Heterogeneous Banks, Liquidity Risk and the Distribution of Banks' Liquidity*

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Abstract

This paper studies banks' size expansion, sorting of banks by type and how banks make decisions by internalising ex-ante and ex-post liquidity regulations. I first propose a theoretical model of heterogeneous banks in an incomplete market economy in which I design liquidity regulations that account for uninsurable deposit withdrawal shocks. The characterisation results show that ex-ante liquidity regulations allocate liquidity to large banks and lead to banks' size expansion through liquidity that originates from the inflow of deposits. The necessary conditions for the expansion in banks' size are the withdrawal effect on deposits that has to be internalised by banks and the measurability of the expanded set. I then show how the sorting of banks by type arises. Since the distribution of banks is part of the equilibrium, I find that 2.2 percent of banks are constrained at the medium liquidity constraint. I study the effect of liquidity regulations on liquidity crises. Ex-ante liquidity regulations mitigate the severity of liquidity crises by reducing the contraction in banks' liquidity to deposits by 24.03 percent less relative to ex-post liquidity regulations.

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The theoretical economy supports ex-ante liquidity regulations for banks to withstand a sudden outflow of deposits and counteract liquidity crises.
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1 Introduction

Banks' liquidity is the main instrument for banks to prevent maturity mismatch and counteract the liquidity risk. Stricter liquidity requirements on banks' balance sheet were imposed subsequent the 2007-08 financial crisis.[1] This event had in some circumstances the aim to affect the distribution of banks in the system. In particular, following the global financial crisis the holding of liquid assets in banks' balance sheet increased significantly and banks' liquidity became widely distributed across the banking sector.[2] Using micro level banks data, I find that concentration in banks' liquid assets in the US decreased by approximately 6.8% in 2010 post financial crisis.

I document novel facts that are important to understand banks' liquid assets choices. First, the bottom percentiles of the liquid assets distribution of large and small banks experienced about 2.9% rise in liquid assets by 2010 whilst the upper tail of the distribution of large banks experienced about 3.5% decay in liquid assets, more than two times larger than the fall in the top percentiles of small banks' liquid assets. Second, tighter liquidity requirements contributed to the rise in banks' liquid assets in the bottom percentiles of the distribution of all banks following the financial crisis.

In this paper I examine the determinants of banks' liquidity allocations and endogenous banks' size expansion induced by liquidity regulations by addressing the question of whether the timing of the imposition of the liquidity regulation becomes relevant for the efficient allocation of banks' liquidity in the market. The regulation policy is the liquidity requirement imposed in a specific time, prior or post the occurrence of the disturbance, as a function of the inflow or outflow of deposits. I then study the timing of the intervention of the liquidity regulation on banking crises induced by deposit outflows and the effect of this type of liquidity regulation on banks' incentive.

The reliance of banks on short term debt makes them exposed to a short term liquidity problem. I place emphasis on the design and implementation of liquidity regulations that command a remunerated buffer of liquidity as function of deposits. With small and medium banks having a less diversified liability structure, the absence of any liquidity regulation makes them unable to internalise the effect of a shortfall of deposits and leave them exposed to a high interest rate cost on deposits which reduces their ability to accumulate liquidity and expand in size.

I think to the banking crisis, arising because of the withdrawal of deposits, as a bank run

[1]The liquidity requirement I analyse is the liquidity as function of deposits that is imposed by the Fed on commercial banks balance sheet. In the US, Dodd-Frank Act required the largest banking institutions to hold 9.5% of their assets in liquid assets such as cash, government bonds and similar assets with low risk profile, however I focus on the former liquidity regulation. Appendix K provides more details on the liquidity regulation inspected in the model.

[2]Ennis and Wolman (2015) provide evidence on the distribution of bank reserves and its relation with other components of bank balance sheet using a sample for insured and uninsured banks. They find that reserves did not substitute other form of liquid assets and did not constrain banks to expand the size of their balance sheet. They document a high concentration of reserves at the end of 2008 with the Top 10 banks holding 50% of reserves and the Top 100 banks holding 80% of reserves, approximately. On the other hand, banks located at the bottom 20% of assets had less than 10% of reserves.

that needs not be connected to banks' balance sheet fundamentals, such as a loss on earning outcomes.[3] In this regard, my model generates a liquidity crisis not only because of the withdrawal of deposits but also, because of insufficient cashflows able to cover period operating costs. Therefore, there is no physical bank run similarly to Holmström and Tirole (1998). The outflow of deposits generates losses that however do not render banks insolvent. Furthermore, by analysing contracts that prevent runs and by focussing on short term liquidity I share with Diamond and Dybvig (1983) the feature of isolating long term assets from the banking programme. However, a distinctive feature of my model relative to them is that the optimal liquidity contract does not involve risk sharing and bank runs do not materialise, because of the upfront liquidity buffer by banks as self-insurance device. Therefore, I study the effects of liquidity regulation generated by liquidity frictions. Moreover, in contrast to Diamond and Rajan (2001) in my model there is a shortage of banks' liquidity due to incomplete markets, as a result this creates a role and a reason for a liquidity buffer.

The paper highlights the importance of banks' liquidity as endogenous factor able to engender a stable contract and liquidity regulations in tempering liquidity crisis in the deposit market. Consequently, my paper focusses on banks' prudence interpreted as the ability of banks in building a large buffer of liquidity in anticipation to a crisis and holding long term assets that are not subject to early redemption. In the wake of the recent turmoils of Silicon Valley Bank and liquidity requirements as function of deposits set to null, the risk to financial stability driven by a profit deterioration due to a drop in lending volumes or a withdrawal of deposits, require bankers adopting a prudent bank liquidity management. Therefore, my paper inspects how the liquidity buffer induced by the ex-ante intervention of liquidity regulations can be a viable instrument for perfectly solvent but illiquid banks in counteracting a liquidity crisis. In this regard, my model provides the ground for banks led by rational bankers able to meet any deposit withdrawal through the accumulation of liquid assets without the need of having to sell their assets.

I confront the question of the impact of heightened liquidity constraints by showing that incomplete markets in the banking sector reduces the severity of liquidity crises. Since banks care about the economic consequences of deposits withdrawals, they accumulate liquidity to provide it when needed. Therefore, the policy implication I draw is that regulators can maintain stability in the market by raising liquidity requirements giving incentives towards prudent banks' liquidity management. Furthermore, the same regulation can reduce inequality in banks' liquid assets thereby contributing to financial stability. The aftermath comes because liquidity concentrates towards small and medium banks who are more reliant on deposits and vulnerable to a crisis. The main methodological contribution of the paper is the characterisation of the properties of the endogenous banks' size expansion, banks' sorting by type and their inequality along the liquidity dimension in the size distribution of banks in addition to the assessment of losses of

[3] The liquidity crisis is thought as an idiosyncratic event instead of a crisis triggered by an aggregate shock. Therefore, in this paper I abstract from aggregate uncertainty.

other types of liquidity regulations from ex-ante intervention of liquidity regulations.

Specifically, I propose a model that provides a connection between the concentration of banks' liquidity in banks' distribution, inequality in this banks' asset and liquidity regulations to counteract liquidity crises. I then study the impact of ex-ante and ex-post liquidity regulations, as well as the absence of liquidity regulations on the distribution of banks' liquidity, the sorting of banks by type, the accumulation of banks' liquid assets and endogenous banks' size expansion. I achieve this result because the liquidity regulation needs to be conceived as function of the change in deposits. The four key assumptions of the model are *i)* incomplete markets, *ii)* idiosyncratic withdrawal risk on deposits, *iii)* assets characterised as liquid and illiquid assets and *iv)* a bank size dependent liquidity constraint in the level of deposits that gives incentives to banks to insure by building a buffer of liquidity. Liquid assets are identified as cash, treasury securities and trading assets held by banks yielding a low return, and illiquid assets by loans having a commensurately higher return. By accumulating liquid assets banks insure against the liquidity risk. It is by virtue of the sorting of banks by type and the liquidity constraint that the distribution of banks' size is dependent on the distribution of banks' liquid assets.

I conduct three experiments. First, I examine whether the behavior of banks' liquidity held by large banks is different from banks' liquidity owned by small banks. Second, I study a tightening of the liquidity constraint on banks' liquid assets inequality. Third, I study the sorting of banks by type and its relationship with banks' size expansion and deposits. In the first experiment, I find that a sizable fraction of small banks acts at the constraint. Specifically 2.2% of banks are constrained until the medium liquidity constraint. This is in line with the data that shows a fraction 1.8% of banks acting at the medium liquidity constraint. In the second experiment, I find that a tightening of the liquidity constraint makes banks' liquidity more equally distributed. In the third experiment, I find that the larger the banks, the lower their deposits and the larger their liquidity and size expansion. However, while for small banks it holds that the smallest banks can expand in size, for large banks it occurs that the least large banks expand in size, posing endogenously an upper bound on the expansion of banks' size. Ex-ante liquidity regulations lead large banks to accumulate liquidity more than small banks, by contrast, ex-post liquidity regulations reallocate liquidity towards small and medium banks.

The second part of this paper studies liquidity crises due to heightened deposit withdrawal risk. During a crisis, deposit contracts leading banks to lessen the accumulation of liquidity. The lower the banks' precautionary behavior, the larger the fall in banks' liquidity and the larger the contraction in banks' balance sheet size and dividends. In this part my main finding is that the intervention of ex-ante liquidity regulations prior to the crisis reduces the contraction in banks' liquidity to deposits and dividends by 24.03% and 0.579%, respectively, whereas banks' profits fall 23.41% less relative to an economy in which the regulator intervenes after the occurrence of the shock and significantly reduce the probability and episodes of liquidity crises by more than threefold relative to an economy without liquidity regulations.

The contribution of this paper is sixfold. First, it provides a theoretical model that explains the creation of a stable contract through incomplete markets in which bank heterogeneity is encapsulated in the amount of deposits that banks own. I demonstrate necessary conditions for the existence of the minimum liquidity requirement and the characterisation of an illiquidity phase in banks' balance sheets. The liquidity regulation embedded through a liquidity requirement represents the fraction of deposits banks must hold in liquid assets against any sudden decline in liquidity. I designate this liquidity regulation in terms of change in deposit thresholds instead of a liquidity ratio, since it becomes necessary to have a direct effect on the total cost on short term debt thereby affecting banks' liquidity incentive. While this liquidity requirement is banks' size dependent, the minimum liquidity requirement does not vary across banks as it ensures a minimum liquidity is met by all banks thereby limiting self-insurance.

The second contribution of the paper is to examine how disparities across banks' liability structure and bank size affect differences in banks' decisions of liquid asset choices. The third key contribution of the paper is that the model highlights the importance of ex-ante liquidity regulations in shaping the distribution of banks' liquidity and tempering the inequality along banks' liquid assets. I analyse the impact of the liquidity regulation on banks' decision rules and show that the value of banks rises with the size of deposits. I theoretically characterise the expansion of banks' size through liquidity arising because of the inflow of deposits. In addition, I demonstrate that a stronger increase in the liquidity requirement reduces the inequality along banks' liquid assets.

Fourth, I analyse banks' decision rules as a function of the interest rate on liquidity. I demonstrate the nonmonotonic behavior of banks' dividends consumption and banks' liquidity which become imperfect substitute in a neighborhood of the stationary liquidity rate. Fifth, I study the relationship between the sorting of banks by type and the size of banks. The sixth contribution of the paper is to study liquidity crises that originate in the deposit market and that generates temporary dividends losses. I assume liquidity crises are rare events that can occur suddenly anytime; therefore they are almost disconnected from credit conditions (Gorton, 2014). I confront liquidity crises in the baseline economy with liquidity crises in a disaster economy by embedding a small probability of an illiquidity disaster state to occur. Therefore, I examine the interplay between liquid assets and deposits and the extent to which precautionary banks' behavior and this type of liquidity regulation temper liquidity crises arising in the deposit market.

In the model banks bear significant deposits uncertainty in their balance sheet liabilities. I focus on the role of the liquidity requirement in affecting total banks' assets where bank size is proxied with the level of total assets. The analysis puts at center stage the banking sector as financial intermediaries interconnected through deposit withdrawal risk in the market of liquid assets with precautionary liquidity motive.

The economy is subject to incomplete markets on the side of the banking sector in the same style of Bewley (1986), Huggett (1993) and Aiyagari (1994). Banks are subject to a liquidity constraint and exposed to deposit withdrawal shock against which they cannot insure. The financial friction in the model arises as impossibility for banks to reduce liquidity beyond the level imposed by the liquidity requirement. Short term debt is the only source of funding in this model, and banks hold equity through accumulated retaining earnings. In my model, all transactions require the use of liquidity, hence creditors need to meet all payment needs in cash and short term liquid assets. The liquidity constraint imposes a minimum level of liquid assets that banks must hold to cover a fraction of deposits. The idiosyncratic withdrawal shock, that occurs in the intraperiod, calls banks to undertake precautionary behavior in liquid assets and to smooth their resources over time. Therefore, banks may experience a withdrawal of deposits that affects their total level of assets and liabilities. The distinguishing feature of the model is the liquidity regulation in terms of change in deposit thresholds that becomes state and size dependent since the deposit withdrawal shock affects the liquidity requirement.[4]

I distinguish banks heterogeneous in the liquidity and size dimensions and within them I distinguish between inflow banks of deposit withdrawal and outflow banks bearing the deposit withdrawal. The model features an intraperiod distortion which distorts banks' intermediation profits and the incentive of banks in holding liquidity. This feature arises since only banks experiencing the outflow of deposits have to bear the payment of the interest rate wedge on the deposits rolled over. The central implication of this theory is that exposure to substantial deposit withdrawal and the inability to verify the individual deposit withdrawal, lead outflow banks to maturity mismatch and illiquidity phases that expose them to a liquidity problem. The riskiness in banks' balance sheets becomes profitable only for inflow banks who are willing to be exposed to more withdrawal risk. Therefore, they have incentives to expand the size of their balance sheets through liquid assets.

I discipline my model by reconciling features of the US banking sector. My model provides a right skewed distribution of banks' liquid assets and deposits. The central prediction is that the state dependency makes the distribution concentrated at the varying liquidity constraint, thereby amplifying responses to a deposit withdrawal shock. I show that the model reproduces well distributional moments of banks' liquid assets. Importantly, the 1st percentile of banks' liquid assets distribution, a non targeted moment in the model, holds 1.1% of liquid assets almost consistent with 1.7% in the data. The model delivers a Gini coefficient for banks' liquid assets of 10.5% smaller than 38.9% in the data. I show that the model nests Holmström and Tirole (1998) model where banks are subject to a liquidity regulation ex-post only. In this version of the model banks' liquidity reallocates towards small banks, thereby leading to less inequality than the one observed in the data.

[4] I design the state and size dependent liquidity constraint in accordance with the Federal Reserve Act that sets a liquidity requirement which varies across banks according to their amount of deposits. Following the 2019-2020 pandemic the liquidity requirement was set to zero across all types of banks heterogeneous in their deposits. I provide further details on the regulation in Appendix K.

Motivated by these results, I examine three equilibrium properties. First, an increase in the liquidity requirement that reflects an 11% rise in banks' reserve of the magnitude experienced in 2009q2 in banks' balance sheet. I find that banks widen their precautionary liquidity motive since they anticipate a possible withdrawal of deposits which makes the distribution of banks' liquidity more equally distributed, thereby reducing the Gini coefficient to 8.2%.

Second, I study the nonmonotonic behavior of banks' liquidity. Banks' dividends consumption and liquidity decisions rules can be substitute or complement depending on whether the liquidity constraint is binding or not. I demonstrate that when the liquidity constraint begins to bind strongly banks' liquidity changes sign of movement prior to which it becomes complement to banks' dividends consumption. This result hinges on outflow banks who become more exposed to the deposit withdrawal risk prior to its realisation than inflow banks. Since the outflow of deposits becomes costly for banks experiencing it, any increase in the interest rate on liquidity leads them to simultaneously reduce liquidity and dividends. My reinterpretation of this complementarity result is tied to the types of each heterogeneous bank and the state dependent liquidity constraint.

Third, in analysing the sorting of banks by type and the expansion in banks' size I find that in equilibrium large banks with small deposits accumulate more liquid assets than small banks. However, while for large banks the expansion in size arises because of their better initial conditions in owning liquidity, small banks can expand in size as consequence of a stationary shock to deposits which is the main driver for their expansion. The sorting of banks by type can be explained by the difficulties of small banks in obtaining other sources of funding relative to large banks, leading them to hold larger retained earning and deposits than large banks. Therefore, any equilibrium entails smaller liquidity for small banks, so that small banks will expand in size only because of an idiosyncratic change in deposits by which they will prefer to rise their liquidity.

In the evaluation of the transitional dynamics of liquidity crises to the individual deposit withdrawal shock I find that the idiosyncratic shock leads to a 4.66% decay in banks' liquidity since the withdrawal shock reduces banks' precautionary behavior which in turn worsen the other components of banks' balance sheet. As short term debt becomes more costly for banks, it motivates a stronger liquidity constraint. I conduct an event study analysis along banks' liquid assets distribution to further characterise the dynamics of the model and its nonlinearities. The model generates liquidity crises following a shock to the deposit withdrawal risk. My main finding is that ex-ante liquidity regulations lead banks to accumulate more liquidity, since banks can better hedge idiosyncratic outflows of deposit shocks the system becomes more resilient. As a result, the probability of a liquidity crisis decreases relative to an economy in which banks are less patient and relative to a non regulated economy, thereby fostering financial stability.

To reconcile a larger volatility of liquidity as observed in the data and to match the observed fall in inequality in banks' liquidity in 2010 post financial crisis, I introduce an illiquidity crisis disaster state. In particular, I consider a rare and unexpected illiquidity event scenario with a probability of illiquidity disaster to occur. The central prediction of this experiment is that the model gives rise to an amplification effect such that the illiquidity disaster state raises banks' precautionary behavior for all banks. It delivers a 4.22% reduction in the inequality of the distribution of banks' liquidity from the baseline model which takes place because the largest rise in liquidity is driven by small banks−banks located in the bottom percentiles of the banks' liquidity distribution. At the same time, I show that along a sequence of crisis intensity probabilities, as the crisis probability rises the prevailing equilibrium is one in which liquidity is low and the liquidity constraint is binding. This result arises because, as the occurrence of a crisis becomes more likely, banks need to decumulate liquid assets and pay dividends to cope with the hurdles of the crisis, despite the binding liquidity constraint.

I find that the disaster state mitigates responses to the deposit withdrawal shock since it enables banks to insure high enough in anticipation to the crisis. The illiquidity episode I examine suggests that banks overaccumulate liquidity strongly at the onset of the crisis which prevents a sizable fall in banks' dividends. This episode is associated with a steadily lower contraction in the liquidity to deposit ratio. Moreover, the illiquidity crisis with the disaster state underscores the ability of the model to generate tempered liquidity crises.

Outline. The rest of the paper is organised as follows. Section 2 discusses the related literature. Section 3 documents evidence on the distribution of banks' liquidity. Section 4 presents the heterogeneous banks model and theoretically analyses equilibrium properties and outcomes. Section 5 presents model results, analyses the nonlinear responses of liquidity crises with and without an illiquidity disaster state and conducts a sensitivity analysis. Section 6 concludes. The Appendix contains proofs of propositions, additional theoretical properties and additional model results and details.

2 Related Literature

My paper is related to the literature of the financial sector in macroeconomics. This literature however has focussed either on a homogeneous financial sector or on a heterogeneous financial sector that however aggregates to a representative bank.[5] I focus on heterogeneous banks.

It relates to the pathbreaking works of Diamond and Dybvig (1983) and Holmström and Tirole (1998) by giving a central role to liquidity. However, while in Holmström and Tirole (1998) the liquidity demand comes from firms who need to raise funds to finance an investment, in my model I focus on banks' liquidity incentives towards the implementation of an investment project. Specifically, in my model banks are subject to liquidity regulations ex-ante to the deposit outflow, as a result the deposit disturbance affects banks' liquidity incentive prior the occurrence

[5] Examples include Adrian and Shin (2010), Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Gertler, Kiyotaki and Queralto (2012), Adrian and Shin (2014), Brunnermeier and Sannikov (2014), Fernández-Villaverde, Hurtado and Nuño (2023), Gertler, Kiyotaki and Prestipino (2020).

of the disturbance. Moreover, I formally characterise liquidity frictions in an infinite dynamic environment through liquidity regulations which creates fractions of banks' networth that allow me to assess the losses of other types of liquidity regulations relative to the intervention of the ex-ante regulation along the size distribution of banks. Additionally, relative to them my paper studies the sorting of banks and banks' expansion through deposits that they do not analyse.

Furthermore, as the main role and reason of existence of banks in my model is to transform illiquid assets in short term liabilities, I share with Diamond and Dybvig (1983) illiquid assets as a completely fixed activity. However, despite to them, in my model the idiosyncratic nature of deposits engenders a stable contract because of banks' incentives towards liquidity and their needs to allocate networth through time across different states of nature without the requirement of suspension of convertibility of deposits. This occurs since banks care about the economic consequences of a liquidity crisis through the imposition of the liquidity requirement which cause them to self-insure. In contrast to Diamond and Dybvig (1983) and Holmström and Tirole (1998), my model offers an economy with dynamic stochastic deposits in infinite time in which the decision of banks to accumulate liquidity depends on a dynamic and bank size dependent liquidity constraint which generates endogenous banks' size expansion across banks.

By analysing liquidity regulations, my model connects with Farhi and Tirole (2009) and Di Tella (2017, 2019). However, while Farhi and Tirole (2009) analyses liquidity regulation on the aggregate amount of liquidity and the optimal liquidity allocation generated by the social planner, my paper studies liquidity regulations on each heterogeneous banks and losses from the liquidity requirement, consequently the model does not give rise to risk sharing across banks. Moreover, the explicit point of investigation of Di Tella (2017) is the propagation of balance sheet recessions on complete short term contracts driven by the stochastic volatility of uncertainty shocks. They show that the increase in idiosyncratic risk driven by uncertainty shocks leads productive agents to raise aggregate risk ex-ante to shield stochastic investments, which in turn weaken balance sheets. The mechanism arises because of a financial constraint that is endogenously generated by a moral hazard problem between agents and intermediaries. In my environment, the liquidity regulation imposed on banks ex-ante induces banks to accumulate more liquidity that acts as a protection against financial crises driven by idiosyncratic risk on deposits. I show that the mechanism allows banks to expand in size, in contrast to ex-post liquidity regulations.

Likewise, Jeanne and Korinek (2020) study the optimal interactions between ex-ante and ex-post macroprudential regulations through the lenses of a social planner. While in their model, the ex-ante policy intervention is a cup on deposits set as a collateral constraint related to the price of the asset which leads to overborrowing, my model designs a liquidity requirement related to the size of banks' deposits and focusses on the desirability of ex-ante liquidity regulations and its distributive effects across large and small banks. The designation of my liquidity regulation leads to overaccumulation of banks' liquidity.[6] Furthermore, while their model features stochastic income with a representative banker, my model highlights the source of idiosyncratic uncertainty in deposits with heterogeneous bankers. In my model ex-ante regulations target the upper tail of the banks' distribution, while ex-post regulations reallocate liquidity towards small banks; if in addition to the ex-ante regulation the regulator tights the liquidity requirement then ex-ante regulations have the same distributive effects of ex-post regulations.

I share with Amador and Bianchi (2023, 2024) the feature of bankers having utility over dividend payments, however while they focus on the implications of credit easing policies on bank runs and welfare driven by limited commitment by the government, in my model the liquidity regulation is meant to create a precautionary motive in the overaccumulation of liquidity which does not lead to the insurgence of bank runs because liquidity leads to a stable contract. Furthermore, they inspect the scope of the regulation rather than the design and the implementation of the liquidity regulation that I delineate in this paper. Similarly to the them, my paper emphasises the need of ex-ante prudential intervention and requires tightening the liquidity requirement to reduce the exposure to deposit risk, whereas their model invokes tighter capital requirement because of the pecuniary externality generated by the general equilibrium effects on price. In their model market incompleteness is due to the idiosyncratic nature of banks' productivity whereby banks may default because of self-fulfilling shock or because of a fundamental shock. In either case, the economy features ex-post one single bank.

This paper contributes to the literature of heterogeneous banks. Bianchi and Bigio (2022) build a monetary model in which heterogeneous banks are subject to liquidity risk. In their model, banks can buffer the risk by having a precautionary shielding of reserves and having access to the interbank market. They make assumptions about the agents' underlying preferences whereby the distribution of banks becomes degenerate allowing a simple characterisation of the bank liquidity management. It becomes the key modelling assumption because the resulting value function turns out homogeneous such that the corresponding portfolio allocations become linear in equity with equity the only individual state variable. This device implies that they can solve for a homogeneous bank model and therefore, the monetary policy implications they draw with regards to conventional and quantitative easing policies are unaffected by bank heterogeneity. I follow them in modelling banks as having liquid and illiquid assets, however in my model aggregation does not occur since the bank budget constraint remains ex-post nonlinear due to the liquidity constraint therefore the composition of banks' balance sheet matters.[7]

De Fiore, Hoerova and Uhlig (2019) also study the liquidity management problem of heterogeneous banks tied up to the central bank liquidity provisions to study the decrease in un-

[6]Other papers that study ex-ante macroprudential interventions, however through the lenses of pecuniary externalities, include Lorenzoni (2008), Jeanne and Korinek (2010), Stein (2012) and Dávila and Korinek (2018).

