest rates and exchange rates. Below, we investigate the sensitivity of our results to the size and the sign of $\theta_i'\theta_{i,t}^{\Psi}$ and $(\theta_{i,t}^{\Psi})'\theta_{-i,t}^{\Psi}$.

Tables 2-7 report the key moments for equilibrium quantities in the two models, with and without intermediation frictions. Each column in those tables corresponds to a different value for the parameter s^{Ψ} defining the correlation between output and demand shocks. The three pairs of tables correspond to ρ^{Ψ} in (34) taking values $-0.3, +0.3$, and 0.

We refer to the model without frictions as "Frictionless." The results for this model are reported in Tables 2, 4, and 6. The results for the model with frictions are reported in Tables 3, 5, and 7, respectively. Finally, Tables 16 and 17 report the results in the presence of country asymmetry. See Appendix D for details.

As usual, in these Tables, we use lowercase letters to denote log-quantities.

5.1 Standard International Marco Moments

First, we discuss several standard equilibrium moments. The autocorrelation of exchange rates is close to zero in both models, which is consistent with the data. The volatility of

Table 1: Parameter Choices for the Simulated Moments

Variable Definitions	**Symbols**	**Values**	**Targeted Moments**
Preferences	$\{\beta_i\}_{i=H,F}$	0.9	Trade-to-GDP ratio 0.2
Time discount	δ	0.03	—
Drift of $\mathcal{M}_{i,t}$	μ	0.03	—
Size of Supply shocks	$\bar{\sigma}^{\mathcal{M}}$	0.014	$\text{std}(dg_H) = 2\%$
Supply shocks correlation	$\rho^{\mathcal{M}}$	0.7	$\text{corr}(dg_H, dg_F) = 0.35$
Correlated demand shocks	σ_t^{Ψ}	$\exp(4\,x_t^2) - 1$	—
Demand shocks correlation	ρ^{Ψ}	0.3	$\text{std}(de) = 10\%$
Idiosyncratic demand shocks	$\bar{\sigma}^{\Psi}$	0.095	$\text{corr}(dc_H, dc_F) = 0.3$
Output-demand correlation	s^{Ψ}	-0.4	$\text{corr}(dc_H, dc_F) = 0.3$
Mean-reversion of x_t	κ^x	0.36/12	$\text{ac1}(r_H - r_F) = 0.95$
Volatility parameter of x_t	σ^x	0.09/2	$\text{std}(r_H - r_F) = 0.6\%$

exchange rates, consumption, and output are comparable across the two models and achieve a reasonable match with their empirical counterparts.

Next, the Backus and Smith (1993) puzzle (the negative correlation between log exchange rates and the log consumption growth differentials) is also explained by both models. The reason is that consumption growth in each country is affected by two components, one related to the shocks to nominal output, θ_i; the other related to the demand (time discount) shocks, θ_i^{Ψ}. While output shocks push exchange rates and consumption growth differentials in the same direction, demand shocks push them in opposite directions. If the latter effect is sufficiently strong, it dominates the former, leading to the empirically observed negative correlation.

The above results are not sensitive to the value of the s^Ψ parameter, controlling the output-demand correlation in (34) and indexing the columns in all Tables 2-7. By contrast, s^Ψ plays a key role in determining the cross-country correlations of consumption growth, $\mathrm{corr}_t(d\log\bar{C}_{H,t}, d\log\bar{C}_{F,t})$. All Tables 2-7 show that this cross-country correlation flips its sign together with the sign of s^Ψ. The underlying mechanism can be understood from formula $\bar{C}_{i,t} \;=\; \Psi_{i,t}M_{i,t}^{-1}C_{i,0}$, implying that

$$\bar{C}_{i,t}^{-1}d\bar{C}_{i,t} \;=\; *dt \;+\; (\underbrace{\theta_{i,t}^{\Psi}}_{demand\ shocks} \;+\; \underbrace{\eta_{i,t}}_{risk\ premium})'dB_t\,.$$

In the frictionless model, (26) implies that consumption exposure to shocks is given by $\theta_i \;+\; (1-\pi_{i,t})(\theta_{i,t}^\Psi - \theta_{-i,t}^\Psi)$. As a result, under the orthogonality assumptions that $\theta_i'\theta_{-i,t}^\Psi = 0$ (which follows directly from (34)), we have

$$\begin{aligned}
&\mathrm{corr}_t(d\log\bar{C}_{i,t}, d\log\bar{C}_{-i,t}) \;=\; \theta_i'\theta_{-i} \\
&-(1-\pi_{-i,t})\theta_i'\theta_{i,t}^\Psi - (1-\pi_{i,t})\theta_{-i}'\theta_{-i,t}^\Psi \\
&-\;(1-\pi_{i,t})(1-\pi_{-i,t})\|\theta_{i,t}^\Psi - \theta_{-i,t}^\Psi\|^2\,.
\end{aligned} \tag{35}$$

The first term in (35) is the cross-country output correlation, which is positive in the data and is set to be 35% in our calibration to monthly data. Absent international trade, $\pi_{i,t} =$

$\pi_{-i,t} = 1$, and consumption coincides with output. However, when international trade is sufficiently large and $\theta_i' \theta_{i,t}^{\Psi} = \theta_{-i}' \theta_{i,t}^{\Psi}$ is sufficiently positive, the sign of the correlation flips in both models.

We now discuss the cross-country correlations between interest rates, $\mathrm{corr}(r_H, r_F)$, and their changes, $\mathrm{corr}(dr_H, dr_F)$. In the frictionless model (Tables 2, 4, and 6), these correlations are both negative, close to -1. This stands in stark contrast with the empirically observed positive correlations, as well as the outcomes of the frictional model (Tables 3, 5, and 7), which also generate realistic positive correlations. The underlying economic mechanism can be seen directly from (27), which can be rewritten as

$$r_{i,t}^C \;=\; \mu_i \;+\; \delta \;+\; \|\theta_i\|^2 - \pi_{i,t} \theta' \theta_t^{\Psi} \,. \tag{36}$$

By Lemma 2, $\pi_{H,t}$ ($\pi_{F,t}$) is monotone increasing (decreasing) in $\mathcal{Q}_t$ and, hence, (27) implies that $\mathcal{Q}_t$ always pushes $r_{H,t}^C$ and $r_{F,t}^C$ in opposite directions. A similar mechanism is at play in the frictional model.

In the setting of (34), $\theta' \theta_t^{\Psi}$ is independent of x_t, so that only $\mathcal{Q}_t$ drives the frictionless interest rates. By contrast, x_t shocks have a major impact on the interest rates in the frictional model, moving the two interest rates in the same direction and producing a positive correlation. The reason is that, in the frictional model, interest rates are driven mainly by intermediation frictions, as can be seen from (31). The demand for risky assets in the D2C markets is driven by the amount of risks that need to be shared. Large x_t produces a large demand, and this demand pushes up intermediation rents and, through (31), also leads to an increase in both interest rates. As a result, interest rates and their changes are positively correlated across countries in the frictional model.

We complete this section with a discussion of equity returns and their correlations. Consistent with the classic equity premium puzzle, our equity risk premia (ERP) and equity Sharpe Ratios (equity SR) are too low in all of the Tables, consistent with the classic equity premium puzzle: There is not enough variation in the SDF to generate sizable risk premia with logarithmic preferences. Realistic equity risk premia and realistic Sharpe ratios can be achieved in our model through the introduction of more complex ingredients such as, e.g., long-run risk and recursive preferences (as in Colacito and Croce (2011)) or habit formation (as in Dahlquist et al. (2023)). However, in Appendix D, we show that we can also achieve quantitatively realistic equity risk premia and Sharpe Ratios in our model. Namely, when preference shocks are large enough and are strongly correlated with stock cash flows, households require large premia for holding stocks. Note also that while the cross-country stock return correlations reported in Appendix D are still too low, this happens because our stocks (Lucas trees) are short-term assets. The return correlation for long-term consumption claims is significantly higher.

5.2 The UIP and the currency risk premium

We now discuss the joint behavior of exchange rates and interest rates. Absent risk premia, expected exchange rate changes should move one-to-one with the interest rate differential, as captured by the formula (30), with the UIP predicting a coefficient of zero in the Fama

(1984) regression

$$(r_{H,t} - r_{F,t})dt - \log \mathcal{E}_{t+dt} + \log \mathcal{E}_t \;=\; \alpha + \beta^{\text{Fama}} (r_{H,t} - r_{F,t})dt + \varepsilon_{t+dt} \,. \tag{37}$$

On the left-hand side, we have the monthly excess return of investing in the home short-term bond markets while borrowing in the foreign short-term bond markets. Contrary to what UIP predicts, the empirically observed regression coefficient β^{Fama}, known as the Fama-beta, is not just different from zero. It is typically above 1, capturing the fact that the country with high short-term rates tends to see its exchange rate appreciate in the short run. This is commonly known as the UIP puzzle. Another closely related puzzle concerns the historically elevated Sharpe ratio of the carry trade strategy, taking positions in the currencies proportional to the interest rate differential and exploiting the negative coefficient in the Fama regression. We define the Carry Trade returns as

$$Carry_{t+dt} \;=\; \text{sign}(r_{H,t} - r_{F,t})\big((r_{H,t} - r_{F,t})dt - \log \mathcal{E}_{t+dt} + \log \mathcal{E}_t\big) \,., \tag{38}$$

We first note that, by (36), interest rates in the frictionless model are constant whenever $\theta'\theta^{\Psi}_t = 0$ (that is, when $s^{\Psi} = 0$ in (34)). In the symmetric setting of our calibration, we thus get $r^C_H = r^C_F$ and, hence, regression (37) cannot be estimated and carry trade returns (38) are identically zero. As Tables 2, 4, and 6 show, a nonzero $\theta'\theta^{\Psi}_t$ does help somewhat. However, there is a caveat: The size and the sign of the Carry SR and β^{Fama} in the frictionless model are highly sensitive to the size and the sign of $\theta'\theta^{\Psi}_t$. This stands in stark contrast to the behavior of the frictional model, where this sensitivity is small. The underlying economic mechanism is based on Lemma 2, which shows how the international trade (the $\pi_{i,t}$ variables) depends on the key endogenous state variable, $\mathcal{Q}_t$. In the Appendix (Figures 2-5), we illustrate the dependence of the key quantities (interest rate differential, the currency risk premium, CIP deviations, and the currency return) on $\mathcal{Q}_t$. As we can see, in the frictional model, the currency risk premium and the interest rate differential depend in the same manner on $\mathcal{Q}_t$, monotone increasing in the bulk of the $\mathcal{Q}_t$ distribution. The reason is that these quantities are dominated by the same intermediation friction, which responds monotonically to the demand for international goods, as captured by $\mathcal{Q}_t$. By contrast, in the frictionless model, a large and positive $\theta'\theta^\Psi$ leads to a complete breakdown of the carry trade so that the CRP SR becomes negative. As Figure 5 shows, the interest rate differential is negatively related to $\mathcal{Q}_t$ when $\theta_i'\theta_i^\Psi > 0$. This can also be seen directly from (36): $r^C_{i,t} - r^C_{-i,t} = const + (\pi_{-i,t} - \pi_{i,t})\theta'\theta^\Psi_t$, and, by Lemma 2, $\pi_{H,t} - \pi_{F,t}$ is monotone increasing in $\mathcal{Q}_t$.

For our base calibration, corresponding to $\rho^\Psi = 0.3$ and $s^\Psi = -0.4$, both frictionless (Table 4) and frictional (Table 5) produce a positive Carry Trade SR and a positive coefficient in the Fama regression. However, the Sharpe ratio in the frictional model is two times higher, while the coefficient in the Fama regression is almost 50 times lower than that in the frictionless model. The reason is that, without frictions, interest rates do not move enough, and their extremely low volatility in the frictionless model leads to an implausible Fama β above 800.

The extant literature has pointed to the balance sheet constraints of intermediaries as an important factor behind the Carry Trade Risk premia. See, e.g., Brunnermeier et al. (2008); Gabaix and Maggiori (2015). Our results imply that the market power of intermediaries could drive a non-trivial fraction of the CRP Sharpe ratio (about 5-10%). The fact that the Carry Trade can be so profitable in a model with risk-neutral, unconstrained intermediaries with zero net positions is surprising and suggests that market frictions originating from intermediaries' price discrimination can be responsible for a quantitatively significant fraction of empirically observed anomalies in the foreign exchange markets.

Carry Trade returns (38) can be broken down into two components,

$$Carry_{t+dt} \;=\; \underbrace{|r_{H,t}-r_{F,t}|dt}_{interest\ rate\ spread} \;+\; \underbrace{\operatorname{sign}(r_{H,t}-r_{F,t})(-\log\mathcal{E}_{t+dt}+\log\mathcal{E}_t)}_{FX\ risk}\,. \tag{39}$$

Empirical evidence implies that both components contribute positively to the Carry Risk premium. As one can see from Figure 2, this is perfectly consistent with our model because the expected exchange rate returns relate negatively to the interest rate differential. However, in the data, about 35% of the premium originates from the interest rate spread:

$$carry\ ratio \;=\; \frac{mean(|r_{H,t}-r_{F,t}|)}{mean(Carry)} \;\approx\; 0.35\,.$$

As we can see from Tables 2, 4, 6, the frictionless model is unable to match the empirically observed contributions because interest rates are not volatile enough: Interest rate volatility and carry ratio are respectively 50 and 100 times lower than their empirically observed counterparts. By contrast, our model does generate a realistic exchange rate volatility, which ends up dominating the carry premium (39).

In our base calibration, mean interest rates are identical across countries, with the interest rate differential mean reverting towards zero. By contrast, real-world countries feature highly asymmetric interest rate behavior. E.g., rates in Australia are always higher than interest rates in Japan. In Appendix D, we show how, by introducing asymmetric shock exposures, we can achieve systematic differences in interest rates (with one country always having higher interest rates). This more realistic calibration allows us to generate a carry ratio that closely agrees with its empirical counterpart (see Tables 16 and 17).

The problem with the low volatility of interest rates is particularly severe for the frictionless model. As we can clearly see from Figures 2-5, this model is incapable of generating realistic fluctuations in the interest rate differential (even in the extreme scenarios of very large or very small $\mathcal{Q}_t$, $r_{H,t} - r_{F,t}$ stays around five basis points). Furthermore, in the frictionless model, Carry breaks down in the tails of $\mathcal{Q}_t$ because the link between interest rate differential and exchange rates flips its sign. By contrast, these figures clearly show that, in the frictional model, we have (1) Carry works even in the tails (interest rate differential and exchange rates are always positively related) and (2) quantitatively realistic rate fluctuations occur when the volatility shock, $|x_t|$, is sufficiently large. Thus, to quantitatively match the joint behavior of interest rates and exchange rates, we need a stochastic process for $|x_t|$ that takes large values with a high probability. This can be achieved by changing the persistence and volatility coefficients, κ^x and σ^x, in Assumption 3. We consider four alternative specifications for the choice of (κ^x, σ^x) in Tables 8-11 in the Appendix. When κ^x is low, and σ^x is high, $|x_t|$ stays large for prolonged periods of time, allowing the model to achieve realistic interest rate volatility about one tenth of the observed Carry Sharpe Ratio and match the Fama β. However, this quantitative success comes at the cost of boosting the consumption volatility to 3% and reducing consumption correlation to nearly 0%. Finally, we note that the asymmetric calibration in Appendix D achieves a significantly higher Carry Sharpe Ratio by allowing for systematic differences in interest rates across countries. See Tables 16 and 17.

The ability of our model to better match the observed empirical moments relies on the joint dynamics of x_t and $\mathcal{Q}_t$ shocks. The former generates demand for risk sharing. The latter generates non-trivial trade dynamics due to the home bias in consumption ($\beta_i > 0.5$). Table 12 reports the results of the calibration when one of these channels (demand shocks or home bias) is shut down. By Lemma 2, absent home bias, $\mathcal{Q}_t = 1$ in this case and exchange rate equals $\mathcal{E}_t = \frac{\mathcal{M}_{H,t}}{\mathcal{M}_{F,t}}$. Intermediation frictions are thus irrelevant to the exchange rate dynamics. A similar phenomenon takes place when we shut down the demand shocks. Indeed, as long as $\theta^\Psi_{H,t} = \theta^\Psi_{F,t}$, there are no real motives for risk sharing between the two countries' households, and Proposition 3 shows that intermediation frictions have no impact on equilibrium behavior. Finally, we note that the volatility of $\theta^\Psi_{i,t}$ in our model is generated by fluctuations in x_t. Shutting down x_t by setting it to zero essentially eliminates volatility in interest rates and makes them negatively correlated, like in the frictionless model because, as we explain above, $\mathcal{Q}_t$ always moves the two rates in opposite directions, and we need large

x_t fluctuations to overcome this. The absence of x_t also significantly reduces the Carry Risk premium and leads to an explosion of the Fama beta.

5.3 The CIP deviations

We complete this section with a discussion of the behavior of CIP deviations. The average CIP deviation of -0.11% reported in Table 5 is broadly consistent with the empirically observed CIP deviations of around -0.21% on average (documented, e.g., in Avdjiev et al. (2019)).[12] Figure 2 also shows that when $|x_t|$ is large, our model can produce very large CIP deviations, comparable to those observed during the financial crisis. In a calibration where x_t is volatile (Table 10), average CIP deviations can take even larger values.

Our model can also shed some light on the joint dynamics of the CIP deviations and exchange rates. A surprising empirical regularity documented by Avdjiev et al. (2019) is that changes in $CIP_{H,t}$ are negatively correlated with contemporaneous changes in the exchange rate (defined as the Dollar Index): In the regression

$$\mathrm{CIP}_{H,t+dt} - \mathrm{CIP}_{H,t} \;=\; \alpha + \beta^{\mathrm{CIP}} \left(\log \mathcal{E}_{t+dt} - \log \mathcal{E}_t\right) + \varepsilon_{t+dt}\,, \tag{40}$$

the estimated β^{CIP} coefficient is negative and large, around -2.7, while the R^2 is also quite high, around 2%. Avdjiev et al. (2019) argue that changes in the Dollar index capture global financial conditions and are related to intermediary balance sheets; hence, the tight link with the CIP deviations in (40). Table 5 shows that our model is able to replicate these empirical

[12] We follow Avdjiev et al. (2019) and define the empirical counterpart of $CIP_{H,t}$ as the average CIP deviation of non-US countries against the foreign country $F = US$.

findings. The underlying mechanism is different and can be understood through the lens of Proposition 4. The latter shows explicitly how the join dynamics of CIP deviations and exchange rates emerge through the risk premia responsible for UIP deviations, whereby the same risk premium component in (30) that drives UIP deviations also drives the CIP deviations, leading to the negative correlation documented in Avdjiev et al. (2019). As such, rather than being two distinct phenomena, UIP and CIP deviations emerge as two sides of the same coin in our framework. Altogether, Table 5 suggests that intermediary market power might be responsible for a sizable part of the empirically observed UIP and CIP deviations and their complex joint dynamics.

It is also important to note that the empirically observed large CIP deviations only appeared after the great financial crisis. As Du et al. (2019a); Rime et al. (2022) show, this phenomenon is related to tighter capital requirements and balance sheet constraints. However, this tightening has also been associated with a significant drop in the competitiveness of FX markets documented, for example, in Moore et al. (2016). The tighter banking regulations increased intermediation's fixed costs, leading to the exit of many players. The associated increase in the market power of remaining FX market makers might be responsible for higher markups and, as a result, larger CIP deviations. See, e.g., Wallen (2020); Hau et al. (2017) for empirical evidence.

Table 2: Frictionless Model. This table presents simulated moments for a frictionless model with negatively correlated demand shocks ($\rho^{\Psi} = -0.3$). The first two columns display estimates of the data moments and their associated standard errors, where applicable. The subsequent five columns present the simulated moments for the frictionless model across various values of s^{Ψ}.

