Health and Economic Inequality during Pandemics: A Heterogeneous Agent Perspective

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Outline

- Motivation: Empirical Facts
- General Equilibrium Model

- The Covid-19 pandemic highlighted **inequality of health outcomes**:
  - by age, income (deprivation indices in UK), ethnicity, spatial, gender, household structure, etc.
- This was clearly recognized during the pandemic and was tracked by Covid policymakers.
- **ONS (UK) data**: infection rate for the deprived area is higher

Empirical Facts — Health Inequality Increased

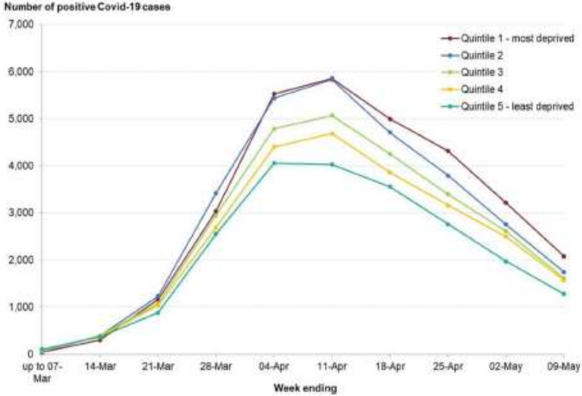
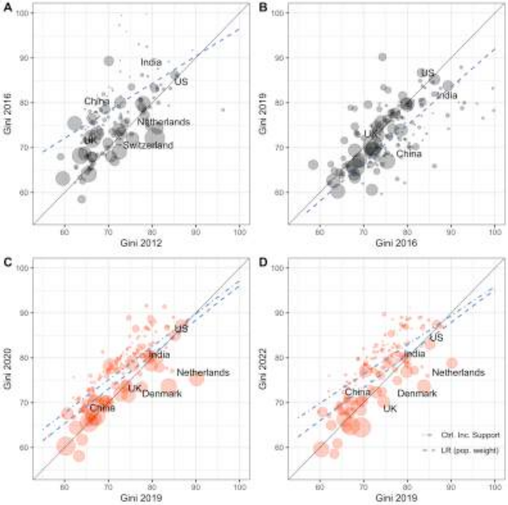


Figure 1: Health Inequality in UK

- Concern that the pandemic could perpetuate **Economic inequality**.
- Income Gini has increased during the pandemic (Chen and Krieger, 2021; Stantcheva, 2022). Inequality Empirical
- Evidence on wealth Gini index by Global Wealth Report (Credit Suisse, 2012-2022)

Empirical Facts

Wealth Equality Worsened without Income Support



Income Support (later)

Worsened health and wealth equality could be correlated.

- Possible bridge between wealth and health: Individual Health Policy
  - Rich people act more preventive to the disease
  - $\Rightarrow$ s.t. lower infection risk
- UK Data: lower tier local authorities
  - Community (Google) mobility is negatively correlated with income.

Empirical Facts — Health Policy

Focus on Oct 2022, when all social restrictions were removed

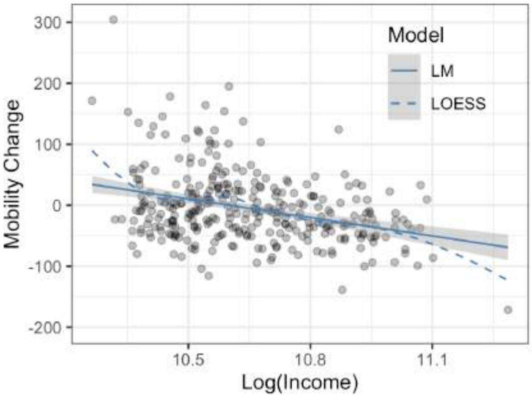


Figure 2: Mobility Change and Income (Oct 2022)

Panel Regression

This paper

We model the dynamics of **health and wealth inequality** during the pandemic.

- Heterogeneous Agent model (*à la* Achdou et.al, 2022) + Disease transmission (SIRS model)
- understand the co-determination and co-evolution of health and economic inequality.
  - **Health inequality**: infection rates disparities[4].
  - **Economic inequality**: income / wealth inequality More
- Opt. individual health policy
  - Preventive (Precautionary) policy
  - Treatment or recuperative (ex-post) policy
- Gov. Income Support Scheme

[4] we do not model mortality in this paper consistent with later period of the pandemic where this has declined. We shut down all avenues of heterogeneity: WFH, gender, age, household structure, spatial issues, occupations, unemployment etc.

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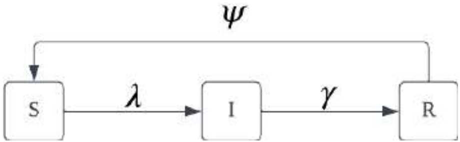
Takeaways

- Model matches epidemiological dynamics and health policy
- Temporary increase in income inequality
- Persistent increase in wealth inequality
- Income support
  (Aggregate trade-off between health and wealth)
  - Rising inequality can be turned-around by income support
  - might discourage peoples' spend on health and induce higher infection

Model Setup — State Variables

model overview **Individual State variables:**

- a: wealth for individuals
  - Continuously distributed in the interval $[\underline{a}, \bar{a}]$
- h: health status for individual (Epidemiological Compartments)
  - **susceptible** $\mathcal{S}$: individuals without immunity; will be infected if contacting with virus
  - **infective** $\mathcal{I}$: individuals carry and be able to transmit virus
  - **recovered** $\mathcal{R}$: individuals recovered from infection with immunity
- Motion of individual health status: SIRS dynamics



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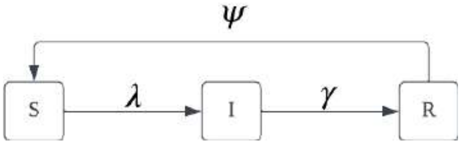

Model Setup — Individual Income

- Function z maps individuals' health status to productivity

$$z : h \to [0, 1]; \quad z(\mathcal{S}) = z(\mathcal{R}) > z(\mathcal{I}) \tag{1}$$

- The productivity $z(h)$ changed stochastically according to the epidemiological motion
  - Idiosyncratic term with Poisson process generates heterogeneity
  - Let $g(a, h)$ to be the joint distribution

Model Setup — Health Expenditure

- Idiosyncratic shock in Aiyagari (1994) is exogenous and uninsured.
- Idiosyncratic term $z(h)$ here is partially insured by two types of health expenditure for individuals
  - **Prevention expenditure** $m_{\mathcal{P}}$:
    - consumption-reduction action for reducing the probability of future infection
    - e.g. self-isolation, facial mask, PCR test etc.
  - **Treatment expenditure** $m_{\mathcal{T}}$:
    - ex-post consumption-reduction action for better and faster health/productivity recovery
    - e.g. supplement, medicine, nourishment, living condition etc.