[7]I draw attention to the liquidity market. Seminal studies on the impact of monetary policy on bank liquidity and bank lending were originally developed by Bernanke and Blinder (1988), Bernanke and Gertler (1995), Kashyap and Stein (1994), Kashyap and Stein (1995), Cúrdia and Woodford (2010), Cúrdia and Woodford (2016), Farhi and Werning (2016), He and Krishnamurthy (2019).

collaterized funds and increase in the threshold of accepted collateral in the interbank market. They find that liquidity risk of banks leads to less aggregate lending and output. However in their model they do not keep track of the distribution of banks, whose problem is shown to be linear in the bank's endogenous state variable. Therefore, the quantitative arguments are on aggregate variables and not sensitive to the heterogeneous measure of banks.[8] I take this one step further since idiosyncratic shocks persevere in the aggregate allowing me to make predictions about unequally distributed liquidity, which bank is mostly affected and liquidity crises to which I try to contribute.

I share with Corbae and D'Erasmo (2013, 2021, 2022) the idiosyncratic nature of deposits where in their models they endogenize the size of banks. They do so by creating a model of banks entry and exit dynamics in which banks' size correlates with banks market power. Their model features an oligopolistic competitive banking sector where followers banks internalise the actions of the leader bank. They analyse the impact of US bank regulation policies on bank risk and banks' market structure. Specifically they study capital and liquidity requirements imposed by Dodd-Frank Act and the liquidity requirement is related to the Liquidity Coverage Ratio. I differ in the type and the scope of the regulation being considered which in my model is the liquidity as a function to deposit regulation set by the Fed that has significant different implications in terms of designation, banks' size dependency and scale. I provide more details on the regulation in Appendix K. Moreover, their paper do not study the accumulation of banks' liquidity. My central contribution is the study of the regulation on banks' liquidity in the pool of heterogeneous banks. I examine the interplay between liquid assets and deposits and the extent to which the precautionary behavior of banks tempers liquidity crises arising in the deposit market. My model features positive cashflows value in banks' resources where earlier studies that investigate this feature of the model include Cooley and Quadrini (2001) and Hennessy and Whited (2007). These studies however focus on the financial problem from the firms' perspective.

Jamilov and Monacelli (2020) develop a heterogeneous banks model with financial frictions building on the models of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011).[9] Their model contains market power in the credit market with endogenous entry and idiosyncratic risk in the return of capital to study economic crises and recessions driven by technology shocks. I take a different stand from them since my incomplete markets economy hinges on the idiosyncratic source of deposit funding with a focus on the liquidity structure of banks' balance sheet.

I build on the literature of banking crises and liquidity. Boissay, Collard and Smets (2016) studies banking crises in which banks are heterogeneous in their intermediation efficiency. In their model a banking crisis arises due to the interbank market freezes with the procyclicality

[8] See also Arce et al. (2020) and Piazzesi, Rogers and Schneider (2019) for the integration of the interbank market with nominal rigidities in a New Keynesian context in which heterogeneous banks do not affect macroeconomic aggregates.

[9] Models that incorporate exogenous heterogeneity in banks' asset returns include Clerc et al. (2015), Elenev, Landvoigt and Van Nieuwerburgh (2018), Gete and Melkadze (2020).

of banks' balance sheets. Moral hazard in the interbank market creates severe recessions.[10] For the purpose of my paper, the main source of liquid liabilities in my model are deposits and internal equity. I offer a mechanism by which the idiosyncratic nature of deposits can trigger a liquidity crisis. Liquidity crises in my model are due to the occasionally binding liquidity constraint. In this regard, this paper is related to the studies of financial crises with occasionally binding constraints (Mendoza, 2010; Brunnermeier and Sannikov, 2014; Bianchi, 2016; Bianchi and Mendoza, 2018).[11]

Other recent contributions to the heterogeneous banks literature include Coimbra and Rey (2023) who analyse financial stability and systemic risk by developing a Value-at-Risk model with heterogeneity in the time variation of bank's leverage; and Begenau and Landvoigt (2021) who develop a quantitative model with regulated and unregulated banks to study capital requirements for commercial and shadow banking sectors. These models however do not feature ex-post heterogeneous banks.

3 Motivating Evidence on Banks' Liquidity

In this section I present evidence on the US banking sector that motivate the premises of the paper on the reduction of banks' liquid assets inequality and its relation to liquidity crises. I consider large banks as the largest 10 banks in the asset distribution and all remaining banks are considered small banks to control for the size distribution of banks.

I use balance sheet data from the Consolidated Report of Condition and Income submitted to the Federal Reserve by banks and I collect a panel of banks from 1998 to 2010. Specifically, I analyse the cross-sectional change in banks' liquidity pre and post the 2008 financial crisis and I report statistics on banks' liquid assets distribution that I use to assess the model. I mainly focus on the time period post financial crisis to inspect the impact of the liquidity constraint on liquidity crisis. Therefore, I analyse the change in distribution from 2008 to 2010. My measure of banks' liquidity is the composition of cash and balances of depository institutions, federal funds sold, securities held to maturity and trading assets. They constitute the assets side of a bank's balance sheet. Appendix L provides details on the liquidity regulation and liquidity requirements according to the threshold of exemption and nonexempt deposits for banks enacted by the Federal Reserve Act and International Banking Act.

Figure 1 Panels A and B present the distribution of banks' liquid assets to deposits for small banks−blue histogram, and large banks−red histogram, in 2008 and 2010, respectively. I consider the liquidity to deposits the measure for the liquidity requirement which I use to

[10]Liquidity hoarding in the interbank market with market break down is studied in Heider, Hoerova and Holthausen (2015). Calomiris, Heider and Hoerova (2015) develop a model of bank liquidity requirements where the liquidity requirement is imposed on cash.

[11]Financial stability and bank capital requirements are analysed in Rios-Rull, Takamura and Terajima (2020) who consider a partial equilibrium model with heterogeneity in banks' default rate. I differ from this work since bank heterogeneity in my model is captured in the quantity of banks' liquidity and uninsurable idiosyncratic deposits rather than in banks' asset returns and risk taking behaviors.

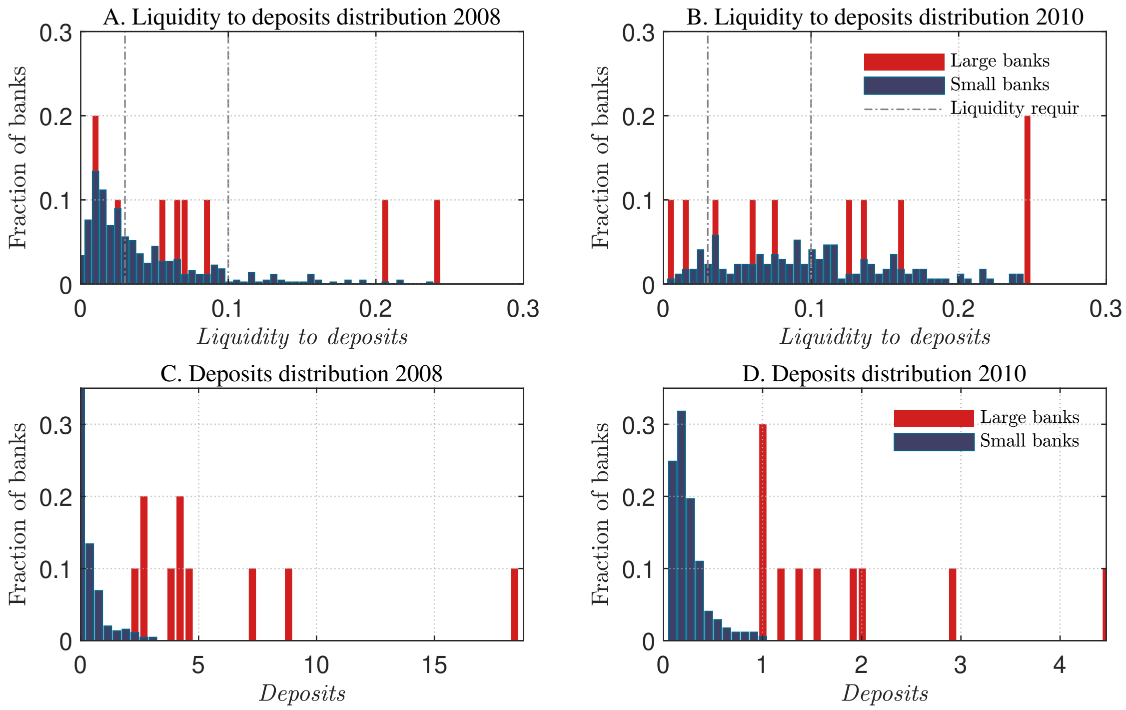

Fig. 1.—Distribution banks' deposits and liquidity to deposits requirement.

Note. Red bars represent the distributions of liquidity to deposits of large banks, whereas blue bars the distributions of small banks. The grey lines represent the liquidity requirement of 3% and 10% for banks in the nonexempt category of deposits. For panels C and D the x-axis reports data in millions of dollars. Data source is Consolidated Reports of Condition and Income. Appendix L contains details on the liquidity regulation.

assess the theoretical model below. Two observations emerges. First, large banks tend to hold higher liquid assets to deposits than small banks. Following the financial crisis stronger liquidity requirements on banks' liquid assets had the effect of making the distribution of liquid assets less concentrated. Second, at the onset of the crisis, approximately 14% of small banks were concentrated at the left tail of the liquid assets to deposits distribution. During the crisis a considerable number of small banks failed or merged with other banks (Corbae and D'Erasmo, 2021), as a result the number of banks decreased and their fraction reached less than 6%. However, large banks were involved in mergers more than small banks. While the fraction of small banks holding liquid assets declined after the crisis, the increase in deposit thresholds to meet the liquidity requirements, raised the amount of liquidity in their balance sheets. As a result, by 2010 a fatter right tail in the distribution of small banks appeared.

The distribution of deposits in Panel C and D of Figure 1 for small banks becomes fatter by the end of 2010 with the fraction of banks falling by 41% and the majority of these banks concentrated below $1 million of deposits. Since the fraction of small banks holding liquid assets fell by 49.4% exceeding the change in the fraction for deposits of 41%, this suggests a 1.2% rise in the fraction of banks in the liquidity to deposits distribution. By contrast, the fraction of large banks holding more than $1 million dollars rises to 30% by 2010, indicating a 0.5% reduction

Table 1.—
Distribution banks' liquid assets to deposits in the US

		Percentiles in the liquid assets to deposits distribution							
		Top distribution					Bottom distribution		
	Gini coefficient	1^{st}	5^{th}	10^{th}	20^{th}	30^{th}	85^{th}	94^{th}	99^{th}
Small banks									
2008 data	0.389	0.017	0.085	0.170	0.341	0.511	0.744	0.898	0.983
2010 data	0.264	0.014	0.071	0.143	0.286	0.428	0.786	0.914	0.986
Large banks									
2008 data	0.319	0.015	0.076	0.152	0.305	0.455	0.772	0.909	0.985
2010 data	0.267	0.014	0.070	0.140	0.280	0.420	0.790	0.916	0.986
All banks									
2008 data	0.437	0.018	0.091	0.182	0.365	0.547	0.726	0.891	0.982
2010 data	0.369	0.016	0.084	0.167	0.335	0.502	0.749	0.899	0.983

Note. Distribution of banks' liquid assets to deposits for small, large and all banks, respectively. Data source and further details are provided in Appendix J.

in the liquidity to deposits distribution. This fact highlights that small banks engaged towards precautionary liquidity buffers more strongly than large banks.[12]

Table 1 reports statistics for small banks, large banks and all banks. The distribution of large banks considers the largest 10 banks in the asset distribution, while small banks correspond to all the rest of banks when sorted by assets. This sorting implies that large banks correspond to those banks with assets larger than $4, 3 millions in 2008 and $1, 1 millions in 2010. Table 1 confirms the two observations above. Comparing the inequality in banks' liquidity to deposits in 2008 and 2010 for small banks, the distribution became less concentrated with a 26.4% Gini coefficient by the end of 2010 and an overall 12.5% change. The wealthiest small banks in terms of liquidity to deposits in the 10th percentile of the distribution held 17% of liquid assets in 2008 and the smallest 85th percentile of small banks held 74.4% of liquidity to deposits.

By 2010 there was a considerable rise in the amount of liquidity to deposits in the bottom percentiles of the distribution of small banks and a contemporaneous decay of banks' liquid assets in the top percentiles of the distribution of small banks. These findings are consistent with previous empirical studies on banks' liquidity that find close effects on the distribution of small banks, and larger effects for large banks (Ennis and Wolman, 2015).[13]

Likewise, banks' liquid assets inequality for large banks experienced a fall from 31.9% in

[12]The liquidity requirement based on cash to assets ratio presents a similar pattern since I sort banks by assets.

[13]Any differences are due to the type of sample as well as the measure of banks' liquidity considered. Ennis and Wolman (2015) use a sample of insured and uninsured banks, I only use insured banks under the Federal Deposit Insurance Corporation, following Corbae and D'Erasmo (2021) and den Haan, Sumner and Yamashiro (2007). Uninsured banks may have a tendency towards self-insuring in liquid assets higher than insured banks, thereby skewing the distribution of large banks. This result is also in line with Afonso et al. (2019) who obtain a fatter and skewed distribution when considering insured banks.

TABLE 2.—

Banks' liquid assets and deposits

	All banks	Large banks, top 10	Small banks
Liquidity			
Mean 2008	41,849	713,189	26,863
Median 2008	7,713	286,910	7,454
St. dev. 2008	180,588	837,467	94,408
Mean 2010	31,131	184,657	22,256
Median 2010	14,535	180,589	13,488
St. dev. 2010	57,738	154,256	28,859
Deposits			
Mean 2008	507,170	6,132,477	381,607
Median 2008	195,835	4,428,802	193,000
St. dev. 2008	1,203,315	4,910,351	520,740
Mean 2010	295,180	1,804,416	207,941
Median 2010	174,079	1,403,128	167,045
St. dev. 2010	472,573	1,113,578	177,025
Liquidity to deposits			
Mean 2008	0.066	0.108	0.065
Median 2008	0.039	0.082	0.039
St. dev. 2008	0.076	0.110	0.075
Mean 2010	0.109	0.109	0.109
Median 2010	0.096	0.099	0.096
St. dev. 2010	0.076	0.089	0.076

NOTE. Statistic values of liquid assets and deposits in dollars amount and levels for liquidity to deposits ratio. Data source is provided in appendix J.

2008 to 26.7% in 2010 as measured by the Gini coefficient. I find a similar pattern for the change in the bottom percentiles of large banks' liquidity distribution. Whilst the upper tail of the distribution of large banks significantly decreased in 2010, the bottom 85th percentile of large banks' liquid assets increased from 77.2% in 2008 to 79% in 2010. Consistently, the bottom 94th percentile of large banks experienced a 0.7% rise in liquid assets. The theoretical model below can reconcile this pattern. I show that liquidity in the bottom distribution of banks' liquid assets increases driving down the inequality in its distribution.

Table 2 reports mean, median and standard deviation of liquid assets and deposits in dollars amount between banks and the liquidity to deposits in 2008 and 2010, respectively. The average and median values are shown for all banks, the top 10 banks and small banks that constitute the rest of banks when sorting by assets the liquidity to deposits distribution. The average amount of liquid asset in 2008 of large banks is $713,189 that is twenty-six times the average liquidity of small banks who hold $26,863 of liquid assets. This is less than almost fourteen times their average deposits in the same year. The average stock of deposits of large banks is

TABLE 3.—
Within and between banks' heterogeneity

	Within heterogeneity, large banks	Within heterogeneity, small banks	Between heterogeneity, amongsts banks
94/10 in 2008	5.991	5.271	0.720
94/10 in 2010	6.542	6.402	0.140
94/10 difference	0.551	1.131	0.580

NOTE. Ratio 94th to 10th for large and small banks between 2008 and 2010. The last row reports the difference in ratios from the 2010 year for within heterogeneity and from 2008 for between heterogeneity. Values are reported in levels.

remarkably high and equals $6,132,477, that is almost twelve times the average stock of deposits between all banks. There was a notable fall in the average banks' liquidity and deposits by 2010 with changes equal to $10,718 for all banks, $528,532 and $4,607 for large and small banks, respectively. The decay reflects the fact that some large banks failed during the financial crisis and hence exited the economy. Since the distribution of liquid assets appears to be skewed to the left, the average masks the impact of extreme values which might drive down the average value of liquid assets.

While the average amount of deposits for banks in the bottom tail of the distribution decreases to $207,941 relative to its level in 2008 and their average amount of liquidity falls to $22,256, their median liquid assets rises to $13,488. The rise in the median liquid assets of small banks guided the rise in the median value of all banks to $14,535. This is because many small banks merged with other banks and because of the rise in deposits thresholds. Moreover, small and medium banks constitute the vast majority of banks in the sample. Therefore, the computation of the median reveals that liquidity in the bottom percentiles of the distribution of small banks increased more than that of large banks with the distribution becoming more skewed to the left. This fact indicates that analysing the banks' liquidity distribution through the lenses of the first moment in place of the second moment might be misleading. Furthermore, consider the liquidity to deposits ratio in Table 2. It confirms the same pattern of the median banks' liquidity increasing for small banks and all banks. However, in contrast to the nominal mean value of liquid assets and deposits that both fall post financial crisis, the mean values of liquidity to deposits ratio rise indicating that liquidity contributed more pronounceable than the fall in deposits in rising the liquidity to deposits ratio.

While the distribution of liquidity to deposits for all banks, in the last two rows of Table 1, corroborates these findings of a reduction in inequality brought by a rise in liquidity to deposits in the bottom tail of the distribution of large and small banks, it cannot guide on the heterogeneity driving the rise, *i.e.*, the rise coming separately from large banks and small banks. Indeed, the bottom 85th and 94th percentiles of the distribution of large banks experienced about 2.6% cumulative rise in liquidity to deposits, accompanied by a 3.5% fall in liquidity to deposits in the 30th percentile of the top distribution of large banks, more than two times lower than the fall in the top distribution of small banks' liquid assets. Instead, the cumulative rise in banks' liquidity to deposits in the bottom percentiles of small banks distribution was almost three times larger than the rise in large banks' liquidity to deposits. Consequently, the distribution of banks' liquidity to deposits became less concentrated. Furthermore, the same distribution in 2010 became slightly more volatile than 2008 pre financial crisis distribution. The volatility of liquidity to deposits of small and all banks in 2010 did not change significantly from the corresponding 2008 values, but the mean of this distribution rose in 2010 post crisis to 10.9% for small and all banks relative to the 2008 values, *i.e.*, 6.6% and 6.5%.

Therefore, there is a non negligible amount of within and between group inequality between large and small banks in banks' liquidity to deposits. To further assess the between and within heterogeneity I compute the ratios between the 94th and 10th percentiles for large and small banks in 2008 and 2010 and the difference in these ratios across banks. Table 3 presents the associated findings. The 94th to 10th ratio for large banks rose from 5.99 in 2008 to 6.54 in 2010 accounting for a significant difference of 55% between the wealthiest large banks and bottom tail of large banks. By contrast, within heterogeneity across small banks experienced the little change in the 94th to 10th ratio. It accounted for 113% rise in banks' liquidity to deposits in the bottom tail distribution of small banks. The rise reflects the severity of liquidity requirements post financial crisis.

To inspect the between heterogeneity, between large and small banks, I compute the difference in changes in the two ratios above. The overall difference between the change in the 94th to 10th ratio for small banks and that for large banks accounted for 58.1% rise change. This means that the bottom upper tail of the distribution varied more than the bottom lower tail distribution. While the first two columns of Table 3 detect the within heterogeneity for large and small banks across years, the rows indicate the heterogeneity between large and small banks for the same year. Indeed, differences between large and small banks were 72% in 2008 and reduced to 14% in 2010, suggesting a reduction in inequality in banks' liquidity post financial crisis as confirmed by the difference in changes in the two ratios within banks. Therefore, the 94th to 10th ratio helps to explain the increase of liquidity in the bottom tails of both distributions relative to the upper tails of the distributions of large and small banks. Since small banks experienced the largest change in the liquidity to deposit ratio compared to large banks, it is important to focus on these types of banks in the model.

The third observation that emerges is that the rise in banks' liquidity in the bottom tail of banks' liquidity distribution and the trending down of banks' liquidity for all other banks is consistent with the fall in inequality in banks' liquidity in 2010. These findings support the observation that banks' liquidity became less unequally distributed. The model I present below is able to reconcile this feature of the data.

These findings suggest that small banks' liquidity concentrated within a large fraction of

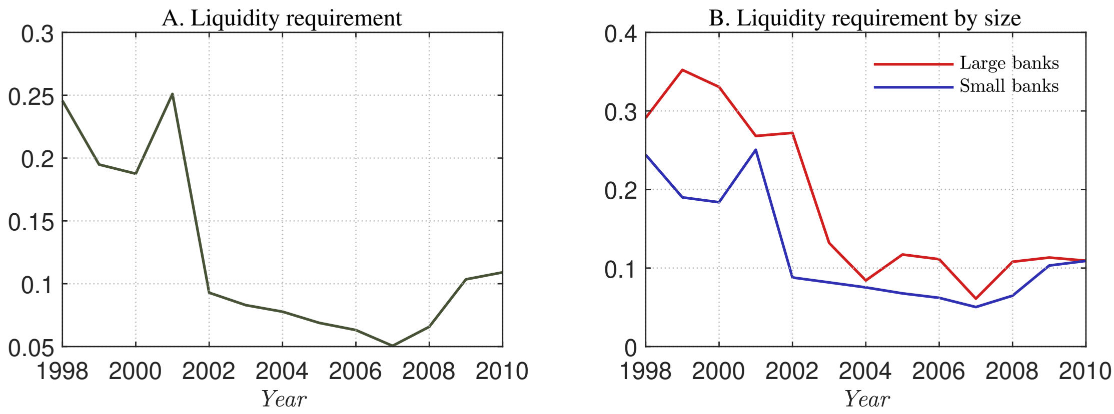

FIG. 2.—Liquidity requirement from micro banks data.

NOTE. The liquidity requirement is defined as the liquidity to deposits ratio. Dark grey line is the average liquidity requirement of all banks. Red line is the liquidity requirement of large banks identified as the top 10 banks sorted by assets, blue line is the liquidity requirement of small banks. Values are expressed in levels. Data source is Consolidated Reports of Condition and Income. Further details are provided in appendix J.

small banks is an indication that banks are not well insured against a shock, therefore a large inequality in banks' liquidity can make banks in distress during a financial crises. I build this connection between the distribution of banks' liquidity and liquidity crises in the theoretical model below. The liquidity regulation I implement as function of deposits explain the rise in liquid assets in the bottom percentiles of small banks and can reconcile these features present in the data. The subsequent question I address is whether liquidity induces differences in banks' value. The heterogeneity suggests that an increased deposit threshold was one of the dominant factors post financial crisis. If liquidity requirements become more severe, then the precautionary banks' behavior towards liquid assets rises. All else equal, stronger buffer of banks' liquid assets are conducive to more resilient banks in hedging liquidity risk thereby mitigating the effects of banking crises in the deposits market. As a consequence, I inspect the connection between the distribution of banks' liquidity and liquidity regulations in the form of liquidity requirements affecting liquidity crises.

The center of the paper is on the liquidity regulation in terms of the liquidity requirement that requires all banks in the US to have an amount of liquid assets as a function of deposits. I assume that my computed measure of liquidity from micro banks data is used to satisfy the liquidity requirement. Figure 2 Panels A and B illustrate the evolution of the liquidity requirement defined as liquidity to deposits ratio by bank size in the period 1998 to 2010. I take for each bank the last quarter for each year as representative for the year and I compute the average across banks for each year. Panel A depicts the average distribution of the liquidity requirement for all banks and Panel B the average liquidity requirement of the top 10 banks ranked by assets that constitutes large banks and all remaining banks are classified as small banks.

The general pattern shows a marked steadily decay in the liquidity requirement of all banks after the year 2001 of approximately 19.50%. Pre financial crisis the path reverts to a permanent higher level. The change in pattern at the outset of the crisis reflects the rise in liquidity more than the rise in deposits thresholds that were imposed subsequent the 2008 financial crisis.

The fourth fact that emerges is that the liquidity requirement of large banks exceeds that of small banks in all considered years. Large banks hold approximately an average of 18.08% of liquid assets in the banking system while the bottom tail of the distribution hold an average of 12.08% of liquid assets. While large banks contributed mostly to the increase in the liquidity requirement pre financial crisis, this is not the case after the financial crisis. Indeed, post financial crisis the rise in small banks' liquid assets and their smaller change in deposits had a role in guiding the rise in the liquidity requirement. I compute the correlation between liquid assets and deposits which results in 64.15% in 2008 and 81.46% in 2010. Therefore, the fifth observation is that liquid assets tend to increase when deposits rise.

To summarise, these five observations delineate considerable heterogeneity in banks' liquidity and regularities in the holdings of banks' liquid assets where any sudden change may result in a liquidity crisis. I study these observations in the theoretical model below.

4 The Model

This section describes the model economy with idiosyncratic deposit risk subject to a liquidity constraint. Individual banks cannot exchange liquidity between themselves to insure against any deposit withdrawal risk. I show the interaction between market incompleteness, liquidity constraint and the relation between banks' size expansion and sorting by type.

I first present and study the equilibrium properties of the model in a partial equilibrium economy where interest rates are determined exogenously. This version is important to capture the main insight and intuition of a deposit withdrawal shock and liquidity constraint in affecting bank optimal choices towards liquidity and dividends consumption. The state of the economy is then represented by the joint distribution of deposits and liquid assets.