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	13.19	13.38	13.17	12.27	10.96
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	6.64	6.76	6.66	6.21	5.54
std($d\bar{c}_H$)(%)	1.49	(0.10)	2.45	2.64	2.83	3.09	3.23
std($d\bar{c}_F$)(%)	1.60	(0.11)	2.44	2.65	2.86	3.12	3.27
std(de)/std($d\bar{c}_H$)	6.32	—	5.20	4.21	3.38	3.01	2.96
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.59	0.65	0.73	0.77	0.80
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	0.18	0.04	-0.07	-0.19	-0.29
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	-0.71	-0.68	-0.68	-0.74	-0.84
std($r_H - r_F$)(%)	0.69	(0.04)	0.01	0.00	0.00	0.00	0.01
std($r_H - r_F$)/std(de)	0.07	—	0.00	0.00	0.00	0.00	0.00
std(r_H)(%)	1.12	(0.07)	0.00	0.00	0.00	0.00	0.00

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
corr(r_H, r_F)	0.81	(0.03)	-0.96	-0.96	—	-0.96	-0.96
corr(dr_H, dr_F)	0.59	(0.06)	-0.86	-0.85	—	-0.85	-0.86
ac1($r_H - r_F$)	0.96	(0.09)	0.99	0.99	—	0.99	0.99
ac1(r_H)	0.98	(0.09)	0.99	0.99	—	0.99	0.99
Fama-β	2.18	(1.25)	459.99	817.62	—	-680.95	-362.99
carry SR(%)	37.23	(18.41)	1.00	1.26	—	-1.06	-0.71
carry i-diff (%)	1.21	(0.39)	0.02	0.01	—	0.01	0.02
carry (%)	3.46	(9.28)	0.12	0.15	—	-0.09	-0.06
carry ratio (%)	34.97	(18.41)	0.09	0.06	—	-0.06	-0.09
std(carry) (%)	9.28	(0.60)	13.19	13.37	—	12.28	10.96
std(carry i-diff) (%)	0.39	(0.03)	0.00	0.00	—	0.00	0.00
mean(CIP$_H$)(%)	-0.21	(0.30)	0.00	0.00	0.00	0.00	0.00
CIP-β	-2.64	(0.68)	—	—	—	—	—
t CIP	-3.87	—	—	—	—	—	—
R^2 CIP(%)	2.00	—	—	—	—	—	—
ERP$_H$(%)	3.48	(12.74)	0.49	0.36	0.19	0.01	-0.13
equity SR$_H$(%)	27.26	(18.41)	6.16	4.47	2.31	0.10	-1.58
ERP$_F$(%)	2.01	(15.85)	0.47	0.34	0.17	-0.00	-0.14

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
equity SR_F(%)	12.66	(18.41)	5.93	4.23	2.14	-0.02	-1.72
corr(ERP_H, ERP_F)	0.85	(0.03)	0.35	0.35	0.35	0.35	0.35

Table 3: Frictional Model. This table presents simulated moments for a frictional model with negatively correlated demand shocks ($\rho^{\Psi} = -0.3$). The first two columns display estimates of the data moments and their associated standard errors, where applicable. The subsequent five columns present the simulated moments for the frictional model across various values of s^{Ψ}.

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	11.31	11.33	10.97	10.11	9.03
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	5.69	5.71	5.54	5.11	4.56
std($d\bar{c}_H$)(%)	1.49	(0.10)	2.18	2.37	2.57	2.79	2.93
std($d\bar{c}_F$)(%)	1.60	(0.11)	2.16	2.36	2.56	2.79	2.94
std(de)/std($d\bar{c}_H$)	6.32	—	4.90	3.99	3.25	2.88	2.73
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.69	0.72	0.78	0.82	0.85
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	0.22	0.06	-0.05	-0.14	-0.22
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	-0.64	-0.63	-0.65	-0.71	-0.81
std($r_H - r_F$)(%)	0.69	(0.04)	0.09	0.14	0.16	0.14	0.08
std($r_H - r_F$)/std(de)	0.07	—	0.01	0.01	0.01	0.01	0.01
std(r_H)(%)	1.12	(0.07)	0.03	0.05	0.06	0.05	0.03

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
corr(r_H, r_F)	0.81	(0.03)	0.56	0.74	0.82	0.83	0.85
corr(dr_H, dr_F)	0.59	(0.06)	0.56	0.61	0.60	0.64	0.70
ac1$(r_H - r_F)$	0.96	(0.09)	0.96	0.95	0.95	0.95	0.95
ac1(r_H)	0.98	(0.09)	0.96	0.95	0.95	0.95	0.96
Fama-β	2.18	(1.25)	17.13	8.19	5.75	6.13	9.03
carry SR(%)	37.23	(18.41)	1.85	2.49	2.55	2.24	1.69
carry i-diff (%)	1.21	(0.39)	0.08	0.09	0.08	0.07	0.04
carry (%)	3.46	(9.28)	0.20	0.23	0.19	0.17	0.13
carry ratio (%)	34.97	(18.41)	0.61	0.66	0.46	0.47	0.23
std(carry) (%)	9.28	(0.60)	11.32	11.33	10.97	10.11	9.02
std(carry i-diff) (%)	0.39	(0.03)	0.02	0.04	0.05	0.04	0.02
mean(CIP$_H$)(%)	-0.21	(0.30)	-0.16	-0.17	-0.17	-0.15	-0.13
CIP-β	-2.64	(0.68)	-0.21	-0.37	-0.51	-0.45	-0.27
t CIP	-3.87	—	-3.06	-2.36	-2.24	-2.21	-2.50
R^2 CIP(%)	2.00	—	5.37	4.73	5.79	5.59	4.94
ERP$_H$(%)	3.48	(12.74)	0.49	0.36	0.19	0.01	-0.12
equity SR$_H$(%)	27.26	(18.41)	6.18	4.48	2.34	0.13	-1.52
ERP$_F$(%)	2.01	(15.85)	0.47	0.34	0.17	-0.00	-0.14

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
equity SR_F(%)	12.66	(18.41)	5.92	4.23	2.17	-0.00	-1.74
corr(ERP_H, ERP_F)	0.85	(0.03)	0.35	0.35	0.35	0.35	0.35

Table 4: Frictionless Model. This table presents simulated moments for a frictionless model with positively correlated demand shocks ($\rho^{\Psi} = 0.3$). The first two columns display estimates of the data moments and their associated standard errors, where applicable. The subsequent five columns present the simulated moments for the frictionless model across various values of s^{Ψ}.

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	11.81	11.40	10.80	10.00	9.20
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	5.97	5.77	5.46	5.05	4.66
std($d\bar{c}_H$)(%)	1.49	(0.10)	2.19	2.28	2.48	2.78	3.02
std($d\bar{c}_F$)(%)	1.60	(0.11)	2.17	2.30	2.49	2.80	3.02
std(de)/std($d\bar{c}_H$)	6.32	—	5.29	4.18	3.21	2.81	2.80
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.66	0.74	0.81	0.84	0.85
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	0.31	0.12	-0.04	-0.17	-0.28
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	-0.67	-0.62	-0.63	-0.72	-0.85
std($r_H - r_F$)(%)	0.69	(0.04)	0.01	0.00	0.00	0.00	0.01
std($r_H - r_F$)/std(de)	0.07	—	0.00	0.00	0.00	0.00	0.00
std(r_H)(%)	1.12	(0.07)	0.00	0.00	0.00	0.00	0.00

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
corr(r_H, r_F)	0.81	(0.03)	-0.97	-0.97	—	-0.97	-0.97
corr(dr_H, dr_F)	0.59	(0.06)	-0.89	-0.89	—	-0.90	-0.89
ac1($r_H - r_F$)	0.96	(0.09)	0.99	0.99	—	0.99	0.99
ac1(r_H)	0.98	(0.09)	0.99	0.99	—	0.99	0.99
Fama-β	2.18	(1.25)	473.60	872.88	—	-687.48	-360.27
carry SR(%)	37.23	(18.41)	0.60	0.83	—	-0.55	-0.44
carry i-diff (%)	1.21	(0.39)	0.01	0.01	—	0.01	0.01
carry (%)	3.46	(9.28)	0.08	0.09	—	-0.04	-0.04
carry ratio (%)	34.97	(18.41)	0.08	0.05	—	-0.04	-0.08
std(carry) (%)	9.28	(0.60)	11.81	11.40	—	10.00	9.20
std(carry i-diff) (%)	0.39	(0.03)	0.00	0.00	—	0.00	0.00
mean(CIP$_H$)(%)	-0.21	(0.30)	0.00	0.00	0.00	0.00	0.00
CIP-β	-2.64	(0.68)	—	—	—	—	—
t CIP	-3.87	—	—	—	—	—	—
R^2 CIP(%)	2.00	—	—	—	—	—	—
ERP$_H$(%)	3.48	(12.74)	0.50	0.36	0.19	0.01	-0.13
equity SR$_H$(%)	27.26	(18.41)	6.28	4.56	2.31	0.07	-1.65
ERP$_F$(%)	2.01	(15.85)	0.48	0.34	0.17	-0.01	-0.14

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
equity $\mathrm{SR}_F(\%)$	12.66	(18.41)	5.99	4.29	2.14	-0.08	-1.82
corr(ERP_H, ERP_F)	0.85	(0.03)	0.35	0.35	0.35	0.35	0.35

Table 5: Frictional Model. This table presents simulated moments for a frictional model with positively correlated demand shocks ($\rho^{\Psi} = 0.3$). The first two columns display estimates of the data moments and their associated standard errors, where applicable. The subsequent five columns present the simulated moments for the frictional model across various values of s^{Ψ}.

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	10.23	9.75	9.07	8.29	7.58
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	5.18	4.94	4.59	4.20	3.83
std($d\bar{c}_H$)(%)	1.49	(0.10)	2.02	2.13	2.33	2.58	2.78
std($d\bar{c}_F$)(%)	1.60	(0.11)	2.02	2.14	2.33	2.58	2.77
std(de)/std($d\bar{c}_H$)	6.32	—	4.91	3.92	3.08	2.65	2.52
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.76	0.81	0.85	0.88	0.89
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	0.38	0.17	0.01	-0.12	-0.21
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	-0.53	-0.52	-0.57	-0.67	-0.81
std($r_H - r_F$)(%)	0.69	(0.04)	0.04	0.06	0.07	0.06	0.04
std($r_H - r_F$)/std(de)	0.07	—	0.00	0.01	0.01	0.01	0.00
std(r_H)(%)	1.12	(0.07)	0.02	0.03	0.03	0.03	0.02

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
corr(r_H, r_F)	0.81	(0.03)	0.47	0.74	0.85	0.86	0.88
corr(dr_H, dr_F)	0.59	(0.06)	0.53	0.67	0.67	0.71	0.76
ac1($r_H - r_F$)	0.96	(0.09)	0.97	0.96	0.95	0.95	0.95
ac1(r_H)	0.98	(0.09)	0.97	0.96	0.95	0.96	0.96
Fama-β	2.18	(1.25)	37.30	17.53	11.21	10.68	15.99
carry SR(%)	37.23	(18.41)	1.14	1.48	1.43	1.60	1.04
carry i-diff (%)	1.21	(0.39)	0.05	0.04	0.03	0.03	0.02
carry (%)	3.46	(9.28)	0.12	0.13	0.11	0.11	0.07
carry ratio (%)	34.97	(18.41)	0.37	0.30	0.16	0.24	0.08
std(carry) (%)	9.28	(0.60)	10.23	9.75	9.06	8.30	7.57
std(carry i-diff) (%)	0.39	(0.03)	0.01	0.02	0.02	0.02	0.01
mean(CIP$_H$)(%)	-0.21	(0.30)	-0.13	-0.11	-0.11	-0.10	-0.09
CIP-β	-2.64	(0.68)	-0.13	-0.23	-0.34	-0.31	-0.18
t CIP	-3.87	—	-3.41	-2.27	-2.02	-2.02	-2.64
R^2 CIP(%)	2.00	—	5.55	3.40	4.13	4.17	4.55
ERP$_H$(%)	3.48	(12.74)	0.50	0.36	0.19	0.01	-0.13
equity SR$_H$(%)	27.26	(18.41)	6.25	4.56	2.34	0.08	-1.62
ERP$_F$(%)	2.01	(15.85)	0.48	0.34	0.17	-0.00	-0.14

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
equity SR_F(%)	12.66	(18.41)	5.98	4.32	2.15	-0.05	-1.78
corr(ERP_H, ERP_F)	0.85	(0.03)	0.35	0.35	0.35	0.35	0.35

Table 6: Frictionless Model. This table presents simulated moments for a frictionless model with uncorrelated demand shocks ($\rho^{\Psi} = 0.0$). The first two columns display estimates of the data moments and their associated standard errors, where applicable. The subsequent five columns present the simulated moments for the frictionless model across various values of s^{Ψ}.

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	12.58	12.54	12.16	11.31	10.17
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	6.34	6.33	6.15	5.70	5.15
std($d\bar{c}_H$)(%)	1.49	(0.10)	2.32	2.46	2.66	2.94	3.12
std($d\bar{c}_F$)(%)	1.60	(0.11)	2.31	2.48	2.68	2.96	3.14
std(de)/std($d\bar{c}_H$)	6.32	—	5.25	4.21	3.30	2.93	2.89
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.62	0.69	0.76	0.80	0.82
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	0.23	0.07	-0.06	-0.18	-0.29
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	-0.70	-0.66	-0.66	-0.74	-0.85
std($r_H - r_F$)(%)	0.69	(0.04)	0.01	0.00	0.00	0.00	0.01
std($r_H - r_F$)/std(de)	0.07	—	0.00	0.00	0.00	0.00	0.00
std(r_H)(%)	1.12	(0.07)	0.00	0.00	0.00	0.00	0.00

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
corr(r_H, r_F)	0.81	(0.03)	-0.97	-0.97	—	-0.97	-0.97
corr(dr_H, dr_F)	0.59	(0.06)	-0.87	-0.87	—	-0.87	-0.88
ac1($r_H - r_F$)	0.96	(0.09)	0.99	0.99	—	0.99	0.99
ac1(r_H)	0.98	(0.09)	0.99	0.99	—	0.99	0.99
Fama-β	2.18	(1.25)	465.89	840.95	—	-681.39	-360.51
carry SR(%)	37.23	(18.41)	0.66	1.11	—	-0.63	-0.46
carry i-diff (%)	1.21	(0.39)	0.02	0.01	—	0.01	0.02
carry (%)	3.46	(9.28)	0.08	0.12	—	-0.05	-0.04
carry ratio (%)	34.97	(18.41)	0.08	0.05	—	-0.05	-0.08
std(carry) (%)	9.28	(0.60)	12.58	12.54	—	11.31	10.17
std(carry i-diff) (%)	0.39	(0.03)	0.00	0.00	—	0.00	0.00
mean(CIP$_H$)(%)	-0.21	(0.30)	0.00	0.00	0.00	0.00	0.00
CIP-β	-2.64	(0.68)	—	—	—	—	—
t CIP	-3.87	—	—	—	—	—	—
R^2 CIP(%)	2.00	—	—	—	—	—	—
ERP$_H$(%)	3.48	(12.74)	0.49	0.36	0.19	0.01	-0.13
equity SR$_H$(%)	27.26	(18.41)	6.22	4.53	2.31	0.09	-1.61
ERP$_F$(%)	2.01	(15.85)	0.48	0.34	0.17	-0.00	-0.14

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
equity SR_F(%)	12.66	(18.41)	5.96	4.25	2.14	-0.04	-1.77
corr(ERP_H, ERP_F)	0.85	(0.03)	0.35	0.35	0.35	0.35	0.35

Table 7: Frictional Model. This table presents simulated moments for a frictional model with uncorrelated demand shocks ($\rho^{\Psi} = 0.0$). The first two columns display estimates of the data moments and their associated standard errors, where applicable. The subsequent five columns present the simulated moments for the frictional model across various values of s^{Ψ}.

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	10.85	10.66	10.18	9.36	8.38
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	5.47	5.38	5.15	4.72	4.24
std($d\bar{c}_H$)(%)	1.49	(0.10)	2.10	2.24	2.45	2.69	2.85
std($d\bar{c}_F$)(%)	1.60	(0.11)	2.09	2.25	2.45	2.69	2.86
std(de)/std($d\bar{c}_H$)	6.32	—	4.91	3.97	3.19	2.79	2.65
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.72	0.76	0.81	0.84	0.87
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	0.28	0.10	-0.03	-0.13	-0.22
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	-0.60	-0.59	-0.62	-0.70	-0.81
std($r_H - r_F$)(%)	0.69	(0.04)	0.06	0.10	0.11	0.09	0.06
std($r_H - r_F$)/std(de)	0.07	—	0.01	0.01	0.01	0.01	0.01
std(r_H)(%)	1.12	(0.07)	0.03	0.04	0.05	0.04	0.02
corr(r_H, r_F)	0.81	(0.03)	0.53	0.74	0.83	0.85	0.87

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
corr(dr_H, dr_F)	0.59	(0.06)	0.55	0.63	0.62	0.66	0.73
ac1($r_H - r_F$)	0.96	(0.09)	0.96	0.95	0.95	0.95	0.95
ac1(r_H)	0.98	(0.09)	0.96	0.95	0.95	0.95	0.96
Fama-β	2.18	(1.25)	24.24	11.39	7.78	7.92	11.53
carry SR(%)	37.23	(18.41)	1.44	2.11	1.89	1.86	1.33
carry i-diff (%)	1.21	(0.39)	0.06	0.06	0.06	0.05	0.03
carry (%)	3.46	(9.28)	0.15	0.20	0.14	0.14	0.11
carry ratio (%)	34.97	(18.41)	0.48	0.49	0.29	0.34	0.14
std(carry) (%)	9.28	(0.60)	10.85	10.66	10.18	9.35	8.38
std(carry i-diff) (%)	0.39	(0.03)	0.02	0.03	0.03	0.03	0.02
mean(CIP$_H$)(%)	-0.21	(0.30)	-0.14	-0.14	-0.14	-0.13	-0.11
CIP-β	-2.64	(0.68)	-0.18	-0.31	-0.43	-0.39	-0.23
t CIP	-3.87	—	-3.19	-2.32	-2.15	-2.14	-2.56
R^2 CIP(%)	2.00	—	5.35	4.11	5.04	4.92	4.73
ERP$_H$(%)	3.48	(12.74)	0.49	0.36	0.19	0.01	-0.12
equity SR$_H$(%)	27.26	(18.41)	6.23	4.54	2.34	0.12	-1.58
ERP$_F$(%)	2.01	(15.85)	0.48	0.34	0.17	-0.00	-0.14
equity SR$_F$(%)	12.66	(18.41)	5.96	4.27	2.15	-0.02	-1.76

Continued on next page

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
corr(ERP_H, ERP_F)	0.85	(0.03)	0.35	0.35	0.35	0.35	0.35

6 Conclusions

We introduce an imperfectly competitive intermediation sector into a standard, international monetary model à la Lucas (1982). We show that one simple friction, whereby intermediaries exploit their market power and charge endogenous markups for providing households with access to foreign securities, can generate rich behavior of risk premiums, exchange rates, and CIP deviations. We solve the model in continuous time, which allows us to analyze all quantities and economic mechanisms through explicit, closed-form expressions.

We show how intermediation markups help account for several known puzzles in exchange rate behavior, including the joint dynamics of deviations from CIP and UIP. In particular, our calibration exercise suggests that both the size and the dynamics of the empirically observed CIP deviations can be partially explained by real demand pressures, with risk-neutral intermediaries holding zero net positions. While intermediary balance sheet constraints are undoubtedly among the key driving forces behind the joint behavior of exchange rates and interest rates, investigating the role of these constraints in the presence of imperfect competition in the intermediation sector is an important direction for future research.

Our model is flexible and can be easily modified to account for more complex real-world features. In particular, incorporating sticky prices and realistic monetary policy and inflation dynamics is crucial for understanding the impact of intermediation frictions on macroeconomic dynamics. In a production economy with financially constrained firms, intermediation frictions would also play the additional role of directly affecting the real side of the economy. We leave these important questions for future research.

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A D2C Bargaining

The following assumption formalizes the D2C bargaining protocol.

Assumption 4 *At the time t, each customer is matched with an intermediary and requests price quotes for all one-period-ahead state-contingent claims. In continuous time, we denote this period by $t + dt$. The intermediary quotes a country-specific D2C pricing kernel $M_{i,t,t+dt}$, $i = H, F$ in the local currency of country i and has full bargaining power in choosing $M_{i,t,t+dt}$: If the customer rejects the offer, they can only trade country i tree and country i one-period risk-free bonds in the country i centralized exchange with another country i households and (a continuum of) intermediaries. The quotes are binding: After receiving the quote, the customer chooses an optimal bundle of state-contingent claims. The intermediary sells this bundle to the customer at the quoted price. See Figure 1.*

Intermediaries in our model are essentially match-makers. Modeling intermediaries this way captures the significant shifts in dealers' market-making business models in the aftermath of the GFC (CGFS (2014, 2016)). The "principal" model, where dealer banks use balance sheet capacity to accommodate client trading demands, has given way to a model where they primarily match clients wishing to trade in opposite directions (see, e.g., Adrian et al. (2017a)). In particular, trading foreign stocks can also be done only through intermediaries. This assumption allows us to capture the fact that trading and owning foreign stocks often involves significant intermediation. Similarly, short-selling a stock (both local and foreign) always involves intermediation. The short seller must go to an intermediary, who must locate a stock owner to borrow the stock. See, e.g., Duffie et al. (2005). In this context,

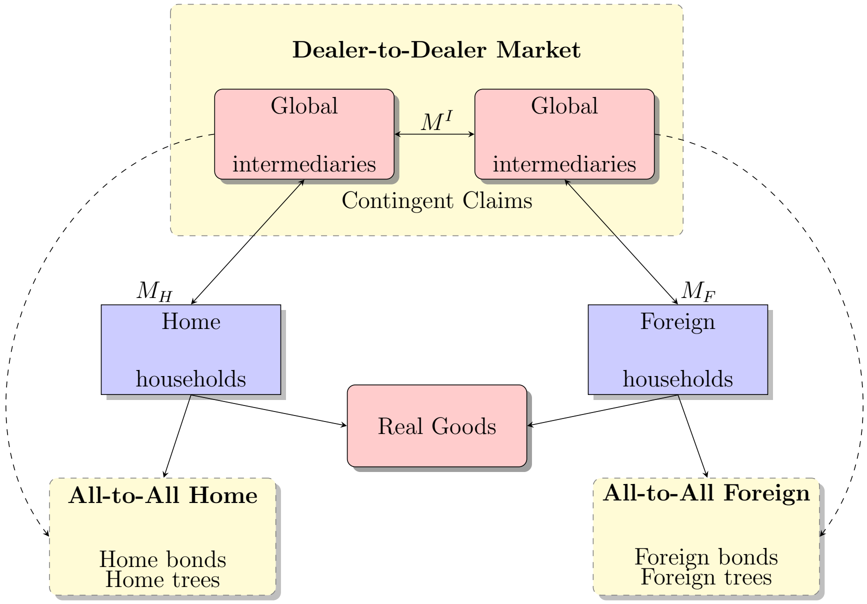

Figure 1: Graphical description of market structure in our model. RFQ denotes the request-for-quote protocol commonly used in D2C segments of OTC markets.

it is essential to note that market segmentation in our model is not fully endogenous: the critical assumption that drives our results is that households have direct access to domestic assets but not to foreign ones.