More

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More

Model Setup — Infection

Disease Transmission:

- The disease is transmitted by infectious contact: Susceptible individual becomes infected when contact with infective individual

Contact Rate:

- Individuals contact with others with a rate α
  - Higher expenditure on prevention $m_{\mathcal{P}}$, lower contact rate.
  - $\alpha(m_{\mathcal{P}})$ is a decreasing function

$$\alpha(m_{\mathcal{P}}) : \mathbb{R}_+ \to \mathbb{R}_+$$
$$\text{with} \quad \alpha' < 0; \alpha'' > 0, \alpha(0) = \bar{\alpha}; \alpha(\infty) = \underline{\alpha}$$

Model Setup — Infection

Infection Process:

- Given individuals are continuously distributed
- The infection probability for susceptible individuals with preventive expenditure $m_{\mathcal{P}}$ is

$$\lambda = \alpha(m_{\mathcal{P}})\zeta \tag{2}$$

  - ζ is the social average infectious contact rate[5]
  - ζ perceived and taken as given in the individual maximization problem.
- In equilibrium, the perception about the average infective contact rate is in fact the true value.

$$\zeta = \int \alpha(m_{\mathcal{P}})\mathbb{1}(h = \mathcal{I})g(a,h)d\mu \tag{3}$$

[5] It is a matching process for susceptible and infective group. Similar setup could be referred to Hethcote (2009) for continuous age group

Model Setup — Recovery

Recovery Process:

- Recovery rate γ for the infective group is increasing with treatment expenditure $m_{\mathcal{T}}$
- $\gamma(m_{\mathcal{T}})$ is an increasing function

$$\gamma(m_{\mathcal{T}}) : \mathbb{R}_+ \to \mathbb{R}_+$$
$$\text{with} \quad \gamma' > 0; \gamma'' < 0; \gamma(0) = \underline{\gamma}; \gamma(\infty) = \bar{\gamma}$$

Model Setup — Individual Problem

$$\max_{c,m_{\mathcal{P}},m_{\mathcal{T}}} \quad \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\frac{c^{1-\sigma}}{1-\sigma} - \chi \mathbb{1}(h = \mathcal{I}) \right] dt$$
$$s.t. \quad \dot{a} = ra + wz(h) - c - m_{\mathcal{P}} - m_{\mathcal{T}}$$
$$h \in \{\mathcal{S}, \mathcal{I}, \mathcal{R}\} \quad \text{Poisson with intensities } \alpha(m_{\mathcal{P}})\zeta,\ \gamma(m_{\mathcal{T}}),\ \psi$$
$$a \geq 0 \tag{4}$$

- a and $\dot{a}$: asset and its differentiation w.r.t. time t
- r and w: interest rate and wage rate
- $\chi \geq 0$ is the level of disutility of being infected.
    - $\chi = 0$: infection is a pure income shock for individuals

Model Setup — Aggregate Variables

Competitive production landscape

- r, w are given by the profit optimization problem of the representative firm

$$\max_{K,L} \Pi = AF(K,L) - rK - \delta K - wL$$
$$\text{F.O.C. yields} \quad r = MPK, \quad w = MPL \tag{5}$$

- K and L are the aggregate capital and labour demand in the economy
- In equilibrium, aggregate demand = aggregate supply

$$K = \int a g(a,h) d\mu$$
$$L = \int z(h) g(a,h) d\mu \tag{6}$$

model overview

HACT

The model is a **Mean Field Game**[6]

- Solve the model by **H**eterogeneous-**A**gent-**C**ontinuous-**T**ime (HACT) dynamic programming (PDE View Point) HACT
  - Hamiltonian-Jacobian-Bellman Equation (HJB)
  - Kolmogorov Forward Equation (KF)
  - Market clearing conditions (MCC)

Parameterization Parameterization

- Calibrate to latter evidence of Omicron
- The model match the data of basic reproduction number R_0; UK infection rate after 2023.

[6] Mean-field game theory is the study of strategic decision making by small interacting agents in very large populations. Lasry and Lions (2007); Huang, Malhamé and Caines (2006). . The Nash Equilibrium is to find (1) Best Response $\mathrm{BR}: g^* \mapsto (c^*, m^*_{\mathcal{P}}, m^*_{\mathcal{T}})$; (2) Probability Behaviour $\mathrm{PB}: (c^*, m^*_{\mathcal{P}}, m^*_{\mathcal{T}}) \mapsto g^*$

Stationary Equilibrium — Baseline Model

- **Definition** (Stationary Equilibrium)
  - Choice variables $\{c, m_{\mathcal{P}}, m_{\mathcal{T}}\}$ solves the HJB equation
  - Value function $v(a, h)$ does not change over time $\partial_t \mathbf{V} = 0$
  - Distribution does not change over time $\partial_t \mathbf{g} = 0$
  - Market cleared $\mathcal{F}(\mathbf{g}) = 0$
- Transitional dynamics (later)

Stationary Equilibrium — Baseline Model

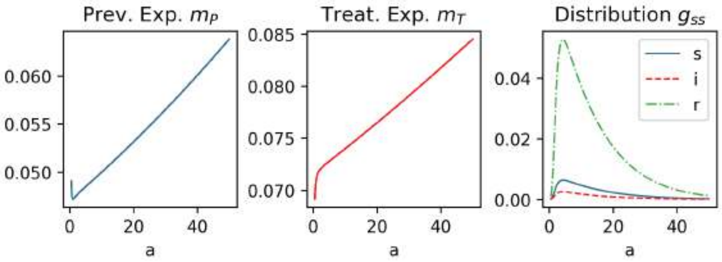

Figure 3: Baseline Model ($\chi = 0.3$)

- Health Policy:
  - Wealthier individuals spend more on both Preventive and Recuperation
- The stationary wealth distribution is skewed
- Consumption and savings Policy

Stationary Equilibrium — Baseline Model

How health policy affects wealth distribution?

- When (partially) shutdown health expenditure, equality improved

Model	R_0	agg.Capital	agg.Income	Wealth Gini
Baseline	9.236	14.447	1.838	0.412
Exog. Disease	216.0	13.869	1.763	0.365
$m_{\mathcal{P}}$ only	10.427	13.966	1.775	0.37
$m_{\mathcal{T}}$ only	104.282	14.393	1.831	0.407
Aiyagari	-	9.962	1.152	0.296

- reason: wealthier cannot mitigate the future risk of infection

Comparative Study — Health Policy

- Change the health punishment χ in

$$U = u(c) - \chi \mathbb{1}(h = \mathcal{I}) \tag{7}$$

  - $\chi \downarrow$, value loss of being infected $\downarrow$
  - $\chi = 0$: infection is a pure income shock

- Comparative Study:
  Poorest 25% (below $\mathtt{Q}_1$) v.s. Richest 25% (over $\mathtt{Q}_3$) Big Table

(a) Prev. Exp. | (b) Infection Rate (%)