4.1 The Environment

I model an infinite horizon closed economy. Time is discrete and indexed by t. There is a continuum of bankers with unit mass indexed by the subscript, $i \in [0, 1]$. Each banker is in charge of a bank.

At the beginning of each period t, the level of liquid assets, $\ell_{i,t}$, illiquid assets, s, deposits, $d_{i,t}$, and individual bank profits, $\mathcal{P}_{i,t}$, for bank i enter from the previous period. Deposits are insured liquid liabilities characterizing the non contingent debt contract. Illiquid assets are an illiquid project with a loan contract, s, in which the bank invests and is held fixed. Liquidity in the model constitutes reserves and short term assets. Banks cannot issue equity but they can

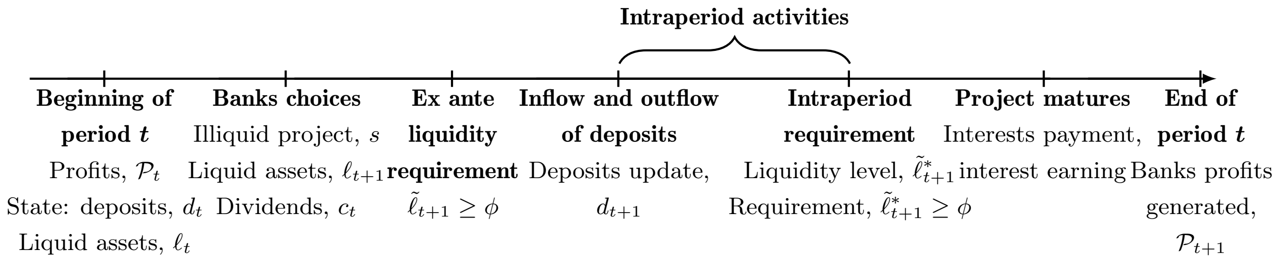

FIG. 3.— Timeline of events

retain earnings. Therefore, banks allocates equity and deposits to liquid assets, illiquid assets and dividends income, $c_{i,t}$. The size of a bank varies with the width of the distributional deposit withdrawal shock as well as the amount of liquid assets in a bank's balance sheet.

Liquid assets earn a gross interest rate, r_ℓ, illiquid assets earn a gross interest rate given by r_s, and deposits pay the gross interest rate r_d. I assume $r_\ell < r_d < r_s$ because−in reality−loans are risky and the interest rate on liquidity is smaller than the interest rate on deposits.

To capture the unpredictability of the circulation of deposits through the banking system, I assume that deposits follow an exogenous Markov process. The Markov chain is given by $\psi(d_{t+1}|d_t)$, whose stationary probability distribution is specified as $\lambda(d_t)$. It characterises the mass of banks in each deposit level at any specific time t with initial probability distribution λ_0. Each deposit level is drawn from the finite set D and d^t represents the specific history of realisations of the deposit process from date 0 up to date t. The process is the same for all banks. In the initial period the beginning of time liquid assets, ℓ_0, is nonnegative and belonging to the set $L \subset \mathbb{R}_+$. I define with B the space of all admissible banker individual states such that $B = (D \times L)$ with initial state given by b_0. The probability measure for idiosyncratic deposits is then given by $\mu^t(b_0, d^t)$. Therefore, the beginning of period banks' equity is given by

$$a_{i,t}(d^t; b_0) = \ell_{i,t}(d^{t-1}; b_0) + s + \mathcal{P}_{i,t}(d^t; b_0) - d_{i,t}, \tag{1}$$

which equals liquid assets, illiquid assets and profits, $\ell_{i,t}(d^t; b_0) + s + \mathcal{P}_{i,t}$, net of funding inflows, $d_{i,t}$. Since dividends cannot exceed banks' net income, then it must hold that $c_{i,t}(d^t; b_0) \in [0, \sum_{d^t \in \mathcal{D}^t} \mu^t(b_0, d^t)\mathcal{P}_{i,t}(d^t; b_0)]$.[14] Therefore, there is limited liability on the side of the banking sector given that dividends must be nonnegative. Given the initial state and history of deposits dependence, bankers have preferences over dividend incomes represented by

$$\sum_{t=0}^{\infty} \beta^t \sum_{d^t \in \mathcal{D}^t} \mu^t(b_0, d^t) u(c_{i,t}(d^t; b_0)), \tag{2}$$

in which, $c_{i,t}$, is bank i dividends and they have utility over the consumption of dividend incomes. I maintain assumptions of bankers' utility function for an interior solution to be nonnegative

[14] I follow Corbae and D'Erasmo (2021) in setting the maximum dividends income in accordance with the Code of Federal Regulations under Title 12 Banks and Banking.

for each history of the deposit process, such that bankers' utility is a real valued function $u : \mathbb{R} \longrightarrow \mathbb{R}_+$, where $u(c_{i,t})$ is twice continuously differentiable with $u_c(c_{i,t}) > 0$ and $u_{cc}(c_{i,t}) < 0$ $\forall c_{i,t} \in \mathbb{R}_+$; and the $\lim_{c_{i,t}\downarrow 0} u_c(c_{i,t}) \leq \infty$ and $\lim_{c_{i,t}\uparrow\infty} u_c(c_{i,t}) = 0$ ensuring bank consumption is a concave function and $c_{i,t} : B \longrightarrow \mathbb{R}_+$. I assume bankers are risk averse with concave utility over dividends incomes to capture the fact that the vast majority of banks are medium and small banks. The parameter $\beta \in [0, 1]$ is the discount factor with which bankers discount future dividends. Given that banks' preferences are increasing in dividends, it must hold that

$$a_{i,t}(d^t; b_0) + d_{i,t+1} = \ell_{i,t+1}(d^t; b_0) + s + c_{i,t}(d^t; b_0). \tag{3}$$

The realization of the shock occurs before any interest on liquid and illiquid assets is earned and interest on deposits is paid. Therefore, once the realization of the shock occurs deposits update to $d_{i,t+1}$. However, the deposit disturbance occurs after banks have made their optimal decisions over dividends consumption, $c_{i,t}$, and liquidity, $\ell_{i,t+1}$. Consequently, I define the ex-ante regulation as the intraperiod liquidity requirement with $\tilde{\ell}_{i,t+1}$, where

$$\tilde{\ell}_{i,t+1} = \ell_{i,t}(d^{t-1}; b_0) + (d_{i,t+1} - d_{i,t}), \tag{4}$$

and $d_{i,t+1}$ and $d_{i,t}$ denote the intraperiod level of deposits. The state contingent liquidity constraint is given by

$$\ell_{i,t}(d^{t-1}; b_0) + (d_{i,t+1} - d_{i,t}) \geq \phi, \quad \forall d^t \in D^t, \quad t = 0, 1, ... \tag{5}$$

where, ϕ, is the exogenous liquidity requirement that imposes a lower bound on banks' liquidity.[15] The value of $d_{i,t+1}$ is the value of deposits from the perspective of time t prior that banks have made their optimal choice of liquidity.[16] Moreover, from the perspective of beginning of t period no change in the liquidity buffer occurs. Therefore, by the end of the period banks must have enough liquidity to cover the liquidity constraint and thus, the inflow or outflow of deposits until the liquidity constraint is slack. Within the intraperiod banks internalise the liquidity constraint such that the ex-ante bank budget constraint is given by,

$$c_{i,t}(d^t; b_0) + s + \ell_{i,t+1}(d^t; b_0) - d_{i,t+1} = r_s s + r_\ell \tilde{\ell}_{i,t+1} - r_d d_{i,t+1}, \quad \forall d^t \in D^t, t = 0, 1, ..., \tag{6}$$

where, $\ell_{i,t+1}(d^t; b_0)$, is the value of ℓ_{t+1} chosen in period t with sequence of d^t individual history

[15] Note that since I am interested in the intraperiod change in liquidity which is important for banks for internalisation and computation of initial resources, I do not conceive the liquidity requirement as a ratio and instead I conceive ϕ in the liquidity constraint as being the ratio of liquidity to deposits. In Appendix C I solve the model with a liquidity ratio and discuss its implications.

[16] This is an exogenous liquidity constraint tied to bank net liquidity. I therefore do not make the constraint dependent on illiquid assets for liquidity to be at least greater than a fraction of illiquid assets, s. The reason is that being illiquid assets exogenous in this model making the constraint dependent on s, for example as $\ell_{i,t+1}(d^t; b_0) + (d_{i,t+1} - d_{i,t}) - \phi(1 + s) \geq 0$, would deliver the same model implication of a tightening of the liquidity requirement. I demonstrate this aspect in Appendix B.

of realisations from 0 to t. The right hand side of equation (6) captures the value of future profits from the perspective of beginning of t period. Therefore, the end of intraperiod liquidity requirement that banks need to satisfy is $\tilde{\ell}^{*}_{i,t+1} = \ell_{i,t+1}(d^t, b_0) + (d_{i,t+1} - d_{i,t})$, namely with the future end of intraperiod level of liquidity. The fixed illiquid assets in the budget constraint captures forms of rigidities in banks' balance sheet such as fixed capital and it raises the incentive of banks to accumulate liquid assets. Consequently, since I focus on short term commitments I assume illiquid assets, s, to be fixed as it is difficult for banks to adjust their portfolio based on long term loans in the short period.

At the end of the period after the realisation of the shock profits are generated, $\mathcal{P}_{i,t+1} = r_s s + r_\ell \tilde{\ell}^{*}_{i,t+1} - r_d d_{i,t+1}$, with the chosen level of liquidity and the realised value of deposits. Therefore, substituting equation (1) one period ahead inside equation (3), the end of period banks' equity is given by

$$a_{i,t+1}(d^t, b_0) = a_{i,t}(d^t, b_0) + \mathcal{P}_{i,t+1}(d^t, b_0) - c_{i,t}(d^t, b_0), \quad \forall d^t \in D^t, t = 0, 1, ..., \tag{7}$$

where equation (7) specifies the evolution of next period equity as increasing in profits and decreasing in dividends payments. A generic bank i contract therefore specifies the fixed level of illiquid assets as investment project, the amount of liquidity banks require, the size of dividends consumption, the non-contingencies states of deposits, the profits banks obtain from operating and the distribution of banks' liquidity.

The liquidity constraint in equation (5) is both state dependent and bank size dependent, in which banks' size grows with the level of banks' liquid assets, $\ell_{i,t+1}$, and the width of the deposit withdrawal, $(d_{i,t+1} - d_{i,t})$. It occasionally binds because of the idiosyncratic disturbance affecting banks' assets. As a consequence of this assumption large banks are the owners of a lower amount of deposits that translates into more liquid assets in contrast to small banks, therefore, both factors: liquid assets and deposits contribute to the expansion in size of each heterogeneous bank.[17] The explanation for this fact follows since liquidity, which is the asset in the model, depends on the realization of the deposit withdrawal shock, the larger the bank the larger liquidity in the their balance sheet to cover future realisation of deposits.

Timeline. The timeline of the events is presented in Figure 3. In a given period

1. Bank i enters the period with its previous period liquid assets, $\ell_{i,t}$, loans, s, dividends, $c_{i,t}$, deposits $d_{i,t}$ and profits, $\mathcal{P}_{i,t}$.
2. Banks decide how much liquidity, $\ell_{i,t+1}$, to save and how many dividends to issue, $c_{i,t}$.
3. *Intraperiod activities*

   i. The deposit shock is realised, banks experience an inflow/outflow of deposits. Therefore,

[17] This assumption corresponds to the banking fact empirically documented by Corbae and D'Erasmo (2021, 2022) that points towards small banks holding lower deposits than big banks in absolute terms. In relative terms it holds that large banks in the same cohort of the largest banks have lower deposits than small banks in the same cohort of large banks and they are subject to less volatile funding inflows.

deposits update to $d_{i,t+1}$.

ii. Banks must satisfy the liquidity requirement, $\tilde{\ell}^*_{i,t+1} = \ell_{i,t+1} + (d_{i,t+1} - d_{i,t}) \geq \phi$ and the deposit shock can be met by increasing or decreasing $\ell_{i,t+1}$, and by using internal equity.

4. The loan project, s, matures. Interest rate on deposits are paid, interest rates on liquid and illiquid assets are earned, banks' profits, $\mathcal{P}_{i,t+1}$, and equity, $a_{i,t+1}$, are generated.

4.2 Equilibrium Analysis

By substituting the definition of $\tilde{\ell}_{i,t+1}$ in equation (4), the budget constraint can be written as,

$$c_{i,t}(d^t; b_0) + s + \ell_{i,t+1}(d^t; b_0) - d_{i,t+1} = r_s s + r_\ell(\ell_{i,t}(d^{t-1}; b_0) + (d_{i,t+1} - d_{i,t})) - r_d d_{i,t+1}, \quad (8)$$

solving and rearranging terms, equation (8) becomes

$$c_{i,t}(d^t; b_0) + s + \ell_{i,t+1}(d^t; b_0) - d_{i,t+1} = r_s s + r_\ell \ell_{i,t}(d^{t-1}; b_0) - r_d d_{i,t} + \underbrace{(r_\ell - r_d)(d_{i,t+1} - d_{i,t})}_{\text{inflow/outflow effect}}. \quad (9)$$

The timing assumption is such that banks experiencing the outflow of deposits pay the interest rate on those deposits. The bank specific liquidity constraint makes banks to climb a deposit ladder represented by the deposit withdrawal that is internalised in their individual problem. This means that there are two competing effects related to banks' size expansion. First, while an inflow of deposits tights the liquidity requirement, it also raises inflow banks' resources and networth in their budget constraint which in turn leads to accumulation of banks' liquidity and banks' size expansion. By contrast, an outflow of deposits by reducing banks' networth it lessens the liquidity requirement. This leads outflow banks to rise their liquidity thereby expanding in size.[18] Bank networth is defined as the sum of banks' liquidity and illiquid assets.

The *inflow/outflow effect* in equation (9) captures the effect of the intraperiod level of the liquidity requirement that banks face and it captures the fact that any withdrawal of deposits is costly to banks since the interest rate on liquidity drives down the interest rate on deposits which raises the total cost on short term debt $r_d d_{i,t} + (r_\ell - r_d)(d_{i,t+1} - d_{i,t})$. In particular, if by the end of the period banks experience an inflow of deposits, the current level of profits, $\mathcal{P}_{i,t}$, increases with a positive net interest rate margin on the change in deposits. By contrast, if banks experience an outflow of deposits the width of the deposit withdrawal, $(d_{i,t+1} - d_{i,t})$, is affected by the negative effect of the net interest rate margin. This occurs because while the outflow effect becomes positive, outflow banks are paying the interest rate wedge alongside the outflow that is transferred to inflow banks. In this case, banks can use their liquid assets when

[18] The deposit ladder in my environment does not affect the probabilities of moving across each bank type and each bank is subject to the same transition probability matrix in contrast to Corbae and D'Erasmo (2020, 2022).

the realisation of the shock is low. As small and medium banks hold more deposits than large banks they experience the largest cost on short term debt further preventing their ability to save in liquidity. The presence of ex-ante liquidity regulation as in this environment provides a rational of why small and medium banks do not hold enough liquidity.

Let $v(\ell_{i,t}, d_{i,t+1}, d_{i,t})$ denote the expected lifetime value of dividends of banks beginning the period with states $b_0 \in B$ and, its corresponding vector be $v_\ell(\ell_{i,t}, d_{i,t+1}, d_{i,t}) \in \mathcal{V} : \mathbb{R}^2_+ \to \mathbb{R}$ such that $\{v(\ell_{i,t}, d_{i,t+1}, d_{i,t}) : \{\ell_{i,t}, d_{i,t+1}, d_{i,t}\} \in B\}$ where $\mathcal{V}$ is the set of all continuous real valued functions satisfying $L \times D \to \mathbb{R}$. Given the sequences of measurable functions $c_{i,t}(d^t; b_0) : B \to \mathbb{R}_+$ and $\ell_{i,t+1}(d^t; b_0) : D^t \to L \in \mathbb{R}_+$ describing the bank plan, the program of the individual bank is to maximise equation (2) subject to equation (5) and (9) taking as a given the interest rates r_ℓ, r_d, r_s, and initial bank state, $b_0 = (\ell_0, d_0)$. The value of a bank is then given by

$$v(\ell_{i,t}, d_{i,t+1}, d_{i,t}) = \max_{\left\{\ell_{i,t+1} \geq \ell^* \in \mathbb{R}_+, c_{i,t} \in [0, \sum_{d^t \in \mathcal{D}^t} \mu^t(b_0, d^t) \mathcal{P}_{i,t+1}(d^t; b_0)]\right\}} u(c_{i,t}) + \beta \sum_{d^t \in \mathcal{D}^t} \mu^t(b_0, d^t) v(\ell_{i,t+1}, d_{i,t+2}, d_{i,t+1}) \tag{10}$$

$$c_{i,t}(d^t; b_0) + s + \ell_{i,t+1}(d^t; b_0) - d_{i,t+1} = r_s s + r_\ell \ell_{i,t}(d^{t-1}; b_0) - r_d d_{i,t} + (r_\ell - r_d)(d_{i,t+1} - d_{i,t}) \tag{11}$$

$$a_{i,t+1}(d^t; b_0) = \ell_{i,t+1}(d^t; b_0) + s + \mathcal{P}_{i,t+1}(d^t; b_0) - d_{i,t+1}, \tag{12}$$

$$\tilde{\ell}^*_{i,t+1} = \ell_{i,t+1}(d^t, b_0) + (d_{i,t+1} - d_{i,t}), \tag{13}$$

$$\tilde{\ell}^*_{i,t+1} \geq \phi, \quad c_{i,t}(d^t, b_0) \geq 0, \tag{14}$$

where I have rewritten the individual banks' programme as a function of the individual states $\{\ell_{i,t}, d_{i,t+1}, d_{i,t}\}$ by making use of equation (12) that is the bank balance sheet identity.

Let ℓ^* be the global minimum attainable level of bank liquidity. Any constrained banks will hold an amount of liquidity in the interval $\ell_{i,t+1} \in [\ell^*, \bar{\ell}_i]$, where $\bar{\ell}_i$, denotes the maximum constrained level of bank liquidity which is bank type dependent. Any unconstrained bank will be such that, $\ell_{i,t+1} > \bar{\ell}_i \in L$.

The impossibility to trade liquid assets amongsts banks within a period implies that a known result in economies with incomplete markets (Bewley, 1986; Huggett, 1993; and Aiyagari, 1994) is that, $\beta r_\ell < 1$, to give rise to a well defined distribution of banks' liquidity and to make banks owing precautionary behaviors towards liquid assets. This condition, therefore, guarantees the existence of stationary long run values of liquid assets. Importantly, it establishes the existence of an upper bound on the accumulation of liquid assets by banks ensuring an inverse relation between banks' dividends consumption smoothing and bank total networth. Banks can then grow in size given their accumulation of liquidity.

In equilibrium, the market for liquid assets clears. Denoting by $\Upsilon_t(\ell_{i,t+1}, d_{i,t+1}, d_{i,t}) : \mathfrak{B} \to [0, 1]$ the cross sectional distribution over banks' liquidity and the idiosyncratic state for deposits and, the global minimum liquidity requirement with ℓ^*, then

$$\int_{\ell^*}^{\infty} \sum_{d^t \in D^t} \ell_{i,t+1}(d^t; b_0)\mu^t(b_0, d^t) d\Upsilon(\ell_{i,t+1}, d_{i,t+1}, d_{i,t}) = \mathcal{L}, \quad t \geq 1, \tag{15}$$

with $\mathcal{L} \in [0, \infty)$ being the aggregate level of liquidity.

The analysis has assumed a strictly positive interest rate on liquidity, r_ℓ. Relaxing this assumption does not exclude the incentive for banks to hold liquid assets since the only requirement for stationarity on liquid assets is an upper bound implied by $\beta r_\ell < 1$. Also note that with $r_s > r_\ell$, then $\ell_{i,t+1}(d^t; b_0) > 0$ still holds because of banks' precautionary behavior whilst illiquid assets are still exogenous. Eliminating any difference between the two assets would make banks indifferent such that $r_s = r_\ell$. Furthermore, risk sharing in this environment does not occur and would require one period economy in which banks commit on their promises to share deposit contracts between them. Appendix B contains further theoretical results that exploit the properties of the model and shed light on the bank heterogeneity and liquidity friction features.

Definition 1 A *stationary competitive equilibrium* for this economy is a set of function sequences $\{c_{i,t}(d^t; b_0)\}_{t=0}^{\infty}$ and $\{\ell_{i,t+1}(d^t; b_0)\}_{t=0}^{\infty}$, value function $\{v_{i,t}(d^t; b_0)\}_{t=0}^{\infty}$, $\forall b_0 \in B$ and for each $d_t \in D^t$, sequence of probability measures $\{\mu^t(b_0, d^t)\}_{t=0}^{\infty}$, strictly positive constant interest rates r_ℓ, r_s and r_d, initial distribution over liquid assets and deposits, $\Upsilon(\ell_{i,t+1}, d_{i,t+1}, d_{i,t})$, such that for each history $d^t \in D^t$ and $b_0 \in B$:

1. $c_{i,t}(d^t; b_0)$, $\ell_{i,t+1}(d^t; b_0)$ and $v_{i,t}(d^t; b_0)$ solve the bank optimisation programme, given the interest rates r_ℓ, r_s and r_d and probability measures $\{\mu^t(b_0, d^t)\}_{t=0}^{\infty}$.
2. Consistency condition for the probability measure satisfies its law of motion with transition probability matrix $\psi_{i,j}$; therefore, $\mu^t(b_0, d^t) = \mu^{t-1}(b_0, d^{t-1})\psi_{i,j}$.

4.3 Equilibrium Characterisation

The amount of liquidity that banks need to have in the balance sheet is controlled by the liquidity requirement that ensures a nonnegative amount of liquidity in case of a deposit withdrawal. Recall I identify with ℓ^* the global minimum liquidity requirement, that is the lowest liquidity requirement admissible by the model. The lower this value, the larger the assets banks can use to save in liquidity. The balance sheet identity of the model at time t is $\ell_{i,t}(d^{t-1}; b_0) = d_{i,t} + \mathcal{P}_{i,t} - s$ and I use it as a corner solution. In particular, I consider a case in which in each period banks finance liquidity and illiquid assets through deposits and banks do not retain profits for equity, $\mathcal{P}_{i,t} = 0$.

Proposition 1 provides a necessary condition for the existence of ℓ^* and shows that this value, ℓ^*, is not bank size dependent but the liquidity requirement is because of the nature of the liquidity constraint. A liquidity requirement closes or equal to zero implies that banks have less liquidity in their balance sheet and the liquidity constraint is more likely to bind. The intuition follows since a liquidity requirement closes to zero gives more margins to banks to use more deposits base in loans and dividends and keep less in required liquidity. Therefore, more liquidity can be spread in the economy.

Proposition 1 *The global minimum level of liquidity requirement, ℓ^*, is independent of bank size and does not vary across banks. Moreover, the global liquidity requirement is a nonnegative value.*

Proof. In Appendix **??**. □

Consequently, since Proposition 1 shows that the global minimum liquidity requirement equals the liquidity requirement, ϕ, then it ensures that $\phi \in [0, 1]$. Therefore any deposit withdrawal bounces around the banking system between banks and with banks holding a nonnegative amount of liquidity.

I can then characterise the value of banks receiving an inflow of deposits with the value of banks experiencing an outflow of deposits as $v^{in}(\tilde{\ell}_{i,t+1}, d_{i,t+1}, d_{i,t})$ and $v^{out}(\tilde{\ell}_{i,t+1}, d_{i,t+1}, d_{i,t})$, respectively. Both are defined as the value of current and expected dividends consumption of inflow and outflow banks. They are given by

$$v^{in}(\tilde{\ell}_{i,t+1}, d_{i,t+1}, d_{i,t}) = u(c^{in}_{i,t}) + \beta \sum_{d^t \in D^t} v^{in}(\ell_{i,t+1}, d_{i,t+2}, d_{i,t+1})\psi(d_t, dd_{t+1}) \tag{16}$$

$$v^{out}(\tilde{\ell}_{i,t+1}, d_{i,t+1}, d_{i,t}) = u(c^{out}_{i,t}) + \beta \sum_{d^t \in D^t} v^{out}(\ell_{i,t+1}, d_{i,t+2}, d_{i,t+1})\psi(d_t, dd_{t+1}). \tag{17}$$

While I identify with $c^{in}_{i,t}$ and $c^{out}_{i,t}$ dividends consumption of inflow and outflow banks, there is still a pool of heterogeneous banks within each type of banks. Within the intraperiod the size of banks deposits is given by the deposit withdrawal that is $(d_{i,t+1} - d_{i,t}) > 0$ for inflows banks and $(d_{i,t+1} - d_{i,t}) < 0$ for outflow banks. Specifically, the width of the deposits withdrawal in terms of the maximum and minimum level drives bank size expansion. Let j be the superscript index identifying the type of a bank, $j = \{in, out\}$. This leads to the next proposition.