The assumption of monopolistic competition in the D2C segment is made for tractability reasons and can be relaxed; for example, our results can easily be adjusted to allow for a different bargaining protocol with a bargaining power below one, such as the Nash protocol that is commonly used in the literature on OTC markets. See, Duffie et al. (2005), Duffie et al. (2007), Lagos and Rocheteau (2009), and Atkeson et al. (2015). The post-crisis regulatory environment (based on the Dodd-Frank Act) is designed to move bilateral relationship trading to electronic platforms. For example, the trading of standardized interest rate swaps has largely moved to swap execution facilities (SEFs). Yet an all-to-all market, such as in equities markets, remains a distant reality. Most D2C transactions are executed via an RFQ protocol, equivalent to an electronic form of OTC trading. The original two-tier market structure thus shows remarkable persistence, with a D2D segment at the market's core, as in our model. The same is true for fixed-income and FX markets. See Collin-Dufresne et al. (2016), Bech et al. (2016), and Moore et al. (2016). However, some papers (see, e.g., Petersen and Rajan (1995)) argue that monopolistic competition in the intermediation sector is a closer approximation to reality due to switching and related costs. See, also, Sharpe (1997), Kim et al. (2003), Bolton et al. (2016), Brunnermeier and Koby (2016), Duffie and Krishnamurthy (2016), and Acharya and Plantin (2016).

B Proofs for Continuous Time

Proof of Proposition 1. Please refer to the discussions in the main text. Q.E.D.

Proof of Lemmas 2 and 3. From the market clearing conditions of the money markets we have

$$\begin{aligned} 1 &= \pi_{H,t} + \frac{1-\beta_F}{\beta_F}\mathcal{Q}_t \pi_{F,t}\,, \\ 1 &= \frac{1-\beta_H}{\beta_H}\frac{1}{\mathcal{Q}_t}\pi_{H,t} + \pi_{F,t}\,, \end{aligned}$$

whereas

Solve for $\pi_{H,t}$ and $\pi_{F,t}$ we get

$$\begin{aligned}
\pi_{H,t} &= \Big(\frac{\beta_F}{1-\beta_F} - \mathcal{Q}_t\Big)\Big(\frac{\beta_F}{1-\beta_F} - \frac{1-\beta_H}{\beta_H}\Big)^{-1}, \\
\pi_{F,t} &= \Big(\frac{\beta_H}{1-\beta_H} - \frac{1}{\mathcal{Q}_t}\Big)\Big(\frac{\beta_H}{1-\beta_H} - \frac{1-\beta_F}{\beta_F}\Big)^{-1}.
\end{aligned}$$

Here, we can see that when $\beta_H > \frac{1}{2}$, and $\beta_F > \frac{1}{2}$, the second term in the brackets are positive, i.e.,

$$\begin{aligned}
\frac{\beta_F}{1-\beta_F} - \frac{1-\beta_H}{\beta_H} &> 0\,, \\
\frac{\beta_H}{1-\beta_H} - \frac{1-\beta_F}{\beta_F} &> 0\,.
\end{aligned}$$

Hence, as we know that in equilibrium $\pi_{H,t}$ and $\pi_{F,t}$ are positive, we can deduce that

$$\mathcal{Q}_t \in \Big(\frac{1-\beta_H}{\beta_H}, \frac{\beta_F}{1-\beta_F}\Big)\,.$$

Therefore,

$$1 = \pi_{H,t} + \frac{1-\beta_F}{\beta_F}\mathcal{Q}_t\pi_{F,t} < \pi_{H,t} + \pi_{F,t}\,,$$

and

$$\frac{1}{2} < \pi_t \triangleq 1 + \frac{1}{2}(1 - \pi_{H,t} - \pi_{F,t}) < 1\,.$$

Q.E.D.

Proof of Proposition 2. We apply Ito's lemma to the money market clearing conditions:

$$\begin{aligned}\beta_H \bar{C}_{H,t} + (1-\beta_F)\bar{C}_{F,t}\mathcal{E}_t &= \mathcal{M}_{H,t}\,, \\ (1-\beta_H)\bar{C}_{H,t}/\mathcal{E}_t + \beta_F \bar{C}_{F,t} &= \mathcal{M}_{F,t}\,.\end{aligned} \tag{41}$$

Recall also that

$$-\frac{dM_{i,t}}{M_{i,t}} = r_{i,t}dt + (\eta_{i,t})' dB_t, \;\; -\frac{dM^I_{i,t}}{M^I_{i,t}} = r^I_{i,t}dt + (\eta^I_{i,t})' dB_t\,,$$

Differentiating (41), we get

$$\begin{aligned}\beta_H d\bar{C}_{H,t} + (1-\beta_F)d(\bar{C}_{F,t}\mathcal{E}_t) &= d\mathcal{M}_{H,t} = \mathcal{M}_{H,t}(\mu_H dt + \theta'_H dB_t) \\ (1-\beta_H)d(\bar{C}_{H,t}/\mathcal{E}_t) + \beta_F d\bar{C}_{F,t} &= d\mathcal{M}_{F,t} = \mathcal{M}_{F,t}(\mu_F dt + \theta'_F dB_t)\,,\end{aligned} \tag{42}$$

where

$$\bar{C}_{i,t} = \Psi_{i,t}\, M_{i,t}^{-1}\, \bar{C}_{i,0}\,,$$

and

$$\frac{d\Psi_{i,t}}{\Psi_{i,t}} = -\delta dt + (\theta^{\Psi}_{i,t})' dB_t\,,$$

so that, by the Ito formula,

$$d\bar{C}_{i,t} \;=\; d\Psi_{i,t}\, M_{i,t}^{-1}\, \bar{C}_{i,0} \;-\; \Psi_{i,t}\, M_{i,t}^{-2} dM_{i,t}\, \bar{C}_{i,0} \;+\; 0.5(2\Psi_{i,t} M_{i,t}^{-3} d\langle M_{i,t}\rangle - 2M_{i,t}^{-2} d\langle \Psi_{i,t}, M_{i,t}\rangle)$$

$$=\; \bar{C}_{i,t}(-\delta dt + (\theta_{i,t}^{\Psi})' dB_t + (r_{i,t}dt + (\eta_{i,t})' dB_t) \;+\; \|\eta_{i,t}\|^2 dt \;+\; \eta_{i,t}' \theta_{i,t}^{\Psi} dt)\,,$$

whereas, by the international no-arbitrage constraint, i.e., $\frac{\mathcal{E}_t}{\mathcal{E}_0} = \frac{M_{F,t}^I}{M_{H,t}^I}$, so that

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} \;=\; \frac{dM_{F,t}^I}{M_{F,t}^I} - \frac{dM_{H,t}^I}{M_{H,t}^I} - \frac{dM_{F,t}^I}{M_{F,t}^I}\frac{dM_{H,t}^I}{M_{H,t}^I} + \big(\frac{dM_{H,t}^I}{M_{H,t}^I}\big)^2$$

$$=\; (r_{H,t} - r_{F,t})dt + (\eta_{H,t}^I - \eta_{F,t}^I)' dB_t \;-(\eta_{H,t}^I)' \eta_{F,t}^I dt \;+\; \|\eta_{H,t}^I\|^2 dt\,.$$

Therefore,

$$d(\bar{C}_{F,t}\mathcal{E}_t) \;=\; d\bar{C}_{F,t}\mathcal{E}_t \;+\; \bar{C}_{F,t} d\mathcal{E}_t \;+\; d\langle \bar{C}_{F,t}, \mathcal{E}_t\rangle$$

$$=\; \bar{C}_{F,t}\mathcal{E}_t \Bigg(-\delta dt + (\theta_{F,t}^{\Psi})' dB_t + (r_{F,t}dt + (\eta_{F,t})' dB_t) \;+\; \|\eta_{F,t}\|^2 dt \;+\; \eta_{F,t}' \theta_{F,t}^{\Psi} dt$$

$$+\; (r_{H,t} - r_{F,t})dt + (\eta_{H,t}^I - \eta_{F,t}^I)' dB_t \;-(\eta_{H,t}^I)' \eta_{F,t}^I dt \;+\; \|\eta_{H,t}^I\|^2 dt$$

$$+\; (\theta_{F,t}^{\Psi} + \eta_{F,t})'(\eta_{H,t}^I - \eta_{F,t}^I)dt \Bigg)$$

$$=\; \bar{C}_{F,t}\mathcal{E}_t \Bigg(-\delta + r_{F,t} + \;\|\eta_{F,t}\|^2 \;-\; \eta_{F,t}' \theta_{F,t}^{\Psi} + \;(r_{H,t} - r_{F,t}) - (\eta_{H,t}^I)' \eta_{F,t}^I$$

$$+\; \|\eta_{H,t}^I\|^2 + \;(\theta_{F,t}^{\Psi} + \eta_{F,t})'(\eta_{H,t}^I - \eta_{F,t}^I) \Bigg) dt$$

$$+\; \bar{C}_{F,t}\mathcal{E}_t \Bigg((\theta_{F,t}^{\Psi}) + (\eta_{F,t}) + (\eta_{H,t}^I - \eta_{F,t}^I) \Bigg)' dB_t\,,$$

and, similarly,

$$
\begin{aligned}
d(\bar{C}_{H,t}/\mathcal{E}_t) \; &= \; \Big(d\bar{C}_{H,t}\,\mathcal{E}_t^{-1} \; - \; \bar{C}_{H,t}\,\mathcal{E}_t^{-2} d\mathcal{E}_t + \; 0.5(2\bar{C}_{H,t}\mathcal{E}_t^{-3} d\langle \mathcal{E}_t\rangle - 2\mathcal{E}_t^{-2} d\langle \bar{C}_{H,t}, \mathcal{E}_t\rangle) \Big) \\
&= \; (\bar{C}_{H,t}/\mathcal{E}_t) \Big(\underbrace{-\delta dt + (\theta^{\Psi}_{H,t})' dB_t + (r_{H,t} dt + (\eta_{H,t})' dB_t) \; + \; \|\eta_{H,t}\|^2 dt \; + \; \eta'_{H,t}\theta^{\Psi}_{H,t} dt}_{\bar{C}^{-1}_{H,t} d\bar{C}_{H,t}} \\
&\underbrace{-(r_{H,t} - r_{F,t})dt - (\eta^I_{H,t} - \eta^I_{F,t})' dB_t \; + (\eta^I_{H,t})'\eta^I_{F,t} dt \; - \; \|\eta^I_{H,t}\|^2 dt}_{-\mathcal{E}_t^{-1} d\mathcal{E}_t} \\
&+ \underbrace{\|\eta^I_{H,t} - \eta^I_{F,t}\|^2 dt}_{d\langle \mathcal{E}_t\rangle} \; - \; \underbrace{(\eta^I_{H,t} - \eta^I_{F,t})'(\theta^{\Psi}_{H,t} + \eta_{H,t}) dt}_{d\langle \mathcal{E}_t, \bar{C}_{H,t}\rangle} \Big) \\
&= \; (\bar{C}_{H,t}/\mathcal{E}_t) \Big(-\delta + r_{H,t} + \|\eta_{H,t}\|^2 + \eta'_{H,t}\theta^{\Psi}_{H,t} - (r_{H,t} - r_{F,t}) + (\eta^I_{H,t})'\eta^I_{F,t} \\
&- \; \|\eta^I_{H,t}\|^2 + \|\eta^I_{H,t} - \eta^I_{F,t}\|^2 - (\eta^I_{H,t} - \eta^I_{F,t})'(\theta^{\Psi}_{H,t} + \eta_{H,t}) \Big) dt \\
&+ \; (\bar{C}_{H,t}/\mathcal{E}_t) \Big(\theta^{\Psi}_{H,t} + \eta_{H,t} - (\eta^I_{H,t} - \eta^I_{F,t}) \Big)' dB_t \,.
\end{aligned}
$$

Substituting these expressions into (42), we get

$$\beta_H \bar{C}_{H,t}(-\delta dt + (\theta^{\Psi}_{H,t})' dB_t + (r_{H,t} dt + (\eta_{H,t})' dB_t) \ + \ \|\eta_{H,t}\|^2 dt \ + \ \eta'_{H,t}\theta^{\Psi}_{H,t} dt)$$

$$+ (1-\beta_F)\bar{C}_{F,t}\mathcal{E}_t\Bigg(-\delta + r_{F,t} + \ \|\eta_{F,t}\|^2 \ + \ \eta'_{F,t}\theta^{\Psi}_{F,t} + \ (r_{H,t} - r_{F,t}) - (\eta^I_{H,t})'\eta^I_{F,t}$$

$$+ \ \|\eta^I_{H,t}\|^2 + \ (\theta^{\Psi}_{F,t} + \eta_{F,t})'(\eta^I_{H,t} - \eta^I_{F,t})\Bigg) dt$$

$$+ \ (1-\beta_F)\bar{C}_{F,t}\mathcal{E}_t\Bigg((\theta^{\Psi}_{F,t}) + (\eta_{F,t}) + (\eta^I_{H,t} - \eta^I_{F,t})\Bigg)' dB_t$$

$$= \ \mathcal{M}_{H,t}(\mu_H dt + \theta'_H dB_t)$$

$$(1-\beta_H)(\bar{C}_{H,t}/\mathcal{E}_t)\Bigg(-\delta + r_{H,t} + \|\eta_{H,t}\|^2 + \eta'_{H,t}\theta^{\Psi}_{H,t} - (r_{H,t} - r_{F,t}) + (\eta^I_{H,t})'\eta^I_{F,t}$$

$$- \ \|\eta^I_{H,t}\|^2 + \|\eta^I_{H,t} - \eta^I_{F,t}\|^2 - (\eta^I_{H,t} - \eta^I_{F,t})'(\theta^{\Psi}_{H,t} + \eta_{H,t})\Bigg) dt$$

$$+ \ (1-\beta_H)(\bar{C}_{H,t}/\mathcal{E}_t)\Big(\theta^{\Psi}_{H,t} + \eta_{H,t} - (\eta^I_{H,t} - \eta^I_{F,t})\Big)' dB_t$$

$$+ \beta_F \bar{C}_{F,t}(-\delta dt + (\theta^{\Psi}_{F,t})' dB_t + (r_{F,t} dt + (\eta_{F,t})' dB_t) \ + \ \|\eta_{F,t}\|^2 dt \ + \ \eta'_{F,t}\theta^{\Psi}_{F,t} dt)$$

$$= \ \mathcal{M}_{F,t}(\mu_F dt + \theta'_F dB_t)\,.$$

Recall now the definition of $\pi_{i,t} \ = \ \frac{\beta_i \bar{C}_{i,t}}{\mathcal{M}_{i,t}}$, $i = H, F$ and $\mathcal{Q}_t = \frac{\mathcal{M}_{F,t}\mathcal{E}_t}{\mathcal{M}_{H,t}}$. Equating the diffusion coefficients, we get

$$\theta_i \ = \ \pi_{i,t}(\theta^{\Psi}_{i,t} + \eta_{i,t}) + (1-\pi_{i,t})(\theta^{\Psi}_{-i,t} + \eta_{-i,t} + \eta^I_{i,t} - \eta^I_{-i,t})\,,$$

while for the drift terms, we get

$$
\begin{aligned}
&\pi_{H,t}\Bigg(- \delta + r_{H,t} + \ \|\eta_{H,t}\|^2 + \eta'_{H,t}\theta^{\Psi}_{H,t}\Bigg) \\
&+ (1-\pi_{H,t})\Bigg(- \delta + r_{H,t} + \ \|\eta_{F,t}\|^2 \ + \ \eta'_{F,t}\theta^{\Psi}_{F,t} - (\eta^I_{H,t})'\eta^I_{F,t} \\
&+ \ \|\eta^I_{H,t}\|^2 + \ (\theta^{\Psi}_{F,t} + \eta_{F,t})'(\eta^I_{H,t} - \eta^I_{F,t})\Bigg) \ = \ \mu_H \\
&(1-\pi_{F,t})\Bigg(- \delta + r_{F,t} + \|\eta_{H,t}\|^2 + \eta'_{H,t}\theta^{\Psi}_{H,t} + (\eta^I_{H,t})'\eta^I_{F,t} \\
&- \ \|\eta^I_{H,t}\|^2 + \|\eta^I_{H,t} - \eta^I_{F,t}\|^2 - (\eta^I_{H,t} - \eta^I_{F,t})'(\theta^{\Psi}_{H,t} + \eta_{H,t})\Bigg) \\
&+ \ \pi_{F,t}\Bigg(- \delta + r_{F,t} + \ \|\eta_{F,t}\|^2 + \eta'_{F,t}\theta^{\Psi}_{F,t}\Bigg) \ = \ \mu_F
\end{aligned}
$$

Now note that without intermediation friction, $\eta_{i,t} = \eta^I_{i,t}$, we can then solve for the risk premia directly from the above linear system of equations

$$
\eta_{i,t} \ = \ \eta^I_{i,t} \ = \ \theta_i - \theta^{\Psi}_i + (1-\pi_{i,t})(\theta^{\Psi}_{i,t} - \theta^{\Psi}_{-i,t})\,.
$$

Then, the short rates are obtained in a similar fashion by matching the coefficients for dt

terms, so that we get

$$
\begin{aligned}
r_{i,t} \;=\; r^I_{i,t} \;&= \delta + \mu_i - \pi_{i,t}\eta'_{i,t}(\eta_{i,t} + \theta^{\Psi}_{i,t}) - (1-\pi_{i,t})(\eta_{-i,t})'(\eta_{-i,t} + \theta^{\Psi}_{-i,t}) \\
&- \; (1-\pi_{i,t})(\eta^I_{i,t} - \eta^I_{-i,t})'(\eta_{-i,t} + \theta^{\Psi}_{-i,t} + \eta^I_{i,t}) \\
&= \; \delta + \mu_i - \pi_{i,t}\eta'_{i,t}(\eta_{i,t} + \theta^{\Psi}_{i,t}) \; - \; (1-\pi_{i,t})(\eta_{-i,t} - \eta^I_{-i,t} + \eta^I_{i,t})'(\eta_{-i,t} + \theta^{\Psi}_{-i,t}) \\
&- \; (1-\pi_{i,t})(\eta^I_{i,t} - \eta^I_{-i,t})'\eta^I_{i,t} \\
&= \; \delta + \mu_i - \pi_{i,t}\eta'_{i,t}(\eta_{i,t} + \theta^{\Psi}_{i,t}) \; - \; (1-\pi_{i,t})(\eta_{-i,t} - \eta^I_{-i,t})'(\eta_{-i,t} + \theta^{\Psi}_{-i,t}) \\
&- \; (1-\pi_{i,t})(\eta^I_{i,t} - \eta^I_{-i,t} + \eta_{-i,t} + \theta^{\Psi}_{-i,t})'\eta^I_{i,t} \\
&= \; \delta \; + \; \mu_i \; + \pi_{i,t}(-\eta^I_{i,t} + \eta^I_{i,t} - \eta_{i,t})'(\eta_{i,t} + \theta^{\Psi}_{i,t}) \; - \; (1-\pi_{i,t})(\eta_{-i,t} - \eta^I_{-i,t})'(\eta_{-i,t} + \theta^{\Psi}_{-i,t}) \\
&- \; (1-\pi_{i,t})(\eta^I_{i,t} - \eta^I_{-i,t} + \eta_{-i,t} + \theta^{\Psi}_{-i,t})'\eta^I_{i,t} \\
&= \; \delta \; + \; \mu_i \; + \; \pi_{i,t}\mathcal{I}_{i,t} \; + \; (1-\pi_{i,t})\mathcal{I}_{-i,t} \\
&- \; \pi_{i,t}(\eta^I_{i,t})'(\eta_{i,t} + \theta^{\Psi}_{i,t}) \\
&- \; (1-\pi_{i,t})(\eta^I_{i,t} - \eta^I_{-i,t} + \eta_{-i,t} + \theta^{\Psi}_{-i,t})'\eta^I_{i,t} \\
&= \; \delta \; + \; \mu_i \; + \; \pi_{i,t}\mathcal{I}_{i,t} \; + \; (1-\pi_{i,t})\mathcal{I}_{-i,t} \\
&- \; (\eta^I_{i,t})'\Big(\pi_{i,t}(\eta_{i,t} + \theta^{\Psi}_{i,t}) + (1-\pi_{i,t})(\eta^I_{i,t} - \eta^I_{-i,t} + \eta_{-i,t} + \theta^{\Psi}_{-i,t}) \\
&= \; \delta \; + \; \mu_i \; + \; \pi_{i,t}\mathcal{I}_{i,t} \; + \; (1-\pi_{i,t})\mathcal{I}_{-i,t} \\
&- \; (\eta^I_{i,t})'\Big(\pi_{i,t}(\eta^I_{i,t} + (\eta_{i,t} - \eta^I_{i,t})) + \theta^{\Psi}_{i,t}) + (1-\pi_{i,t})(\eta^I_{i,t} - \eta^I_{-i,t} + \eta_{-i,t} + \theta^{\Psi}_{-i,t})\Big) \\
&= \; \delta \; + \; \mu_i \; + \; \pi_{i,t}\mathcal{I}_{i,t} \; + \; (1-\pi_{i,t})\mathcal{I}_{-i,t} \\
&- \; \|\eta^I_{i,t}\|^2 \; - \; (\eta^I_{i,t})'(\pi_{i,t}\theta^{\Psi}_{i,t} + (1-\pi_{i,t})\theta^{\Psi}_{-i,t}) \\
&- \; (\eta^I_{i,t})'\Big(\pi_{i,t}(\eta_{i,t} - \eta^I_{i,t}) + (1-\pi_{i,t})(\eta_{-i,t} - \eta^I_{-i,t})\Big)
\end{aligned}
$$

When $\eta_{i,t} = \eta^I_{i,t},\ \eta_{-i,t} = \eta^I_{-i,t}$, we get the required formula. Q.E.D.