χ	(a) $a < \mathtt{Q}_1$	(a) $a > \mathtt{Q}_3$	(a) diff	(b) $a < \mathtt{Q}_1$	(b) $a > \mathtt{Q}_3$	(b) diff
0	0.03	0.031	0.001	4.44	4.3	-0.13
0.1	0.036	0.039	0.003	4.36	4.2	-0.17
0.3	0.048	0.057	0.009	4.23	4.02	-0.21

Transitional Dynamics — Simulating Pandemic

- Transitional dynamics to the stationary distribution
  - Evolution of distribution and aggregate variables given an initial distribution $g_0(a, h)$
- Vaccination
- Government Income Support

Transitional Dynamics — Simulating Pandemic

- Construct initial distributions with compartmental composition of
  - Same initial infection population: 0.5%
  - **Different recovery population**: $\{0, 34\%, 68\%\}$
- Recovered group:
  - Individuals with immunity.
  - Recovered group in the initial distribution (pre-existing immunity)
  - could be used to interpret **vaccination**[7]:
    higher vaccination rate $\Rightarrow$ larger pre-existing immunity
- 68%: Fully Vaccinated Population at before Omicron B.A.1 wave.
  - Infection dynamics fits UK data of Omicron B.A.1 wave. Dynamic Fit

[7] Federico, Ferrari and Torrente (2022)

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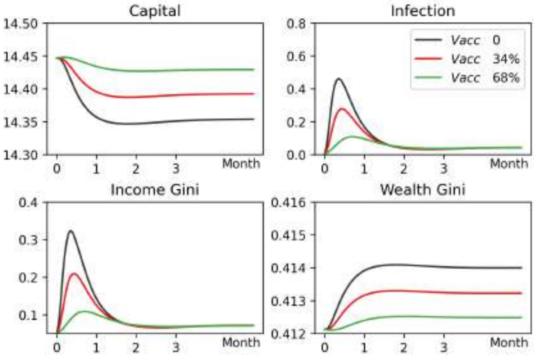


Figure 4: Transitional Path — Aggregate Variables (selected)

- Wealth and income equality is worsened in the pandemic
- Persistency is different

Other State Variables Disease Mutation

Transitional Dynamics — Simulating Pandemic

How wealth equality worsened? Figures

- We track the dynamics of the distribution

$$\partial_t g(a, h) \tag{8}$$

  by Kolmogorov Forward Equation
- There are more poor people compared to the pre-pandemic stage.

Why more poor people? Mechanism

- Poor spend less on health $\Rightarrow$ Higher infection rate $\Rightarrow$ less income/savings contribution in the distribution

Transitional Dynamics — Income Support

- Government cover part of the income lost by infection
- Lump-sum transfer τ per infected individual

$$ra + wz(h) + \tau \mathbb{1}(h = \mathcal{I}) \tag{9}$$

- Government budget constraint

$$\int \tau \mathbb{1}(h = \mathcal{I}) g(a, h) d\mu \leq \mathcal{B} \tag{10}$$

  - $\mathcal{B}$ exogenous
  - Abstracted from budget financing

Transitional Dynamics — Income Support

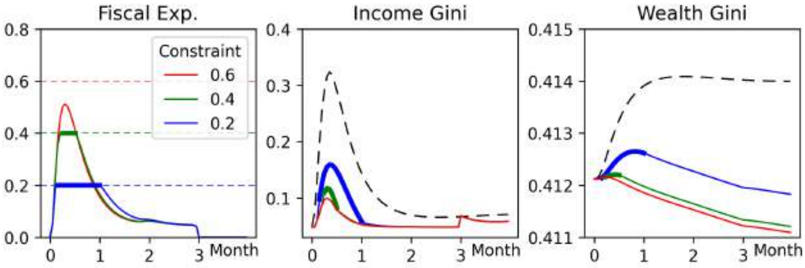


Figure 5: Income Support

- black dash: no income support

Transitional Dynamics — Income Support

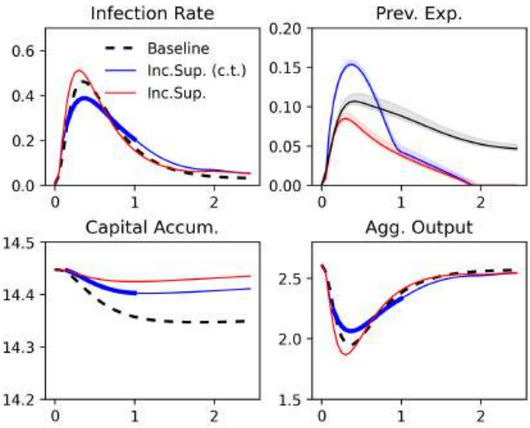


Figure 6: Income Support (cont.)

- Unconstrained support discourage preventive expenditure $\Rightarrow$ infection $\uparrow \Rightarrow$ output $\downarrow$

Transitional Dynamics — Other Simulations

Other Income Support Plans

- Transfer to lower 25% of the wealth distribution
- General (non-targeted) Support Plan Other Support

Temporary Shock

- MIT shock[8]: unanticipated temporary shock to infectivity MIT Shock

Weaker Disease

- The latter variants of Omicron is weaker that it induces smaller drop of productivity
- Simulate the dynamics but with $z(\mathcal{I}) = 0.6$ Weaker

Long Covid

- Assume productivity does not fully recover after infection
- $z(\mathcal{R}) = 0.8$ Long Covid

[8] Krugman and Blanchard pioneered these shocks when graduate students at MIT

Concluding Remarks

We extend the representative-agent epidemiological economic model to a heterogeneous-agent framework.

Key Conclusions

- The policy functions for prevention and treatment expenditure are increasing & more elastic with higher wealth
- In the stationary equilibrium, infection rate for the poor individuals is higher
- Income and wealth equality is worsened during the pandemic
- Income support for infection improves equality
- But unconstrained support discourages production

Discussion

- Generate increase in income inequality based on optimal policy functions on response to infectious diseases.
- The mechanism is different from Hall and Jones (2007) that focuses on mortality.
- We abstract from other mechanisms that can also increase inequality:
  - unemployment; sectorial heterogeneity (Chetty et.al 2022); remote working and digital devices (Stantcheva, 2022); drop in capital/wealth return (Gupta et.al, 2022; Kartashova and Zhou, 2021), *etc.*
- The dynamics in our paper is driven by the disease and optimal policies

===============*Thanks* ===================

Income inequality

Empirical evidence on inequality after Covid-19

- Observation: Income Gini index increased during the pandemic.