Proposition 2 *Let $(d_{i,t+1} - d_{i,t})$ be the size of the inflow and outflow of banks. Then $v^{in}(\tilde{\ell}_{i,t+1}, d_{i,t+1}, d_{i,t}) > v^{out}(\tilde{\ell}_{i,t+1}, d_{i,t+1}, d_{i,t})$ and, the value of a bank $v^j(\tilde{\ell}_{i,t+1}, d_{i,t+1}, d_{i,t})$ is increasing in the size of banks' deposits.*

Proof. In Appendix **??**. □

Proposition 2 highlights that the nature of the idiosyncratic deposit withdrawal shock contributes to the expansion in banks' value. The reason is that any increase in deposits tights the liquidity constraint which leads banks to save more in liquidity, increasing their networth and expanding in size. The expansion in banks' size occurs since the direct effect of the inflow of deposits on the liquidity requirement expands the set of liquidity, which in turns raises banks' networth. Then, gathering deposits through an idiosyncratic event reflects the ability of banks to improve their value and expand in size.

Proposition 3 establishes and demonstrates the size expansion of individual banks. Let $\delta_{\tilde{d}} = (d_{i,t+2} - d_{i,t+1})$ be the units of deposits which translates in δ_ℓ units of liquid assets. Both units range in the sets $D_\delta := [\underline{\delta}_d, \overline{\delta}_d]$ and $L_\delta := [\underline{\delta}_\ell, \overline{\delta}_\ell]$, respectively. Lastly, let $D_{\tilde{d}}$ and $L_{\tilde{\ell}}$ be the composite sets of deposits and liquidity for inflow banks receiving δ_ℓ units of liquid assets.

Proposition 3 *The inflow of deposits leads to the expansion in banks' size by expanding deposits and subsequently the liquid asset sets. If* $(d_{i,t+2} - d_{i,t+1}) > 0$ *and* $(d_{i,t+2} - d_{i,t+1}) > (d_{i,t+1} - d_{i,t})$ *then* $D \subseteq D_{\tilde{d}}$ *and* $\ell^1_{i,t+1} > \ell^0_{i,t+1}$*, therefore* $L \subseteq L_{\tilde{\ell}}$*.*

Proof. In Appendix **??**. □

Since the value function is increasing in liquidity, by increasing available resources the inflow of deposits expands the set of liquid assets in a banks' balance sheet, thereby raising their size.

With the theoretical properties above, I can prove the existence of a stationary recursive competitive equilibrium. To set up the proposition, define with $(Tv)(\ell_{i,t}, d_{i,t+1}, d_{i,t})$ the operator which ensures $v(\ell_{i,t}, d_{i,t+1}, d_{i,t}) \in B$ is the fixed point of the contraction mapping $T : B \to B$ of banks' programme.

Proposition 4 *There exists a recursive competitive equilibrium such that* $v^*(d^t, b_0)$ *is the unique banks' value function with* $v^*(d^t, b_0) = Tv^*(d^t, b_0)$*. The unique value function* $v^*(d^t, b_0)$ *is continuous, increasing and bounded in liquid assets* $\ell_{i,t}(d^t, b_0)$ *for inflow and outflow banks, it is decreasing in deposits* $d_{i,t}$ *and increasing in the inflow/outflow effect* $(d_{i,t+1} - d_{i,t})$*. The contraction mapping leads to a unique decision correspondence that is compact valued and upper hemicontinuous.*

Proof. In Appendix **??**. □

The results in Proposition 4 guarantees the existence of a well defined equilibrium. Key to prove the proposition is the continuity of banks' value of expected cashflow function and the satisfaction of Blackwell sufficient conditions for a contraction. Let $\mathcal{S}(B, \mathcal{D}(D))$ be space of probability measures with $(B, \mathcal{D}(D))$ the measurable space and $\mathcal{D}(D)$ the Borel sigma algebra. The next proposition ensures the existence of an invariant distribution which is unique.

Proposition 5 *There exists a unique invariant distribution* $\Upsilon^*(\ell_{i,t+1}, d_{i,t+1}, d_{i,t})$ *specified over the measurable selection with contraction mapping* $T^* : \mathcal{S}(B, \mathcal{D}(D)) \to \mathcal{S}(B, 2^{\mathcal{B}} \times \mathcal{D}(D))$ *such that* $\Upsilon^*(\ell_{i,t+1}, d_{i,t+1}, d_{i,t}) = T^*\Upsilon^*(\ell_{i,t+1}, d_{i,t+1}, d_{i,t})$*.*

Proof. In Appendix **??**. □

By proving existence and uniqueness of the invariant distribution, Proposition 5 describes the evolution of the distribution of banks over their states.

The unique equilibrium shows sorting of banks by type whereby banks with less deposits hold more liquidity than small banks, who own liquidity the least. This implies that small banks' dividends are greater and the mass of banks in the industry is smaller. Let $v_{\ell_{i,t},d_{i,t}}(\ell_{i,t}, d_{i,t+1}, d_{i,t})$ be the value of banks at a given pair of current liquidity and dividends $(\ell_{i,t}, d_{i,t})$, holding unrealised the future level of deposits.

Proposition 6 *In any equilibrium the model features negative sorting of banks by type such that* $v_{\bar{\ell},\underline{d}}(\ell_{i,t}, d_{i,t+1}, d_{i,t}) \geq v_{\bar{\ell},\bar{d}}(\ell_{i,t}, d_{i,t+1}, d_{i,t})$ *and* $v_{\ell^*,\bar{d}}(\ell_{i,t}, d_{i,t+1}, d_{i,t}) \geq v_{\ell^*,\underline{d}}(\ell_{i,t}, d_{i,t+1}, d_{i,t})$. *Moreover, in any ex-post equilibrium of the model after the realisation of deposits it follows that* $v_{\bar{\ell},\bar{d},d_{t+1}=\bar{d}}(\ell_{i,t}, d_{i,t+1}, d_{i,t}) - v_{\bar{\ell},\underline{d},d_{t+1}=\underline{d}}(\ell_{i,t}, d_{i,t+1}, d_{i,t}) < 0$ *and* $v_{\ell^*,\underline{d},d_{t+1}=\underline{d}}(\ell_{i,t}, d_{i,t+1}, d_{i,t}) - v_{\ell^*,\bar{d},d_{t+1}=\bar{d}}(\ell_{i,t}, d_{i,t+1}, d_{i,t}) < 0$.

Proof. In Appendix **??**. □

The proposition shows that the type of banks is inversely related to the holding of liquidity, therefore high deposits is associated with small banks. In the first part it shows that large banks with small deposits prefer more liquidity than small banks who prefer more dividends consumption. Moreover, banks with more deposits will have stochastically more deposits in the next period. Second, the negative sorting result holds also in the stochastic equilibrium after the idiosyncratic realisation of the shock.

I now present three theoretical results brought by the heterogeneous banks model. First, I show that the equilibrium exhibits a reduction in banks' liquid assets inequality as the liquidity constraint becomes more stringent. Second, I consider how banks' liquidity decision rule responds to different interest rates on liquidity. I show that the resulting decision rule is nonlinear and admits a point of inflection. Third, I analyse the relationship between the sorting of banks by type and banks' size expansion. I show that even small banks can expand in size as a consequence of an idiosyncratic change in deposits.

The distribution of banks' liquid assets is sensitive to the liquidity requirement. This becomes relevant since any change in the liquidity requirement affects the degree of inequality of banks' liquidity. To isolate the effect on banks' liquid assets inequality, let $\eta(\ell_i)$ and $\mu(\eta(\ell_i))$ represent the cumulative proportion of banks and the Gini coefficient−a measure of dispersion of inequality−respectively. Let L be the set of banks' liquidity choices specified above. I show that an increase in the liquidity requirement leads to an increase in the fraction of liquid assets of constrained banks in the lower left tail of the distribution and a decline in the inequality in banks' liquidity.

Proposition 7 *Tightening the liquidity requirement makes banks' liquidity less unequally distributed. For each* $\phi_2 \geq \phi_1$ *if* $\phi_1 \in L$ *then also* $\phi_2 \in L$*, then* $\mu_2(\eta(\ell_i)) < \mu_1(\eta(\ell_i))$.

Proof. In Appendix **??**. □

The intuition for the result in Proposition 7 follows since a more stringent liquidity requirement requires more banks to increase their amount of liquid assets. This means that a larger population fraction of constrained banks needs to meet the liquidity requirement. This population fraction contains the share of medium banks that prior to the rise in the liquidity requirement were unconstrained banks. Banks that turn from unconstrained to constrained want to keep liquidity equal and above the their previous level. Consequently, the precautionary liquidity motive rises.

Therefore, the more stringent liquidity requirement is not only a mere shift of the distribution of banks' liquidity assets, but−because it changes the constrained bank's programme of liquidity and dividends−it impacts the incentive for banks to invest in liquidity. This depends on their exposure to the the risk of a deposit withdrawal and banks with little liquidity−constrained banks−are heavily exposed to the risk of a deposit withdrawal. The liquidity imposed by the tighter liquidity requirement gives banks more protection against the deposit withdrawal. Moreover, since a tighter liquidity requirement imposes banks to hold more liquidity, then the value of a bank rises.

To gauge more understanding, it is important to emphasize that the population fraction of banks that rises belongs to the bottom percentile of liquid assets distribution, that is constrained small and medium banks. This fraction of banks increases more than the upper tail−top percentiles of banks' liquid assets distribution, which by contrast declines. This is because the upper tail of the distribution contains unconstrained banks who are less exposed to the risk of a deposit withdrawal which depends on their amount of liquid assets. Since they are unconstrained, their liquidity constraint may not bind despite the tighter liquidity requirement. For those unconstrained banks whose liquidity requirement becomes binding instead, the imposition of a tighter liquidity requirement may cause them to turn constrained thereby reducing the top percentiles of banks. Therefore, there is a reallocation of liquidity between banks. As a result, banks' liquid assets distribution becomes less concentrated towards few banks−the upper tail of the distribution−and inequality in banks' liquidity declines.

Turning to the equilibrium along a sequence of interest rates on liquidity, there are different values of the interest rate on liquidity that are consistent with the properties of the model. Proposition 8 shows that banks' liquidity is expanding with the interest rate on liquidity and banks' dividends decline. The model features complementarity between banks' decision rules around the inflection point whereby liquidity and dividends decrease jointly. This complementarity result follows from outflow banks who value less dividends consumption and liquidity subsequent the deposit withdrawal. In particular, outflow banks reduce liquidity and dividends around the inflection point since they are paying the wedge on deposit withdrawal. This shrinks their dividends and outweighs the benefit of inflow banks to increase liquidity driving down the total effect on banks' liquidity.

Proposition 8 *Let r_ℓ^* be the value of the interest rate on liquidity that solves the stationary equilibrium. For a given finite sequence of the interest rate on liquidity $\{r_{\ell,j}\}_{j=1}^{n}$, it is always optimal for banks to hold liquid assets if the value of the interest rate on liquidity is not too small. In a small neighborhood of r_ℓ^* banks' liquidity reverts and becomes non monotonic such* *that at r_ℓ^* banks' liquidity admits a point of inflection.*

Proof. In Appendix **??**. □

Proposition 8 states that banks' liquidity and dividends consumption are substitute for a given interest rate on liquidity, and in a small enough neighborhood of the liquidity rate they become complement due to the occasionally binding liquidity constraint. In other words, banks reduce consumption and liquidity jointly despite the positive liquidity rate in an illiquidity event that put them tight at the liquidity constraint and therefore, they must use their liquid assets to cover their illiquidity.

The next result pertains to the sorting of banks by type and how it relates to the banks' size expansion. Proposition 9 demonstrates that small banks with large deposits and therefore, worse initial conditions because they hold more idiosyncratic risk, can also expand in size. By contrast, for large banks a weaker result holds. In particular, the least large banks can expand in size while the largest banks bear the deposit withdrawal. This result further supports the limit on the expansion in size of large banks proved in Proposition 3 and confirms the existence of an upper bound on the expansion of banks' liquid assets set. Let $v_{\ell^*,\underline{d},d_{t+1}}(\ell_{i,t},d_{i,t+1},d_{i,t})$ be the least small banks and $v_{\ell^*,\overline{d},d_{t+1}}(\ell_{i,t},d_{i,t+1},d_{i,t})$ be the smallest banks. Moreover, assume $v_{\overline{\ell},\underline{d},d_{t+1}}(\ell_{i,t},d_{i,t+1},d_{i,t})$ defines the largest banks and $v_{\overline{\ell},\overline{d},d_{t+1}}(\ell_{i,t},d_{i,t+1},d_{i,t})$ defines the least large banks.

Proposition 9 *If $d_{i,t+1} = \underline{d}$ endogenous banks' size expansion arises for inflow banks $v_{\ell^*,\overline{d},d_{t+1}=\underline{d}}(\ell_{i,t},d_{i,t+1},d_{i,t})$ and $v_{\overline{\ell},\overline{d},d_{t+1}=\underline{d}}(\ell_{i,t},d_{i,t+1},d_{i,t})$ such that $v_{\ell^*,\underline{d},d_{t+1}=\overline{d}}(\ell_{i,t},d_{i,t+1},d_{i,t}) - v_{\ell^*,\overline{d},d_{t+1}=\underline{d}}(\ell_{i,t},d_{i,t+1},d_{i,t}) < 0$ and $v_{\overline{\ell},\underline{d},d_{t+1}=\overline{d}}(\ell_{i,t},d_{i,t+1},d_{i,t}) - v_{\overline{\ell},\overline{d},d_{t+1}=\underline{d}}(\ell_{i,t},d_{i,t+1},d_{i,t}) < 0$. Therefore, each bank with a given liquidity level holds more deposits than any other bank with smaller liquidity. If $d_{i,t+1} = \overline{d}$ then an outflow of deposits occurs for outflow banks such that $v_{\ell^*,\underline{d},d_{t+1}=\overline{d}}(\ell_{i,t},d_{i,t+1},d_{i,t}) - v_{\overline{\ell},\underline{d},d_{t+1}=\overline{d}}(\ell_{i,t},d_{i,t+1},d_{i,t}) < 0$ and $v_{\ell^*,\overline{d},d_{t+1}=\underline{d}}(\ell_{i,t},d_{i,t+1},d_{i,t}) - v_{\overline{\ell},\overline{d},d_{t+1}=\underline{d}}(\ell_{i,t},d_{i,t+1},d_{i,t}) < 0$. The extended set $D_{\tilde{d}}$ is measurable and the state space admits the extended measurable space defined as $(D_{\tilde{d}} \times L_{\tilde{\ell}}, \mathcal{D}(D_{\tilde{d}}))$.*

Proof. In Appendix **??**. □

The aftermath of Proposition 9 comes primarily from the fact that with an endogenous rise in deposits banks can operate with a expanded balance sheet. The inflow of deposits tights the liquidity constraint for the smallest banks and least large banks leading them to raise their liquidity and therefore their size. The tightening of the liquidity constraint comes from the inflow of deposits as opposite to a rise in ϕ shown in Proposition 7. The sorting of banks by type whilst negative is beneficial in this economy since even small banks can expand in size. This result explains therefore the need of ex-ante liquidity regulation even in the absence of deposit outflows and the designation of the liquidity requirement in the level of deposits more than the ratio which has to be controlled only once the change in deposits has been taken into account in the regulation.

4.4 Ex-post Liquidity Regulation and Equivalence

In this subsection I consider an environment in which the regulator imposes a liquidity regulation ex-post the occurrence of the disturbance. This means banks do not need to internalise the ex-ante liquidity regulation and they only need to meet an ex-post liquidity requirement. I explain the relation with Holmström and Tirole (1998) model and discuss how the present model differ from Holmström and Tirole (1998) in which the liquidity requirement, in a three period environment, does not affect banks' distribution. I retain all the other model features described in previous subsections taking into account that the economy is a decentralised environment where banks obtains the liquidity regulation sets by the government as external description of the model environment.

The interpretation of this assumption is that the timing at which the intervention of the regulation occurs, becomes relevant for the attentiveness of banks towards liquidity needs. Since with the ex-post regulation banks become less attentive towards liquidity needs, this leads to a reallocation of banks' liquidity. In particular, by not internalising the ex-ante liquidity requirement the distortion in banks' budget constraint and profits does not arise anymore. Consequently, a reallocation of banks' liquidity occurs since banks belonging to the bottom percentiles of the liquidity distribution end up holding more liquidity than the baseline economy in which the regulation is imposed prior to the deposit outflow. The end of the intraperiod banks' budget constraint is now given by

$$\tilde{c}_{i,t}(d^t; b_0) + s + \tilde{\ell}^{**}_{i,t+1}(d^{t+1}; b_0) - d_{i,t+1} = r_s s + r_\ell \tilde{\ell}^{**}_{i,t}(d^t; b_0) - r_d d_{i,t}, \quad \forall d^{t+1} \in D^{t+1}, t = 0, 1, ..., \tag{18}$$

which makes clear that there is no direct effect of the inflow/outflow of deposits on intermediation profits. Furthermore, it needs to take into account the new maximiser of banks' liquidity $\tilde{\ell}^{**}_{i,t+1}(d^{t+1}; b_0)$, optimal value of dividends consumption $\tilde{c}_{i,t}(d^t; b_0)$ and associated probability measure $\tilde{\mu}^{t+1}(b_0, d^{t+1})$ while keeping unchanged the transition probability matrix $\psi(d_{t+1}, d_t)$.[19]

Let $\tilde{v}(\tilde{\ell}_{i,t}, d_{i,t+1}, d_{i,t})$ and $\tilde{\mathcal{P}}_{i,t}(d^t; b_0)$ be the expected lifetime value of dividends and profit function of banks not internalising the liquidity regulation at the beginning of the intraperiod. Then, banks solve the following optimal dynamic programme

$$\begin{aligned} \tilde{v}(\tilde{\ell}_{i,t}, d_{i,t+1}, d_{i,t}) = & \max_{\{\tilde{\ell}^{**}_{i,t+1} \geq \ell^* \in \mathbb{R}_+, \tilde{c}_{i,t} \in [0, \sum_{d^{t+1} \in \mathcal{D}^{t+1}} \mu^{t+1}(b_0, d^{t+1}) \tilde{\mathcal{P}}_{i,t+1}(d^{t+1}; b_0)]\}} u(\tilde{c}_{i,t}) \\ & + \beta \sum_{d^{t+1} \in \mathcal{D}^{t+1}} \tilde{\mu}^{t+1}(b_0, d^{t+1}) \tilde{v}(\tilde{\ell}_{i,t+1}, d_{i,t+2}, d_{i,t+1}) \end{aligned} \tag{19}$$

[19]The dependency of decision rule $\tilde{\ell}^{**}_{i,t+1}(d^t; b_0)$ on initial condition and deposit history is used to identify the endogenous optimal decision rule, in contrast to ℓ^* that is the global minimum liquidity requirement specified in Subsection 4.3.

$$\tilde{c}_{i,t}(d^t;b_0) + s + \tilde{\ell}^{**}_{i,t+1}(d^{t+1};b_0) - d_{i,t+1} = r_s s + r_\ell \tilde{\ell}^{**}_{i,t}(d^t;b_0) - r_d d_{i,t} \tag{20}$$

$$\tilde{a}_{i,t+1}(d^{t+1};b_0) = \tilde{\ell}^{**}_{i,t+1}(d^{t+1};b_0) + s + \tilde{\mathcal{P}}_{i,t+1}(d^{t+1};b_0) - d_{i,t+1}, \tag{21}$$

$$\tilde{\tilde{\ell}}_{i,t+1} = \tilde{\ell}^{**}_{i,t+1}(d^{t+1},b_0) + (d_{i,t+1} - d_{i,t}), \tag{22}$$

$$\tilde{\tilde{\ell}}_{i,t+1} \geq \phi, \quad \tilde{c}_{i,t}(d^t,b_0) \geq 0, \tag{23}$$

where banks have not taken into account the ex-ante liquidity regulation but only ex-post, and the budget constraint in equation (20) is completely unaffected by the distortion of the deposit withdrawal. Therefore, the optimal choice of liquidity is in turn independent of the inflow/outflow effect. The profit function is now $\tilde{\mathcal{P}}_{i,t}(d^t;b_0) = r_s s + r_\ell \tilde{\ell}^{**}_{i,t}(d^t;b_0) - r_d d_{i,t}$. Let $\tilde{\Upsilon}_t(\tilde{\ell}^{**}_{i,t+1}, d_{i,t+1}, d_{i,t}) : \mathfrak{B} \to [0,1]$ be the associated cross sectional distribution over banks' liquidity and the idiosyncratic state for deposits then

$$\int_{\ell^*}^{\infty} \sum_{d^t \in D^t} \tilde{\ell}^{**}_{i,t+1}(d^{t+1};b_0)\tilde{\mu}^{t+1}(b_0,d^{t+1})d\tilde{\Upsilon}(\tilde{\ell}^{**}_{i,t+1}, d_{i,t+1}, d_{i,t}) = \tilde{\mathcal{L}}, \quad t \geq 1, \tag{24}$$

equation (24) clears the market for liquid assets.

There are three point worth noting. First, banks experiencing the inflow/outflow of deposits can be any bank along the size distribution including the small and least profitable banks. Since the inflow/outflow has not effect on current resources and retained earnings, expansion in banks' value does not arise anymore. Therefore, banks are less likely to expand in size through liquidity. Proposition 3 no longer holds under this modification. My model presented in the previous subsections helps to smooth the overaccumulation in banks' liquidity, thereby generating more inequality as observed in the data.

Second, the modification of this assumption leads the baseline model to nest Holmström and Tirole (1998) environment as a special case, however in an infinite time and dynamic context. In fact, without the intervention of the liquidity regulation ex-ante, the liquidity constraint is less likely to bind which in turn leads to high aggregate liquidity. In this case, an inflow of deposits lessens the liquidity constraint and no bank finds itself at the lowest liquidity requirement. Third, the central equivalence result of the paper proves the equivalence between the baseline equilibrium with the ex-ante liquidity regulation and size dependent liquidity constraint under a tightening of the liquidity requirement and, the equilibrium in which the intervention of the liquidity regulation occurs ex-post only, therefore banks do not internalise it ex-ante to the deposit shock. Since a tightening of the liquidity requirement ϕ leads banks to store more liquidity as proved in Proposition 7, small and medium banks become less constrained which makes the baseline and ex-post regulation equilibria to coincide. However, while in the former case more equality is brought by more liquidity being held by constrained medium and small banks who needs liquidity the most, in the latter case the less binding liquidity constraint makes the distribution of banks' liquidity more equally distributed.

Moreover, while both equilibria are inefficient, in the first case the inefficiency is due to market incompleteness and uninsurable liquidity risk, in the ex-post regulation equilibrium the inefficiency arises since the ex-post distribution of banks' liquidity is inefficient: the liquidity market is unable to allocate liquidity efficiently. Consequently, inflow banks with low deposit withdrawal shocks hold more liquidity than what they need whereas outflow banks bearing large deposit withdrawals do not save enough since they lack sufficient liquidity instruments. In this instance, the only way for outflow banks to face the deposit withdrawal is to use their whole retained earnings. As a result, they will start the following period with zero dividends consumption. Relative to Holmström and Tirole (1998), my model can explain differences of liquidity regulations along the size distribution of banks. Indeed, the reallocation of banks' liquidity rests on inflow banks located in the bottom percentiles of the liquidity distribution.

Definition 2 A *stationary competitive equilibrium in the ex-post regulation* for this economy is a set of function sequences $\{\tilde{c}_{i,t}(d^t;b_0)\}_{t=0}^{\infty}$ and $\left\{\tilde{\ell}^{**}_{i,t+1}(d^{t+1};b_0)\right\}_{t=0}^{\infty}$, value function $\{\tilde{v}_{i,t}(d^t;b_0)\}_{t=0}^{\infty}$, $\forall b_0 \in B$ and for each $d_{t+1} \in D^{t+1}$, sequence of probability measures $\{\tilde{\mu}^{t+1}(b_0, d^{t+1})\}_{t=0}^{\infty}$, strictly positive constant interest rates r_ℓ, r_s and r_d, initial distribution over liquid assets and deposits, $\tilde{\Upsilon}(\tilde{\ell}^{**}_{i,t+1}, d_{i,t+1}, d_{i,t})$, such that for each history $d^{t+1} \in D^{t+1}$ and $b_0 \in B$:

1. $\tilde{c}_{i,t}(d^t;b_0)$, $\tilde{\ell}^{**}_{i,t+1}(d^{t+1};b_0)$ and $\tilde{v}_{i,t}(d^t;b_0)$ solve the bank optimisation programme specified in equation (19), (20), (21), (22), (23), given the interest rates r_ℓ, r_s and r_d and probability measures $\{\tilde{\mu}^{t+1}(b_0, d^{t+1})\}_{t=0}^{\infty}$.
2. Consistency condition for the probability measure satisfies its law of motion with transition probability matrix $\psi_{i,j}$; therefore, $\tilde{\mu}^{t+1}(b_0, d^{t+1}) = \tilde{\mu}^t(b_0, d^t)\psi_{i,j}$.

I can now prove the existence of a stationary recursive competitive equilibrium.

Proposition 10 *There exists a recursive competitive equilibrium in the ex-post regulation economy such that $\tilde{v}^*(d^t, b_0)$ is the unique banks' value function with $\tilde{v}^*(d^t, b_0) = T\tilde{v}^*(d^t, b_0)$. The unique value function $\tilde{v}^*(d^t, b_0)$ is increasing and bounded in liquid assets $\tilde{\ell}^{**}_{i,t}(d^t, b_0)$ for inflow and outflow banks. The contraction mapping leads to a unique policy correspondence that is compact and upper hemicontinuous. The unique invariant distribution features $\tilde{\Upsilon}^*(\tilde{\ell}^{**}_{i,t+1}, d_{i,t+1}, d_{i,t}) = \tilde{T}^*\tilde{\Upsilon}^*(\tilde{\ell}^{**}_{i,t+1}, d_{i,t+1}, d_{i,t})$.*

Proof. In Appendix **??**. □

Recall that banks cannot raise external funds, as a result they only self finance themselves with liquidity and by retaining internal equity. Since the deposit withdrawal in the baseline economy leads to lower aggregate liquid assets in equilibrium, the losses from internalising liquidity friction are larger than the ex-post regulation equilibrium. This occurs since the willingness to save in liquidity in the economy with ex-ante regulation is greater than the baseline economy. However, since I am focussing on short term contracts, the liquidity losses of the baseline economy are smaller than the losses implied by a long term liquidity contract.