Proof of Proposition 3 and of Related Results in Section 4.3. The steps are similar to the proof for Proposition 2 except for $\eta_{i,t} \neq \eta^I_{i,t}$. By Proposition 1,

$$\eta_{i,t} \;=\; \frac{\Gamma}{2+\Gamma}\,\eta^I_{i,t} + \frac{2}{2+\Gamma}\,\big[\frac{1}{2}(\eta^I_{i,t} - \theta^\Psi_{i,t} + \lambda_{i,t}\theta_i)\big],\ \textit{with}\ \lambda_{i,t} \;=\; \frac{(\eta^I_{i,t} + \theta^\Psi_{i,t})'\theta_i}{\|\theta_{i,t}\|^2}\,, \tag{43}$$

whereas

$$\theta_i \;=\; \pi_{i,t}(\theta^\Psi_{i,t} + \eta_{i,t}) + (1-\pi_{i,t})(\theta^\Psi_{-i,t} + \eta_{-i,t} + \eta^I_{i,t} - \eta^I_{-i,t})\,. \tag{44}$$

Recall that

$$\pi_t \;=\; 0.5 + \frac{1}{2}((1-\pi_{i,t}) + (1-\pi_{-i,t})) \;=\; \frac{1}{2}(3 - \pi_{i,t} - \pi_{-i,t})\,.$$

Taking the difference of the (44), we get

$$\begin{aligned}\theta_i - \theta_{-i} \;&=\; \pi_{i,t}(\theta^\Psi_{i,t} + \eta_{i,t}) + (1-\pi_{i,t})(\theta^\Psi_{-i,t} + \eta_{-i,t} + \eta^I_{i,t} - \eta^I_{-i,t})\\ -\;& \pi_{-i,t}(\theta^\Psi_{-i,t} + \eta_{-i,t}) - (1-\pi_{-i,t})(\theta^\Psi_{i,t} + \eta_{i,t} + \eta^I_{-i,t} - \eta^I_{i,t})\end{aligned}$$

implying that

$$\eta^I_{i,t} - \eta^I_{-i,t} \;=\; \frac{1}{2\pi_t - 1}\left(\theta_i - \theta_{-i} - (\pi_{i,t} + \pi_{-i,t} - 1)(\theta^\Psi_{i,t} - \theta^\Psi_{-i,t} + \eta_{i,t} - \eta_{-i,t})\right), \tag{45}$$

Taking the difference of (43), we get

$$
\begin{aligned}
\eta_{i,t} - \eta_{-i,t} &= \frac{1+\Gamma}{2+\Gamma}\left(\eta^I_{i,t} - \eta^I_{-i,t}\right) + \frac{1}{2+\Gamma}\left[\left(\theta^\Psi_{-i,t} - \theta^\Psi_{i,t} + \lambda_{i,t}\theta_i - \lambda_{-i,t}\theta_{-i}\right)\right] \\
&= \frac{1+\Gamma}{2+\Gamma}\frac{1}{2\pi_t - 1}\left(\theta_i - \theta_{-i} - (2-2\pi_t)(\theta^\Psi_{i,t} - \theta^\Psi_{-i,t} + \eta_{i,t} - \eta_{-i,t})\right) \\
&+ \frac{1}{2+\Gamma}\left[\left(\theta^\Psi_{-i,t} - \theta^\Psi_{i,t} + \lambda_{i,t}\theta_i - \lambda_{-i,t}\theta_{-i}\right)\right].
\end{aligned}
$$

Let

$$
\alpha_t = \frac{1+\Gamma}{2+\Gamma}\frac{2-2\pi_t}{2\pi_t - 1}.
$$

Then,

$$
\begin{aligned}
\eta_{i,t} - \eta_{-i,t} &= \frac{1}{1 + \frac{1+\Gamma}{2+\Gamma}\frac{2-2\pi_t}{2\pi_t-1}}\left(\frac{1+\Gamma}{2+\Gamma}\frac{1}{2\pi_t - 1}\left(\theta_i - \theta_{-i} - (2-2\pi_t)(\theta^\Psi_{i,t} - \theta^\Psi_{-i,t})\right)\right. \\
&\left. + \frac{1}{2+\Gamma}\left[\left(\theta^\Psi_{-i,t} - \theta^\Psi_{i,t} + \lambda_{i,t}\theta_i - \lambda_{-i,t}\theta_{-i}\right)\right]\right) \\
&= \frac{1}{1 + \frac{1+\Gamma}{2+\Gamma}\frac{2-2\pi_t}{2\pi_t-1}}\left(\frac{1+\Gamma}{2+\Gamma}\frac{1}{2\pi_t - 1}\left(\theta_i - \theta_{-i}\right)\right. \\
&+ \left(\frac{1+\Gamma}{2+\Gamma}\frac{2-2\pi_t}{2\pi_t - 1} + \frac{1}{2+\Gamma}\right)[\theta^\Psi_{-i,t} - \theta^\Psi_{i,t}] \\
&\left. + \frac{1}{2+\Gamma}\left[\lambda_{i,t}\theta_i - \lambda_{-i,t}\theta_{-i})\right]\right) \\
&= \frac{1}{1+\alpha_t}\left(\frac{1}{2-2\pi_t}\alpha_t\left(\theta_i - \theta_{-i}\right)\right. \\
&+ \left(\alpha_t + \frac{1}{2+\Gamma}\right)[\theta^\Psi_{-i,t} - \theta^\Psi_{i,t}] \\
&\left. + \frac{1}{2+\Gamma}\left[\lambda_{i,t}\theta_i - \lambda_{-i,t}\theta_{-i})\right]\right)
\end{aligned}
$$

Then, by (45), we have

$$
\begin{aligned}
\eta^I_{i,t}-\eta^I_{-i,t} \;&=\; \frac{1}{2\pi_t-1}\Bigg(\theta_i-\theta_{-i}-(2-2\pi_t)(\theta^\Psi_{i,t}-\theta^\Psi_{-i,t}+\eta_{i,t}-\eta_{-i,t})\Bigg)\\
&=\; \frac{1}{2\pi_t-1}\Bigg(\theta_i-\theta_{-i}-(2-2\pi_t)\Bigg(\theta^\Psi_{i,t}-\theta^\Psi_{-i,t}+\frac{1}{1+\alpha_t}\Bigg(\frac{1+\Gamma}{2+\Gamma}\,\frac{1}{2\pi_t-1}\Bigg(\theta_i-\theta_{-i}\\
&+\; (\alpha_t+\frac{1}{2+\Gamma})[\theta^\Psi_{-i,t}-\theta^\Psi_{i,t}]\\
&+\frac{1}{2+\Gamma}\left[\lambda_{i,t}\theta_i-\lambda_{-i,t}\theta_{-i})\right]\Bigg)\Bigg)\Bigg)\Bigg)\\
&=\; \frac{1}{2\pi_t-1}\Bigg(\frac{1}{1+\alpha_t}(\theta_i-\theta_{-i})-(2-2\pi_t)(\theta^\Psi_{i,t}-\theta^\Psi_{-i,t})\Big(1-\frac{1}{1+\alpha_t}(\alpha_t+(2+\Gamma)^{-1})\Big)\\
&-\; (2-2\pi_t)\frac{1}{1+\alpha_t}\frac{1}{2+\Gamma}\left[\lambda_{i,t}\theta_i-\lambda_{-i,t}\theta_{-i})\right]\Bigg)\\
&=\; \frac{1}{2\pi_t-1}\Bigg(\frac{1}{1+\alpha_t}(\theta_i-\theta_{-i})-(2-2\pi_t)(\theta^\Psi_{i,t}-\theta^\Psi_{-i,t})\frac{1+\Gamma}{2+\Gamma}\frac{1}{1+\alpha_t}\Big)\\
&-\; (2-2\pi_t)\frac{1}{1+\alpha_t}\frac{1}{2+\Gamma}\left[\lambda_{i,t}\theta_i-\lambda_{-i,t}\theta_{-i})\right]\Bigg)\\
&=\; \frac{1}{2\pi_t-1}\frac{1}{1+\alpha_t}(\theta_i-\theta_{-i})-\frac{\alpha_t}{1+\alpha_t}(\theta^\Psi_{i,t}-\theta^\Psi_{-i,t})\\
&-\frac{1}{1+\Gamma}\frac{\alpha_t}{1+\alpha_t}\left[\lambda_{i,t}\theta_i-\lambda_{-i,t}\theta_{-i})\right].
\end{aligned}
$$

By (44), we have

$$
\begin{aligned}
\theta_i \;&=\; \pi_{i,t}(\theta^{\Psi}_{i,t} + \eta_{i,t}) + (1-\pi_{i,t})(\theta^{\Psi}_{-i,t} + \eta_{-i,t} + \eta^{I}_{i,t} - \eta^{I}_{-i,t}) \\
&=\; \eta_{-i,t} \;+\; \pi_{i,t}\theta^{\Psi}_{i,t} + (1-\pi_{i,t})\theta^{\Psi}_{-i,t} \\
&+\; \pi_{i,t}(\eta_{i,t} - \eta_{-i,t}) \\
&+ (1-\pi_{i,t})(\eta^{I}_{i,t} - \eta^{I}_{-i,t}) \\
&=\; \eta_{-i,t} \;+\; \pi_{i,t}\theta^{\Psi}_{i,t} + (1-\pi_{i,t})\theta^{\Psi}_{-i,t} \\
&+\; \pi_{i,t}\Bigg(\frac{1}{1+\alpha_t}\Bigg(\frac{1}{2-2\pi_t}\alpha_t\Bigg(\theta_i - \theta_{-i}\Bigg) \\
&+\; (\alpha_t + \frac{1}{2+\Gamma})[\theta^{\Psi}_{-i,t} - \theta^{\Psi}_{i,t}] \\
&+ \frac{1}{2+\Gamma}\left[\lambda_{i,t}\theta_i - \lambda_{-i,t}\theta_{-i}\right]\Bigg)\Bigg) \\
&+ (1-\pi_{i,t})\Bigg(\frac{1}{2\pi_t - 1}\frac{1}{1+\alpha_t}(\theta_i - \theta_{-i}) - \frac{\alpha_t}{1+\alpha_t}(\theta^{\Psi}_{i,t} - \theta^{\Psi}_{-i,t}) \\
&- \frac{1}{1+\Gamma}\frac{\alpha_t}{1+\alpha_t}\left[\lambda_{i,t}\theta_i - \lambda_{-i,t}\theta_{-i})\right]\Bigg) \\
&=\; \eta_{-i,t} \;+\; \pi_{i,t}\theta^{\Psi}_{i,t} + (1-\pi_{i,t})\theta^{\Psi}_{-i,t} \\
&+\; (\theta_i - \theta_{-i})\left(\frac{\pi_{i,t}\alpha_t}{(1+\alpha_t)(2-2\pi_t)} + \frac{(1-\pi_{i,t})}{(2\pi_t - 1)(1+\alpha_t)}\right) \\
&+\; [\theta^{\Psi}_{-i,t} - \theta^{\Psi}_{i,t}]\left(\pi_{i,t}\frac{1}{1+\alpha_t}(\alpha_t + \frac{1}{2+\Gamma}) \;+\; (1-\pi_{i,t})\frac{\alpha_t}{1+\alpha_t}\right) \\
&+\; \left[\lambda_{i,t}\theta_i - \lambda_{-i,t}\theta_{-i}\right]\left(\pi_{i,t}\frac{1}{1+\alpha_t}\frac{1}{2+\Gamma} - (1-\pi_{i,t})\frac{1}{1+\Gamma}\frac{\alpha_t}{1+\alpha_t}\right).
\end{aligned}
$$

Therefore, from

$$\eta_{i,t} \;=\; \frac{\Gamma}{2+\Gamma}\,\eta^{I}_{i,t} + \frac{2}{2+\Gamma}\,\big[\frac{1}{2}(\eta^{I}_{i,t} - \theta^{\Psi}_{i,t} + \lambda_{i,t}\theta_i)\big]\,,$$

we get

$$\eta^{I}_{i,t} \;=\; \frac{2+\Gamma}{1+\Gamma}(\eta_{i,t} + \frac{1}{2+\Gamma}(\theta^{\Psi}_{i,t} - \lambda_{i,t}\theta_i))$$

In the limit as $\Gamma \to \infty$, we get

$$\alpha_t \;=\; \frac{2-2\pi_t}{2\pi_t - 1},\;\; \alpha_t + 1 \;=\; \frac{1}{2\pi_t - 1},\;\; \frac{\alpha_t}{\alpha_t + 1} \;=\; 2 - 2\pi_t\,,$$

and

$$\begin{aligned}
\theta_i \;&=\; \eta_{-i,t} \;+\; \pi_{i,t}\theta^{\Psi}_{i,t} + (1-\pi_{i,t})\theta^{\Psi}_{-i,t} \\
&+\; (\theta_i - \theta_{-i})\left(\frac{\pi_{i,t}\alpha_t}{(1+\alpha_t)(2-2\pi_t)} + \frac{(1-\pi_{i,t})}{(2\pi_t-1)(1+\alpha_t)}\right) \\
&+\; [\theta^{\Psi}_{-i,t} - \theta^{\Psi}_{i,t}]\left(\pi_{i,t}\frac{1}{1+\alpha_t}\alpha_t \;+\; (1-\pi_{i,t})\frac{\alpha_t}{1+\alpha_t}\right) \\
&=\; \eta_{-i,t} \;+\; \pi_{i,t}\theta^{\Psi}_{i,t} + (1-\pi_{i,t})\theta^{\Psi}_{-i,t} \\
&+\; (\theta_i - \theta_{-i})\frac{\alpha_t}{(1+\alpha_t)(2-2\pi_t)} \\
&+\; [\theta^{\Psi}_{-i,t} - \theta^{\Psi}_{i,t}]\frac{\alpha_t}{1+\alpha_t} \\
&=\; \eta_{-i,t} \;+\; \pi_{i,t}\theta^{\Psi}_{i,t} + (1-\pi_{i,t})\theta^{\Psi}_{-i,t} \\
&+\; (\theta_i - \theta_{-i}) \\
&+\; [\theta^{\Psi}_{-i,t} - \theta^{\Psi}_{i,t}](2-2\pi_t)
\end{aligned}$$

so that

$$\begin{aligned}
\eta_{-i,t} \;&=\; \theta_{-i} - \pi_{i,t}\theta^{\Psi}_{i,t} - (1-\pi_{i,t})\theta^{\Psi}_{-i,t} \;-\; [\theta^{\Psi}_{-i,t} - \theta^{\Psi}_{i,t}](\pi_{i,t} + \pi_{-i,t} - 1) \\
&=\; \theta_{-i} - \pi_{i,t}\theta^{\Psi}_{i,t} - (1-\pi_{i,t})\theta^{\Psi}_{-i,t} \\
&=\; \theta_{-i} - (1-\pi_{-i,t})\theta^{\Psi}_{i,t} - \pi_{-i,t}\theta^{\Psi}_{-i,t},
\end{aligned}$$

consistent with Proposition 2. In the case when $\Gamma = 0$, we get

$$\alpha_t \;=\; \frac{1-\pi_t}{2\pi_t - 1},\; \alpha_t + 1 \;=\; \frac{\pi_t}{2\pi_t - 1},\; \frac{\alpha_t}{\alpha_t + 1} \;=\; \frac{1-\pi_t}{\pi_t}\;,\; \alpha_t + 0.5 \;=\; 0.5\frac{1}{2\pi_t - 1}\,,$$

and

$$
\begin{aligned}
\eta_{-i,t} \;=\; & \theta_i \;-\pi_{i,t}\theta^{\Psi}_{i,t} - (1-\pi_{i,t})\theta^{\Psi}_{-i,t} \\
- \;& (\theta_i - \theta_{-i})\left(\frac{\pi_{i,t}\alpha_t}{(1+\alpha_t)(2-2\pi_t)} + \frac{(1-\pi_{i,t})}{(2\pi_t-1)(1+\alpha_t)}\right) \\
- \;& [\theta^{\Psi}_{-i,t} - \theta^{\Psi}_{i,t}]\left(\pi_{i,t}\frac{1}{1+\alpha_t}(\alpha_t + \frac{1}{2+\Gamma}) \;+\; (1-\pi_{i,t})\frac{\alpha_t}{1+\alpha_t}\right) \\
- \;& \big[\lambda_{i,t}\theta_i - \lambda_{-i,t}\theta_{-i}\big]\left(\pi_{i,t}\frac{1}{1+\alpha_t}\frac{1}{2+\Gamma} - (1-\pi_{i,t})\frac{1}{1+\Gamma}\frac{\alpha_t}{1+\alpha_t}\right) \\
=\; & \theta_i \;-\pi_{i,t}\theta^{\Psi}_{i,t} - (1-\pi_{i,t})\theta^{\Psi}_{-i,t} \\
- \;& (\theta_i - \theta_{-i})\left(\frac{\pi_{i,t}}{2\pi_t} + \frac{(1-\pi_{i,t})}{\pi_t}\right) \\
- \;& [\theta^{\Psi}_{-i,t} - \theta^{\Psi}_{i,t}]\left(\pi_{i,t}\frac{1}{2\pi_t} \;+\; (1-\pi_{i,t})\frac{1-\pi_t}{\pi_t}\right) \\
- \;& \big[\lambda_{i,t}\theta_i - \lambda_{-i,t}\theta_{-i}\big]\left(\pi_{i,t}\frac{2\pi_t-1}{2\pi_t} - (1-\pi_{i,t})\frac{1-\pi_t}{\pi_t}\right) \\
=\; & \theta_{-i} - \theta^{\Psi}_{-i,t} \\
+ \;& (\theta_i - \theta_{-i})\frac{1-\pi_{-i,t}}{2\pi_t} \\
- \;& [\theta^{\Psi}_{-i,t} - \theta^{\Psi}_{i,t}]\left(-\pi_{i,t} + \pi_{i,t}\frac{1}{2\pi_t} \;+\; (1-\pi_{i,t})\frac{1-\pi_t}{\pi_t}\right) \\
- \;& \big[\lambda_{i,t}\theta_i - \lambda_{-i,t}\theta_{-i}\big]\left(\pi_{i,t}\frac{2\pi_t-1}{2\pi_t} - (1-\pi_{i,t})\frac{1-\pi_t}{\pi_t}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
0.5\eta^I_{-i,t} &= \eta_{-i,t} + 0.5(\theta^\Psi_{-i,t} - \lambda_{-i,t}\theta_{-i}) \\
&= \theta_{-i} - 0.5\theta^\Psi_{-i,t} - 0.5\lambda_{-i,t}\theta_{-i} \\
&+ (\theta_i - \theta_{-i})\frac{1-\pi_{-i,t}}{2\pi_t} \\
&- \ [\theta^\Psi_{-i,t} - \theta^\Psi_{i,t}]\left(- \pi_{i,t} + \pi_{i,t}\frac{1}{2\pi_t} \ + \ (1-\pi_{i,t})\frac{1-\pi_t}{\pi_t}\right) \\
&- \ \left[\lambda_{i,t}\theta_i - \lambda_{-i,t}\theta_{-i}\right]\left(\pi_{i,t}\frac{2\pi_t - 1}{2\pi_t} - (1-\pi_{i,t})\frac{1-\pi_t}{\pi_t}\right).
\end{aligned}
$$