Citation Countries	Method	Without policy response	With policy response (Overall effect)
Almeida et al. (2020) EU (27)	Simulating effect of policies	+3.6%	-0.7%
Aspachs et al. (2020) Spain	Evolution over time	+24.4% (0.560)	-23.21% (0.430)
Brunori et al. (2020) Italy	Simulating effect of policies	+0.67% (0.3396)	-0.67% (0.3396)
Clark et al. (2020) DE, ES, FR, IT, SE	Evolution over time	+2.17% (0.322)	-2.48% (0.322)
Li et al. (2020) Australia	Comparison market and post-tax and transfers income	+3.33% (0.539)	-7.57% (0.330)
O'Donoghue et al. (2020) Ireland	Comparison market and post-tax and transfers income	+20.64% (0.499)	-6.62% (0.317)
Palomino et al. (2020) EU (29)	Simulating effect of policies	+3.5% to +7.3%	NA

[9]

back

[9] Table summarized by Stantcheva (2022)

Epi-Econ Model

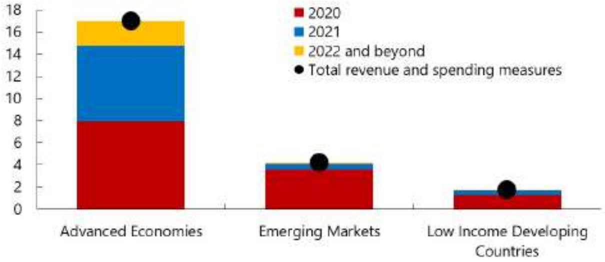
Introducing Heterogeneity

- Age heterogeneity (standard in epidemiology literature):
  - 2 groups: Acemoglu, et al. (2021).
  - Fabbri, Gozzi, and Zanco (2021) more general approach.
- Heterogeneity in contact in industries
  - Andersen, et al. (2020), Pichler, et al. (2020), Haw, et al. (2021).
- Wealth heterogeneity
  - Greg Kaplan and Moll (2020): Lock down policy experiment - exogenous disease and policies.
  - Angelopoulos et.al (2021): Non-compartmental model.

Income Support

Diminishing fiscal support in EMDEs in response to COVID-19

(percent of 2020 GDP)



Sources: IMF Fiscal Monitor database of Country Fiscal Responses to COVID-19 and IMF staff calculations.
Note: Includes revenue and spending measures.

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Income and Mobility

$$Mob_{i,t} = \theta_i + \eta_t + \sum_{\tau \neq \text{Feb2020}} \beta^{(\tau)} \log(I_i) \times T_t^{(\tau)} + \varepsilon_{i,t} \quad (11)$$

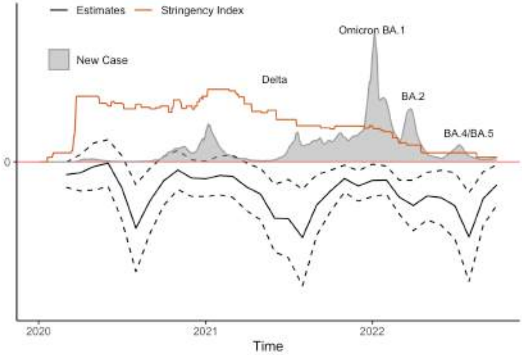

Figure 7: Income and Mobility

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More on Health Expenditure

- Health expenditure: opportunity cost
- General cost/price for health
  - **Preventive**: any consumption reduction action for reducing future infection risk
    - ★ Precautionary expenditure for health/productivity risk
    - ★ e.g. self-isolation, facial mask, PCR test etc.
  - **Treatment**: expenditure increases recuperation rate and which reduces consumption
    - ★ Ex-post expenditure for better and faster health/productivity recovery
    - ★ e.g. supplement, medicine, better source of nourishment, better living condition etc.

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More on Health Expenditure

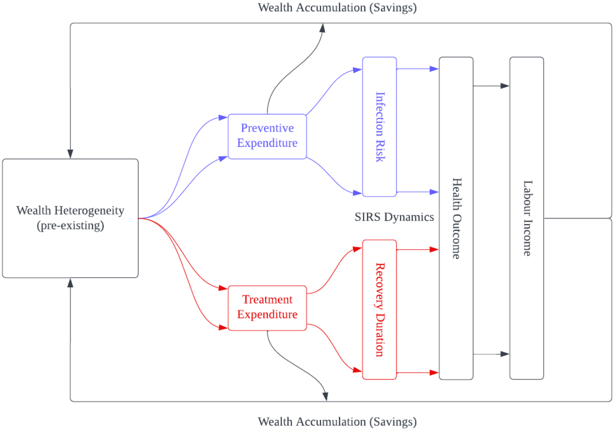
Other ways to endogenize contact rate

- (Eichenbaum et.al, 2021 RFS) consumption-based contact rate
  - Higher consumption $\rightarrow$ Higher contact/infection rate
- (Glover et.al, 2023 JMonE) mitigation policy of luxury worker: those instructed not to work
  - More luxury worker $\rightarrow$ lower contact and production
- Lockdown (Acemoglu et.al, 2021 AER: Insights, Goenka et.al, 2023 ET)
  - Lower contact rate with lower labour participation
- **Key**: trade-off between health and consumption
  - $\alpha(C_t)$
  - $\alpha(L_t)$: $C_t = w_t L_t$
- Our model: health outcome $\leftrightarrow^m$ consumption

back

back Model

Model Overview



back model begin back model end

HACT — HJB

- Let $v(a,h)$ to be the value function. The HJB for the individual problem reads

$$\begin{aligned}\rho v(a,h) = \max_{c,m_{\mathcal{P}},m_{\mathcal{T}}} \quad & u(c) - \chi \mathbb{1}_{(h=\mathcal{I})} \\ & + \partial_a v(a,h)[wz(h) + ra - c - m_{\mathcal{P}} - m_{\mathcal{T}}] \\ & + \Lambda^{h'}(m_{\mathcal{P}}, m_{\mathcal{T}}, h)[v(a,h') - v(a,h)] \\ & + \partial_t v(a,h)\end{aligned} \tag{12}$$

- $\Lambda^{h'}(m_{\mathcal{P}}, m_{\mathcal{T}}, h)$ is the probability of transiting to other health status
  - Poisson intensity defined before.
  - i.e. Infection probability; recovery probability; reinfection probability
- First Order Conditions F.O.C.