The larger inequality generated by the baseline economy is due to a reallocation of liquidity towards medium and large banks, as compared to the reallocation in the economy with ex-post regulation which by contrast, reallocates liquidity towards more small and medium banks in the liquidity distribution. Proposition 11 demonstrates the magnitude of losses from the ex-ante liquidity regulation relative to the economy with ex-post regulation and that they are strictly increasing in deposits. Let $\eta_{i,t}(\ell_{i,t}, d_{i,t}, d_{i,t+1})$ define the losses in terms of current and expected banks' value.

Proposition 11 *The losses $\eta_{i,t}(\ell_{i,t}, d_{i,t}, d_{i,t+1})$ from the ex-ante intervention of liquidity regulation are larger than ex-post regulation. Therefore, $\ell_{i,t+1}(d^t, b_0) \geq \tilde{\ell}^{**}_{i,t+1}(d^{t+1}, b_0)$ and $\eta_{i,t}(\ell_{i,t}, d_{i,t}, d_{i,t+1}) \leq 0$. Moreover, ex-post liquidity regulation reallocates liquidity towards the smallest banks, thereby $\mu_j(\eta(\ell_i)) \geq \mu_j(\eta(\tilde{\ell}^{**}_i))$ and inequality in banks' liquid assets is lower than the baseline economy.*

Proof. In Appendix **??**. □

The result of Proposition 11 follows since the liquidity friction is still present in the economy in which the intervention of liquidity regulation is ex-post. Therefore, the losses from the allocation of liquidity to banks that need it the least persist in the ex-post intervention economy, although in a smaller magnitude than the baseline economy. Furthermore, a regulator who considers the accumulation of liquidity of large banks more than the accumulation of small banks needs to regulate liquidity ex-ante imposing a liquidity requirement prior to the realisation of the shock. This ex-ante intervention rises inequality in banks' liquidity but reduces the losses of a liquidity crisis. By contrast, when the focus of the regulator is the accumulation of small banks' liquidity, then ex-post intervention of liquidity regulation reallocates liquidity towards the smallest banks thereby reducing inequality in banks' liquid assets.

The economy with ex-post liquidity regulation creates an incentive for small and medium banks to hold high future liquid assets. This incentive makes the ex-post equilibrium to be equivalent to a tighter liquidity requirement in the baseline equilibrium. Let $\tilde{\Upsilon}(\tilde{\ell}^{**}_{i,t+1}, d_{i,t+1}, d_{i,t})$ be the distribution of liquid assets and deposits in the economy with the ex-post-liquidity regulation. Proposition 12 demonstrates the equivalence between the ex-post regulation and the baseline economy that is subject to a tightening of the liquidity requirement.

Proposition 12 *The equilibrium of the ex-post regulation economy is an equilibrium of the tighter economy with ex-ante and ex-post intervention of liquidity regulations. The equilibrium of the tighter economy with ex-ante intervention of liquidity frictions is an equilibrium of the ex-post regulation economy. Then, $\ell_{i,t+1}(d^t; b_0) = \tilde{\ell}^{**}_{i,t+1}(d^{t+1}; b_0)$ and $c_{i,t}(d^t; b_0) = \tilde{c}_{i,t}(d^t; b_0)$. Furthermore, $\tilde{\Upsilon}(\tilde{\ell}^{**}_{i,t+1}, d_{i,t+1}, d_{i,t}) \leq \Upsilon(\ell_{i,t+1}, d_{i,t+1}, d_{i,t})$.*

Proof. In Appendix **??**. □

The result in Proposition 12 follows since in the ex-ante regulated economy the liquidity constraint is binding therefore it must be that the marginal propensity of banks towards current dividends consumption is larger than future dividends consumption. Since the liquidity constraint will also bind in the ex-post regulated economy, the two economies are equivalent and the lowest current dividends consumption allows liquidity to reallocate towards the bottom tail of the distribution.

The main takeway of this subsection is that the ex-ante intervention of liquidity regulation is superior to an environment in which banks do not internalise it at the beginning of the intraperiod. This is due for two reasons. First, the baseline model leads to a banks' size expansion through the accumulation of banks' liquidity driven by the inflow of deposits and the sorting of banks by type documented empirically. Second, being banks subject to ex-ante regulation it generates the more inequality in banks' liquidity as observed in the data since the least largest inflow banks accumulate liquidity the most. Therefore, the marginal utility of dividends consumption into the present becomes large and dividends fall. As a result, in the equilibrium with ex-post liquidity regulation small banks end up having zero dividends and the present value of cashflow distribution concentrates towards large banks, while liquidity reallocates towards small and medium banks.

In Appendix C, I consider a version of the liquidity constraint as being the ratio of the deposit withdrawal. Similarly to the ex-post regulation economy, in the economy with a liquidity ratio not only banks cannot expand in size anymore, but additionally banks are never liquidity constrained. Therefore, these results show the advantage of the baseline economy which imposes ex-ante liquidity regulations to banks leading them to internalise the deposit distortion prior and post the intraperiod of the deposit withdrawal.

4.5 Equilibrium with No Liquidity Regulation

This subsection considers the economy that abstracts from liquidity regulations. Similarly to the ex-post regulated economy the distortion in deposits in banks' budget constraint does not arise anymore. The banks' dynamic programme solves

$$\begin{aligned}\tilde{v}(\tilde{\ell}_{i,t}, d_{i,t+1}, d_{i,t}) = & \max_{\left\{\tilde{\ell}^{**}_{i,t+1} \geq \ell^* \in \mathbb{R}_+, \tilde{c}_{i,t} \in [0, \sum_{d^{t+1} \in \mathcal{D}^{t+1}} \mu^{t+1}(b_0, d^{t+1}) \tilde{\mathcal{P}}_{i,t+1}(d^{t+1}; b_0)]\right\}} u(\tilde{c}_{i,t}) \\ & + \beta \sum_{d^{t+1} \in \mathcal{D}^{t+1}} \mu^{t+1}(b_0, d^{t+1}) \tilde{v}(\tilde{\ell}_{i,t+1}, d_{i,t+2}, d_{i,t+1})\end{aligned} \tag{25}$$

$$\tilde{c}_{i,t}(d^t; b_0) + s + \tilde{\ell}^{**}_{i,t+1}(d^{t+1}; b_0) - d_{i,t+1} = r_s s + r_\ell \tilde{\ell}^{**}_{i,t}(d^t; b_0) - r_d d_{i,t} \tag{26}$$

$$\tilde{a}_{i,t+1}(d^{t+1}; b_0) = \tilde{\ell}^{**}_{i,t+1}(d^{t+1}; b_0) + s + \tilde{\mathcal{P}}_{i,t+1}(d^{t+1}; b_0) - d_{i,t+1}, \tag{27}$$

$$\tilde{c}_{i,t}(d^t, b_0) \geq 0. \tag{28}$$

The assumption of no regulation makes the feasible choice of banks to be unrestricted over the liquidity requirement, therefore Proposition 1 does not hold in this environment and the global minimum level of liquidity can be below $\tilde{\ell}^{**}_{i,t+1} = \phi$. It follows that banks may fall below the liquidity requirement to satisfy their optimal condition over liquidity and dividends. Without small and medium banks liquidity constrained, the liquidity contract generated by this economy is less likely to engender a stable contract in the intraperiod.

Proposition 13 *Suppose there is no liquidity regulation. The deposit wedge* $(d_{i,t+1} - d_{i,t})$ *does not affect banks' incentive towards liquidity and banks' continuation value, then i)* $\tilde{v}(\tilde{\ell}_{i,t}, d_{i,t+1}, d_{i,t}) < v(\ell_{i,t}, d_{i,t+1}, d_{i,t})$ *with* $(r_\ell - r_d)d_{i,t+1} - r_\ell d_{i,t} \lesseqgtr -r_d d_{i,t}$ *and, ii)* $\tilde{\ell}^{**}_{i,t+1}(d^{t+1}; b_0) < \phi$ *as well as* $\eta(\ell_i) \leq \phi$ *that is the optimal choice of banks' liquidity and the fraction of banks fall below the liquidity requirement, consequently banks are never liquidity constrained.*

Proof. In Appendix **??**. □

Proposition 13 elucidates that the absence of the regulation decreases the interest rate on deposits and raise the total cost on short term debt. As the constraint set is now empty banks are indifferent between accumulating liquidity or paying dividends and small and medium banks are misleadingly treated as unconstrained banks. Consequently, medium and small banks who need to store more liquidity hold less than under the ex-ante liquidity regulation. In fact since nothing prevents banks from trading liquidity contracts more often, they accumulate less liquidity and are willing to forgo current dividends for future dividends payments and use current equity to pay any outflow of deposits. Moreover, by not affecting directly banks' liquidity incentives with inflow of deposits there does not need to be a limit on the expansion of the liquid assets set since under these circumstances banks cannot expand in size. Liquidity regulations set ex-ante to the deposit shortfall guarantee that small and medium banks protect themselves against a sudden outflow of deposits.

The emphasis then is how a more equal distribution is brought under no regulation and ex-ante regulation with a tightening of the liquidity requirement. In the no regulation economy banks become indifferent in holding liquidity therefore small and medium banks found themselves below the liquidity requirement, likewise large banks reduce their incentive towards liquidity. In the ex-ante liquidity regulation economy small and medium banks at the liquidity constraint are forced to hold more precautionary liquidity against the outflow of deposits. Since in this latter case, all banks are holding relatively more liquidity the banking system becomes less vulnerable to liquidity crises driven by a deposit shortfall. As a proof of concept Appendix H illustrates the corresponding distributions.

The baseline theoretical model pronounces the need of ex-ante liquidity regulation and the introduction of liquidity requirements even in periods of no deposit outflows. My theory therefore discovers aftermaths that support the recent findings of Amador and Bianchi (2024) theory who however inspect the need for equity buffer and minimum capital requirements in a three period general equilibrium with bank runs. My theory however predicts that banks do not need to sell their assets to pay depositors in case of an outflow of deposits because the ex-ante regulation makes them sufficiently prudent towards liquidity. This occurs because banks take into account the rise of the cost on short term debt due to the liquidity regulation which creates a stable contract. Therefore, it becomes relevant to account not only for the design of the liquidity regulation which needs to take place in the deposit threshold but also for the timing of the imposition of the liquidity regulation. The model advocates the need of ex-ante liquidity regulations in addition to the requirements set by Basel III regulation since it reduces the exposure to losses in banks' value.

4.6 Extension with Stochastic Liquidity Rate

In the baseline economy banks' choices are unaffected by changes in the interest rate on liquidity, r_ℓ. In Appendix L, I develop the model to incorporate a stochastic liquidity rate consistent with the view that the interest rate on liquidity needs to vary. I assume that banks can experience a positive interest rate on liquidity with probability $\pi_{r_\ell^+}$ and a negative interest rate on liquidity with probability $\pi_{r_\ell^-}$, which I call the good state and the bad state, respectively. The joint transition probability between deposits and the interest rate on liquidity reflects interactions between the two idiosyncratic variations. Liquidity and dividends consumption decision rules depend on a larger self-insurance motive since banks have to withstand a negative remuneration on liquidity if the bad state occurs. In this version of the model I show that the main results about the sorting of banks by type and the endogenous banks' size expansion continue to hold. Furthermore, I show that the spread of variation in dividends consumption for each bank is wider than the baseline economy and that the probability of a liquidity crisis occurring rises. In the quantitative analysis, I choose the baseline economy because of its technical advantages.

5 Quantitative Analysis

This section presents model parameterisation, calibration and results of the heterogeneous bank model. Section 5.2 shows comparative statistics that analyse equilibrium properties of the model. Section 5.3 examines the dynamics of liquidity crises engendered by a deposit withdrawal.

5.1 Model Parameterization

One period is a quarter. I calibrate the model using the US from 2008q4 to end of 2019. I choose this time period since it is the period after the financial crisis in which the Federal Reserve started paying the interest rate on excess reserves balances in October 2008.[20]

[20] The Fed announced the normalisation of its balance sheet in June 14, 2017 by starting a gradual reduction of securities (Federal Open Market Committee, Addendum to the Policy Normalization Principles and Plans).

TABLE 4.—
Composition of liquid and illiquid assets and liquid liabilities

US assets and liabilities data					
Liquid assets		Deposits liabilities		Illiquid assets	
Total reserves	0.184	Deposits commercial banks	0.538	Commercial and industrial loans	0.052
		Demand deposits	0.053	Loans and leases	0.237
				Real estate loans	0.119
				Commercial real estate loans	0.053
		Other liquid deposits[21]	0.409	Consumer loans	0.037
				Bank credit	0.325
Total	0.184	Total	1	Total	0.823

NOTE. Components of liquid and illiquid assets and composition of deposits in the liability of a bank balance sheet. The values of liquid assets and deposits are computed as proportion of total deposits whilst values of illiquid assets are computed as proportion of total liquid and illiquid assets. Sources of the data are in Appendix J.

I consider two subsets of parameters. The first subset consists of parameters taken directly from the data and or the literature, while the second subset of parameters consists of those parameters that are generated from the model in order to match moments data targets. Table 4 reports composition of liquid assets, illiquid assets and deposits. The target measure of banks' liquidity is defined by total reserves held by depository institutions.

Deposits are made by the following categories: deposits for all commercial banks, demand deposits, checking accounts and saving accounts. I decided to not include time deposits since although large and small time deposits are predominantly retail deposits they are subject to a different idiosyncratic shock that affects the interest rate. Indeed in the US saving deposits are not subject to a liquidity requirement. The largest source of deposits in banks' balance sheet is commercial deposits and it constitutes 53.8% of total deposits.

I define illiquid assets as the sum of commercial and industrial loans, bank credit, loans and leases in bank credit, real estate loans, commercial real estate loans and consumer loans in banks' balance sheet.[22]

Summary of the external and calibrated parameters is presented in Table 5.

The functional form for banker preferences is,

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}, \quad \gamma > 0.$$

I set the constant relative risk aversion coefficient to $\gamma = 1$. For the US, the gross interest rate on deposits, r_d, is set to 0.688%, according to Drechsler, Savov and Schnabl (2017).[23]

[21] This voice contains the sum of checking accounts and saving accounts that after May 2020 were consolidated under the same voice Other liquid assets from the Board of Governors of the Federal Reserve database.

[22] Appendix J dispenses data sources for each data variable.

[23] A close estimate is provided by Corbae and D'Erasmo (2021) using the average cost of funds for a different

Table 5.—External and Calibrated Parameters

	US Calibration		
Parameter	Interpretation	Value	Source/Steady state target
	A. External parameters		
γ	Risk aversion parameter	1	Literature
s	Illiquid assets	0.823	Observed, $\tilde{s} = s/(s+\ell)$
r_ℓ	Liquidity rate	1.001	Annualised rate excess reserves 0.0673%
r_d	Deposit rate	1.006	Drechsler et al. (2017)
r_s	Loan interest rate	1.109	Assigned
	B. Calibrated parameters		
β	Subjective discount factor	0.99896	Constraint, $\beta r_\ell < 1$
ϕ	Liquidity requirement	0.118	Match target, $\ell/d, 0.184$
ρ_d	Persistence deposit withdrawal shock	0.944	Match annual target, ρ_d
σ_d	Standard deviation deposits	0.05	Match annual target, σ_d

Note. This table reports parameters and calibration of the model for the US economy, whose calculations and targets are reported in the main text.

The gross interest rate on liquidity, r_ℓ, is set to 1.001 consistent with annualised nominal excess reserve rate in the selected period of 0.0673%. The subjective discount factor, β, is parameterized to guarantee the existence of a stationary equilibrium, thus to satisfy the condition $\beta r_\ell < 1$ and ensuring that banks trade off liquid assets with dividends consumption. For my purpose I use a target for liquidity that reflects the aggregate liquidity to deposits ratio.

The larger the β, the larger the increase in banks' liquidity holdings as banks strongly want to insure against the liquidity risk. Given the interest rate on liquidity, any value of β that still satisfies the condition $\beta r_\ell < 1$ but is lower than the value that leads banks to stay closer to their constraint would attain a weak liquidity holding. The reason being that in the absence of a liquidity requirement, banks would always want to hold an indefinitely small amount of liquidity.[24]

The liquidity constraint parameter, ϕ, is calibrated to 0.118 to match the long run average liquidity to deposit ratio, 0.184, in the selected sample period. I therefore make use of two parameters including β for the identification of the calibration. Specifically while only the parameter ϕ is used to match the moment, the discount factor, β, is then calibrated accordingly. In particular, I find the value of β making sure that it satisfies the constraint $\beta r_\ell < 1$.

Deposit process dynamics. The deposit process deserves particular attention since deposits constitute the main funding source for banks. Therefore, one of the most important parameter is the frequency at which the deposit withdrawal shock hits each bank balance sheet. The model is at quarterly frequency. I have annual data on deposits. I thus calibrate the standard deviation and the persistence of the quarterly deposit process to match annual moments. The size of the shock adds to the parameter that determines the precautionary behavior of banks in holding

time period from the US Call Reports.

[24] Appendix J provides data sources of each variable.

liquid assets.

The deposit withdrawal shock, represented by the Markov chain $\psi(d_{t+1}|d_t)$, is the approximation of an AR(1) process in the log of deposits since the empirical literature points towards a log function for the characterization of the deposit withdrawal shock (Corbae and D'Erasmo, 2021). Hence the functional form is parameterized to be

$$\log(d_{i,t+1}) = (1-\rho_d)\eta_d + \rho_d \log(d_{i,t}) + \xi_{i,t+1},$$

in which $\xi_{i,t+1}$ is iid and normally distributed $\mathcal{N}(0, \sigma_\xi)$. I use the values of mean, autocorrelation and standard deviation of innovation, $\{\eta_d, \rho_r, \sigma_\xi\}$, as estimated in Corbae and D'Erasmo (2021) and therefore I target the annual persistence of the idiosyncratic process, ρ_d, to 0.882 and the standard deviation of deposits, σ_d, to 0.00366. Since my model is calibrated as quarterly I derive time-aggregated annual parameters of quarterly standard deviation and autocorrelation of the discrete deposit process so as to match the annual data moments counterparts.[25] I internally calibrate these two parameters. This procedure yields values for the standard deviation of deposits, σ_d, and persistence, ρ_d, equal to 0.002 and 0.944, respectively. Since banks only partially insure against the idiosyncratic shock I adjust the deposit volatility to meet ± 0.05 size, while still matching moments for deposits, in order for banks to have enough precautionary liquidity motive.

The idiosyncratic process is discretised using Tauchen and Hussey (1991) for the finite state space representation of the Markov process for $d_{i,t}$ with three states $d_j \in \{d_h, d_m, d_l\}$, in which $\{h, m, l\}$ states for high, medium and low states of the deposit withdrawal shock such that $\{d_l < d_m < d_h\}$. I impose further assumptions of the transition probability matrix such that the fraction of big banks is different from the fraction of small banks respecting symmetry in the reduced form transition probability matrix and that the probabilities of moving from the medium state to the high and low state are the same. The mean deposit is normalised to 1.

The liquidity requirement imposes an additional assumption on how I treat the output of the Tauchen and Hussey discretisation. Specifically, a bank cannot switch from the high to the low state immediately, but it has to pass from the medium state assigning zero probability to the remaining states appearing in the same subset of states. The specification of the deposit withdrawal, $(d_{i,t+1} - d_{i,t})$, in the liquidity requirement leads me to modify the 3×3 transition probability matrix. To guarantee its existence I impose a 9×9 transition probability matrix. The transition probability matrix encodes the following exogenous states

$$\begin{aligned}&\Big[\Big\{d_{t-1}^h, d_t^h\Big\}, \Big\{d_{t-1}^m, d_t^h\Big\}, \Big\{d_{t-1}^l, d_t^h\Big\},\\ &\Big\{d_{t-1}^h, d_t^m\Big\}, \big\{d_{t-1}^m, d_t^m\big\}, \Big\{d_{t-1}^l, d_t^m\Big\}, \Big\{d_{t-1}^h, d_t^l\Big\}, \Big\{d_{t-1}^m, d_t^l\Big\}, \Big\{d_{t-1}^l, d_t^l\Big\}\Big] \in D.\end{aligned}$$

Denoting with p_{11} and p_{22} the probabilities of moving from each deposit state to the next

[25] Further calibration details and the time-aggregation phase are provided in the technical Appendix G.1.

period deposit state, the reduced form transition probability matrix is given by,

$$\begin{bmatrix} p_{11} & 1-p_{11} & 0 \\ \frac{1-p_{22}}{2} & p_{22} & \frac{1-p_{22}}{2} \\ 0 & 1-p_{22} & p_{22} \end{bmatrix},$$

reflecting the assumptions I imposed above. I then modify the output of the transition probability matrix. In particular, as a consequence of the two assumptions: that banks cannot move directly from the low to the high state and viceversa and that the fraction of small banks is different from the fraction of large banks the derived transition probability matrix is a sparse matrix. It is not symmetric anymore, while maintaining symmetry of the reduced form transition probability matrix presented above. In particular, the stationary probability distribution is an orthornormal but is not symmetric since it is not invariant to any permutation of the transition probability matrix.[26]

Illiquid assets. Finally, the aggregate level of illiquid assets, s, is computed as a the ratio of illiquid assets to the sum of liquid and illiquid assets, and set to 0.823.[27] I set the interest rate on loans to 10.9% which is slightly larger than what the empirical evidence document (Bianchi and Bigio, 2022; Del Negro et al., 2017) for the illiquidity premium on illiquid assets, but necessary to ensure the existence of a well definite equilibrium.

5.2 Results

To understand the precautionary behavior of banks and banks' liquidity distribution I present the stationary equilibrium banks' decision rules in which the idiosyncratic deposit withdrawal hits the banking economy.

Figure 4 displays decision rules for banks' dividends and liquid assets as a function of the initial level of banks' liquidity. Small banks live in state $\{d^h_{t-1}, d^h_t\}$. Likewise, medium and large banks live in states $\{d^m_{t-1}, d^m_t\}$ and $\{d^l_{t-1}, d^l_t\}$, respectively. Since the constraint is state dependent, for each realization banks may end up in one of the three states given the previous level of deposits. A novel aspect of the model is the timing of the liquidity trading, in which at the onset of the next period banks must hold enough liquid assets in order to face the change in liquidity needs arising from a temporary liquidity shock.

The varying binding region identifies the relevant constrained regions. This region lies between boundaries $\ell_{i,t+1} \in [\ell^*, \overline{\ell}_l)$, where ℓ^* represents the global minimum liquidity requirement, ϕ. Constrained medium banks stay within the region $\ell_{i,t+1} \in [\ell^*, \overline{\ell}_m)$, constrained small banks face a larger left region $\ell_{i,t+1} \in [\ell^*, \overline{\ell}_l)$ due to their precautionary behavior and large banks optimising in the entire region would always choose a level of liquid assets $\ell_{i,t+1} > \ell^*$ whilst smoothing dividends income.

[26] Further calibration details on the discretisation of the exogenous process are available in Appendix G.

[27] For the average level of illiquid assets I use the series reported in Table 4 with data sources provided in Appendix J.

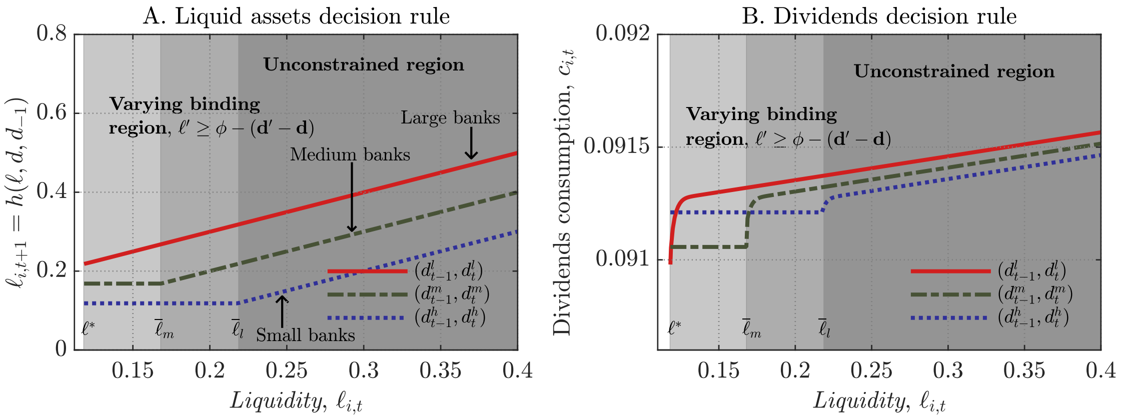

FIG. 4.—Banks' decision rules.