By direct calculation,

$$
-\pi_{i,t} + \pi_{i,t}\frac{1}{2\pi_t} \ + \ (1-\pi_{i,t})\frac{1-\pi_t}{\pi_t} \ = \ \frac{-1+\pi_{-i,t}}{2\pi_t}
$$

and

$$
\pi_{i,t}\frac{2\pi_t - 1}{2\pi_t} - (1-\pi_{i,t})\frac{1-\pi_t}{\pi_t} \ = \ \frac{1-\pi_{-i,t}}{2\pi_t}\,.
$$

Substituting, we arrive at the expression from Proposition 3, which represents equilibrium risk premia as

$$
\eta^I_{i,t} = \underbrace{(2-\lambda_{i,t})\theta_i - \theta^\Psi_{i,t}}_{domestic\ risk} + \underbrace{\frac{1-\pi_{i,t}}{\pi_t}\big(\theta^\Psi_{i,t} - \theta^\Psi_{-i,t} + (\lambda_{i,t}-1)\theta_i - (\lambda_{-i,t}-1)\theta_{-i}\big)}_{risk\ sharing}\,.
$$

where the dynamics of the Lagrange multipliers are given by

$$\lambda_{i,t} \;=\; 1+\frac{1}{\sigma_i^2}(\theta^{\Psi}_{i,t}-\theta^{\Psi}_{-i,t})'\Xi_{i,t} \;\; with \;\Xi_{i,t} \;=\; \frac{\theta_i+\frac{1-\pi_{-i,t}}{2-\pi_{i,t}}\rho\frac{\sigma_i}{\sigma_{-i}}\theta_{-i}}{\frac{2-\pi_{-i,t}}{1-\pi_{i,t}}-\frac{1-\pi_{-i,t}}{2-\pi_{i,t}}\rho^2}.$$

Then recall that

$$\lambda_{i,t} \;=\; \frac{(\eta^I_{i,t}+\theta^{\Psi}_{i,t})'\theta_i}{\|\theta_i\|^2}$$

Substitute the expressions of $\eta^I_{i,t}+\theta^{\Psi}_{i,t}$ into the conditions that determine the Lagrange multipliers, we obtain

$$\begin{aligned}
&\lambda_{i,t}\|\theta_i\|^2\big(1-\frac{1-\pi_{i,t}}{2\pi_t}\big)\\
=\;\; &\|\theta_i\|^2+\frac{1-\pi_{i,t}}{2\pi_t}\big((\theta^{\Psi}_{i,t}-\theta^{\Psi}_{-i,t})'\theta_i-\lambda_{-i,t}(\theta_{-i})'\theta_i-(\theta_i-\theta_{-i})'\theta_i\big).
\end{aligned}$$

We can then determine $\lambda_{i,t}$ as claimed. Note that when assumption 2 holds, we have that $\lambda_{i,t}=\lambda_{-i,t}=1$.

Finally, for the case of a general Γ, we have

$$\begin{aligned}
\eta_{-i,t} \;&=\; \theta_{-i}-\theta^{\Psi}_{-i,t}\\
&-\;(\theta_i-\theta_{-i})\left(-1+\frac{\pi_{i,t}\alpha_t}{(1+\alpha_t)(2-2\pi_t)}+\frac{(1-\pi_{i,t})}{(2\pi_t-1)(1+\alpha_t)}\right)\\
&-\;[\theta^{\Psi}_{-i,t}-\theta^{\Psi}_{i,t}]\left(-\pi_{i,t}+\pi_{i,t}\frac{1}{1+\alpha_t}(\alpha_t+\frac{1}{2+\Gamma})\;+\;(1-\pi_{i,t})\frac{\alpha_t}{1+\alpha_t}\right)\\
&-\;\big[\lambda_{i,t}\theta_i-\lambda_{-i,t}\theta_{-i}\big]\left(\pi_{i,t}\frac{1}{1+\alpha_t}\frac{1}{2+\Gamma}-(1-\pi_{i,t})\frac{1}{1+\Gamma}\frac{\alpha_t}{1+\alpha_t}\right)
\end{aligned}$$

so that

$$\eta^I_{i,t} \;=\; \frac{2+\Gamma}{1+\Gamma}(\eta_{i,t} + \frac{1}{2+\Gamma}(\theta^\Psi_{i,t} - \lambda_{i,t}\theta_i))$$

implies

$$\begin{aligned}
\frac{1+\Gamma}{2+\Gamma}\eta^I_{-i,t} \;&=\; \eta_{-i,t} + \frac{1}{2+\Gamma}(\theta^\Psi_{-i,t} - \lambda_{-i,t}\theta_{-i}) \\
&=\; \theta_{-i} - (\frac{1+\Gamma}{2+\Gamma}\theta^\Psi_{-i,t} + \frac{1}{2+\Gamma}\lambda_{-i,t}\theta_{-i}) \\
&-\; (\theta_i - \theta_{-i})\Bigg(-1 + \frac{\pi_{i,t}\alpha_t}{(1+\alpha_t)(2-2\pi_t)} + \frac{(1-\pi_{i,t})}{(2\pi_t - 1)(1+\alpha_t)}\Bigg) \\
&-\; [\theta^\Psi_{-i,t} - \theta^\Psi_{i,t}]\Bigg(-\pi_{i,t} + \pi_{i,t}\frac{1}{1+\alpha_t}(\alpha_t + \frac{1}{2+\Gamma}) \;+\; (1-\pi_{i,t})\frac{\alpha_t}{1+\alpha_t}\Bigg) \\
&-\; \big[\lambda_{i,t}\theta_i - \lambda_{-i,t}\theta_{-i}\big]\Bigg(\pi_{i,t}\frac{1}{1+\alpha_t}\frac{1}{2+\Gamma} - (1-\pi_{i,t})\frac{1}{1+\Gamma}\frac{\alpha_t}{1+\alpha_t}\Bigg)
\end{aligned}$$

Similarly, by matching the drift terms, we obtain that

$$\begin{aligned}
r_{i,t} \;=\; r^I_{i,t} \;&= \delta + \mu_i - \pi_{i,t}(\eta_{i,t})'(\eta_{i,t} + \theta^\Psi_{i,t}) - (1-\pi_{i,t})(\eta_{-i,t})'(\eta_{-i,t} + \theta^\Psi_{-i,t}) \\
&-\; (1-\pi_{i,t})(\eta^I_{i,t} - \eta^I_{-i,t})'(\eta_{-i,t} + \theta^\Psi_{-i,t} + \eta^I_{i,t})\,.
\end{aligned}$$

Given assumption 2 and assuming that $\Gamma = 0$, we can direct compute the D2C and D2D risk premia

$$\begin{aligned}
\eta_{i,t} \;&=\; \theta_i - \theta^\Psi_{i,t} + \frac{1-\pi_{i,t}}{2\pi_t}(\theta^\Psi_{i,t} - \theta^\Psi_{-i,t})\,, \\
\eta^I_{i,t} \;&=\; \theta_i - \theta^\Psi_{i,t} + \frac{1-\pi_{i,t}}{\pi_t}(\theta^\Psi_{i,t} - \theta^\Psi_{-i,t})\,.
\end{aligned}$$

To compute the short rates, we shall first compute

$$(\eta_{i,t})'(\eta_{i,t} + \theta^{\Psi}_{i,t}) \;=\; -(\theta^{\Psi}_{i,t})'(\theta^{\Psi}_{i,t} - \theta^{\Psi}_{-i,t})\frac{1-\pi_{i,t}}{2\pi_t} + \|\theta_i\|^2 + \big(\frac{1-\pi_{i,t}}{2\pi_t}\big)^2\|\theta^{\Psi}_{i,t} - \theta^{\Psi}_{-i,t}\|\,.$$

Then we compute

$$\begin{aligned}
&(\eta^I_{i,t} - \eta^I_{-i,t})'\eta^I_{i,t} \\
=\;& (\theta_i - \theta_{-i})'\theta_i - (\theta^{\Psi}_{i,t})'(\theta^{\Psi}_{i,t} - \theta^{\Psi}_{-i,t})\frac{\pi_t - 1}{\pi_t} + \frac{1-\pi_{i,t}}{\pi_t}\frac{\pi_t - 1}{\pi_t}\|\theta^{\Psi}_{i,t} - \theta^{\Psi}_{-i,t}\|^2\,.
\end{aligned}$$

Lastly we compute

$$\begin{aligned}
&(\eta^I_{i,t} - \eta^I_{-i,t})'(\eta_{-i,t} + \theta^{\Psi}_{-i,t}) \\
=\;& \big(\theta_{-i} + \frac{1-\pi_{-i,t}}{2\pi_t}(\theta^{\Psi}_{-i,t} - \theta^{\Psi}_{i,t})\big)'\big((\theta^{\Psi}_{i,t} - \theta^{\Psi}_{-i,t})\frac{\pi_t - 1}{\pi_t} + \theta_i - \theta_{-i}\big) \\
=\;& -\;\frac{\pi_t - 1}{\pi_t}\frac{1-\pi_{-i,t}}{2\pi_t}\|\theta^{\Psi}_{i,t} - \theta^{\Psi}_{-i,t}\|^2 + (\theta_{-i})'(\theta_i - \theta_{-i})\,.
\end{aligned}$$

Now substitute all these expressions into the short rates we get

$$r_{i,t} \;=\; \mu_i + \delta - \|\theta_i\|^2 + \frac{(1-\pi_{i,t})(1-\pi_{-i,t})}{(2\pi_t)^2}\|\theta^{\Psi}_{i,t} - \theta^{\Psi}_{-i,t}\|^2 + \frac{\pi_t - 1}{\pi_t}\frac{1-\pi_{i,t}}{\pi_t}(\pi_{i,t} - \pi_{-i,t})\|\theta^{\Psi}_{i,t}\|^2\,.$$

The short rate differential has the following simplified representation,

$$r^I_{H,t} - r^I_{F,t} \;=\; -\;(\pi_{H,t} - \pi_{F,t})\frac{1-\pi_t}{\pi_t}\frac{2\pi_t - 1}{2\pi_t}\|\theta^{\Psi}_{H,t} - \theta^{\Psi}_{F,t}\|^2\,,$$

which is negatively related to the annualized expected change of exchange rate state-by-state

$$E_t\Big[d\log\mathcal{E}_t\Big]/dt \;=\; (\pi_{H,t}-\pi_{F,t})\Big(\frac{1-\pi_t}{\pi_t}\Big)^2\|\theta^{\Psi}_{H,t}-\theta^{\Psi}_{F,t}\|^2\,.$$

Meanwhile, the short rate differential in the frictionless model is zero and hence does not co-move with exchange rates. Q.E.D.

Proof of Proposition 4. Let $\mathcal{F}^{(t+dt)}_{F,t}$ be the $t+dt$ forward exchange rate available to an F-household at time t. I.e., upon paying one unit of currency F at time $t+dt$, the contract delivers $\mathcal{F}^{(t+dt)}_{F,t}$ unit of currency H. By no-arbitrage, the forward exchange rate satisfies

$$E_t\big[M_{F,t,t+dt}(\mathcal{F}^{(t+dt)}_{F,t}/\mathcal{E}_{t+dt}-1)\big] \;=\; 0\,.$$

Hence, we obtain[13]

$$\mathcal{F}^{(t+dt)}_{F,t} \;=\; E_t[M_{F,t,t+dt}]/E_t\big[M_{F,t,t+dt}/\mathcal{E}_{t+dt}\big]\,.$$

The classic CIP condition states that the payoff of investing one unit of currency F in the domestic risk-free asset, i.e., $1/P^{(t+dt)}_{F,t}$, shall earn the same amount as exchanging to currency H and then investing in the foreign risk-free asset while simultaneously purchasing the forward contract, i.e., $1\times\mathcal{E}_t/P^{(t+dt)}_{H,t}/\mathcal{F}^{(t+dt)}_{F,t}$. However, in our model, due to the intermediation friction, the two quantities are not equal. Instead, only for the forward exchange rate available in the D2D market $\mathcal{F}^{I,(t+dt)}_{F,t}$, the CIP holds. Hence, an econometrician who observes

[13] Please note that the superscript $(t+dt)$ denotes the maturity date of the risk-free asset.

the forward exchange rate available to the customer F and the risk-free rate available to local customers would conclude that there exists a CIP deviation (see, for example, Avdjiev et al. (2019)). Furthermore, such a deviation is usually measured as (annualized)

$$CIP_{F,t} \;=\; \big(-\log P_{H,t}^{(t+dt)} + \log P_{F,t}^{(t+dt)} + \log \mathcal{E}_t - \log \mathcal{F}_{F,t}^{(t+dt)}\big)/dt\,.$$

A risk-free asset that promises to pay one unit of currency H would be worth

$$P_{F,t}^{H,(t+dt)} \;=\; E_t\Big[M_{F,t,t+dt}/\mathcal{E}_{t+dt}\Big]\,,$$

in the D2C market, in terms of currency F for an F-household. So its time-t price in terms of currency H would be $\mathcal{E}_t P_{F,t}^{H,(t+dt)}$ in contrast to the domestic bond price available to customer H, $P_{H,t}^{(t+dt)} = E_t[M_{H,t,t+dt}] = E_t[M^I_{H,t,t+dt}]$.

Thus, the CIP deviation can be represented as the differential cost of borrowing currency H:

$$CIP_{F,t} \;=\; \big(-\log P_{H,t}^{(t+dt)} + \log(P_{F,t}^{H,(t+dt)} \times \mathcal{E}_t)\big)/dt$$

We now compute the price $P_{F,t}^{H,(t+dt)}$ explicitly,

$$
\begin{aligned}
\mathcal{E}_t P_{F,t}^{H,(t+dt)} &= E_t\Big[e^{-r_{F,t}dt-0.5\|\eta_{F,t}\|^2dt-(\eta_{F,t})'dB_t} \\
&\quad \times\ e^{-(r^I_{H,t}-r^I_{F,t})dt-0.5(\|\eta^I_{H,t}\|^2-\|\eta^I_{F,t}\|^2)dt-(\eta^I_{H,t}-\eta^I_{F,t})'dB_t}\Big] \\
&= e^{-r_{H,t}dt-0.5\|\eta_{F,t}\|^2dt-0.5(\|\eta^I_{H,t}\|^2-\|\eta^I_{F,t}\|^2)dt} \times\ E_t\Big[e^{-(\eta_{F,t}+\eta^I_{H,t}-\eta^I_{F,t})'dB_t}\Big] \\
&= e^{-r_{H,t}dt-(\eta^I_{H,t}-\eta^I_{F,t})'(\eta^I_{F,t}-\eta_{F,t})dt}\,.
\end{aligned}
$$

Hence, the short-rate that an F household has to pay in order to borrow currency H is

$$
r^H_{F,t} \;=\; -\log(\mathcal{E}_t P_{F,t}^{H,(t+dt)})/dt \;=\; r_{H,t} + (\eta^I_{H,t}-\eta^I_{F,t})'(\eta^I_{F,t}-\eta_{F,t})\,.
$$

The formula for H-households is analogous. Q.E.D.

Internet Appendix (For Online Publication)

C Details of the Calibration

We consider the following specifications for the dynamics of the output and demand shocks:

$$
\begin{bmatrix} (\theta_H)' \\ (\theta_F)' \\ (\theta^\Psi_H)' \\ (\theta^\Psi_F)' \end{bmatrix}
=
\begin{bmatrix}
\bar\sigma^{\mathcal{M}} & 0 & \bar\sigma^{\mathcal{M}} & 0 & 0 & 0 \\
0 & \bar\sigma^{\mathcal{M}} & \rho^{\mathcal{M}}\bar\sigma^{\mathcal{M}} & \sqrt{1-(\rho^{\mathcal{M}})^2}\,\bar\sigma^{\mathcal{M}} & 0 & 0 \\
s^\Psi\bar\sigma^\Psi & 0 & \sigma^{\text{carry}} & \sigma^{\text{ERP}} & \sqrt{1-(s^\Psi)^2}\,\sigma^\Psi_t & 0 \\
0 & s^\Psi\bar\sigma^\Psi & \sigma^{\text{carry}} & \sigma^{\text{ERP}} & \sqrt{1-(s^\Psi)^2}\,\rho^\Psi\sigma^\Psi_t & \sqrt{1-(s^\Psi)^2}\sqrt{1-(\rho^\Psi)^2}\,\sigma^\Psi_t
\end{bmatrix}
$$

Here, the parameter $\bar{\sigma}^{\mathcal{M}}$ captures the volatility of the output shock, while $\rho^{\mathcal{M}}$ captures the correlation between the output shock across countries, with the covariance given by $\theta_H'\theta_F = (\bar{\sigma}^{\mathcal{M}})^2\rho^{\mathcal{M}}$. Note that under the current parametric assumption, when $\rho^{\mathcal{M}} = 1$, the output correlation between the two countries will be capped at 0.5. We can adjust the volatilities in the third and fourth columns to achieve even larger cross-country output correlation. Similarly, $\bar{\sigma}^{\Psi}$ captures the idiosyncratic demand shock, and we set $\bar{\sigma}^{\Psi} = 0.095$. Finally, s^{Ψ} captures the correlation between the idiosyncratic output shock and the demand shock, with the covariance given by $\theta_i'\theta_i^{\Psi} = s^{\Psi}\bar{\sigma}^{\Psi}\bar{\sigma}^{\mathcal{M}}$.

Given these assumptions, the volatility of the output shocks can be computed as $\sqrt{2(\bar{\sigma}^{\mathcal{M}})^2}$, and the correlation of the two output shocks is $(\bar{\sigma}^{\mathcal{M}})^2\rho^{\mathcal{M}}/2(\bar{\sigma}^{\mathcal{M}})^2 = \frac{\rho^{\mathcal{M}}}{2}$. We calibrate the parameters $\bar{\sigma}^{\mathcal{M}}$ to match the volatility of nominal GDP (2%) and the correlation between the nominal GDPs of the two countries (0.35), as reported in Table 2 of Itskhoki and Mukhin (2017).

Finally, $\mathrm{ac1}(\cdot)$ denotes the auto-correlation coefficient with one lag; $\mathrm{std}(\cdot)$ denotes the annualized volatility; $\mathrm{corr}(\cdot, \cdot)$ denotes the correlation between the variables of interest; d denotes the first-order difference. Lower-case variables are the logarithms of the corresponding upper-case variables. We simulate 10,000 sample paths starting with $\mathcal{Q}_0 = 1$ and $x_0 = 0$, representing the symmetric steady state. For each sample path, we simulate 80 years of monthly observations, discarding the first 40 years to mitigate stability concerns.

C.1 Data sources

We consider the U.S. as the home country and Euro Area (19 countries) as the foreign country in our calibration exercise. We collect the data on nominal gross domestic output (GDP) and nominal consumption in domestic currency units from IMF International Financial Statistics. The data for Euro Area (19 countries) is from 1995Q1 to 2023Q4, quarterly frequency and seasonally adjusted. As U.S. has longer periods of observations (dating back to 1950s), we have also considered Germany and France together as a proxy for Euro Area dating back to 1980s. The results are similar and hence not reported.

Meanwhile the bilateral quarter-end nominal exchange rates on EUR/USD are downloaded from BIS website. Please note that per our definition of exchange rates, we would measure Euro in the unit of US dollar.

Lastly, we collect the domestic stock price index and short rates (3 months) from OECD Financial Markets Indicators. Again we have less observations for the Euro Area, starting only from 1994Q1 until 2023Q4.

We excluded the four observations in 2020 in our calibrations for the international macro moments, due to the extreme movements in nominal consumption and GDP caused by the outbreak of Covid-19. The effects are summarized in Figure 6.

C.2 The stochastic process for x_t in Assumption 3. Sensitivity to κ^x and σ^x.

As explained in assumption 3, we consider a mean-reverting process x_t for the state that determines the volatility of demand shocks

$$\sigma^{\Psi}_{i,t} \;=\; \exp(4\, x_t^2) - 1\,.$$

We choose $\kappa^x = \frac{0.36}{12}$ and $\sigma^x = \frac{0.09}{2}$ to ensure that $\max|x_t| < \frac{3}{8}$ for most sample paths, corresponding to approximately 75% instantaneous volatility in the demand shocks. These parameter choices also allow us to generate persistence in the country-specific short-term rates and the cross-country interest rate differential.