HACT — KF

- The associated Kolmogorov Forward Equation reads

$$\frac{\partial g(a,h)}{\partial t} = -\frac{\partial}{\partial a}[s(a,h)g(a,h)] - \Lambda^{h'}(m_{\mathcal{P}}, m_{\mathcal{T}}, h)g(a,h) + \Lambda^{h}(m_{\mathcal{P}}, m_{\mathcal{T}}, h'')g(a,h'') \tag{13}$$

- $s(a,h)$ is the saving $s(a,h) = wz(h) + ra - m_{\mathcal{P}} - m_{\mathcal{T}}$

population change $=$population change due to wealth change

$-$ population flows out to the next health status

$+$ population flows in from the previous health status

(14)

HACT — MCC

Market Clearing conditions: Aggregate Demand = Aggregate Supply

- (Assets Market)

$$K = \int a g(a,h) d\mu \tag{15}$$

- (Labour Market)

$$L = \int z(h) g(a,h) d\mu \tag{16}$$

- (Infectious Contact Rate Perception)

$$\zeta = \int \alpha(m^*_{\mathcal{P}}) \mathbb{1}(h = \mathcal{I}) g(a,h) d\mu \tag{17}$$

back

HACT — FDM

- The model could be represented in matrix form
  - (HJB) $\rho \mathbf{V} = u(\mathbf{V}) + \mathcal{A}\mathbf{V} + \partial_t \mathbf{V}$
  - (KF) $\partial_t \mathbf{g} = \mathcal{A}^* \mathbf{g}$
  - (MCC) $\mathcal{F}(\mathbf{g}) = 0$
- Stochastic partial differentiation functions
- Finite Differencing Method (Achdou et.al, 2020) to solve the model [10]
  - FDM presents a unique viscosity solution to PDEs if there is no convex kink

[10] Deep learning neural network could also be applied to solve MFGs (Fernandez-Villaverde and Nuno, 2023)

F.O.C. for HJB

The F.O.C. reads

$$\begin{aligned} c: \quad & u'(c) - \partial_a v = 0 \\ m_{\mathcal{P}}: \quad & -\partial_a v + \frac{\partial \Lambda^{h'}(m_{\mathcal{P}}, m_{\mathcal{T}}, h)}{\partial m_{\mathcal{P}}}[v(a, h') - v(a, h)] = 0 \\ m_{\mathcal{T}}: \quad & -\partial_a v + \frac{\partial \Lambda^{h'}(m_{\mathcal{P}}, m_{\mathcal{T}}, h)}{\partial m_{\mathcal{T}}}[v(a, h') - v(a, h)] = 0 \end{aligned} \tag{18}$$

The first F.O.C. yields $c^* = u'^{-1}(\partial_a v)$.

F.O.C. for HJB

For the second F.O.C., notice that the transition probability Λ is a function of health expenditure m only for the susceptible group $\mathcal{S}$. For the rest of health group, health expenditure will not have impact on their transition probability. Therefore, we have

$$\frac{\partial \Lambda^{h'}(m_{\mathcal{P}}, m_{\mathcal{T}}, h)}{\partial m_{\mathcal{P}}} = 0 \quad \text{for} \quad h \neq \mathcal{S} \tag{19}$$

Hence, for the group $h \neq \mathcal{S}$, we have the optimal health policy

$$m^*_{\mathcal{P}}(a, \mathcal{I}) = m^*_{\mathcal{P}}(a, \mathcal{R}) = 0 \tag{20}$$

For the susceptible group, we have

$$-\partial_a v(a, \mathcal{S}) + \frac{\partial \Lambda^{\mathcal{I}}(m_{\mathcal{P}}, m_{\mathcal{T}}, \mathcal{S})}{\partial m_{\mathcal{P}}}[v(a, \mathcal{I}) - v(a, \mathcal{S})] = 0 \tag{21}$$

F.O.C. for HJB

Recall that the infection probability is assumed as $\Lambda^{\mathcal{I}} = \alpha(m_{\mathcal{T}})\zeta$. So, we have

$$-\partial_a v(a,\mathcal{S}) + \alpha'(m_{\mathcal{P}})\zeta[v(a,\mathcal{I}) - v(a,\mathcal{S})] = 0 \tag{22}$$

which implies the optimal health policy

$$m^*_{\mathcal{P}}(a,\mathcal{S}) = \alpha'^{-1}\left(\frac{\partial_a v(a,\mathcal{S})}{\zeta[v(a,\mathcal{I}) - v(a,\mathcal{S})]}\right) \tag{23}$$

Similarly, for the choice variable $m_{\mathcal{T}}$, we have

$$m^*_{\mathcal{T}}(a,\mathcal{I}) = \gamma'^{-1}\left(\frac{\partial_a v(a,\mathcal{I})}{v(a,\mathcal{R}) - v(a,\mathcal{I})}\right) \tag{24}$$

F.O.C. for HJB

Hence, we have the optimal HJB written as

$$\begin{aligned}\rho v(a,h) = u(c^*) + \partial_a v(a,h)[wz^h(h) + ra - c^* - m_{\mathcal{P}}^* - m_{\mathcal{T}}^*] \\ + \Lambda^{h'}(m_{\mathcal{P}}^*, m_{\mathcal{T}}^*, h)[v(a,h') - v(a,h)] + \partial_t v(a,h)\end{aligned} \tag{25}$$

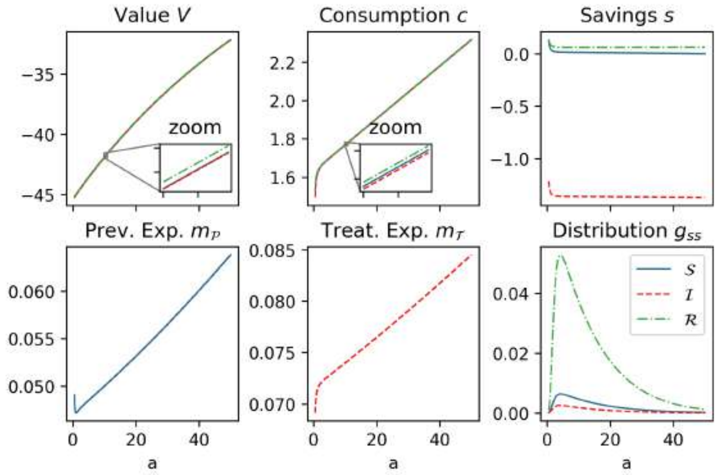
where

$$c^* = u'^{-1}(\partial_a v(a,h,g)) \tag{26}$$

$$m_{\mathcal{P}}^* = \begin{cases} 0; \quad h = \{\mathcal{I}, \mathcal{R}\} \\ \alpha'^{-1}\left(\frac{\partial_a v(a,\mathcal{S})}{\zeta[v(a,\mathcal{I}) - v(a,\mathcal{S})]}\right); \quad h = \mathcal{S} \end{cases} \tag{27}$$

$$m_{\mathcal{T}}^* = \begin{cases} 0; \quad h = \{\mathcal{S}, \mathcal{R}\} \\ \gamma'^{-1}\left(\frac{\partial_a v(a,\mathcal{I})}{v(a,\mathcal{R}) - v(a,\mathcal{I})}\right); \quad h = \mathcal{I} \end{cases} \tag{28}$$

Stationary Equilibrium — Consumption



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Stationary Equilibrium — Dsitribution

Wealth distribution v.s. Income distribution

- Wealth a: state variable
- Income $y = ra + wz(h)$: depends on both wealth and health status
- Income group against state variables.
- Grouped by percentiles: (Low) 25% (Mid) 75% (High)

(a) Low Income Group

	$\mathcal{I}$	$\mathcal{S}$	$\mathcal{R}$
Low a	0.042	0.089	0.748
Mid a	0.081	0.0	0.0
High a	0.04	0.0	0.0