NOTE. The figure graphs banks' liquid assets and banks' dividends consumption decision rules. There are nine states and I only show rules for three states of the process for deposits, $d_j \in \{d_h, d_m, d_l\}$. The lighter grey area, $[\ell^*, \bar{\ell}_m)$, is the constrained region for medium banks, the middle grey area, $[\ell^*, \bar{\ell}_l)$ is the constrained region for small banks, the dark grey area is the unconstrained region. This last region covers the entire domain for large banks. Values are given in level.

Panel A of Figure 4 illustrates liquid assets decision rule that is increasing in the current level of banks' liquidity. It presents the level of liquid assets that sustain the equilibrium level of dividends. The decision rule is nonlinear from constrained banks in the medium and low states that stay bounded at the kink to meet the liquidity requirement until they reach the new state. Therefore, there exists a varying positive value of banks' liquidity larger than the admissible minimum from the liquidity requirement. When this future value of liquidity is below the optimal value dictated by the liquidity constraint banks should consume all resources exhausting the liquidity constraint and set future liquid assets to the minimum imposed by the liquidity constraint. For values larger than this positive value both dividends consumption and future liquid assets are increasing in current liquid assets.

Panel B of Figure 4 shows the dividend consumption decision rule. The function is concave, displaying the most marked non linearity for banks who are close to their liquidity constraint and also hold a low amount of liquid assets. Given that small banks in the model have the largest volatility of deposit funding, they are the banks that have the strongest precautionary incentive to increase the amount of liquid assets in order to not hit the constraint in the next period.[28] Consequently, they build the largest precautionary buffer of liquidity by the end of the period. Indeed they also exhibit the largest concavity in their dividends consumption decision rule. By contrast, for banks away from their liquidity constraint owning a large amount of liquid assets the dividends decision rule is always concave monotonically increasing with no presence of kinks. Although they start with a lower dividends value than constrained small and medium banks, they are never at the constraint. Therefore, unconstrained bankers have a low marginal propensity to issue dividends. Bank dividends decision rule is an increasing function of banks'

[28] As documented in Corbae and D'Erasmo (2021) small banks have larger deposit variance than big banks.

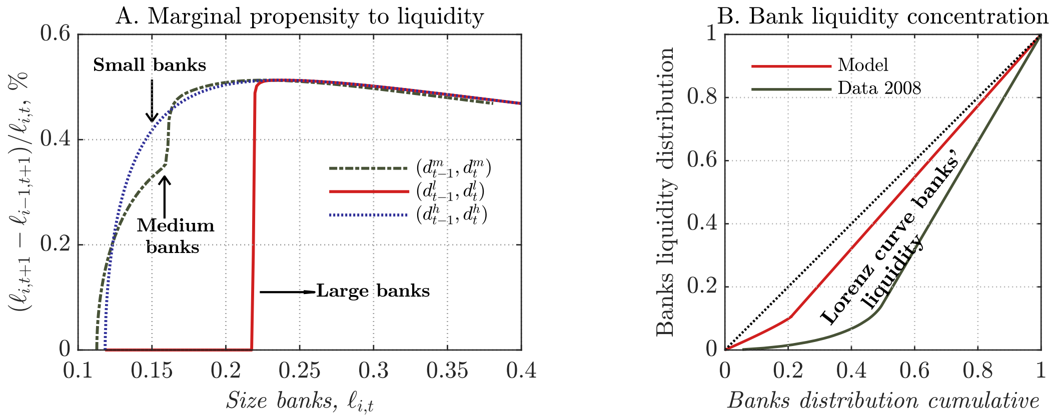

FIG. 5.—Marginal propensity to liquidity and Lorenz curve for bank liquidity.

NOTE. Panel A vertical axis graphs the marginal propensity towards banks' liquidity in percentages and horizontal axis graphs the value of liquidity. Panel B vertical axis graphs the cumulative distribution of banks' liquidity and the horizontal axis the cumulative distribution of individual banks.

assets. Between banks the magnitude of change in dividends is 16 basis points only, thus for almost all banks the incentive to save in liquidity dominates their dividends consumption. In the unconstrained region for all banks when the liquidity constraint is loose, banks can increase profits by levering through deposits.

Figure 5 panel A highlights the incentive banks have towards liquidity between any banks and illustrates some important points.[29] First, I define banks' marginal propensity towards liquidity as $(\ell_{i,t+1} - \ell_{i-1,t+1})/(\ell_{i,t})$ and therefore, the graphs shows the effect of the deposit withdrawal as a function of the size of banks. Second, the graph reveals that banks' marginal propensity to liquidity is increasing in bank size, but this is not true when bank size is large. In this case, as banks grow in size it is optimal for them to reduce their incentive towards liquidity to gain in profitability. Small banks gain more than large banks from saving in liquidity as they care from a possible drop in dividends and because of their higher volatility of deposits. Therefore, their marginal propensity to liquidity rises gradually. This is also the reason why the model delivers lower inequality in the distribution of banks' liquidity. Third, larger bank size is not necessarily correlated with higher marginal propensity to liquidity, especially for large banks their incentive towards liquidity is almost unresponsive at lower beginning of period liquidity and rises steeply and faster for higher beginning of period liquidity suggesting that they have less volatile funding inflow. Indeed, their marginal propensity towards liquidity switches from 0% to 0.52%.

Figure 6 panel A illustrates the cross sectional distribution of banks' liquidity in which the x-axis represents the histogram bins for the liquidity distribution, whilst the y-axis represents the fraction of banks in each liquidity bin. From the distribution, 2.2% percent of banks are liquidity constrained until the medium liquidity constraint. Furthermore, the model reveals that more

[29] I wish to thank Ellen McGrattan for the inspiration she gave me in producing panel A of this figure during many conversations with her.

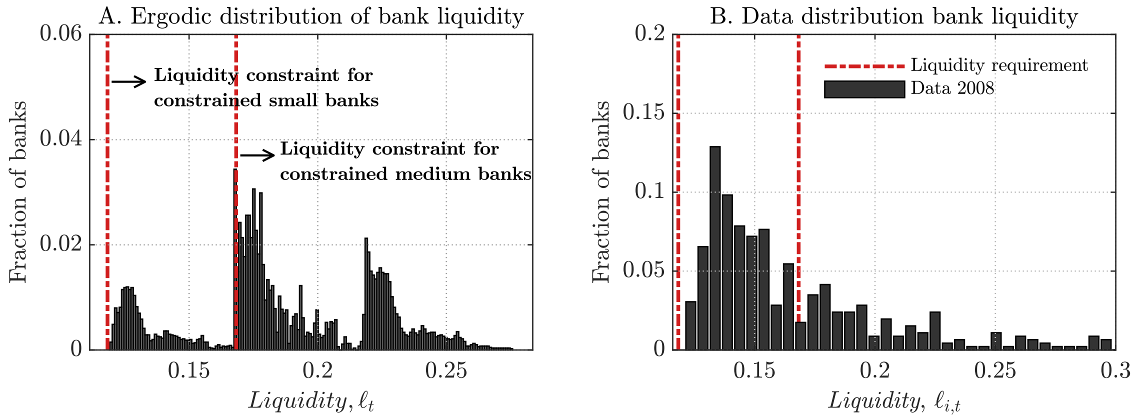

FIG. 6.—Ergodic bank liquidity distribution and data distribution.

NOTE. Panel A and B vertical axis plots the fraction of banks in level and horizontal axis plots the value of liquidity. Panel B plots the distribution of banks' liquidity from 2008 data.

than 2% of banks holds a larger amount of liquid assets. The tail to the right of the distribution is significant of the fact that few banks, precisely top banks in the US, hold a big value of liquid assets. In particular, there is only a small share of banks holding more than $1\frac{1}{4}$ the average level of liquid assets that captures large banks in the model. The distribution has three lower bounds due to the distribution of the idiosyncratic process of deposits and the state dependent liquidity constraint. Both imply the presence of three masses of banks at the high, medium and low state of deposits which makes the distribution more concentrated at the varying constraint. The distribution is skewed to the right, and the largest peak in the distribution is present in the medium state implying that many banks are constrained in this state. In particular, because of the imposition of the liquidity constraint occasionally binding banks tend to switch repeatedly between deposit states. The aggregate level of liquid assets results into being 0.188 very close to the data counterpart of 0.184.

The model however cannot produce the whole distribution of banks' liquidity as in the data. Figure 6 panel B shows the distribution of banks' liquidity from the data in 2008. The model does well in generating the fraction of banks at the medium liquidity constraint and the long right tail of the distribution. In the data the fraction of banks just at the medium liquidity constraint is 1.8% and it is in correspondence of banks with size $1 million deposits when sorted by assets. However, the model cannot generate the fat mass of banks at the lower left tail of liquidity distribution due to the very sparse transition matrix.

To gauge more understanding on the effect of the idiosyncratic process on the distribution of liquid assets in the entire banking system, I compute a standard measure of inequality that is the Gini coefficient. I then construct a Lorenz curve for the distribution of liquid assets that is conducive to the degree of concentration of banks' liquidity in the banking system. Figure 5 panel B shows the result for the corresponding curve that is produced from the model solution−red line, in comparison with the data−dark grey line. Data curve is for small banks. Appendix G.2

Table 6.—
Distribution banks' liquid assets in the US

		Percentiles in the liquid assets distribution							
		Top distribution					Bottom distribution		
	Gini coefficient	1^{st}	5^{th}	10^{th}	20^{th}	30^{th}	85^{th}	94^{th}	99^{th}
Data	0.389	0.017	0.085	0.170	0.341	0.511	0.744	0.898	0.983
Model baseline	0.105	0.011	0.056	0.113	0.226	0.339	0.830	0.932	0.989
Model tighter, ϕ	0.082	0.011	0.055	0.110	0.220	0.330	0.835	0.933	0.989
Model ex-post	0.049	0.010	0.052	0.105	0.211	0.316	0.841	0.936	0.989
Model no regul	0.062	0.010	0.053	0.107	0.214	0.321	0.839	0.936	0.989

Note. Distribution of banks' liquid assets in the US. Data moments in 2008 for all banks are compared in the second row with the model. Moments construction and data source are provided in Appendices G.2 and J, respectively.

provides details on the calculation of the Lorenz curve of banks' liquid assets.

The Lorenz curve is the continuous line below the 45° degree line. The more the curve is directed towards the 45° line the more equally liquid assets are distributed in the banking sector. The curve is away from an equal distribution and approximately 21% of banks own less than 11% of liquid assets. More than a half of the universe of banks own more than half liquidity in the banking sector since they accumulate a significant fraction of liquidity. These banks are mainly the top banks who belong to the upper right tail of the deposit distribution. In the data the same fraction of banks hold less than 46% of liquid assets.

A natural question relates to the three masses of the distribution delivered by the deposit withdrawal shock but that are not present in the Lorenz curve. This is because when sorting liquid assets for the Lorenz curve, the three masses are all close together therefore the three kinks become concentrated in one kink only. The model cannot account for the level of concentration in liquid assets and its Lorenz curve with a Gini coefficient of 10.5% compared to 38.9% in the data. One of the explanations for the lower Gini coefficient than the data is the merger and acquisition feature of the data (Corbae and D'Erasmo, 2020), that the current model does not contain. Introducing it constitutes an avenue of fruitful research.

Table 6 reports data and model moment counterparts on the distribution of banks' liquid assets in the US. The first row gives the data moments for all banks, the second row reports moments produced by the baseline model economy. The third and fourth rows show the corresponding moments for the model with a tighter liquidity requirement and the ex-post regulation economy, respectively. In the last row there are moments for the economy without liquidity regulation. In the data the distribution of banks' liquidity is largely distributed in the banking sector with the smallest banks in the bottom 85th and 94th percentiles of the distribution holding 74.4% and 89.8% of liquid assets, respectively. The top 1st and 5th percentiles of banks hold 1.7% and 8.5% of liquid assets, respectively, while banks in the top 30th percentile hold 51.1%

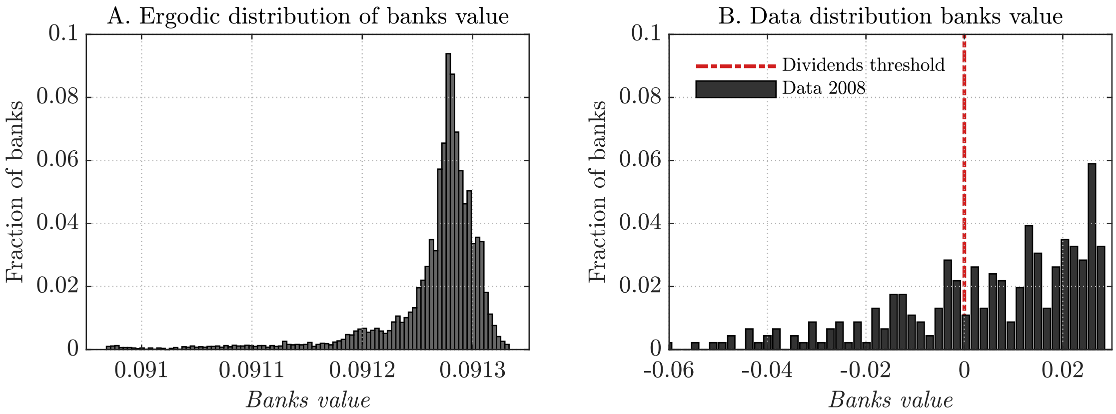

FIG. 7.—Ergodic banks' value distribution and data distribution.

NOTE. Panel A and B vertical axis plots the fraction of banks in level and horizontal axis plots banks' value. Panel B plots the distribution of banks' value from 2008 data.

of total banks' liquidity.

The second row of Table 6 reports moments from the baseline model. The model underestimates the top 5th and 20th of banks holding 5.6% and 22.6% of liquid assets. However, it does a very good success in generating the top 1st and 10th percentiles with 1.1% and 11.3%, respectively, of banks' liquid assets in line with the data. Moreover, while it underestimates the top 30th percentile with banks holding 33.9%, the model does reasonable well in reproducing the 85th percentile in the bottom distribution of banks' liquidity with banks holding 83% of liquid assets. Nonetheless the model does well in replicating the bottom 99th percentiles of banks' liquidity distribution where these banks hold 98.9% of total banks' liquidity in line with 98.3% in the data. In the model the only source of funding is deposits and while bank liquidity constrains the bank ability to expand the size of their balance sheet, it remains widely distributed across the banking system. Furthermore, I have considered a period in which banks were recovering from the turmoils of the crisis when the the interest rate on liquidity started being paid to banks therefore their incentive to hold liquidity was the largest.

Third and fourth rows of Table 6 present moments for the tighter economy and the ex-post regulation economy. Both of them, in contrast to the baseline economy, generate a more equal banks' liquid assets distribution as liquidity reallocates towards the bottom percentiles of the distribution. Both of them are however inconsistent with the 2008 data. Therefore, the baseline model shows the importance of internalising the liquidity friction not only at the beginning of the intraperiod but also ex-post, given its ability to generate moments much closer to the data counterpart. The last row of Table 6 presents the no regulation economy in which case the friction arising from the regulation does not take place. Since this version of the model represents the frictionless environment small banks do not overaccumulate liquidity as much as the ex-post regulated economy, as liquidity is mostly taken over by large banks inequality in banks' liquidity is larger than the ex-post regulated economy. As a proof of concept Appendix

H provides the distribution for liquidity and banks' value under the equilibrium with ex-post liquidity regulation.

The model produces a Gini coefficient of 0.105 below the 0.389 in the data. A sensible fraction of banks holds a large share of liquid assets since I have calibrated not only the size of the deposit shock in the high state but also in the medium state. Moreover, the value of the Gini coefficient for banks' liquidity mainly depends on how many banks are assumed to stay in the low deposit state in addition to the degree of persistence of the deposit process. In an environment in which the deposit withdrawal shock is not highly persistent there is no guarantee that the model can generate the small mass of banks in the low deposit state who accumulates more liquidity than small banks. It then remains upon the ability of the other model parameters to match the empirical Gini coefficient observed in the data.

The fact that liquidity is not very much concentrated means that across banks there is also reallocation of liquidity themselves since individual banks in the model can choose the level of liquidity they want, therefore the reallocating mechanism in the model is at work not only across assets, liquid and illiquid assets, but also within the same asset.[30]

In the model the size of banks through liquidity for all banks' assets induces differences in banks' value. Figure 7 plots the distribution of banks' value according to the model and the data in 2008. To render the data in a consistent comparison with the model, the measure of banks' value I adopt is the present discounted value of future dividends. I recover this measure from micro banks' data as given by net income to deposits.[31] The model successfully replicates the leftskewed distribution as in the data with the vast majority of banks concentrated in the upper part of the banks' value distribution. However, it slightly overpredicts the fraction of banks at the top of the distribution with 9.18% of banks against 5.89% in the data.

While the data distribution of banks' value is characterised by banks falling below the threshold of nonnegative dividends, the threshold is the second inequality constraint in the model therefore all banks are located above this threshold. However, the important point to note is that while this constraint impedes banks from having negative dividends as in the data it does not impede the model from generating the left tail which represents *i)* the ability of banks in internalising the possibility of losses caused by a withdrawal of deposits and *ii)* the knowledge that inflows driven by changes in deposits can be small and infrequent. Furthermore, since banks' size is kept through assets and in the data I rank banks by assets, the banks' value distribution confirms the prediction of Figure 5 panel A, that is large banks have a greater incentive towards gaining in profitability whilst small banks care more about saving in liquidity to avoid from being liquidity constrained in the future. This is also consistent with the prediction of Proposition 2 where deposits enhances banks' value. Consequently, many small banks survive

[30] As empirically documented in Ennis and Wolman (2015) a substantial reallocation of reserves occurred upon and after the 2008 financial crises.

[31] I also used a more conventional measure of banks' value given by the stock price as the price to earnings ratio computed from micro banks' data. The differences are negligible and the distribution provides the same pattern as the one reported in Figure 7. Appendix J provides further data details on the measure.

Table 7.—

Target moments model variations

	Baseline		No liquidity risk	Ex-post regulat	Liquidity ratio	No regulation
Moment	Data	Model	Model	Model	Model	Model
mean, $\tilde{\ell}$	0.184	0.188	0.168	0.279	0.261	0.260
st.dev., $\tilde{\ell}$	0.049	0.033	0.000	0.063	0.073	0.073
skewness, $\tilde{\ell}$	0.046	0.918	0.000	0.919	0.641	0.640
kurtosis, $\tilde{\ell}$	2.107	2.594	0.000	2.597	2.145	2.144

Note. This table reports the target mean for liquidity to deposit ratio in comparison with the target and moments of the key economic variable in the model, obtained by approximating the distribution of liquid assets. Moments are reported in comparison with the data. It also reports the moments for different versions of the model in comparison with the baseline environment. Liquidity to deposit is defined as $\tilde{\ell} = \ell/d$.

in the left tail of the banks' value distribution and the size of banks induces differences in banks' value. Moreover, since many small banks in the data have negative dividends and the interest rate on liquidity was not paid prior to the financial crisis, this indicates that banks were not enough liquid thereby falling with negative dividends. The imposition of a liquidity requirement that has a direct effect on the liability side of banks' balance sheet not only mitigates inequality in banks' size distribution but can also be a viable instrument to improve banks' profitability.

To provide a clear inspection of key moments of the distribution I do a simulation in the cross-section of banks. Table 7 reports moments of the liquidity to deposits ratio and data moment counterparts. The table compares the success of the baseline model against four versions of it. The four versions are the performance of the model when disregarding liquidity risk, the economy when banks do not internalise the ex-ante liquidity friction at the beginning of the intraperiod, the model solved with a liquidity ratio in addition with having banks not internalising the liquidity friction and the economy without regulation. The banks' programme for the liquidity ratio model with additional theoretical results are presented in Appendix C.

The baseline model does a good success in matching the mean and the standard deviation of banks' liquidity to deposit ratio. The model however overpredicts the skewness of the liquidity to deposit ratio with the data pointing towards a fairly symmetric distribution in 2008 whereas the model produces a positively skewed distribution. The reason is due to the calibration of the probabilities of the transition probability matrix in the low and in the high state. Since the probability attached to the high state is larger than the probability in the low state this inevitably leads to more skewness in the distribution. In Subsection 5.3.3 I take care of these two moments by fundamentally changing the nature of the problem and I consider a liquidity crisis embedded as an illiquidity disaster state. Liquidity to deposit ratio shows moderate excess kurtosis in the data, that is captured by the model.

To gauge more predictions on the distribution of banks' liquidity I solve the model by isolating the liquidity risk. This counterfactual experiment entails two consequences. First, the model effectively reduces to a model of a representative bank without uncertainty where the bank dividends consumption saving problem disappears, and second, the inflow/outflow effect in banks' balance sheet vanishes. The third column of Table 7 reports moments for the model without liquidity risk. The model with a representative bank has serious difficulties in producing mean and volatility of liquidity to deposit ratio in line with the data since there is no bank precautionary liquidity motive. The target for banks' liquid assets ventures far away from the data counterpart and results in 0.168 model moment. These difficulties are further exacerbated for skewness and persistence of liquidity to deposit ratio as there is no bank heterogeneity in the economy without risk. Furthermore, by isolating the effect of deposit withdrawal risk, the important insight I obtain from the model is that disregarding the idiosyncratic deposits withdrawal risk has difficulties in generating distributional moments and inequality of banks' liquid assets. The aftermath derives because the economy becomes an economy with complete markets that does not allow the model to explain the degree of inequality along banks' liquid assets.

The last three columns of Table 7 reports moments for the ex-post regulation economy where banks do not internalise the liquidity regulation at the beginning of the intraperiod, the model with a liquidity ratio in place and banks not internalising the liquidity regulation and the model without liquidity regulation. These three versions of the model generate a far high aggregate level of banks' liquidity, as well as volatility and skewness. The aftermath derives from the disappearance of the inflow/outflow effect in banks' budget constraint which gives incentives to small and medium banks to raise their liquidity. Furthermore, in the model solved with a liquidity ratio banks are never liquidity constrained, thereby generating a high value of aggregate banks' liquidity. Likewise, in the economy without liquidity regulation the absence of the friction does not lead banks to overaccumulate liquidity. Nonetheless, since liquidity is now reallocated along the distribution of banks' liquid assets containing banks below the liquidity requirement the aggregate amount of liquidity rises to 26%.

Liquidity constraints. I study an experiment in which I tight the liquidity constraint to match the liquidity to deposit ratio in the data. Episodes of tightening banks' liquidity constraints are normally implemented in crisis periods. I concentrate the attention to liquidity crises in Section 5.3 and here I focus on stationary equilibria. While the exercise is a merely increase in the liquidity constraint, it necessarily forces banks to own more liquidity to insure against the deposit withdrawal risk making on average lower profits. The distribution of US banks' liquid assets becomes more equally distributed with a Gini coefficient of 0.082. The result however needs to be counterbalanced with the target of liquidity to deposits ratio which increases to 0.222.

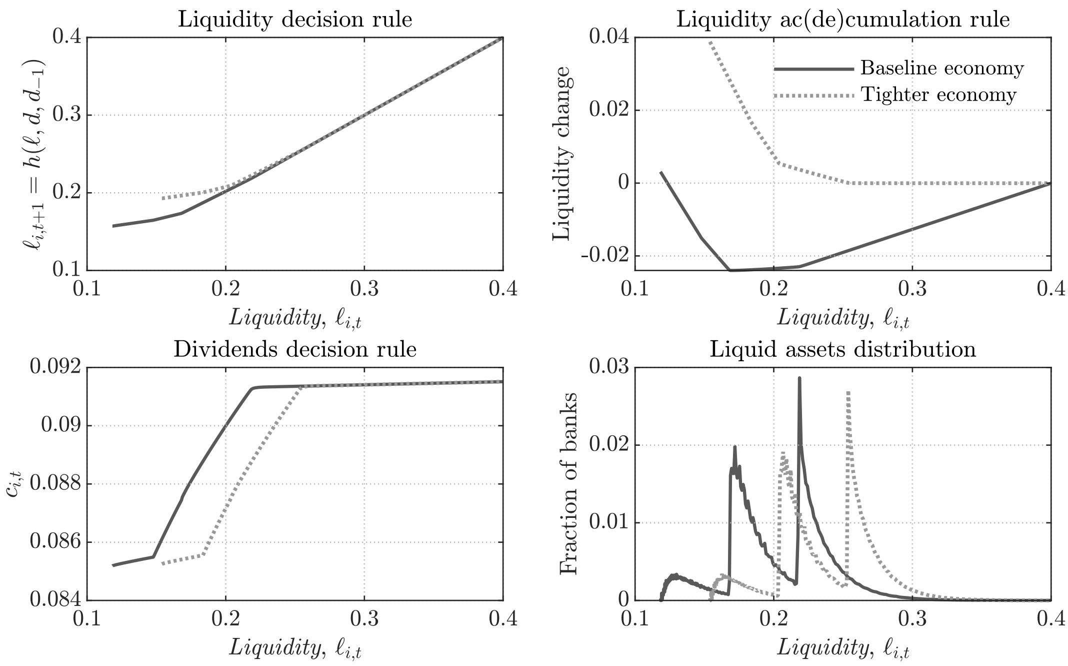

FIG. 8.—Equilibrium to a tighter economy.

NOTE. The stationary equilibrium is implemented from the baseline economy to a more severe liquidity requirement, ϕ, that is set to 0.154. Decision rules are reported in levels.