We consider four combinations of the mean-reversion $\kappa^x \in \{0.02, 0.04\}$ and volatility $\sigma^x \in \{0.03, 0.06\}$ parameters for the demand shock process x_t.

Table 8: Frictional, x_t with $\sigma^x = 0.03$ and $\kappa^x = 0.04$, with a positive covariance for demand shocks: $\rho^\Psi = 0.3$.

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	8.66	6.53	4.23	3.75	4.95
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	4.39	3.31	2.15	1.90	2.51
std($d\bar{c}_H$)(%)	1.49	(0.10)	1.82	1.88	2.05	2.32	2.55
std($d\bar{c}_F$)(%)	1.60	(0.11)	1.83	1.88	2.05	2.32	2.55
std(de)/std($d\bar{c}_H$)	6.32	—	4.81	3.51	2.05	1.60	1.98
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.88	0.94	0.98	0.96	0.94
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	0.73	0.57	0.27	-0.01	-0.17
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	0.03	0.33	0.15	-0.56	-0.83
std($r_H - r_F$)(%)	0.69	(0.04)	0.01	0.00	0.00	0.00	0.00
std($r_H - r_F$)/std(de)	0.07	—	0.00	0.00	0.00	0.00	0.00
std(r_H)(%)	1.12	(0.07)	0.00	0.00	0.00	0.00	0.00
corr(r_H, r_F)	0.81	(0.03)	-0.44	0.58	0.96	0.91	0.91
corr(dr_H, dr_F)	0.59	(0.06)	0.15	0.70	0.90	0.89	0.87

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1($r_H - r_F$)	0.96	(0.09)	0.98	0.97	0.95	0.96	0.96
ac1(r_H)	0.98	(0.09)	0.98	0.97	0.96	0.96	0.97
Fama-β	2.18	(1.25)	190.60	223.21	113.36	24.99	76.28
carry SR(%)	37.23	(18.41)	0.13	0.15	0.01	0.22	0.25
carry i-diff (%)	1.21	(0.39)	0.02	0.01	0.00	0.00	0.00
carry (%)	3.46	(9.28)	0.01	0.01	0.00	0.01	0.01
carry ratio (%)	34.97	(18.41)	0.10	0.05	0.00	0.06	0.02
std(carry) (%)	9.28	(0.60)	8.66	6.53	4.22	3.74	4.95
std(carry i-diff) (%)	0.39	(0.03)	0.00	0.00	0.00	0.00	0.00
mean(CIP$_H$)(%)	-0.21	(0.30)	-0.07	-0.04	-0.02	-0.02	-0.04
CIP-β	-2.64	(0.68)	-0.05	-0.04	-0.05	-0.06	-0.05
t CIP	-3.87	—	-6.82	-2.55	-1.15	-1.50	-4.22
R^2 CIP(%)	2.00	—	13.03	2.25	0.84	1.34	7.29
ERP$_H$(%)	3.48	(12.74)	0.51	0.37	0.19	-0.00	-0.14
equity SR$_H$(%)	27.26	(18.41)	6.36	4.63	2.32	-0.00	-1.72
ERP$_F$(%)	2.01	(15.85)	0.49	0.35	0.17	-0.02	-0.15
equity SR$_F$(%)	12.66	(18.41)	6.15	4.44	2.14	-0.21	-1.93
corr(ERP$_H$, ERP$_F$)	0.85	(0.03)	0.35	0.35	0.35	0.35	0.35

Table 9: Frictional, x_t with $\sigma^x = 0.03$ and $\kappa^x = 0.02$, with a positive covariance for demand shocks: $\rho^\Psi = 0.3$.

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	9.03	7.42	5.85	5.24	5.65
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	4.58	3.76	2.96	2.66	2.86
std($d\bar{c}_H$)(%)	1.49	(0.10)	1.88	1.94	2.11	2.38	2.62
std($d\bar{c}_F$)(%)	1.60	(0.11)	1.88	1.94	2.11	2.38	2.62
std(de)/std($d\bar{c}_H$)	6.32	—	4.87	3.65	2.52	2.04	2.14
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.84	0.91	0.95	0.95	0.93
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	0.64	0.45	0.19	-0.05	-0.19
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	-0.19	-0.05	-0.21	-0.59	-0.82
std($r_H - r_F$)(%)	0.69	(0.04)	0.01	0.01	0.01	0.01	0.01
std($r_H - r_F$)/std(de)	0.07	—	0.00	0.00	0.00	0.00	0.00
std(r_H)(%)	1.12	(0.07)	0.01	0.01	0.01	0.01	0.00
corr(r_H, r_F)	0.81	(0.03)	-0.02	0.62	0.92	0.89	0.89
corr(dr_H, dr_F)	0.59	(0.06)	0.25	0.66	0.80	0.82	0.82

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1($r_H - r_F$)	0.96	(0.09)	0.98	0.97	0.96	0.96	0.96
ac1(r_H)	0.98	(0.09)	0.98	0.97	0.97	0.97	0.97
Fama-β	2.18	(1.25)	122.11	97.93	43.40	18.88	42.69
carry SR(%)	37.23	(18.41)	0.41	0.55	0.50	0.43	0.41
carry i-diff (%)	1.21	(0.39)	0.03	0.01	0.01	0.01	0.00
carry (%)	3.46	(9.28)	0.04	0.04	0.02	0.02	0.03
carry ratio (%)	34.97	(18.41)	0.15	0.10	0.01	0.08	0.02
std(carry) (%)	9.28	(0.60)	9.03	7.42	5.83	5.24	5.65
std(carry i-diff) (%)	0.39	(0.03)	0.00	0.00	0.00	0.00	0.00
mean(CIP$_H$)(%)	-0.21	(0.30)	-0.09	-0.05	-0.04	-0.04	-0.05
CIP-β	-2.64	(0.68)	-0.06	-0.07	-0.11	-0.11	-0.07
t CIP	-3.87	—	-5.85	-2.77	-1.69	-1.93	-4.13
R^2 CIP(%)	2.00	—	11.54	3.05	1.99	2.41	7.43
ERP$_H$(%)	3.48	(12.74)	0.51	0.37	0.19	0.00	-0.14
equity SR$_H$(%)	27.26	(18.41)	6.33	4.61	2.34	0.04	-1.70
ERP$_F$(%)	2.01	(15.85)	0.49	0.35	0.17	-0.01	-0.15
equity SR$_F$(%)	12.66	(18.41)	6.10	4.39	2.15	-0.15	-1.88
corr(ERP$_H$, ERP$_F$)	0.85	(0.03)	0.35	0.35	0.35	0.35	0.35

Table 10: Frictional, x_t with $\sigma^x = 0.06$ and $\kappa^x = 0.02$, with a positive covariance for demand shocks: $\rho^\Psi = 0.3$.

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	13.38	13.63	13.39	12.76	11.83
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	6.77	6.90	6.78	6.44	5.98
std($d\bar{c}_H$)(%)	1.49	(0.10)	2.67	2.88	3.07	3.26	3.34
std($d\bar{c}_F$)(%)	1.60	(0.11)	2.77	3.13	3.43	3.54	3.51
std(de)/std($d\bar{c}_H$)	6.32	—	4.59	3.78	3.17	2.91	2.88
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.59	0.66	0.72	0.76	0.79
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	0.08	0.01	-0.05	-0.12	-0.21
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	-0.66	-0.61	-0.61	-0.68	-0.79
std($r_H - r_F$)(%)	0.69	(0.04)	0.32	0.48	0.54	0.47	0.30
std($r_H - r_F$)/std(de)	0.07	—	0.02	0.03	0.04	0.04	0.03
std(r_H)(%)	1.12	(0.07)	0.09	0.12	0.14	0.12	0.09
corr(r_H, r_F)	0.81	(0.03)	0.63	0.74	0.77	0.78	0.80
corr(dr_H, dr_F)	0.59	(0.06)	0.55	0.55	0.55	0.57	0.60

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1($r_H - r_F$)	0.96	(0.09)	0.95	0.94	0.94	0.94	0.94
ac1(r_H)	0.98	(0.09)	0.95	0.94	0.94	0.94	0.95
Fama-β	2.18	(1.25)	4.11	2.54	2.28	2.32	2.96
carry SR(%)	37.23	(18.41)	3.61	3.78	3.98	3.80	3.62
carry i-diff (%)	1.21	(0.39)	0.20	0.25	0.25	0.22	0.15
carry (%)	3.46	(9.28)	0.35	0.31	0.28	0.30	0.28
carry ratio (%)	34.97	(18.41)	2.08	2.78	1.40	2.40	1.23
std(carry) (%)	9.28	(0.60)	13.38	13.62	13.38	12.76	11.82
std(carry i-diff) (%)	0.39	(0.03)	0.09	0.13	0.15	0.13	0.08
mean(CIP$_H$)(%)	-0.21	(0.30)	-0.27	-0.30	-0.29	-0.28	-0.23
CIP-β	-2.64	(0.68)	-0.43	-0.65	-0.79	-0.73	-0.51
t CIP	-3.87	—	-2.74	-2.37	-2.31	-2.28	-2.43
R^2 CIP(%)	2.00	—	6.69	7.79	8.56	8.65	7.29
ERP$_H$(%)	3.48	(12.74)	0.47	0.35	0.19	0.03	-0.10
equity SR$_H$(%)	27.26	(18.41)	5.96	4.44	2.45	0.35	-1.27
ERP$_F$(%)	2.01	(15.85)	0.46	0.33	0.18	0.02	-0.11
equity SR$_F$(%)	12.66	(18.41)	5.72	4.18	2.22	0.23	-1.38
corr(ERP$_H$, ERP$_F$)	0.85	(0.03)	0.35	0.35	0.35	0.35	0.35

Table 11: Frictional, x_t with $\sigma^x = 0.06$ and $\kappa^x = 0.04$, with a positive covariance for demand shocks: $\rho^\Psi = 0.3$.

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	11.77	11.94	11.60	10.80	9.64
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	5.97	6.05	5.88	5.45	4.87
std($d\bar{c}_H$)(%)	1.49	(0.10)	2.29	2.50	2.73	2.94	3.04
std($d\bar{c}_F$)(%)	1.60	(0.11)	2.29	2.53	2.76	2.96	3.05
std(de)/std($d\bar{c}_H$)	6.32	—	4.84	3.98	3.31	2.96	2.81
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.65	0.68	0.74	0.79	0.83
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	0.17	0.03	-0.06	-0.15	-0.23
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	-0.67	-0.66	-0.67	-0.72	-0.81
std($r_H - r_F$)(%)	0.69	(0.04)	0.12	0.20	0.24	0.20	0.12
std($r_H - r_F$)/std(de)	0.07	—	0.01	0.02	0.02	0.02	0.01
std(r_H)(%)	1.12	(0.07)	0.05	0.07	0.08	0.07	0.04
corr(r_H, r_F)	0.81	(0.03)	0.63	0.77	0.81	0.83	0.85
corr(dr_H, dr_F)	0.59	(0.06)	0.61	0.61	0.60	0.63	0.69

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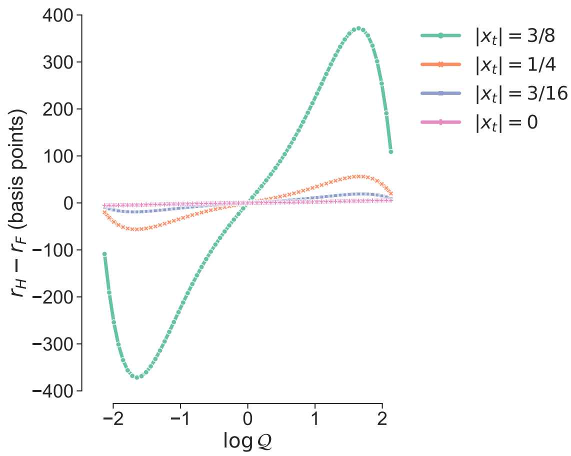
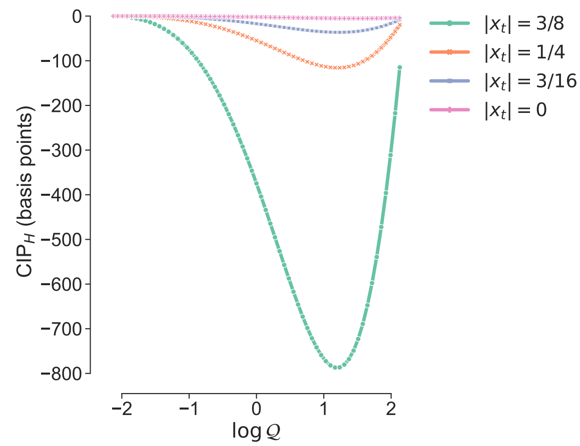
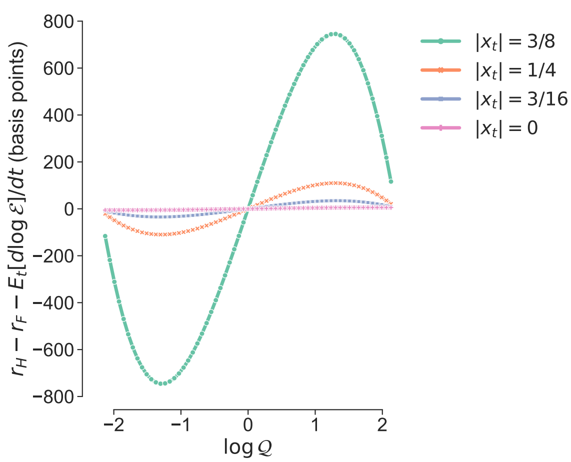
	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1($r_H - r_F$)	0.96	(0.09)	0.95	0.94	0.94	0.94	0.94
ac1(r_H)	0.98	(0.09)	0.95	0.94	0.94	0.94	0.95
Fama-β	2.18	(1.25)	10.96	5.49	4.41	4.77	6.91
carry SR(%)	37.23	(18.41)	2.07	2.36	2.37	2.56	1.94
carry i-diff (%)	1.21	(0.39)	0.10	0.11	0.11	0.09	0.06
carry (%)	3.46	(9.28)	0.21	0.22	0.19	0.20	0.16
carry ratio (%)	34.97	(18.41)	0.82	0.91	0.65	0.77	0.32
std(carry) (%)	9.28	(0.60)	11.77	11.93	11.60	10.80	9.63
std(carry i-diff) (%)	0.39	(0.03)	0.03	0.06	0.07	0.06	0.03
mean(CIP$_H$)(%)	-0.21	(0.30)	-0.19	-0.20	-0.20	-0.18	-0.15
CIP-β	-2.64	(0.68)	-0.28	-0.48	-0.63	-0.57	-0.36
t CIP	-3.87	—	-2.58	-2.16	-2.10	-2.06	-2.21
R^2 CIP(%)	2.00	—	4.68	4.97	6.02	5.76	4.65
ERP$_H$(%)	3.48	(12.74)	0.49	0.36	0.19	0.02	-0.12
equity SR$_H$(%)	27.26	(18.41)	6.15	4.47	2.35	0.20	-1.44
ERP$_F$(%)	2.01	(15.85)	0.47	0.33	0.17	0.00	-0.13
equity SR$_F$(%)	12.66	(18.41)	5.87	4.20	2.18	0.04	-1.66
corr(ERP$_H$, ERP$_F$)	0.85	(0.03)	0.35	0.35	0.35	0.35	0.35



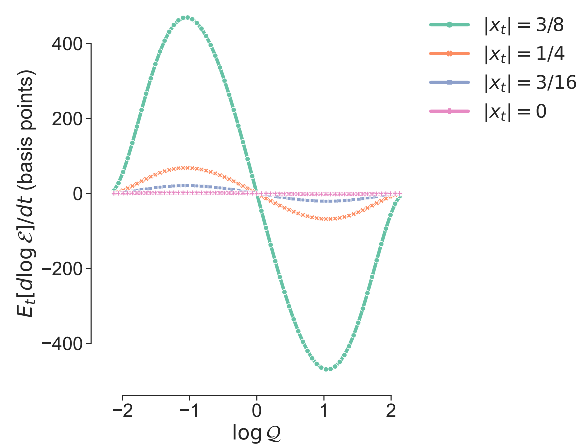
(a) Short rate differential (see (32))



(b) CIP deviations (see (33))

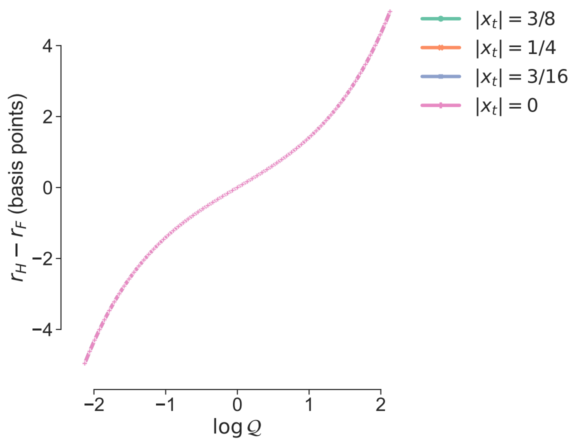


(c) Currency risk premium (see (30))

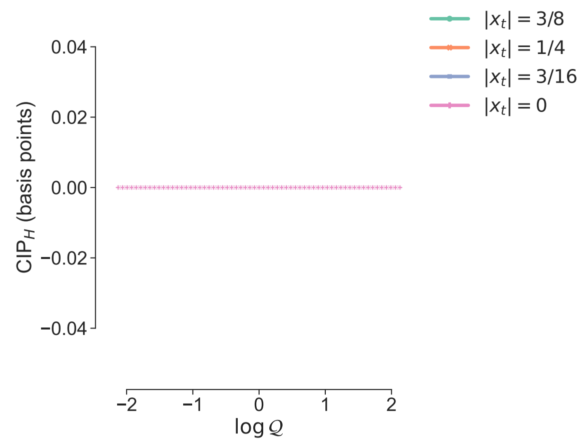


(d) Drift of exchange rates (see (30))

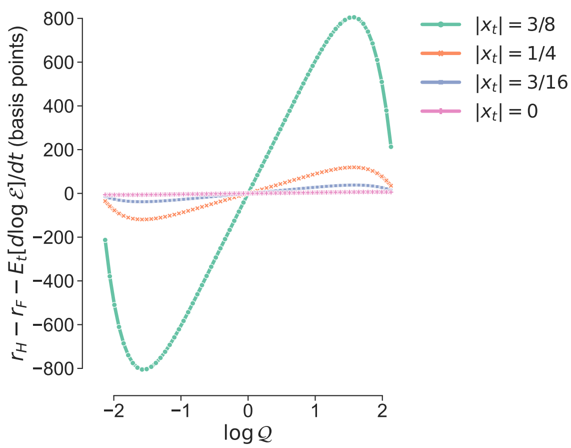
Figure 2: This plot shows under the calibration with $s^{\Psi} = -0.4$ for the frictional model, the variation of short-term rates and currency risk premia as a function of the endogenous state variable $\log \mathcal{Q}_t$ and the exogenous variable x_t. For example, the upper left panel exhibits the short-rate differential as a function of $\log \mathcal{Q}_t$ for various choices of $|x_t| \in \{0, 3/16, 1/4, 3/8\}$.