(b) Middle Income Group

	$\mathcal{I}$	$\mathcal{S}$	$\mathcal{R}$
Low a	0.0	0.007	0.056
Mid a	0.0	0.104	0.833
High a	b	0.0	0.0

(c) High Income Group

	$\mathcal{I}$	$\mathcal{S}$	$\mathcal{R}$
Low a	0.0	0.0	0.0
Mid a	0.0	0.005	0.039
High a	b	0.117	0.839

Notes: 0<b<1e-5

Comparative Study — Health Policy

- Income elasticity of health expenditure
  - Wealth and health status (a, h) for individual is stochastic
  - Take future health expenditure into consideration when calculating elasticity
- Expected health expenditure over a certain period from 0 to τ.

$$M_k(a_0, h_0) = \mathbb{E}\left[\int_0^\tau m_k(a_t, h_t)dt \middle| a_0, h_0\right] \quad k \in \{\mathcal{P}, \mathcal{T}\} \tag{29}$$

- Income Elasticity of Health Expenditure is defined as

$$\begin{aligned} \varepsilon_{M_k,y} &= \frac{\partial M_k(a_0, h_0)}{\partial y}\frac{y}{M_k} \\ &= \frac{M_k(a_0+\Delta, h_0) - M_k(a_0, h_0)}{\Delta}\frac{a_0}{M_k(a_0, h_0)} \\ k &= \{\mathcal{P}, \mathcal{T}\} \end{aligned} \tag{30}$$

- Obtained by the **Feynman-Kac Formula**

Comparative Study — Health Policy

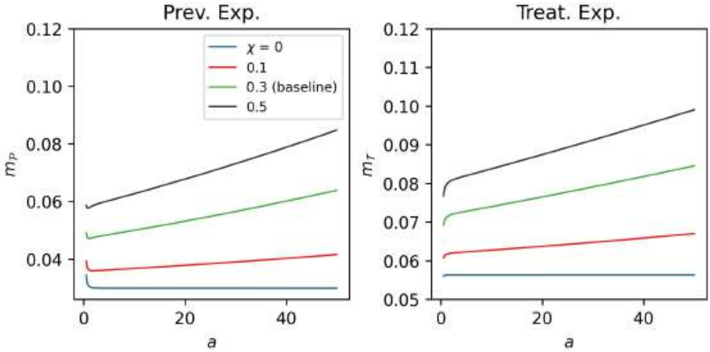

Figure 8: Health Policy: Varying Disutility χ

- Income Elasticity of Health Expenditure is also increasing with wealth
  Elasticity

Comparative Study

Table 2: Comparative Study

(a) Aggregate Variables

χ	0	**0.3**	0.5
Infection Rate			
Aggregate	4.344	4.107	3.97
Bottom 25%	4.444	4.235	4.115
Top 25%	4.307	4.023	3.862
diff.	-0.137	-0.212	-0.253
Capital	14.418	14.447	14.463
Prices			
Wage Rate	1.694	1.694	1.694
Interest Rate	0.014	0.014	0.014
Inequality			
Wealth Gini	0.41	0.412	0.423
Income Gini	0.072	0.071	0.07
Wealth Share			
Bottom 25%	6.72	6.67	6.49
Top 25%	52.89	53.03	54.08
diff	46.18	46.36	47.59

(b) Contol Variables

χ	0	**0.3**	0.5
Consumption			
Aggregate	1.83	1.829	1.827
Bottom 25%	1.682	1.681	1.678
Top 25%	2.052	2.053	2.059
diff.	0.37	0.372	0.38
Preventive Exp.			
Aggregate	0.03	0.052	0.066
Bottom 25%	0.03	0.048	0.06
Top 25%	0.03	0.057	0.074
diff.	-0.0	0.009	0.015
Treatment Exp.			
Aggregate	0.056	0.075	0.085
Bottom 25%	0.056	0.072	0.081
Top 25%	0.056	0.079	0.092
diff.	0.0	0.007	0.01

Comparative Study

Table 4: Comparative Study (cont.)

χ	0	**0.3**	0.5
Expected Income in 3-yr duration			
Bottom 25%	20.5	20.532	20.534
Top 25%	25.0	25.07	25.204
diff.	4.5	4.539	4.669
Labour Income diff.	0.0	0.013	0.02
Capital Income diff.	4.461	4.488	4.612
Expected Savings in 3-yr duration			
Bottom 25%	2.793	2.701	2.755
Top 25%	-1.029	-0.991	-0.916
diff.	-3.822	-3.692	-3.672

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Stationary Equilibrium — Health Policy

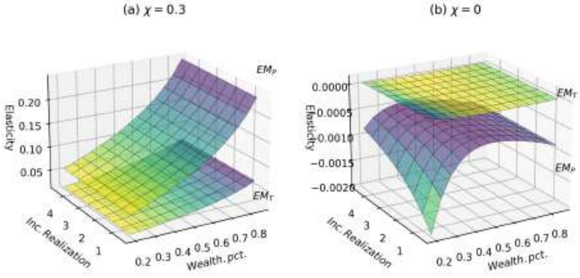

Figure 9: Elasticity of Health Expenditure

- χ: direct health punishment of being sick
- For baseline $\chi = 0.3$, elasticity for both types of expenditure are positive and higher at higher wealth percentiles.
- Under pure income shock $(\chi = 0)$, health expenditure is less elastic at higher wealth percentiles.

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Transitional Dynamics — Fitting the Data

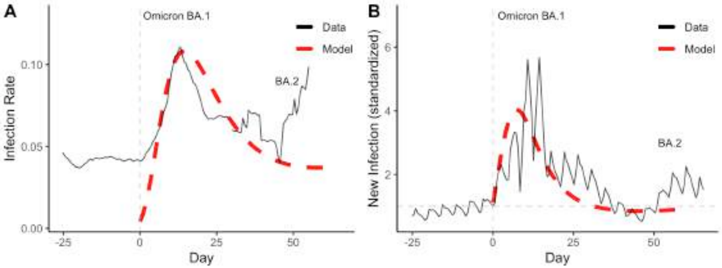

Figure 10: Simulation and Empirical Data

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Transitional Dynamics — Simulating Pandemic

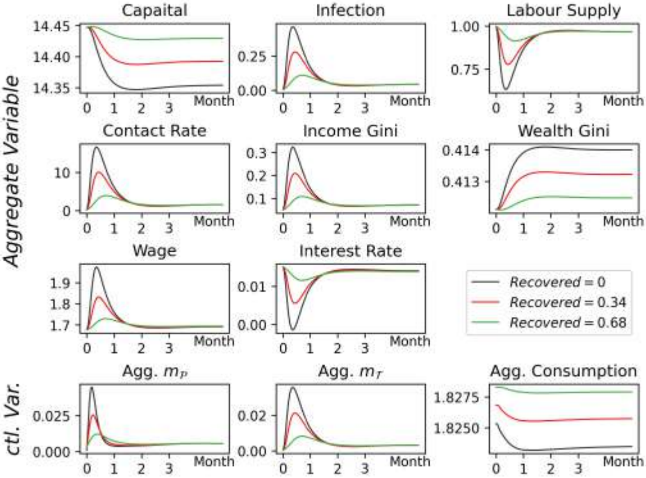

Figure 11: Transitional Path — Aggregate Variables

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Transitional Dynamics — Simulating Pandemic