5.2.1 Steady State Comparative Statics

In this subsection I compute three steady state comparative static experiments. In the first experiment I study more closely the liquidity constraint and inspect the relation between two steady states due to a tightening of the liquidity requirement. Tightening the liquidity requirement requires banks to accumulate more liquidity to hedge liquidity shocks in the future. Banks therefore undertake more prudent behaviors. As banks build more abundant liquidity buffer, the system as a whole become more resilient to the deposit withdrawal shock. The second experiment is the stationary equilibrium of the model given a sequence of the interest rate on liquidity. This experiment allows to shed light on the optimal bank responses for any given change in the interest rate on liquidity. The third experiment shows the sorting of banks by type and its relation to the endogenous banks' size expansion. It demonstrates that small banks can also expand in size.

Tighter bank liquidity. I compute the banking equilibrium with a more severe liquidity requirement along the lines of the previous increase of the liquidity requirement studied in the previous subsection. The liquidity requirement, ϕ, starts with the initial value equals to 0.1183 and passes to the economy with a tighter liquidity constraint of 0.154. The new value of ϕ is chosen so as to imply an increase in banks' liquidity of 11% consistent with the increase in bank liquidity at the beginning of 2009q2 during the financial crisis. This means that it is possible to inspect the optimal path of banks' decision rules by averaging them by the stationary probability

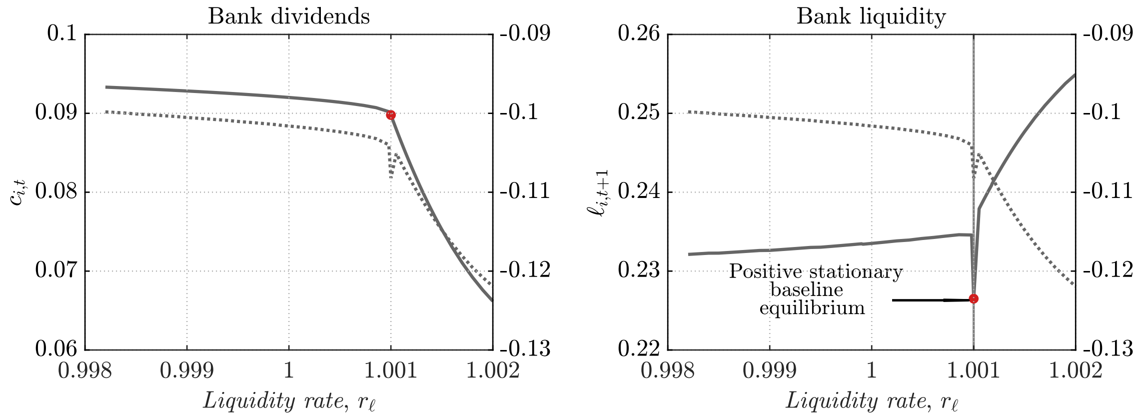

FIG. 9.—Equilibrium along a sequence of interest rate on liquidity, r_ℓ.

NOTE. The liquidity constraint, ϕ, matches the liquidity to deposit ratio for any given level of the interest rate on liquidity. The dotted blue lines represent the end of intraperiod net liquidity constraint, $\phi - (d_{i,t+1} - d_{i,t}) - \ell_{i,t+1} = 0$. Decision rules are reported in levels.

distribution of deposits. Concretely, each decision rule except the distribution of banks' liquidity is multiplied by the invariant distribution of the withdrawal shock, $\sum_{d^t \in D^t} \mu^t(b_0, d^t)$.

Figure 8 contains bank decision rules and the theoretical distribution of banks' liquidity under the more stringent liquidity constraint. The dark grey lines show the results for the baseline economy whereas the dotted grey lines display the economy under the new liquidity requirement. With a larger liquidity requirement banks are forced to accumulate more liquidity.

The top right panel shows in fact that small and medium banks accumulate liquidity to meet the increased liquidity requirement. Whilst large banks with higher networth than small banks tend to decumulate liquid assets because they already have the necessary liquidity to meet the constraint, the largest adjustment occurs for small and medium banks. Motivated by their precautionary behavior small and medium banks need to build a larger buffer of liquidity that is observable to the left of the beginning of period liquidity. Since current banks' dividends consumption for small and medium banks becomes more costly due to the tighter liquidity requirement, dividends consumption tends to fall. Indeed the bottom right panel displays a shift to the right of the distribution of banks' liquid assets, whilst means across the two distributions are the same.

By computing distributional moments for the tighter economy as computed in Table 1, the fraction of banks' liquidity in the bottom tail of the distribution rises whereas that of the upper tail distribution falls. Bank liquidity becomes more widely spread amongst medium and small banks having more liquidity, while their fractions decay. This is the reason why banks' liquid assets become less unequally distributed amongst banks. Large banks whose incentive to accumulate liquidity is the least stay close to the baseline economy and decumulate liquid assets.

Equilibrium along a sequence of r_ℓ. In this second experiment I analyse the stationary equilibrium for different values of the interest rate on liquidity. This comparative static shows the interplay of liquidity and banks' dividends consumption for any given level of the interest rate on liquidity. I allow the liquidity rate to be negative. In particular, I construct an equilibrium in which I fix different levels of the interest rate on liquidity, r_ℓ, and recalibrate the liquidity to deposit ratio with the liquidity requirement, ϕ, for each given level of the interest rate on liquidity, all else equal. In particular, this means that for the given sequence of the liquidity rate I collect the cross-sectional means decision rules of banks' dividends consumption and liquidity and then construct the new decision rules on the basis of the mean values. I am therefore moving across stationary equilibria for different levels of the liquidity constraint.

Figure 9 shows the results. Panel A and B present banks' dividends consumption and liquidity, respectively, as a function of the interest rate on liquidity. The red dot in both panels represents the steady state solution for the baseline economy that was solved with a liquidity rate of 1.001. As expected at low levels of the interest rate on liquidity the incentive for banks to hold liquidity decreases, whilst for high levels of the interest rate on liquidity the incentive for banks to store liquidity rises. This is because in effect bank liquidity is an increasing function of the liquidity rate. Moreover, I document a decreasing path of banks' dividends consumption along the liquidity rate since banks' dividends in the present become more costly than future dividends, therefore banks find it optimal to accumulate liquidity when the liquidity rate rises.

The distortion in banks' intermediation profits manifests itself in the sudden contraction in dividends consumption and liquidity under the baseline equilibrium. Bank decision rules show a kink that occurs exactly when the liquidity constraint begins to bind strongly for constrained small and medium banks since they are experiencing a high cost on total short term debt. This feature resonates the finding of Bianchi and Bigio (2022) who however find bank reserves to be monotonic and the nonmonotonicity occurring for bank credit. In their model it is the capital requirement constraining the bank's ability to expand credit which in turn impacts bank's holding of reserves. In my analysis the increase and decrease in bank liquidity hinges on the liquidity requirement and the distortion on short term debt. Therefore, the liquidity constraint has a direct impact on banks' liquidity that subsequently affects the bank's ability to expand credit. My approach is also different since I have computed the equilibrium for an economy with heterogeneous banks. I therefore do not solve for the representative bank as in Bianchi and Bigio (2022) to begin with.

As anticipated in Proposition 8 banks' liquidity displays a point of inflection occurring before that the liquidity constraint begins to bind. The nonmonotonicity in banks' liquidity now arises since banks find optimal to reduce liquidity because of the less binding liquidity requirement around the inflection point that reduces banks' profits. At that point the benefit for banks to hold liquidity suddenly decreases which happens simultaneously to the fall in dividends consumption. The fall in liquidity is accompanied by the sudden decay of the liquidity constraint which indeed becomes less binding. As a consequence, the effect is that banks' dividends consumption and banks' liquidity become complement. Away from the binding liquidity constraint

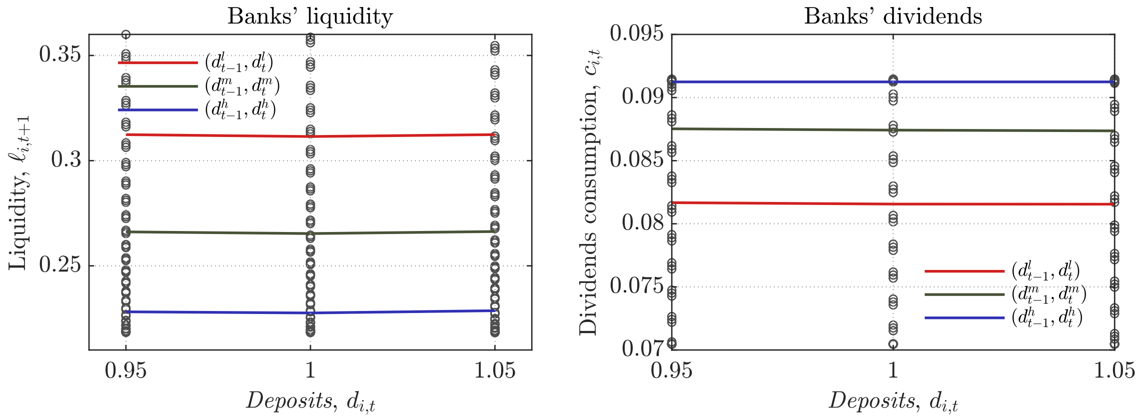

FIG. 10.—Endogenous banks' size expansion

NOTE. This figure depicts the expansion in banks' size through the liquidity and consumption implications of deposits. The figure shows decision rules for three states of the process for deposits, $d_j \in \{d_h, d_m, d_l\}$ only.

banks' dividends consumption and liquidity are substitute and liquidity heavily increases to the right of the inflection point. This is because deposits are purely idiosyncratic, therefore any increase in the interest rate on liquidity makes less costly for banks to insure any deposit withdrawal relative to the cost of borrowing and relative to the return on illiquid assets. As a consequence, the liquidity constraint is binding to the left of the small neighborhood of the inflection point and banks' dividends necessarily contract.

To give more intuition on this result, consider the wedge between bankers' discount factor and the interest rate on liquidity. For low value of the interest rate on liquidity, banks find optimal to roll over their deposits reducing their precautionary liquidity buffer. When this wedge is the largest, that occurs when the liquidity rate is large, bankers save in liquidity up to the liquidity constraint. This occurs at the inflection point. After having stayed a while at the liquidity constraint, bankers strongly save in liquidity in any positive stationary equilibrium. Therefore, the relevance of the liquidity constraint becomes crucial to the right of the inflection point because there are gains for bankers from holding liquidity.

In addition the complementarity result manifests itself in outflow banks who are affected by the liquidity distortion. For these banks the benefit from holding liquidity vanishes around the inflection point and they simultaneously reduce liquidity and dividends as soon as the interest rate wedge rises. It occurs because they are paying the wedge on deposit withdrawal, which shrinks dividends and outweighs the benefit of inflow banks to increase liquidity driving down the total effect on banks' liquidity.

Sorting and endogenous banks' size expansion. One of the central results of the paper is the endogenous banks' size expansion that arises through liquidity and it is driven by the deposit withdrawal risk. Figure 10 presents the effects of a negative deposit shock along the stationary equilibrium for large, medium and small banks in red, grey and blue lines, respectively. In particular, it makes clear the relationship between deposits and the size of banks which is related to the amount of banks' liquidity.

Two results emerges. First, a negative deposit shock to the set of heterogeneous banks leads to a moderate rise in banks' liquidity, which falls before starting to grow once deposits have sufficiently expanded and banks have reached a larger size. The fall in banks' liquidity is counterbalanced by dividends consumption which needs to fall. Second, since there are two competing effects at work: the fall in deposits and the sorting of banks through deposits, as deposits rise small banks increase the amount of liquidity relatively more than large banks in the same cohort of small banks. Therefore, the larger deposits the larger banks' liquidity through which even small banks can expand in size. However, while large banks can expand in size by holding and accumulating liquidity because of their better initial conditions, small banks can expand in size only as a result of a shock to deposits that increases their marginal propensity to accumulate liquidity due to the tighter liquidity constraint and therefore rises their liquidity.

5.3 Illiquidity and Transitional Dynamics

In this subsection I inspect the nonlinear dynamics and conduct the liquidity crisis analysis brought by the model. I study three different experiments of banks' illiquidity. Illiquidity in this environment is interpreted as banks lacking enough liquidity to counteract the deposit withdrawal shock in the market. In the first experiment I analyse the optimal bank responses due to a deposit withdrawal shock. The second experiment characterises a banks' illiquidity episode envisaging a liquidity crisis. In the last experiment I study the liquidity crisis originated by disasterising the economy.

5.3.1 Transitional Dynamics to a Liquidity Shock

I compute the transitional dynamics of the nonlinear economy to show the responses of the endogenous variables to a deposit withdrawal shock. I set the initial value of bank liquidity to its mean value and a value of accumulation of liquidity that implicitly produces 2.26% fall in the liquidity to deposit ratio. I compare this regime with the regime in which the initial value of bank liquidity is left unrestricted. I take these values and feed them back with the deposit withdrawal shock. The incidence of the liquidity crisis therefore becomes independent of initial values but remains endogenous to the model outcomes purified by the initial conditions.

The model is simulated for $100,000$ times for 30 periods and the resulting values are inserted in the endogenous variables to produce the path of the economy. While a typical liquidity crisis arises in the interbank market, my paper provides the ground for the study of liquidity crises that originates in the deposit market. I therefore study how the idiosyncratic withdrawal shock induces banks' precautionary behavior and a negative amplification on banks' balance sheet.

Figure **??** shows the paths of the variables of interest in mean deviations from the liquidity crisis left unrestricted from initial conditions with the liquidity crisis originated by imposing an endogenous 2.26% fall in the liquidity to deposit ratio which corresponds to half standard deviation its mean value. Paths are produced for the baseline economy with grey circled and dotted lines and for the economy with ex-post regulation with black dotted lines.

A deposit withdrawal shock leads to an immediate contraction on impact of all variables. As the deposit withdrawal shock hits, banks need to use their dividends to cover the outflow of deposits. The liquidity constraint tights and profits become negative. The baseline economy with ex-ante regulation reduces the contraction in banks' liquidity to deposits and liquidity by 24.03% and 8.28%, respectively, from the ex-post regulated economy. Consequently, in the baseline economy dividends consumption falls 0.579% less than the ex-post regulated economy. Likewise, the ex-ante regulation prevents a sizable fall in banks' profits by reducing it by 23.41% from the ex-post regulated economy fostering the analysis that ex-ante liquidity regulation in anticipation to the crisis helps to mitigate the economic consequences of a liquidity crisis.

While the accumulation of liquidity rises when the deposit withdrawal shock impacts the economy, the effect is only transitory since banks have to use their liquid funds to cover the outflow. Bank liquidity falls by 4.66% points while reaching the second quarter after the shock. The sizable 4.98% absolute cumulative fall in bank dividends consumption contributes to the significant reduction in bank profitability. Since banks have to counterbalance the precautionary behavior during crisis time, the worsening of bank liquidity in bank balance sheet exacerbates all other components of their balance sheet. In this sense, the liquidity crisis inherits a mechanism similar to the financial accelerator mechanism that worsen the path of the economy. While the financial accelerator is an aggregate phenomena, its underpinning root arises in banks' balance sheet which is individual and heterogeneous across banks. The weakness of bank precautionary behavior due to the crisis propagates to the only source of bank funding which is short term debt and therefore to bank liquidity.

In Appendix L I supplement the model with a stochastic liquidity rate consistent with the wisdom that during financial crises the liquidity rate needs to vary. Therefore, I introduce imperfectly correlated shocks between deposits and the liquidity rate. This imperfection makes banks to strengthen their liquidity outside of crisis times and it effectively widens banks' risk aversion.

5.3.2 Illiquidity Episode Analysis

In this subsection I perform an illiquidity event analysis by which I consider liquidity crises produced by the deposit withdrawal shock. In particular I compare event windows in a simulation of $100,000$ periods in which I discard the first $2,000$ periods. The leading point of this experiment is the design of a liquidity crisis. I identify a liquidity crisis as periods in which the liquidity constraint is binding and bank liquidity falls by half standard deviation of liquidity. I consider event windows with 2 years before the crisis and 5 years and 2 quarters after the crisis which amount to event windows of $t = 30$ periods (quarters) in my simulation. The actual crisis is identified at time 0. Across the whole simulation I then average the history of paths in all event windows.

The model produces 201 liquidity crises and an endogenous probability of occurrence equals to 0.296% which mirrors the empirical fact that liquidity crises, as in the experience of all banking crises, are rare events. The small probability event is also due to the precautionary liquidity motive. This is one of the important results of the illiquidity event analysis where heterogeneous banks and occasionally binding liquidity constraints can reduce the incidence of the crisis. The crisis probability of this heterogeneous banks model is significantly lower than the crisis probability originated in Corbae and D'Erasmo (2021) for a competitive banking sector, where however, in their model the crisis arises in the loan market and liquidity regulations in their model reflect Dodd-Frank Act. It remains also below the crisis probability obtained in Corbae and D'Erasmo (2021) when considering a version of their model with a liquidity requirement capturing the liquidity coverage ratio.

Figure **??** shows the corresponding results. All variables are expressed in relative deviations except bank liquidity that is expressed in absolute deviation. The liquidity crisis is driven by a deposit withdrawal shock that abruptly falls leading to a deposit outflow and posing banks in need to cover the amount of deposits. The pivotal point is that preceding the crisis banks start decumulating liquid assets and contracting dividends, significance of the fact that banks perceive the arrival of the crisis. The size of the crisis is given by the sudden contraction of liquidity to deposits that falls by 7.29% below average for then to stay above its average.

At the outset of the crisis the liquidity constraint strains due to the fall in deposits that requires banks to build stronger liquidity buffer. On impact liquidity falls while dividends consumption rises, but while liquidity recovers after the crisis dividends show troughs post-crisis with the second trough documented in the second period after the crisis. Banks need to use their dividends to cover the collapse of liquid assets but at the same time they need to rebuild their buffer of liquidity. Once the liquidity constraint relaxes, bank dividends unwind and both decision rules return to their pre-crisis levels.

Table 8 reports illiquidity event crisis moments for the baseline economy with ex-ante regulation in comparison with the ex-post regulation economy, the economy with a liquidity ratio and no regulation in place. The table documents that while in the ex-ante regulated economy the probability of occurrence and the number of liquidity crisis exceed the respective moments of the ex-post regulated economy, the contraction of -7.801% in the liquidity to deposits is almost fivefold lower than the contraction in ex-post regulated economy. Likewise, the magnitude of illiquidity crisis produced by the contraction of -0.002% in banks' dividends in the ex-ante regulated economy is significantly lower than the ex-post regulated economy. The results confirm that in the baseline economy since the number of illiquidity events occurring are larger, banks internalise the arrival of the crisis at higher probability leading them to store enough precautionary liquidity. Moreover, small and medium banks insure well enough in anticipation

TABLE 8.—
Event analysis in different regulated economies

Moment	Occurrence, probability, %	Magnitude, $\%\Delta\ell/d$	Magnitude, $\%\Delta c$	Number crisis episodes
	(2)	(3)	(4)	(5)
Baseline, ex-ante reg	0.296	-7.801	-0.002	201
Ex-post regulation	0.123	-33.972	-0.008	56
Liquidity ratio model	0.233	-52.988	-0.116	69
No regulation	6.573	-12.879	-0.007	683

NOTE. Moments for illiquidity event analysis in the economies with ex-ante regulations, ex-post regulations, liquidity ratio and no regulation economy. Moments are reported in percent deviation from the long run mean.

to the crisis such that the subsequent fall in banks' liquidity hits less severely. Consequently, the liquidity to deposits and dividends need to fall by less than the ex-post regulated economy to cover the outflow of deposits. The largest contraction in the ex-post regulation economy is due to the fact that the ex-post regulation does not lead banks who needs liquidity the most to hold liquidity because the interest rate on liquidity does not drive up the cost on total short term debt. Therefore, less small banks are at the lowest liquidity requirement.

In the economy with a liquidity ratio in place the illiquidity event is generated with liquidity falling half standard deviation of liquidity since in this case banks are never liquidity constrained. The illiquidity event produces a much larger probability of liquidity crisis as well as number of illiquidity episodes than ex-post liquidity regulations. The contractions of -52.988% and -0.116% in banks' liquidity to deposits and dividend cashflows, respectively, are larger than the ex-post regulated economy since the banks that are going to cover the outflow are mainly large banks who are not liquidity constrained in the model and can better hedge the deposit outflows.

The last row of Table 8 reports crisis moments for the no regulation economy. In this case liquidity falls to -12.879% and dividends to -0.007% in a larger magnitude than the ex-ante regulated economy in conjunction with a greater rise of probability and episodes of liquidity crisis, threefold larger than the baseline increasing to 6.573% and 683, respectively. The reason is that the absence of the liquidity regulation does not engage banks towards building a buffer of liquidity and all those small and medium banks that were originally saving liquidity and building their precautionary liquidity in the ex-ante regulated economy by predicting the arrival of the crisis, now they fall below the liquidity requirement as nothing prevent them from doing it. The absence of regulation makes then liquidity crises much sever since the crisis is not only more likely to occur but episodes occur at a higher frequency. Therefore, small and medium banks need to internalise the deposit outflow to insure in anticipation to the crisis and this can only occur in an economy regulated ex-ante prior to the deposit shock since the presence of the interest rate on liquidity drives up the cost on short term debt.

I perform a simulation disregarding idiosyncratic risk and considering a representative bank. This modification however faces the cost of making all states recurrent and because the liquidity constraint turns out never binding then a liquidity crisis never occurs. In other words markets become complete. Importantly, the starting point of this paper is the precautionary liquidity motive led by ex-ante liquidity regulations with size dependent liquidity requirements and idiosyncratic withdrawal of deposits.

5.3.3 Liquid Crises in Bank Disaster Economy

This subsection modifies the model economy so as to embed a rare liquidity crisis in the form of a low probability of illiquidity disaster state. The purpose is to assess the economic significance of the model *i)* to produce a more volatile liquidity to deposit ratio with a fatter rightward distribution of bank liquidity and *ii)* a lower Gini coefficient as the value observed in 2010.

From Table 7 the baseline model has difficulties in generating volatility moment of the liquidity to deposit ratio in line with the data. To account for a more volatile and higher right tail of the distribution of the liquidity to deposit ratio I model the illiquidity disaster state by enriching the exogenous deposit process with an illiquidity disaster state. In contrast to the baseline economy, to discipline my model in the disaster economy banks are faced with a worst case scenario that is the possibility of a crisis. The transition probability matrix is a 4×4 probability matrix that, by the nature of the deposit withdrawal, becomes a 16×16 probability matrix. The probability of an illiquidity disaster state, p_c, occurring is constant and I study a counterfactual analysis for the response of the economy with a low probability event with the economy switching to a high probability event. I maintain a constant probability of illiquidity disaster state since my focal point has been the mean of liquidity to deposit ratio.[32] I further impose the assumption that any bank can reach the illiquidity disaster state, differently from the baseline economy.

The novelty is the ability to generate illiquidity phases due to the joint interaction of the disaster state and the occasionally binding liquidity constraint. Furthermore, the disaster state is such that large banks are able to keep their position in the banking sector. In particular, for all other states different from the crisis state, $d_j \neq d_c$, banks optimally choose their amount of dividends consumption and liquidity taking into account the state dependent liquidity constraint.[33]

Banks are aware that a crisis scenario may occur and therefore they must build their buffer of liquidity before that the crisis takes place. Otherwise, in an illiquidity disaster state it may happen that large banks away from the constraint, most likely large banks will undertake precautionary choices of liquidity whereas constrained small and medium banks with low deposits

[32] This is introduced along the lines of Gabaix (2008, 2011, 2012), Barro (2009) and Gourio (2012) who show that constant and time varying disaster risk can account for many asset pricing puzzles such as more volatile price dividends ratio.

[33] Subscript c identifies an illiquidity crisis disaster state.

will start dissaving to face the lack of liquidity violating the liquidity constraint. If banks violate the liquidity constraint they will need to use all their retained earnings worsening their total resources. While also large banks can switch to the illiquidity disaster state, by virtue of the state dependent liquidity constraint there is still a finite mass of banks living in all other states for which $d_j \neq d_c$.

The benchmark liquidity crisis in the disaster economy is calibrated with a probability of banks ending in the illiquidity disaster state, p_c, equal to, 3.91% per quarter. Likewise, the probability to transiting to a non illiquidity disaster state is set to 0.89. The value of the liquidity constraint, ϕ, is such that it matches the first moment of the liquidity to deposit ratio as in the baseline economy. The liquidity crisis disaster economy produces a liquidity requirement, ϕ, equal to 0.082 all else equal. This implies a significant fall of 5.63% in the liquidity constraint in equation (5) from the pure baseline economy analysed in Subsection 5.2.[34]

Figure **??** displays banks' decision rules for liquidity and dividends consumption with the inclusion of the illiquidity crises disaster state in the deposit process that reduces the liquidity constraint. The varying binding region is further expanded by the illiquidity crisis possibility in which the crisis bound, $\bar{\ell}_c$, occurs at the midpoint between the regions of constrained small banks and constrained medium banks, $\bar{\ell}_c \in [\bar{\ell}_m, \bar{\ell}_l]$. This is because in this case every one bank can transit to the crisis state regardless of their size while maintaining existence of a finite distribution of banks.

The qualitative properties of the model are confirmed by the presence of a fraction of banks acting at the constraint. Importantly, small banks represented by the blue dot lines need to build a larger precautionary buffer of liquidity to cover the hurdles of the crisis. Since large banks by nature have been able to build a buffer of liquidity superior than small banks, they are consequently those banks able to hedge fully the disaster innovation. While small banks understand that the possibility of an illiquidity disaster can occur, they are still in run to build their precautionary buffer of liquidity by the time the disaster takes place. The lower liquidity constraint prevents them to expand further their dividends consumption. Consequently, they are the banks that suffer the most from the possibility of a crisis.