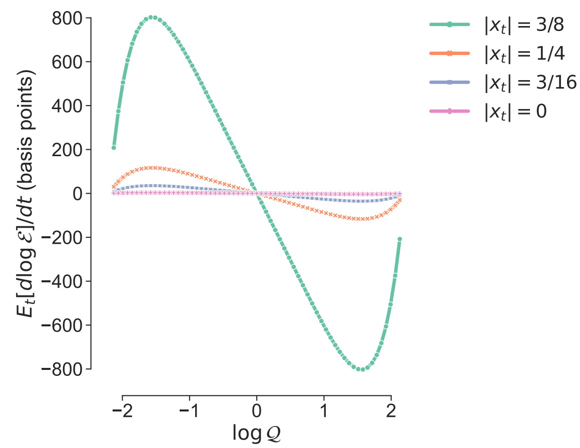
(a) Short rate differential (see (32))



(b) CIP deviations (see (33))

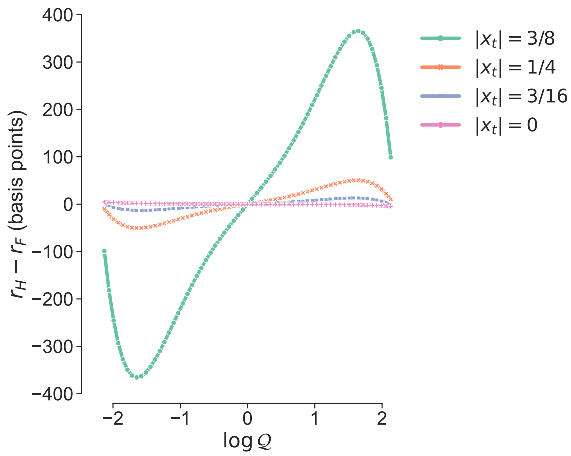


(c) Currency risk premium (see (30))

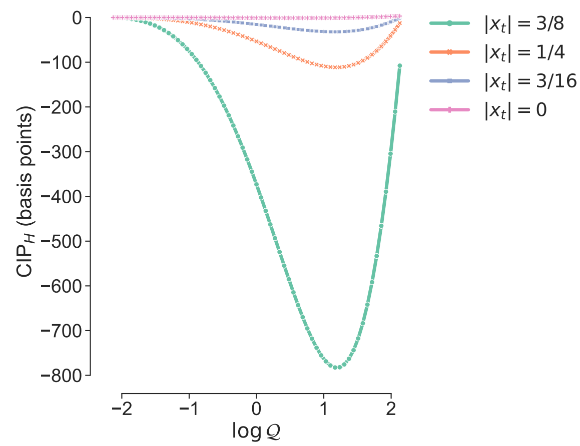


(d) Drift of exchange rates (see (30))

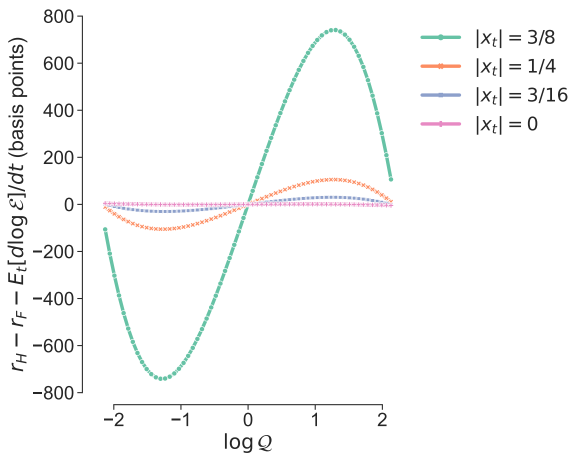
Figure 3: This plot shows under the calibration with $s^{\Psi} = -0.4$ for the frictionless model, the variation of short-term rates and currency risk premia as a function of the endogenous state variable $\log \mathcal{Q}_t$ and the exogenous variable x_t. For example, the upper left panel exhibits the short-rate differential as a function of $\log \mathcal{Q}_t$ for various choices of $|x_t| \in \{0, 3/16, 1/4, 3/8\}$.



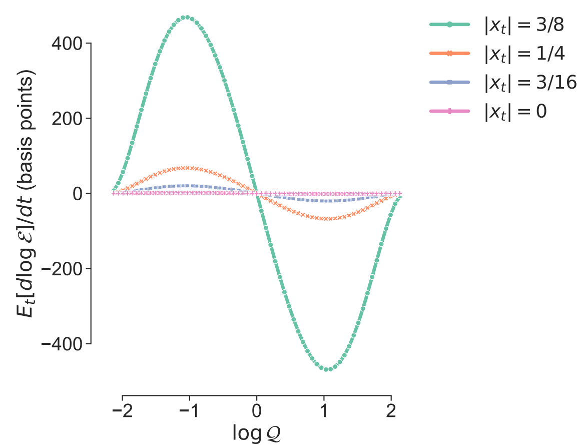
(a) Short rate differential (see (32))



(b) CIP deviations (see (33))

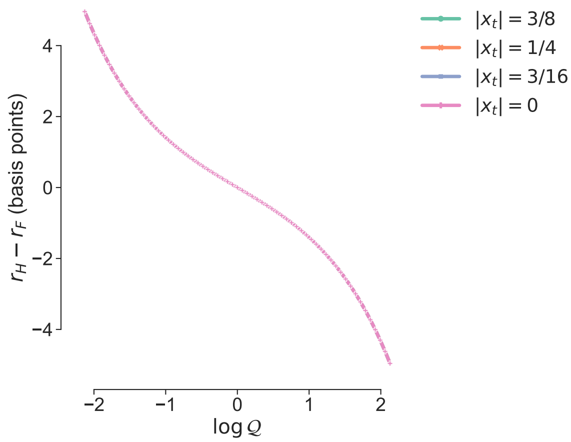


(c) Currency risk premium (see (30))

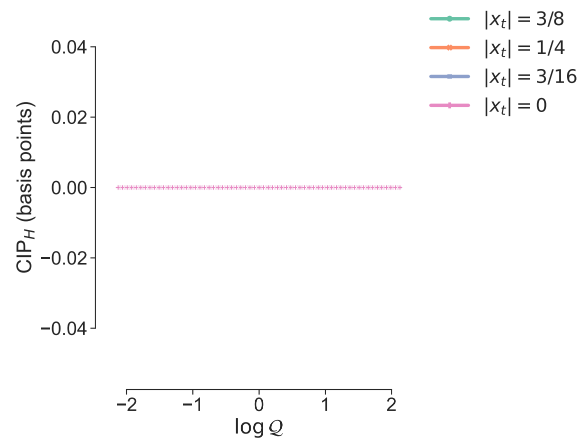


(d) Drift of exchange rates (see (30))

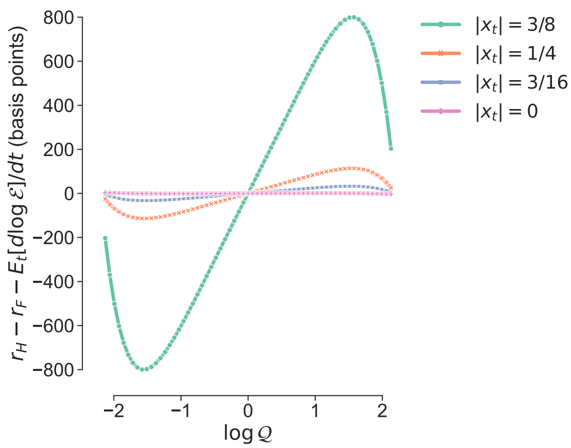
Figure 4: This plot shows under the calibration with $s^{\Psi} = 0.4$ for the frictional model, the variation of short-term rates and currency risk premia as a function of the endogenous state variable $\log \mathcal{Q}_t$ and the exogenous variable x_t. For example, the upper left panel exhibits the short-rate differential as a function of $\log \mathcal{Q}_t$ for various choices of $|x_t| \in \{0, 3/16, 1/4, 3/8\}$.



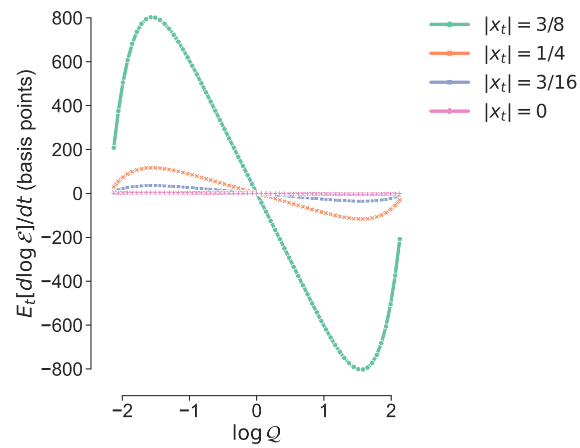
(a) Short rate differential (see (32))



(b) CIP deviations (see (33))



(c) Currency risk premium (see (30))



(d) Drift of exchange rates (see (30))

Figure 5: This plot shows under the calibration with $s^{\Psi} = 0.4$ for the frictionless model, the variation of short-term rates and currency risk premia as a function of the endogenous state variable $\log \mathcal{Q}_t$ and the exogenous variable x_t. For example, the upper left panel exhibits the short-rate differential as a function of $\log \mathcal{Q}_t$ for various choices of $|x_t| \in \{0, 3/16, 1/4, 3/8\}$.

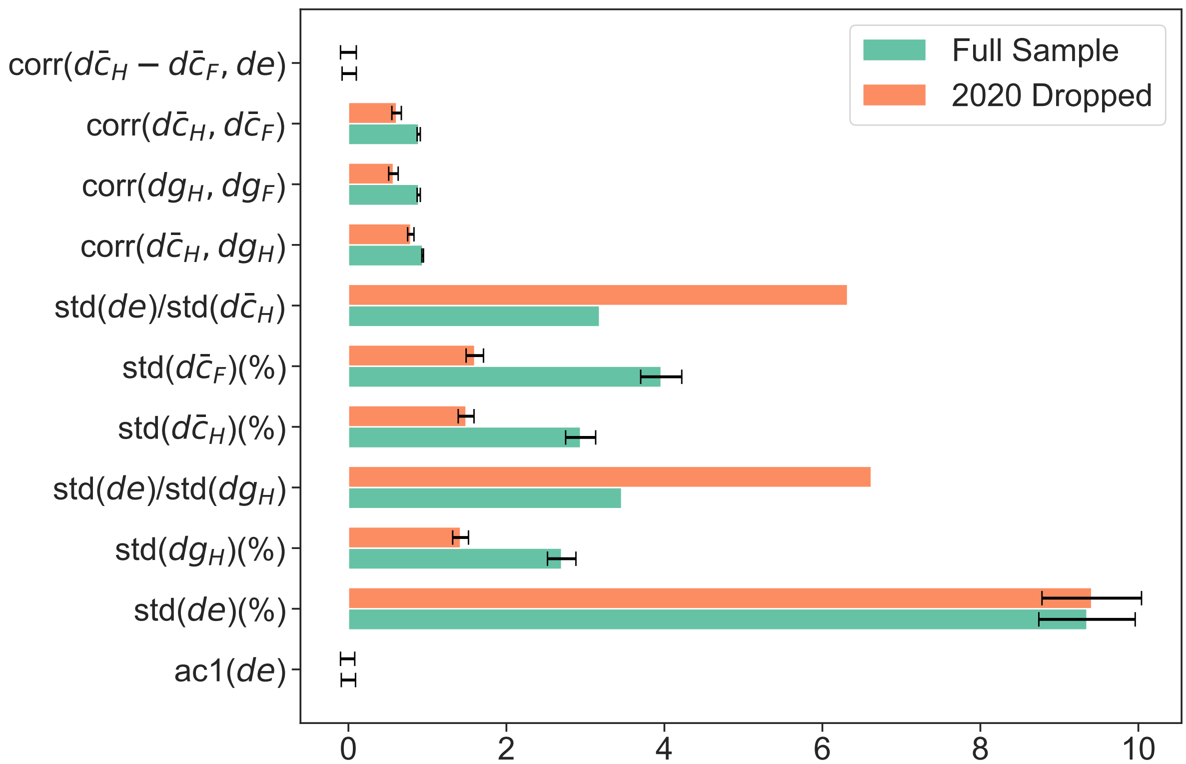


Figure 6: Covid-19 effects on international macro moments.

Table 12: This table presents simulated moments for models with positively correlated demand shocks ($\rho^{\Psi} = 0.3$). The parameter s^{Ψ} is set to -0.4 where applicable. The first two columns display estimates of the data moments and their associated standard errors, where applicable. The subsequent two columns present the simulated moments for the models without home bias ($\beta_H = \beta_F = 0.5$). The last two columns present the simulated moments for the models without demand shocks ($\theta_H^{\Psi} = \theta_F^{\Psi} = 0$).

			No Home-bias		No Demand	
	Data	S.E.	Frictionless	Frictional	Frictionless	Frictional
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	2.26	2.26	2.26	2.26
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	1.14	1.14	1.14	1.14
std($d\bar{c}_H$)(%)	1.49	(0.10)	6.92	3.80	1.98	1.98
std($d\bar{c}_F$)(%)	1.60	(0.11)	6.77	3.78	1.98	1.98
std(de)/std($d\bar{c}_H$)	6.32	—	0.32	0.59	1.14	1.14
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.09	0.30	1.00	1.00
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	-0.68	-0.54	0.35	0.35
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	-0.17	-0.09	1.00	1.00
std($r_H - r_F$)(%)	0.69	(0.04)	0.01	0.00	0.00	0.00
std($r_H - r_F$)/std(de)	0.07	—	0.00	0.00	0.00	0.00

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			No Home-bias		No Demand	
	Data	S.E.	Frictionless	Frictional	Frictionless	Frictional
std(r_H)(%)	1.12	(0.07)	0.00	0.20	0.00	0.00
corr(r_H, r_F)	0.81	(0.03)	-1.00	1.00	—	—
corr(dr_H, dr_F)	0.59	(0.06)	-1.00	1.00	—	—
ac1($r_H - r_F$)	0.96	(0.09)	0.99	0.99	—	—
ac1(r_H)	0.98	(0.09)	0.99	0.96	—	—
Fama-β	2.18	(1.25)	25.76	50.72	—	—
carry SR(%)	37.23	(18.41)	0.49	0.42	—	—
carry i-diff (%)	1.21	(0.39)	0.02	0.01	—	—
carry (%)	3.46	(9.28)	0.01	0.01	—	—
carry ratio (%)	34.97	(18.41)	0.70	0.28	—	—
std(carry) (%)	9.28	(0.60)	2.26	2.26	—	—
std(carry i-diff) (%)	0.39	(0.03)	0.00	0.00	—	—
mean(CIP$_H$)(%)	-0.21	(0.30)	0.00	-0.02	0.00	0.00
CIP-β	-2.64	(0.68)	—	-0.01	—	—
t CIP	-3.87	—	—	-8.43	—	—
R^2 CIP(%)	2.00	—	—	12.62	—	—
ERP$_H$(%)	3.48	(12.74)	0.29	0.33	0.19	0.19

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	Data	S.E.	No Home-bias		No Demand	
			Frictionless	Frictional	Frictionless	Frictional
equity SR_H(%)	27.26	(18.41)	3.64	4.15	2.31	2.31
ERP_F(%)	2.01	(15.85)	0.27	0.31	0.17	0.17
equity SR_F(%)	12.66	(18.41)	3.44	3.92	2.14	2.14
corr(ERP_H, ERP_F)	0.85	(0.03)	0.35	0.35	0.35	0.35

Table 13: Frictionless Model Without x_t Shocks. This table presents simulated moments for a frictionless model with positively correlated demand shocks ($\rho^\Psi = 0.3$). The parameter σ_t^Ψ is set to 0.012 to ensure the volatility of exchange rates is comparable to that in Table 5, which includes stochastic x_t shocks. The first two columns display estimates of the data moments and their associated standard errors, where applicable. The subsequent five columns present the simulated moments for the frictionless model across various values of s^Ψ.

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	11.83	11.47	10.84	10.03	9.23
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	5.96	5.77	5.45	5.04	4.64
std($d\bar{c}_H$)(%)	1.49	(0.10)	2.07	2.26	2.51	2.71	2.85
std($d\bar{c}_F$)(%)	1.60	(0.11)	2.07	2.29	2.53	2.72	2.84
std(de)/std($d\bar{c}_H$)	6.32	—	5.56	4.99	4.32	3.73	3.29
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.73	0.75	0.79	0.83	0.87
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	0.35	0.14	-0.06	-0.21	-0.30
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	-0.62	-0.65	-0.70	-0.77	-0.83
std($r_H - r_F$)(%)	0.69	(0.04)	0.01	0.00	0.00	0.00	0.01
std($r_H - r_F$)/std(de)	0.07	—	0.00	0.00	0.00	0.00	0.00

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
std(r_H)(%)	1.12	(0.07)	0.00	0.00	0.00	0.00	0.00
corr(r_H, r_F)	0.81	(0.03)	-0.97	-0.97	—	-0.97	-0.98
corr(dr_H, dr_F)	0.59	(0.06)	-0.91	-0.91	—	-0.91	-0.91
ac1($r_H - r_F$)	0.96	(0.09)	0.99	0.99	—	0.99	0.99
ac1(r_H)	0.98	(0.09)	0.99	0.99	—	0.99	0.99
Fama-β	2.18	(1.25)	457.22	745.29	—	-647.05	-349.75
carry SR(%)	37.23	(18.41)	-0.01	0.09	—	-0.12	0.00
carry i-diff (%)	1.21	(0.39)	0.01	0.01	—	0.01	0.01
carry (%)	3.46	(9.28)	-0.00	0.01	—	-0.01	0.00
carry ratio (%)	34.97	(18.41)	-0.31	0.03	—	-0.03	0.35
std(carry) (%)	9.28	(0.60)	11.84	11.47	—	10.03	9.23
std(carry i-diff) (%)	0.39	(0.03)	0.00	0.00	—	0.00	0.00
mean(CIP$_H$)(%)	-0.21	(0.30)	0.00	0.00	0.00	0.00	0.00
CIP-β	-2.64	(0.68)	—	—	—	—	—
t CIP	-3.87	—	—	—	—	—	—
R^2 CIP(%)	2.00	—	—	—	—	—	—
ERP$_H$(%)	3.48	(12.74)	0.50	0.37	0.19	-0.00	-0.14
equity SR$_H$(%)	27.26	(18.41)	6.37	4.62	2.31	-0.01	-1.75

Continued on next page

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
$\text{ERP}_F(\%)$	2.01	(15.85)	0.49	0.35	0.17	-0.01	-0.15
equity $\text{SR}_F(\%)$	12.66	(18.41)	6.13	4.46	2.14	-0.18	-1.93
$\text{corr}(\text{ERP}_H, \text{ERP}_F)$	0.85	(0.03)	0.35	0.35	0.35	0.35	0.35

Table 14: Frictional Model Without x_t Shocks. This table presents simulated moments for a frictional model with positively correlated demand shocks ($\rho^{\Psi} = 0.3$). The parameter σ_t^{Ψ} is set to 0.012 to ensure the volatility of exchange rates is comparable to that in Table 5, which includes stochastic x_t shocks. The first two columns display estimates of the data moments and their associated standard errors, where applicable. The subsequent five columns present the simulated moments for the frictional model across various values of s^{Ψ}.

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	10.23	9.77	9.08	8.28	7.56
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	5.15	4.92	4.56	4.16	3.80
std($d\bar{c}_H$)(%)	1.49	(0.10)	1.97	2.13	2.34	2.53	2.67
std($d\bar{c}_F$)(%)	1.60	(0.11)	1.98	2.14	2.35	2.54	2.66
std(de)/std($d\bar{c}_H$)	6.32	—	5.10	4.53	3.87	3.29	2.88
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.81	0.82	0.85	0.88	0.90
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	0.45	0.23	0.02	-0.13	-0.21
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	-0.42	-0.49	-0.58	-0.68	-0.78
std($r_H - r_F$)(%)	0.69	(0.04)	0.02	0.01	0.01	0.01	0.01
std($r_H - r_F$)/std(de)	0.07	—	0.00	0.00	0.00	0.00	0.00
std(r_H)(%)	1.12	(0.07)	0.01	0.01	0.01	0.00	0.00

Continued on next page

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
corr(r_H, r_F)	0.81	(0.03)	-0.96	-0.96	-0.94	-0.91	-0.78
corr(dr_H, dr_F)	0.59	(0.06)	-0.87	-0.85	-0.82	-0.73	-0.43
ac1($r_H - r_F$)	0.96	(0.09)	0.99	0.99	0.99	0.99	0.99
ac1(r_H)	0.98	(0.09)	0.99	0.99	0.99	0.99	0.99
Fama-β	2.18	(1.25)	152.62	152.90	165.27	202.59	276.14
carry SR(%)	37.23	(18.41)	0.17	0.50	0.31	0.55	0.27
carry i-diff (%)	1.21	(0.39)	0.03	0.03	0.03	0.02	0.01
carry (%)	3.46	(9.28)	0.02	0.05	0.03	0.04	0.02
carry ratio (%)	34.97	(18.41)	0.18	0.22	0.16	0.15	0.09
std(carry) (%)	9.28	(0.60)	10.23	9.78	9.08	8.29	7.57
std(carry i-diff) (%)	0.39	(0.03)	0.00	0.00	0.00	0.00	0.00
mean(CIP$_H$)(%)	-0.21	(0.30)	-0.12	-0.12	-0.11	-0.10	-0.09
CIP-β	-2.64	(0.68)	-0.09	-0.10	-0.10	-0.10	-0.09
t CIP	-3.87	—	-74.84	-66.57	-61.80	-61.08	-64.56
R^2 CIP(%)	2.00	—	95.22	93.48	92.05	91.75	92.74
ERP$_H$(%)	3.48	(12.74)	0.50	0.37	0.19	0.00	-0.14
equity SR$_H$(%)	27.26	(18.41)	6.33	4.60	2.31	0.01	-1.73
ERP$_F$(%)	2.01	(15.85)	0.49	0.35	0.17	-0.01	-0.15

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
equity $\mathrm{SR}_F(\%)$	12.66	(18.41)	6.08	4.43	2.14	-0.15	-1.90
corr(ERP_H, ERP_F)	0.85	(0.03)	0.35	0.35	0.35	0.35	0.35

D Asymmetric Countries

In this section, we investigate an alternative calibration, allowing the two countries to have asymmetric shock exposures. We consider the following specifications for the dynamics of the output and demand shocks:

$$\begin{bmatrix} (\theta_H)' \\ (\theta_F)' \\ (\theta_H^\Psi)' \\ (\theta_F^\Psi)' \end{bmatrix} = \begin{bmatrix} \bar{\sigma}^{\mathcal{M}} & 0 & \bar{\sigma}^{\mathcal{M}} & 0 & 0 & 0 \\ 0 & \bar{\sigma}^{\mathcal{M}} & \rho^{\mathcal{M}}\bar{\sigma}^{\mathcal{M}} & \sqrt{1-(\rho^{\mathcal{M}})^2}\bar{\sigma}^{\mathcal{M}} & 0 & 0 \\ s^\Psi\bar{\sigma}^\Psi & 0 & \sigma^{\text{carry}} & \sigma^{\text{ERP}} & \sqrt{1-(s^\Psi)^2}\sigma_t^\Psi & 0 \\ 0 & s^\Psi\bar{\sigma}^\Psi & \sigma^{\text{carry}} & \sigma^{\text{ERP}} & \sqrt{1-(s^\Psi)^2}\rho^\Psi\sigma_t^\Psi & \sqrt{1-(s^\Psi)^2}\sqrt{1-(\rho^\Psi)^2}\sigma_t^\Psi \end{bmatrix} \tag{46}$$

Table 15 reports the calibrated coefficients in (46) (See Appendix C for details). σ_t^Ψ is defined in Assumption 3.