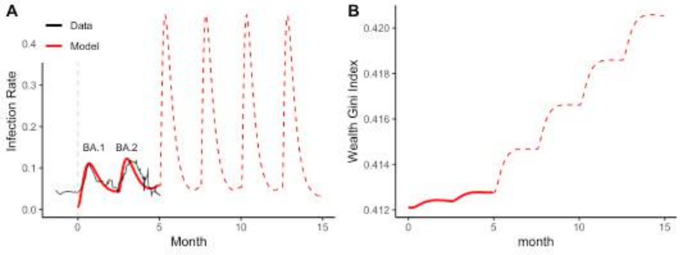

Figure 12: Disease Mutation

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Transitional Dynamics — Simulating Pandemic

There are more poor people after pandemic

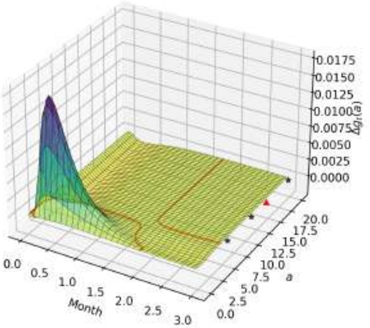


Figure 13: Change in Wealth Distribution

Transitional Dynamics — Health Policy

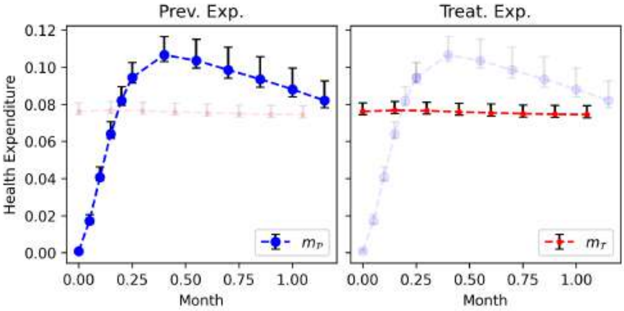
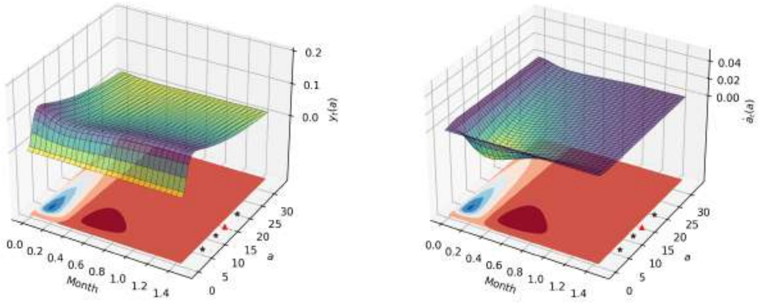


Figure 14: Change in Health Policy

- 25%, 50% and 90% of wealth distribution
- Preventive expenditure ↑ during pandemic
- Health expenditure biased towards the wealthier

Transitional Dynamics — Simulating Pandemic

Poor lose more income and save less



(a) Income Change

(b) Savings Change

Figure 15

Transitional Dynamics — Income Support (other Plans)

- Targeted support for the poor

$$ra + wz(h) + \tau \mathbb{1}(a \leq a_{25\%}) \tag{31}$$

- General (Non-targeted) support for everyone

$$ra + wz(h) + \tau \tag{32}$$

- Compare support plans, holding the binding fiscal constraint

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Transitional Dynamics — Income Support (other Plans)