The figure reveals that precautionary behavior of all constrained banks expands and dividends consumption smoothing for small banks is impeded by more than the baseline economy by the presence of the missing market for liquidity. This is emphasised by the largest concavity of banks' dividends consumption where the percentage change in dividends from the lowest value to the highest value for small banks is 0.118 compared to the baseline economy that produces a percent change in consumption for small banks equal to 0.025. This is further confirmed for large and medium banks, as well as banks in a disaster state while still maintaining a unitary coefficient of risk aversion.

The analyses further reveals two characteristics regarding the distribution of banks' liquid

[34] Further details for the calibration and transition probability matrix with regards to the illiquidity crisis disaster state are available in Appendix I.

TABLE 9.—
Illiquidity disaster crisis mean and std liquidity to deposit ratio

	Mean, ℓ/d				Std, ℓ/d			
	Crises intensity, p_c							
Risk aversion	0.0017	0.0391	0.0751	0.0912	0.0017	0.0391	0.0751	0.0912
$\gamma = 1.00$	0.190	0.184	0.181	0.179	0.032	0.036	0.039	0.040
$\gamma = 1.25$	0.191	0.185	0.181	0.180	0.032	0.036	0.039	0.040
$\gamma = 1.50$	0.192	0.185	0.182	0.181	0.032	0.036	0.039	0.040
$\gamma = 2.00$	0.193	0.187	0.183	0.181	0.031	0.036	0.039	0.040
$\gamma = 4.00$	0.197	0.189	0.185	0.183	0.031	0.036	0.039	0.040

NOTE. This table reports values for first and second moment of liquidity to deposits ratio ℓ/d, by varying the coefficient of relative risk aversion and the probability of a crisis occurring.

assets. Table I.1 in Appendix I presents the results. First, the Gini coefficient decays to 0.063 with a 4.22% reduction from the baseline economy. The decay consistently replicates the decay to 0.264 Gini coefficient observed in 2010 post financial crisis, but cannot reproduce the data counterpart. This is a consequence of the additional precautionary liquidity motive undertaken by all banks during a crisis. More small banks need to meet the liquidity constraint, therefore liquidity in the hands of this group of banks is larger than the baseline economy. The second remark pertains the holding of liquid assets along the percentiles distribution. Banks located in the top 1st and 5th percentiles of the liquid assets distribution now hold 1.1% and 5.4%, respectively, while the top 10th percentile holds 10.8% of liquid assets in the banking sector. The three moments in the data are 1.4%, 7.1% and 14.3%, respectively. That is to say the illiquidity disaster state helps to explain the upper tail of the distribution of liquid assets. The model predicts a sizable quantity of banks' liquidity in the bottom tail of the distribution thereby failing to generate the consistent righthand tail of liquid assets of small banks.

Comparative Statics. Table 9 displays the numerical values for two moments of the liquidity to deposit ratio, means and standard deviation, when varying the probability of a liquidity crisis occurring and the risk aversion coefficient, all else equal. The liquidity constraint, ϕ, is kept constant to the calibrated value of 0.082. The baseline liquidity crisis in a disaster economy is produced with a unitary risk aversion coefficient and a small probability of a liquidity crisis occurring, equal to 0.0391.

The table shows three features of the liquidity crisis in a disaster economy. First, the baseline liquidity disaster crisis successfully delivers first moment of liquidity to deposit ratio equal to the data and a more volatile second moment close to the data counterpart of 0.049. Second, the mean of liquidity to deposit ratio decreases as the probability of an illiquidity disaster increases. More fundamentally, banks are decumulating more liquidity when the probability of a liquidity crisis rises since they need to use they buffer of liquidity during the illiquidity phase. In anticipation of the crisis bankers engage in more accumulation of liquidity that means dividends consumption is low and the marginal utility high. However, as bankers become risk averse their incentive to accumulate liquidity increases. Holding constant the crisis intensity parameter, the average liquidity to deposit ratio increases as bankers become more risk averse because of precautionary motive. I conclude from the table that during an illiquidity crisis the liquidity to deposit rises.

Third, the volatility of the liquidity to deposit ratio increases with the crises intensity and in the majority of the cases it is invariant to changes in the degree of risk aversion. Nonetheless, to achieve a volatility close to the data moment a larger crisis intensity as well as a larger risk aversion coefficient are required. There are reasons for bankers to perceive the more risk which induce them to save more in liquidity in anticipation to the crisis, thereby making the liquidity to deposit ratio more volatile relative to the data. In Appendix I.1 I show a complementary analyses to banks' dividends and liquidity for different values of, p_c, crisis intensity.

Illiquidity Crisis Disaster Dynamics. In this part of the subsection I perform the dynamics of the economy given a one standard deviation idiosyncratic shock to the deposit withdrawal. I perform the same exercise in Figure **??** and therefore compute the liquidity crisis with the imposition of initial conditions for liquidity to deposit ratio to its mean value and the liquidity crisis left unrestricted from initial conditions.

I simulate the model for $100,000$ times for 30 periods and insert the values in the endogenous variables of interest. I solve the model twice, the second time by assuming as initial condition 1.4 the mean of liquidity to deposit that is a 40% rises from the mean in the baseline economy. The reason for doing it is to get a sense of the nonlinear dynamics of the model. The transitional dynamics with initial conditions equal the mean value of liquidity to deposits ratio is the consistent comparison with the responses obtained under the baseline economy in Subsection 5.3.1 without the introduction of the illiquidity disaster state.

Figure **??** illustrates the path of the economic variables. The results with different initial conditions are in line with the baseline economy in terms of expected path. The deposit withdrawal shock triggers an immediate contraction in bank liquidity, consumption and therefore in the liquidity to deposit ratio. The liquidity crisis is generated by the liquidity constraint that is tightening. The transitory effect of the rise in bank liquidity accumulation and profits is preserved on impact while vanishing afterwards since banks need to use their funds to cope with the hurdles of the deposit withdrawal during a crisis.

The first main result that emerges is that the illiquidity crisis state in the deposit process attenuates responses to the deposit withdrawal shock when initial conditions are at the mean value of liquidity to deposit−solid grey lines, that is the consistent comparison with the baseline economy. This is because banks insure high enough in anticipation to the crisis to prevent the large drop in dividends consumption to happen. Indeed, dividends fall by 0.038% only and liquidity to 0.029% in the second quarter. Banks are therefore able to make a smaller use of their liquidity accumulation that falls to 0.035% while approaching the second quarter. Moreover, banks' profits are nearly twice as much as the baseline economy at time 0 since they now contain profits not only of constrained banks, but also of unconstrained banks that can now move to the crisis state. The less dramatic decay in bank liquidity in turns keeps profits bounded at zero when leaving the first quarter, as they are high enough for banks that do not need to make a disproportionate use of their profits against the liquidity crisis.

The second important result in this analysis is that the liquidity crisis brought with initial condition the mean of liquidity to deposits delivers a more volatile liquidity to deposits ratio and profits. To trigger a smoother path of liquidity to deposit and profits I had to increase the value of the initial condition to 1.4 times the mean of liquidity to deposits. This is mainly due for two reasons. First, in the illiquidity disaster crisis all banks are going to transit to the crisis state whereas in the baseline illiquidity crisis banks were constrained to the next adjacent state. Second, as shown in Table 9, the liquidity deposit ratio is more volatile in the illiquidity disaster crisis than the baseline liquidity crisis since by construction the deposit process becomes more volatile under the illiquidity disaster crisis.

The illiquidity disaster crisis with initial conditions increased to 1.4 times the mean of liquidity to deposit produces a rise in the endogenous variables of interest of more pronounced magnitude. The impact effect is a drop of 3.06% in liquidity and 11.68% in the liquidity to deposit ratio. Bank dividends decay to 0.42% and while profits increase nearly as much as the profits with reduced initial conditions, they turn negative to 10.96% since their increase on impact is not enough to cover the fall in liquidity during the crisis. Note however that the attenuated fall in bank dividends is delivered by the illiquidity disaster state irrespective of initial conditions, while the impact response of all other variables is close to the baseline liquidity crises analysed in Subsection 5.3.1. This is significant of the fact that with the knowledge that an illiquidity disaster can occur, banks insure disproportionately to smooth any variation in possible drops in dividends.

Illiquidity Episode Analysis Dynamics. The aim in this part of the subsection is to analyse the episode dynamics of a liquidity crisis with the inclusion of the illiquidity disaster state. I perform the illiquidity episode analysis engendered by the deposit withdrawal shock. As in Subsection 5.3.2, the liquidity crisis occurs when the liquidity constraint is binding and liquidity falls half standard deviation below its mean. In a stochastic simulation of $100,000$ periods, an event window contains 2 years prior to the crises and 5 years and 2 quarters after the crises with the crisis hitting at time 0.

Figure **??** shows the results. The model with the illiquidity disaster state emulates the crisis of the baseline model for bank dividends and the liquidity constraint. The liquidity crisis is set off by the sizeable slump in the deposit withdrawal. The effects of the liquidity crisis manifests itself in the shrinkage of 2.26% below long run mean of the liquidity to deposit ratio.

The illiquidity episode analysis in a disaster economy unfolds two important results. First, in the periods prior to the liquidity crisis, deposits and liquidity accumulation exhibit a smooth decay characterised by less peaks and troughs. During the precrises phase liquidity accumulation

TABLE 10.—

Illiquidity crises from disaster economy

	Baseline crisis illiquidity disaster		High crisis intensity	Large risk aversion	Large size shock
	(2)	(3)	(4)	(5)	(6)
Moment	Data	Model	Model, p_c	Model, γ	Model, σ_c
mean, $\tilde{\ell}$	0.184	0.184	0.184	0.189	0.184
st.dev., $\tilde{\ell}$	0.049	0.036	0.039	0.036	0.042
median, $\tilde{\ell}$	0.189	0.182	0.185	0.185	0.183
skewness, $\tilde{\ell}$	0.046	1.147	1.154	1.153	1.344
Liquidity crises moment					
Occurrence probability, %	–	2.297	2.281	1.419	1.137
Illiquid crises magnitude, % $\Delta\ell/d$	–	-11.967	-10.941	-11.498	-4.383
Illiquid crises magnitude, % Δc	–	-0.612	-0.591	-0.611	-0.991
Number illiquidity episodes	–	$1,171$	$1,170$	946	825

NOTE. Liquidity to deposit is defined as $\tilde{\ell} = \ell/d$. For each parameter variation the corresponding liquidity constraint for the third column is given by 0.082, 0.085 for fourth and fifth columns and 0.033 for the experiment in the sixth column. Values are expressed in levels.

shows a mild decline that is followed at the onset of the crisis by a rise of more than threefold, 7.02%, the increase in the baseline liquidity crisis. This means that banks overaccumulate liquidity prior to the crisis since they now understand that a crisis is coming and to prevent the possibility of significant losses in profits. As soon as the crisis hits, banks decumulate liquidity to cover the deposit withdrawal.

Second, while liquidity and liquidity to deposits contract to 15% and 16.43%, respectively, in the first period postcrises they are characterised by a remarkable increase of 22.02% and 21.24%, respectively, above the long run mean values. Both remain above their trends for a long time, before reverting back to their long run means. The pace of dividends changes is not significantly different from the baseline economy, since banks have been able to insure to smooth any variations in dividends consumption. In other words, banks have responded more strongly with the accumulation of liquidity to the same shock and almost equivalent liquidity constraint that brought the liquidity crisis in a disaster economy than in the baseline economy. Moreover, banks undertake a decumulation in dividends consumption that is fast enough to approach the second quarter after the crises. Note also the small confidence bands around the peaks of the crisis which is due to the low variability of liquidity in a quarter.

The results demonstrate the crucial importance of the illiquidity disaster state in leading banks to have a strong incentive to accumulate liquidity before the crisis takes place, which therefore counterbalances the decline in dividends consumption. This poses the question of how much liquidity can grow. Since in this model liquidity is an asset for banks, there is a link between liquidity and the location of banks along the liquidity distribution. This means that for any bank the growth in liquidity is delimited by their size and by their incentive towards dividends consumption which constitutes their second choice. The rise in bank liquidity therefore reconciles the empirical fact of the rise in reserves in banks' balance sheet following the aftermath of the financial crisis.

Table 10 reports selected moments for the liquidity to deposit ratio in the first half of the table. Given the predictions of Table 9 I consider the second highest intensity of liquidity crisis, 0.0751, and a coefficient of risk aversion equal to 2 as good compromise to generate first and second moment close to the data. I therefore take these two values and compute the liquidity crisis moments produced by the dynamics of an event of a liquidity crisis scenario. These are shown in columns 4 and 5. The last column of Table 10 shows the results by increasing the size of the illiquidity disaster shock. The main difference from Table 9 is that I recompute the liquidity constraint, ϕ, that matches the liquidity to deposit ratio at each single parameter variation one at a time. For the selected parameter variations, the liquidity constraint is 0.085 for the experiments in columns 4 and 5 and it equals 0.033 for the experiment in column 6 with a larger size of the shock.

The first part of Table 10 shows on the one hand, that across all parameter variations only the model with an increase in the probability of the illiquidity disaster crisis and the model with a large size of the illiquidity disaster shock, in conjunction with the baseline illiquidity disaster crisis model can match exactly the first moment of liquidity to deposit ratio. On the other hand, in the illiquidity disaster crisis only a larger size of the shock can produce more variability in the liquidity to deposit ratio. On this aspect therefore the model does a good success since I introduced the illiquidity crisis state as a means to obtain more volatility in the second moment of the liquidity to deposit ratio. To obtain both moments exactly in line with the data I had to increase simultaneously the crisis intensity and the size of the illiquidity disaster shock and recalibrate the liquidity constraint with the target. This procedure resulted in 0.039 as new value for the liquidity constraint.

The second part of Table 10 shows moments dynamics for a liquidity crisis disaster that are computed as average from the long run distribution given the occurrence of the crises. The moments are the probability of occurrence of a liquidity crisis, the change in the liquidity to deposit ratio and dividends and the number of illiquidity crisis episodes occurring for each parameter variation. This part shows that the baseline model with an illiquidity disaster generates a liquidity crisis with a probability of occurrence equals to 2.297%. The liquidity crisis is associated with a fall in bank dividends consumption of 0.612% and a decay of the liquidity to deposit ratio equals to 11.967%. A high crisis intensity and large risk aversion cause a fall in bank dividends and liquidity to deposit ratio of order of magnitude almost identical to the baseline crisis. The main difference is that as bankers become more risk averse the probability of an illiquidity crisis decreases as they want to insure more against the risk of a liquidity crisis and hence of possible dividends drops. Therefore an increase in γ to 2 leads to a 1.419% probability of liquidity crisis.

The economy with a large size of the shock, that is produced in the last column, significantly reduces the size of the fall of liquidity to deposit ratio by almost threefold as lower as the value in the baseline crisis. However, it increases the drop in bank dividends from the baseline crises by a factor of half the decay in dividends under the baseline crisis. Likewise, the probability of an illiquidity crisis occurring is twice as less as the baseline crisis. This is consequence of the lower number of liquidity crises that a larger size of the shock can deliver.

The experiment emphasises the economic significance of increasing the size of the shock. It turns out relevant not only in producing mean and volatility moments of liquidity to deposits almost in line with the data, but also in the ability of the model to generate liquidity crises with low probability of occurrence and attenuated severity in the change in liquidity to deposits.

5.3.4 Sensitivity Analysis

In this subsection I recompute the model for different parameter values to understand which parameters are meaningful in explaining banks' liquidity distribution and liquidity crises.

I mainly focus on those parameters that impact on the precautionary behavior of banks. Therefore I consider robustness exercises on bankers' discount factor, β, risk aversion, γ, and the liquidity constraint, ϕ. For completeness I report cross sectional long run moments under the baseline economy with heterogeneous banks and liquidity crises in addition to the liquidity disaster crises and then perform parameter variations for both economies.

For all parameter variations, given the nature of the model I recalibrate the liquidity constraint to match first moment of liquidity to deposit ratio, except when I change the liquidity constraint since, in this case, another parameter would be required to match the moment. The liquidity constraint is therefore calibrated at $[0.1319, 0.1462]$ for β and γ parameter sensitivity, respectively, under the baseline economy, whereas under the illiquidity crises disaster state is calibrated to 0.0846 for both, β and γ, parameter sensitivity.

Furthermore, since the illiquidity crisis disaster economy analysed above delivered a looser liquidity requirement than the baseline economy to match the mean of liquidity to deposit; when I analyse the parameter sensitivity I make the liquidity constraint loose in the baseline economy to disentangle any difference with the liquidity crises in the disaster economy. Therefore, I set $\phi = 0.062$ relative to the baseline economy.

Table 11 summarises the robustness exercises. The first part of the table provides the parameter sensitivity under the baseline liquidity crises. The second part, instead, dispenses the results for the heterogeneous banks model with the illiquidity crisis disaster state.

As a first remark the baseline heterogeneous banks model of liquidity crises already produces a contained liquidity crisis, as pointed out in Subsection 5.3.2 in terms of liquidity to deposit and

TABLE 11.—
Sensitivity analysis

		Long run and crises moments				
	Gini	mean, ℓ/d	std, ℓ/d	prob.crises	% $\Delta\ell/d$	% Δc
Baseline liquidity crises						
Benchmark values	0.105	0.188	0.033	0.296	-7.801	-0.361
$\beta = .79599$	0.103	0.184	0.029	2.619	-23.279	-0.116
$\gamma = 2$	0.003	0.184	0.017	2.817	-16.977	-0.000
$\phi = .062$	0.145	0.136	0.037	0.291	-15.625	-0.369
Liquidity disaster crises						
Benchmark values	0.063	0.184	0.036	1.338	-11.364	-0.585
$\beta = .79599$	0.054	0.182	0.037	1.477	-13.393	-0.000
$\gamma = 2$	0.061	0.188	0.036	1.420	-11.601	-0.612
$\phi = .062$	0.071	0.163	0.037	1.344	-14.175	-0.586

NOTE. Liquidity to deposit is defined as $\tilde{\ell} = \ell/d$. Prob.crises denotes the liquidity crises occurrence probability expressed in percentages.

dividends decays that are 7.80% and 0.36%, respectively. In the baseline economy the discount factor was calibrated to .99896. Across all parameter variations, the less patient banks are the more severe the liquidity crisis hits since liquidity to deposit falls by 23.28% and dividends consumption by 0.12%. The result is confirmed under the liquidity crisis disaster economy although in a different magnitude. Indeed, banks undertake more precautionary behavior in advance to the crisis which mitigates the drop in liquidity to deposit, 13.39%, and it sharply cancels the drop in consumption, compared to the baseline economy.

The next point I analyse is the change in bank risk aversion not only in the liquidity disaster economy but also in the baseline model. Under the benchmark calibration, risk aversion, γ, was parameterised at unitary value. A more stringent risk aversion equal to 2 raises considerably the likelihood of the liquidity crisis relative to the benchmark values. However, while the increase of the likelihood is of 2.817% in the baseline heterogeneous banks model of liquidity crises, it only rises by 0.082% in the liquidity disaster economy. Note however that while inequality along banks' liquidity assets measured through the Gini coefficient is almost insensitive to the parameter variations but the liquidity constraint in the liquidity disaster economy, it decreases in the baseline model. More fundamentally, as all banks become more risk averse they all increase their precautionary behavior towards liquid assets but small banks are able to store a larger size of liquid assets by virtue of their small dimension. Since more liquid assets are kept by small banks, inequality along banks' liquid assets decreases from 0.105 to 0.003 relative to the benchmark values.

Likewise, there is a remarkable increase in banks' dividends consumption when banks become more risk averse in the baseline liquidity crises from 0.36% to 0.00% as liquid assets become the preferred choice to banks. This dividends consumption effect is reverted in the liquidity disaster crises model. In this case the more risk averse banks are, the larger the drop in dividends consumption. This is because the fact that banks know that a crisis may hit, leads them to store liquidity prior to the crisis. However, liquid assets are not sufficient to cover the hurdles of the crisis. Consequently, by the time the crisis comes banks need to use their dividends. Yet, the fact that banks know that a crisis may come, allows also small banks to store more liquidity thereby helping inequality in liquid asset distribution to stay close to the benchmark values.

As last robustness exercise, I inspect the impact of a relaxed liquidity constraint. As shown above the model gives rise to a less unequally distributed distribution of banks' liquid assets than when the liquidity constraint is more stringent. If I consider a less tighter liquidity requirement, the mechanism works in the opposite direction. As the floor for the liquidity requirement shrinks less constrained banks need to meet the liquidity constraint, then the banks that will insure more against the liquidity risk are large banks. This is because the lower liquidity requirement makes the liquidity constraint less binding. Consequently, the Gini coefficient rises relative to the benchmark values for both economies. Moreover, it is interesting to emphasise that while the probability of a crisis occurring remains fairly small compared to the benchmark values under both economies, the decay in bank liquidity to deposits is more than doubling in the baseline liquidity crisis, 15.625%, relative to the benchmark values whereas the decay in consumption is rather unaffected by the looser liquidity requirement. Therefore, when the liquidity crisis rarely hits this will have a profound impact on bank liquidity and dividends. In the liquidity disaster crises instead, the illiquidity disaster state smoothes mainly the drop in banks' liquidity.

The sensitivity analysis highlights the result that the knowledge of banks that an illiquidity crisis can occur through the illiquidity disaster state tempers the severity of the crises when banks are more risk averse and less patient compared to the baseline liquidity crisis. I conclude from the sensitivity analysis that results are robust and the baseline heterogeneous banks model of liquidity crises provides an adequate laboratory in reducing the impact of a liquidity crisis.

6 Conclusion

This paper developed a model of heterogeneous banks to study the effect of ex-ante liquidity regulations on the sorting of banks by types, bans' size expansion and inequality in banks' liquid assets and liquidity crises. Motivated by the rise in banks' liquidity in the bottom tail of the distribution of banks after the financial crisis and the deposit uncertainty surrounding banks' balance sheet, this paper analyses the implications arising from banks' uninsurable idiosyncratic deposit withdrawal risk and state dependent ex-ante liquidity regulations. Both of them expose banks to significant fluctuations in dividends. Nonetheless, the liquidity regulation I designate in terms of change in deposits thresholds delivers a stable contract that makes banks less prone to financial crises. The designation and timing of the liquidity regulation in this theory provides a rational on the reason of why small and medium banks−the banks who are more exposed to the risk of a deposit outflow−do not hold enough liquidity.

The absence of liquidity regulations when banks' liquidity is remunerated makes short term debt more costly, banks increase their exposure to the risk by reducing their liquidity and liquidity crises are more likely to occur. When ex-ante liquidity regulations are not enacted, the regulator is missing that banks will not internalise the cost on inflow and outflow of deposits, which in turn will not produce the change in deposit threshold necessary to generate a change in banks' incentives and impact the continuation value of banks. By taking into account this effect, ex-ante liquidity regulations in the change of the level of deposits generate the fraction of constrained small and medium banks reliant more on deposit and, an endogenous distribution of banks' liquid assets arises since deposits are qualified by a time and state dependent process.

Liquidity becomes costly for outflow banks since there are gains from inflow banks to expand the size of their balance sheet which occurs because these banks value the positive interest rate wedge on their dividends after the deposit withdrawal shock. I show that a tighter liquidity requirement makes the distribution of banks' liquid assets more equally distributed. Furthermore, I demonstrate the nonmonotonic behavior of banks' liquidity. The reinterpretation for the complementarity between liquidity and dividends rests on the occasionally binding state dependent liquidity constraint and the cost outflow banks face in expanding liquidity. Moreover, I show that while large banks can expand in size because of their better initial conditions, small banks can expand their size following a change in deposits. In equilibrium, the sorting of banks by type implies that large banks with less deposits will prefer more liquid assets than small banks. Therefore, the lower the deposits, the larger banks' liquidity and banks' size expansion.

The analysis underscores the relevance of ex-ante prudential interventions on liquidity in the level of deposits to temper liquidity crises. This type of regulation contains the risk of future deposit outflows thereby limiting contractions in banks' liquidity. In an illiquidity episode ex-ante regulations engender mild liquidity crises due to the endogenous creation of a stable contract which is stronger than ex-post liquidity regulations. As a result, contractions in banks' liquidity and profits lessen significantly. In addition, I propose the model with an illiquidity disaster state. The model predicts sizable gains from the illiquidity disaster state in reconciling the low inequality in banks' liquid assets post financial crisis and low contractions in banks' liquidity due to the deposit withdrawal shock.

To focus on the effect of liquidity regulation on the sorting of banks, banks' size expansion, inequality in banks' liquid assets and the dynamics of liquidity crises, I have abstracted from important features surrounding the banking sector that deserve further exploration. While loans are highly illiquid they constitute the main source of maturity mismatch in banks' balance sheets. The precautionary liquidity motive is then strictly related to the choice of banks' illiquid assets, yet in the model banks do not make a choice over loans. In Gigante (2022) I endogenise banks' illiquid assets to study redistribution effects amongst liquidity and loans which is conducive to distributional concentration in banks' illiquid assets and marginal propensity to lend.

Accumulation of banks' liquidity may be the endogenous outcome of real forces therefore general equilibrium considerations in my current theoretical framework can uncover new insights on the distributional consequences to banks' liquid and illiquid assets. I leave all these important avenues for future research.

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Appendix to
"Heterogeneous Banks, Liquidity Risk and the Distribution of Banks' Liquidity"

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Appendix available separately.