In comparison to the previous version of our calibration (see (34) and (15)), we have added two more shocks, with common exposures, $\sigma^{\text{carry}}, \sigma^{\text{ERP}}$. This adjustment helps us match the magnitudes and differences in equity risk premiums across countries. Additionally, this asymmetric calibration allows us to achieve a higher carry Sharpe ratio (5.1% for the frictionless and 6.8% for the frictional) and a higher carry ratio (39) (17.1% for the frictionless and 23.9% for the frictional, see Tables 16 and 17).

Table 15: Parameter Choices for the Simulated Moments

Variable Definitions	Symbols	Values	Targeted Moments
Preferences	$\{\beta_i\}_{i=H,F}$	0.9	Trade-to-GDP ratio 0.2
Time discount	δ	0.03	—
Drift of $\mathcal{M}_{i,t}$	μ	0.03	—
Size of Supply shocks	$\bar{\sigma}^{\mathcal{M}}$	0.014	$\mathrm{std}(dg_H) = 2\%$
Supply shocks correlation	$\rho^{\mathcal{M}}$	0.7	$\mathrm{corr}(dg_H, dg_F) = 0.35$
Correlated demand shocks	σ_t^{Ψ}	$\exp(4\,x_t^2) - 1$	—
Demand shocks correlation	ρ^{Ψ}	0.3	$\mathrm{std}(de) = 10\%$
Idiosyncratic demand shocks	$\bar{\sigma}^{\Psi}$	0.095	$\mathrm{corr}(dc_H, dc_F) = 0.3$
Output-demand correlation	s^{Ψ}	-0.4	$\mathrm{corr}(dc_H, dc_F) = 0.3$
Mean-reversion of x_t	κ^x	0.36/12	$\mathrm{ac1}(r_H - r_F) = 0.95$
Volatility parameter of x_t	σ^x	0.09/2	$\mathrm{std}(r_H - r_F) = 0.6\%$
Carry demand shocks	σ^{carry}	-0.5	equity $\mathrm{SR}_H = 27.26\%$
ERP demand shocks	σ^{ERP}	0.3	equity $\mathrm{SR}_F = 12.66\%$

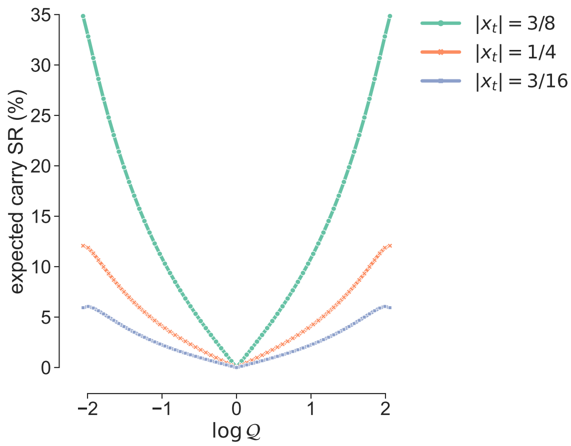
We have

$$r_{i,t} \;=\; r^C_{i,t} \;=\; \delta + \mu_i - \|\theta_i\|^2 + \theta_i'\theta_i^{\Psi}$$

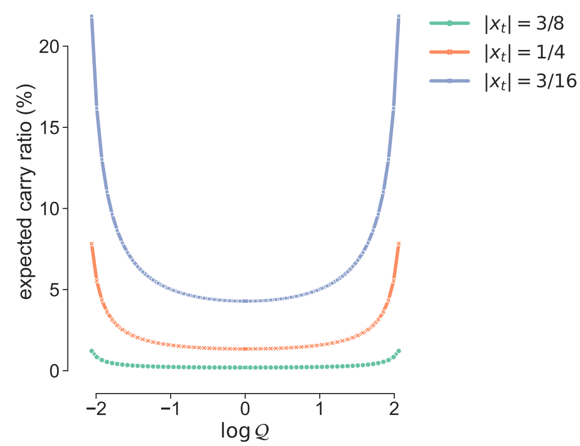
Hence, for country H, we have $r^C_{H,t} = \delta + \mu - (\sigma^{\mathcal{M}})^2 + \sigma^{\mathcal{M}}\sigma^{\Psi}_1$, while for country F, we have that $r^C_{F,t} = \delta + \mu - (\sigma^{\mathcal{M}})^2 + \rho^{\mathcal{M}}\sigma^{\mathcal{M}}\sigma^{\Psi}_1 + \sqrt{1-(\rho^{\mathcal{M}})^2}\sigma^{\mathcal{M}}\sigma^{\Psi}_2$. As long as $\rho^{\mathcal{M}} \neq 1$, we have $r^C_{H,t} \neq r^C_{F,t}$, generating systematic differences in interest rates. Meanwhile, we note that the volatility of the exchange rates is purely driven by $\|\theta_H - \theta_F\| = \sqrt{2-2\rho^{\mathcal{M}}}\sigma^{\mathcal{M}}$, and the expected changes in the exchange rates are determined solely by $r_{H,t} - r_{F,t}$ (see equation (30)). We would, therefore, expect to observe a relatively large expected carry trade return and relatively small volatility due to exchange rate fluctuations. I.e., a high Carry Sharpe ratio.

A large common demand shock $\sigma^{\Psi}_1 = -0.5$ significantly increases the volatility of the country-specific SDF and elevates the correlation of the SDFs across countries. The high volatility of the country-specific SDFs, combined with the negative correlation between demand and supply shocks leads to a high equilibrium equity premium. This premium is determined by the product of the quantity of risk θ_i (volatility of the endowment claim) and the price of risk $\theta_i - \theta^{\Psi}_i$. The low correlation between cross-country equity risk premiums is a consequence of the endowment claim being a short-term claim, meaning its price dynamics are driven solely by the supply shock θ_i.

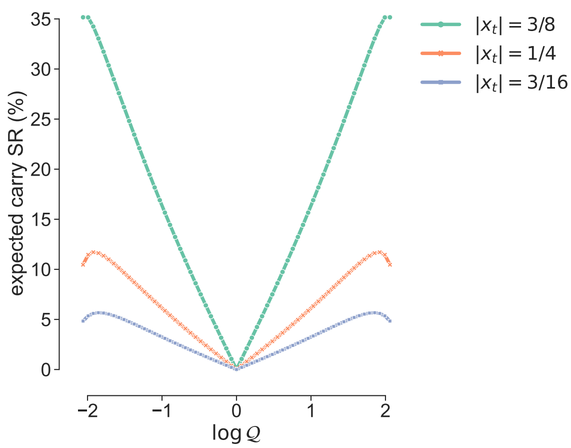
Figure 7 and 8 illustrate that the expected carry Sharpe ratio and the expected carry ratio can be substantially larger for the frictional model, particularly for tail realizations of $\mathcal{Q}_t$ and $|x_t|$ (the latter leads to a large risk-sharing demand).



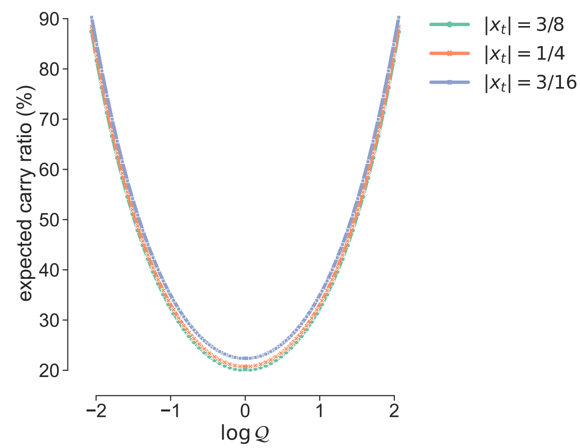
(a) Frictionless carry SR



(b) Frictionless carry ratio

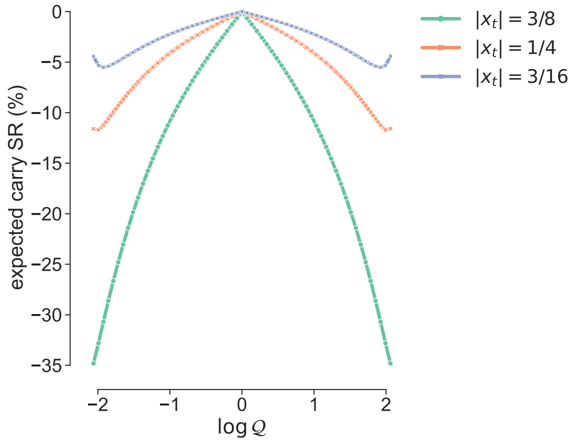


(c) Frictional carry SR

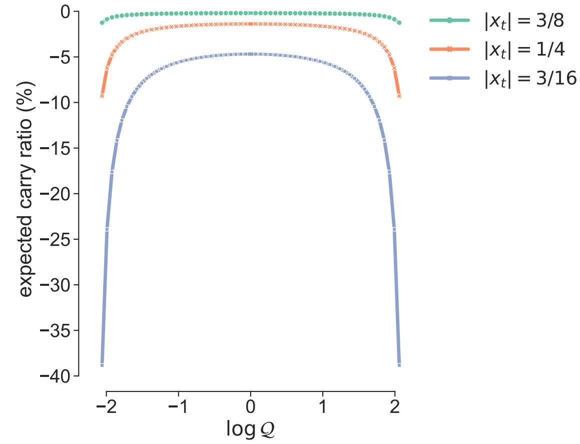


(d) Frictional carry ratio

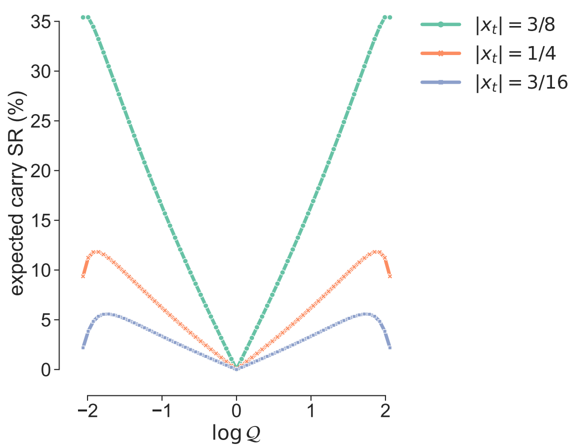
Figure 7: This plot shows the expected Carry Sharpe ratio as well as the expected carry risk premium explained by the interest rates differentials defined in (39) conditional on $\log \mathcal{Q}_t$ and various choices of $|x_t| \in \{3/16, 1/4, 3/8\}$. We consider a negative correlation between the supply and demand shocks ($s^{\Psi} = -0.4$) for both the frictionless (first row) and the frictional model (second row).



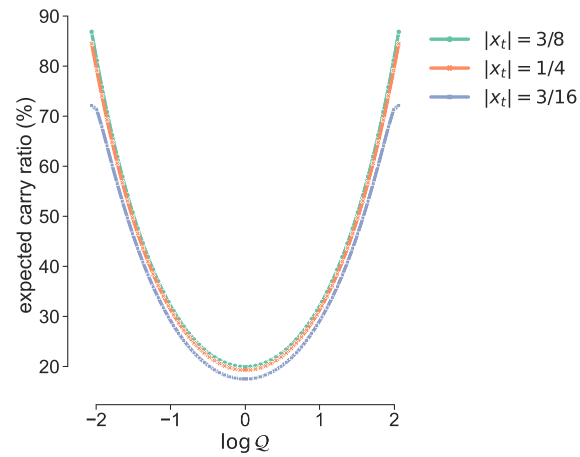
(a) Frictionless carry SR



(b) Frictionless carry ratio



(c) Frictional carry SR



(d) Frictional carry ratio

Figure 8: This plot shows the expected Carry Sharpe ratio as well as the expected carry risk premium explained by the interest rates differentials defined in (39) conditional on $\log \mathcal{Q}_t$ and various choices of $|x_t| \in \{3/16, 1/4, 3/8\}$. We consider a positive correlation between the supply and demand shocks ($s^{\Psi} = 0.4$) for both the frictionless (first row) and the frictional model (second row).

Table 16: Frictionless Model With Perfectly Correlated Carry/ERP Shocks. This table presents simulated moments for a frictionless model with positively correlated demand shocks ($\rho^{\Psi} = 0.3$). The parameter σ^{carry} is set to -0.5 to ensure a positive carry trade premium. The parameter σ^{ERP} is set to 0.3 to match the equity risk premium (ERP). The first two columns display estimates of the data moments and their associated standard errors, where applicable. The subsequent five columns present the simulated moments for the frictionless model across various values of s^{Ψ}.

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	11.81	11.40	10.80	10.00	9.20
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	5.97	5.77	5.46	5.05	4.66
std($d\bar{c}_H$)(%)	1.49	(0.10)	2.19	2.28	2.48	2.78	3.02
std($d\bar{c}_F$)(%)	1.60	(0.11)	2.17	2.30	2.49	2.80	3.02
std(de)/std($d\bar{c}_H$)	6.32	—	5.29	4.18	3.21	2.81	2.80
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.66	0.74	0.81	0.84	0.85
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	0.31	0.12	-0.04	-0.17	-0.28
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	-0.67	-0.62	-0.63	-0.72	-0.85
std($r_H - r_F$)(%)	0.69	(0.04)	0.01	0.00	0.00	0.00	0.01
std($r_H - r_F$)/std(de)	0.07	—	0.00	0.00	0.00	0.00	0.00

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
std(r_H)(%)	1.12	(0.07)	0.00	0.00	0.00	0.00	0.00
corr(r_H, r_F)	0.81	(0.03)	-0.97	-0.97	—	-0.97	-0.97
corr(dr_H, dr_F)	0.59	(0.06)	-0.89	-0.89	—	-0.90	-0.89
ac1($r_H - r_F$)	0.96	(0.09)	0.99	0.99	—	0.99	0.99
ac1(r_H)	0.98	(0.09)	0.99	0.99	—	0.99	0.99
Fama-β	2.18	(1.25)	473.63	873.12	—	-687.44	-360.28
carry SR(%)	37.23	(18.41)	4.59	5.09	6.15	6.48	5.65
carry i-diff (%)	1.21	(0.39)	0.51	0.51	0.51	0.51	0.51
carry (%)	3.46	(9.28)	0.56	0.54	0.51	0.51	0.49
carry ratio (%)	34.97	(18.41)	15.99	17.13	20.04	21.77	21.95
std(carry) (%)	9.28	(0.60)	11.81	11.40	10.80	10.00	9.20
std(carry i-diff) (%)	0.39	(0.03)	0.00	0.00	0.00	0.00	0.00
mean(CIP$_H$)(%)	-0.21	(0.30)	0.00	0.00	0.00	0.00	0.00
CIP-β	-2.64	(0.68)	—	—	—	—	—
t CIP	-3.87	—	—	—	—	—	—
R^2 CIP(%)	2.00	—	—	—	—	—	—
ERP$_H$(%)	3.48	(12.74)	3.31	3.18	3.00	2.82	2.68
equity SR$_H$(%)	27.26	(18.41)	41.54	39.81	37.64	35.35	33.64

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ERP_F(%)	2.01	(15.85)	1.24	1.10	0.93	0.76	0.62
equity SR_F(%)	12.66	(18.41)	15.53	13.88	11.71	9.50	7.77
corr(ERP_H, ERP_F)	0.85	(0.03)	0.35	0.35	0.35	0.35	0.35

Table 17: Frictional Model With Perfectly Correlated Carry/ERP Shocks. This table presents simulated moments for a frictional model with positively correlated demand shocks ($\rho^{\Psi} = 0.3$). The parameter σ^{carry} is set to -0.5 to ensure a positive carry trade premium. The parameter σ^{ERP} is set to 0.3 to match the equity risk premium (ERP). The first two columns display estimates of the data moments and their associated standard errors, where applicable. The subsequent five columns present the simulated moments for the frictional model across various values of s^{Ψ}.

	Data	S.E.	-0.7	-0.4	0	0.4	0.7
ac1(de)	-0.01	(0.09)	-0.00	-0.00	-0.00	-0.00	-0.00
std(de)(%)	9.41	(0.01)	10.19	9.73	9.07	8.33	7.63
std(dg_H)(%)	1.42	(0.10)	1.98	1.98	1.98	1.98	1.98
std(de)/std(dg_H)	6.62	—	5.16	4.92	4.59	4.21	3.86
std($d\bar{c}_H$)(%)	1.49	(0.10)	2.04	2.14	2.33	2.57	2.74
std($d\bar{c}_F$)(%)	1.60	(0.11)	2.01	2.13	2.33	2.60	2.81
std(de)/std($d\bar{c}_H$)	6.32	—	4.88	3.90	3.08	2.67	2.55
corr($d\bar{c}_H, dg_H$)	0.79	(0.04)	0.74	0.81	0.85	0.88	0.89
corr(dg_H, dg_F)	0.57	(0.06)	0.35	0.35	0.35	0.35	0.35
corr($d\bar{c}_H, d\bar{c}_F$)	0.61	(0.06)	0.38	0.17	0.01	-0.12	-0.21
corr($d\bar{c}_H - d\bar{c}_F, de$)	-0.00	(0.10)	-0.54	-0.52	-0.57	-0.67	-0.81
std($r_H - r_F$)(%)	0.69	(0.04)	0.04	0.06	0.07	0.06	0.04
std($r_H - r_F$)/std(de)	0.07	—	0.00	0.01	0.01	0.01	0.00

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
std(r_H)(%)	1.12	(0.07)	0.02	0.03	0.03	0.03	0.02
corr(r_H, r_F)	0.81	(0.03)	0.48	0.74	0.85	0.86	0.88
corr(dr_H, dr_F)	0.59	(0.06)	0.53	0.67	0.67	0.71	0.76
ac1($r_H - r_F$)	0.96	(0.09)	0.97	0.96	0.95	0.95	0.95
ac1(r_H)	0.98	(0.09)	0.97	0.96	0.95	0.96	0.96
Fama-β	2.18	(1.25)	36.05	17.19	11.19	11.01	16.57
carry SR(%)	37.23	(18.41)	6.15	6.75	8.15	8.17	6.79
carry i-diff (%)	1.21	(0.39)	0.52	0.52	0.51	0.51	0.51
carry (%)	3.46	(9.28)	0.65	0.63	0.59	0.54	0.48
carry ratio (%)	34.97	(18.41)	20.16	23.90	29.42	32.13	29.40
std(carry) (%)	9.28	(0.60)	10.19	9.73	9.06	8.33	7.63
std(carry i-diff) (%)	0.39	(0.03)	0.01	0.02	0.02	0.02	0.01
mean(CIP$_H$)(%)	-0.21	(0.30)	-0.12	-0.11	-0.11	-0.10	-0.09
CIP-β	-2.64	(0.68)	-0.13	-0.23	-0.34	-0.31	-0.18
t CIP	-3.87	—	-3.34	-2.24	-2.02	-2.05	-2.69
R^2 CIP(%)	2.00	—	5.38	3.33	4.13	4.24	4.73
ERP$_H$(%)	3.48	(12.74)	3.31	3.18	3.00	2.82	2.68
equity SR$_H$(%)	27.26	(18.41)	41.50	39.82	37.66	35.37	33.66

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	Data	S.E.	-0.7	-0.4	0	0.4	0.7
$\text{ERP}_F(\%)$	2.01	(15.85)	1.24	1.11	0.93	0.76	0.62
equity $\text{SR}_F(\%)$	12.66	(18.41)	15.57	13.87	11.72	9.54	7.81
$\text{corr}(\text{ERP}_H, \text{ERP}_F)$	0.85	(0.03)	0.35	0.35	0.35	0.35	0.35