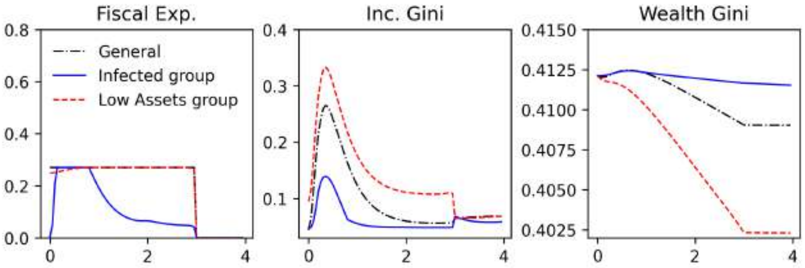

Figure 16: Other Income Support

Transition Dynamics — Other Simulation

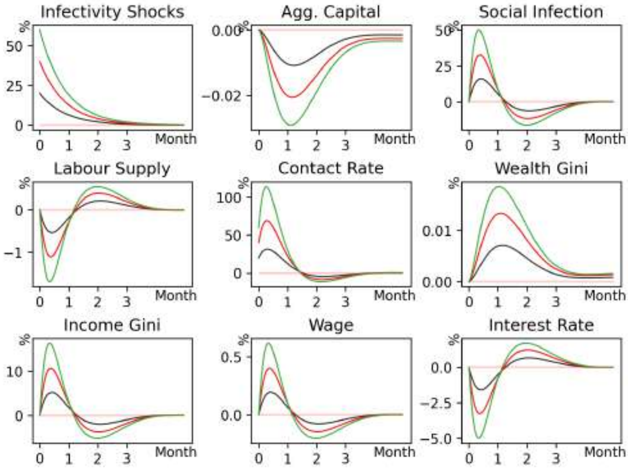

Figure 17: MIT Shock — Infectivity

Transition Dynamics — Other Simulation

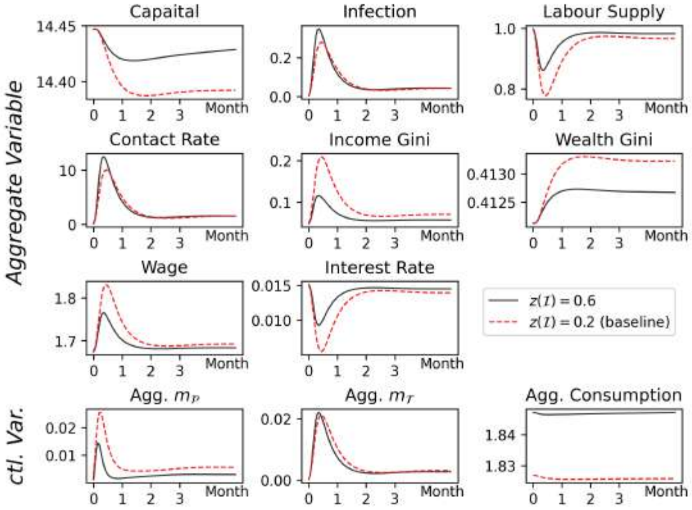

Figure 18: Transitional Dynamics — Lower Productivity Loss

Transition Dynamics — Other Simulation

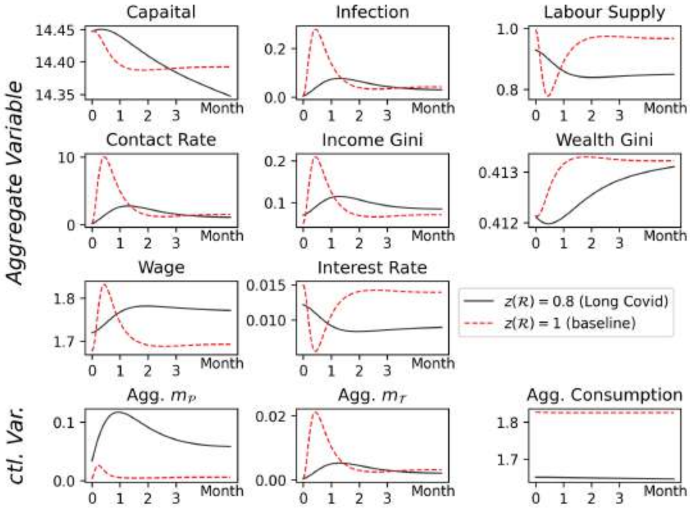

Figure 19: Transitional Dynamics — Long Covid

Parameterization

- It is difficult to calibrate the model as the model abstracts too many channels.
- So we can't use the aggregate variables (e.g. aggregate infection rate) to calibrate the epidemiological part of the model.
- Economic side of the model is standard; the epidemiological side of the model needs to be parameterized using clinical evidence (e.g. average duration to recover or get infected)

Parameterization

Economic Part

- Parameters
  - CRRA utility function: σ; disutility level χ
  - Individual subjective discount rate: ρ
  - Competitive market: TFP A; capital share β; capital depretiation rate δ
- These parameters are standard
  - $\sigma = 2$; $\rho = 0.0138$
  - $A = 1$; $\beta = 0.36$; $\delta = 0.05$
- Parameter $\chi = 0.3$ in the baseline. It would be varied in the comparative study.

Parameterization

Epidemiology Part

- Parameters

$$
\begin{aligned}
\alpha(m_{\mathcal{P}}) &= \epsilon_0(m_{\mathcal{P}} + \epsilon_2)^{\epsilon_1} \\
\gamma(m_{\mathcal{T}}) &= \gamma_U - \eta_0(m_{\mathcal{T}} + \eta_2)^{\eta_1} \\
&\epsilon_1, \eta_1 < 0 \\
&\text{Rinfection: } \psi
\end{aligned} \tag{33}
$$

- Notice $\alpha(0) = \epsilon_0\epsilon_2^{\epsilon_1}$; $\gamma(0) = \gamma_U - \eta_0\eta_2^{\eta_1}$; $\gamma(\infty) = \gamma_U$
- We can also find that $\lim_{m_{\mathcal{P}}\to\infty} \frac{\partial\alpha(m_{\mathcal{P}})}{\partial m_{\mathcal{P}}} \times \frac{m_{\mathcal{P}}}{\alpha(m_{\mathcal{P}})} = \epsilon_1$, which is the maximum elasticity
- In the baseline, we let
    - unit elasticity $\epsilon_1 = \eta_1 = -1$
    - $\epsilon_2 = \eta_2 = 0.005$
    - $\epsilon_0 = 0.18$ so that $\alpha(0) = 36$ (2.5 days of generated duration)
    - $\eta_0 = 0.034$ such that the recovery duration is bounded between 7 and 15 days.
- $\psi = 5/3$ (150 days of generated duration)

Parameterization

Epidemiology Part

- We can roughly calculate the basic reproduction number R_0 at the stationary equilibrium
  - Next few slides introduce how R_0 and R_e is obtained at our heterogeneous agent framework.
  - $R_0^{(SS)} = 9.714$
- Liu and Rocklöv (2022) summarize estimated R_0 of Omicron variants in the recent studies. The Omicron variant has an average basic reproduction number of 9.5 and a range from 5.5 to 24

Parameterization

	Model		Data
	Mean	Median	
Basic Rep. Num. R_0	9.236		9.5 ave., range 5.5-24
Days to Infection	19.183	18.833	-
Days to Recover	7.241	7.244	around 7 to 15
Days to Lose Immunity	150		around 90 to 240
Fraction $\mathcal{S}$	10.8%		-
Fraction $\mathcal{I}$	4.1%		2%-5% after 2023 (UK)
Fraction $\mathcal{R}$	85.1%		77%-80% Feb 2023 (UK)

Notes: (a) Data source: R_0 Liu and Rocklöv (2022) etc.; Days to recover UK Health Security Agency (2023); Days to lose immunity Cagigi et al. (2021); Gilboa et al. (2022) etc.; UK data ONS (2023a). (b) The data of recovery population is proxied by fraction of population with antibody more than 800 ng/ml

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Reproduction Number

- Basic reproduction number R_0 is defined as the average number of secondary infections that occur when one infective is introduced into a completely susceptible host population
- The replacement number R (Effective reproduction number) is defined to be the average number of secondary infections produced by a typical infective during the entire period of infectiousness

In a simple epidemiological model with SIRS dynamics, the motion of infection rate can be written as

$$\dot{i} = \alpha is - \gamma i \tag{34}$$

where α and γ is the contact and recovery rate.

- This expression is governed by $\alpha s - \gamma = \frac{\alpha s}{\gamma} - 1$. This ratio $\frac{\alpha s_0}{\gamma} = \frac{\alpha}{\gamma}$ is defined as R_0
- Time varying ratio $\frac{\alpha s_t}{\gamma}$ is the effective reproduction number R
- Here we have $R_t = R_0 s_t$

Reproduction Number

- Using a similar way, we can define the effective reproduction number
- By Kolmogorov Forward Equation, the net flow of infectious group is

$$\begin{aligned}\dot{i} &= \int \alpha(m_{\mathcal{P}})\zeta g(a,\mathcal{S})da - \int \gamma(m_{\mathcal{T}})g(a,\mathcal{I})da \\ &= \frac{\int \alpha(m_{\mathcal{P}})\zeta g(a,\mathcal{S})da}{\int \gamma(m_{\mathcal{T}})g(a,\mathcal{I})da} - 1\end{aligned} \tag{35}$$

- We can similarly define the effective reproduction number as the first term $R_t = \frac{\int \alpha(m_{\mathcal{P}})\zeta g(a,\mathcal{S})da}{\int \gamma(m_{\mathcal{T}})g(a,\mathcal{I})da}$. $R_t > 1$ implies the aggregate infection rate would increase
- We can obtain $R_0 = \frac{R_t}{s_t} = \frac{\int \alpha(m_{\mathcal{P}})\zeta g(a,\mathcal{S})da}{\int \gamma(m_{\mathcal{T}})g(a,\mathcal{I})da \int \mathbb{1}(h=\mathcal{S})g(a,h)d\mu}$