General Financial Economic Equilibria

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Presentation at AMES Conference
New Delhi, India
January 14 2024

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- Hence the reference to a "General Financial Economic Equilibrium."
- Implications for economic policy are discussed by numerically solving a variety of equilibrium examples.

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- However, in the presence of uncertainty excess demands must be random functions of prices and as a consequence they cannot be equated to zero, to form market clearing prices.
- The fundamental equilibrium equations are no longer valid and neither is the underlying equilibrium concept.

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- Economies exposed to uncertainty must deliver price systems that cannot be state contingent.
- In principle the states are too numerous and diverse to warrant either a full description or enumeration.
- We therefore seek to revise the equilibrium concept in the context of stochastic or random excess demands.

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- Equilibrium prices are defined by equating demands to supplies or excess demands to zero.
- The market may be viewed as an additional and abstract participant determining prices with a view to clearing markets.
- All the information across all economic participants is available to the market in setting prices.

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- The market therefore cannot equate excess demands to zero as they are random functions of the given prices.

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- Similarly the supply functions are

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$$z(p_L, p_U, \omega_D, \omega_S) = S(p_L, \omega_S) - D(p_U, \omega_D),$$

- and for excess net revenue by

$$R(p_L, p_U, \omega_D, \omega_S) = p_U D(p_U, \omega_D) - p_L S(p_L, \omega_S).$$

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- Acceptable risks are defined in ADEH as a convex cone of random outcomes containing the nonnegative outcomes.
- The latter are of course acceptable, by virtue of being devoid of risk.

Cones of Acceptable Risks

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- As a result the prices of the two price economy must satisfy

$$\begin{aligned} \inf_{Q \in \mathcal{M}_i} E^Q \left[z_i(p_L, p_U, \omega_D, \omega_S) \right] &= 0 \\ \inf_{Q \in N_i} E^Q \left[R_i(p_L, p_U, \omega_D, \omega_S) \right] &= 0. \end{aligned}$$

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- We thus have $2n$ equations in $2n$ unknowns defining the two price n commodity equilibrium.

Two Price Equilibria using Distorted Expectations

- When risk acceptability is defined in terms of the probability law of the risk and satisfies comonotone additivity (Kusuoka (2001)) then there exist distorted expectation operators $\mathcal{E}_i$ and $\widetilde{\mathcal{E}}_i$ such that the two price general financial equilibrium (GFEE) may be defined by

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- By virtue of minimizing a nonnegative distance function such an approximate two price equilibrium always exists.
- Of course, it need not be unique but is probably locally so.

Distorted Expectations

- A concave distribution $\Psi(u)$ on the unit interval $0 \leq u \leq 1$ defines the distorted expectation of a risk X with distribution function $F(x)$ by

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\begin{aligned}
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- The greater the concavity of Ψ the greater the weight given to negative outcomes and the lower the weight given to positive outcomes and the lower is the distorted expectation.

Specific Distortions

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- The greater is γ the greater is the concavity.

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$$E^P\left[S_i(p_L,\omega_S)\right] \geq E^P\left[D_i(p_U,\omega_D)\right]$$

and

$$E^P\left[p_{Ui}D_i(p_U,\omega_D)\right] \geq E^P\left[p_{L_i}S_i(p_L,\omega_S)\right]$$

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- So we have that $p_L \leq p_U$.

Marshallian Partial Financial Equilibrium

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- The excess supply is then

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- In a simple model we may take the uncertainties Z_D, Z_S to be normally distributed with zero means variances σ_D^2, σ_S^2 and correlation ρ, ignoring the issues of demand and supply possibly getting negative.

Two Price Net Revenue

- The net revenue is given by

$$R = p_U D(p_U) + p_U Z_D - p_L S(p_L) - p_L Z_S$$

Two Price Toy Equilibria

- The equilibrium equations are

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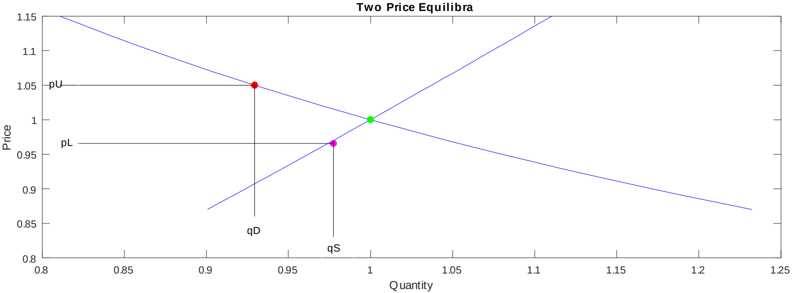
- for a distorted expectation operator $\mathcal{E}$.

A Sample Solution

- For a demand elasticity of 1.5 and a supply elasticity of 0.75 with $\theta, \widetilde{\theta}$ at $0.2, 0.75$ and σ_D, σ_S at $0.2, 0.1$ the Figure displays the two price equilibrium with p_U, p_L at $1.05, 0.97$ and qD, qS at $0.93, 0.98$.


Two Price Equilibra
1.15
1.1
1.05
1
0.95
0.9
0.85
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Price
pU
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qD
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1.2
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Quantity

Figure:

A K-Good Full Employment General Equilibrium

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- where L_k is the labor employed, M_k is a unit expectation positive shock.

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- The labor market must clear with full employment and the wage rate is determined to ensure that

$$\sum_k L_k = \sum_j \overline{L}_j = \overline{L}.$$

Consumer Incomes and Demand for Goods

- The consumers with labor endowments $\overline{L}_j$ have shares in the profits of σ_{jk} and they receive as income

$$\overline{V}_j = w\overline{L}_j + \sum_k \sigma_{jk}\pi_k.$$

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- The demand functions are stochastic reflecting random influences on preferences that prices cannot be made dependent on.

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- The net revenue exposure on account of product k is

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- To induce stochastic or random demands we take the logarithm of the $a_{kj}'s$ to be distributed as multivariate normal, and r_j, the elasticity of substitution to be gamma distributed and independent of the $a_{kj}'s$.

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- The paper provides an extensive analysis of a two good economy.

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- The correlations in the $a_k's$ were 0.5. The stress levels for all five excess supplies were 0.25 and net revenues were 0.5.

Five Good Solution

- For this economy the shortfalls in demand relative to output, the markups for the upper price relative to the lower price and the expected revenues by sector in labor units were as follows.

	1	2	3	4	5
shortfall	0.1152	0.1379	0.1923	0.3620	0.5752
markup	0.2973	0.1921	0	0	0
Exp. Revenue	0.3193	0.0311	−0.1076	−0.1149	−0.1269

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- Later we consider random supplies as well.
- The demand for labor is made random by shifts in the levels of the production functions observable to producers before they make their employment decisions

Stochastic Production Functions

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- The demand for labor is random at the wage w_U, and the total demand is

$$L_D = \sum_k L_k$$

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- for appropriate distorted expectation operators.

Solution of a Particular Five Good Case

- The production function parameters are

$$\alpha = (0.8,\ 0.7,\ 0.6,\ 0.7,\ 0.8).$$
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- The stress levels were 0.25 for the excess supplies and 0.5 for the net revenues in all markets.

The Two Price Equilibrium

- The equilibrium wage levels were

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- The price levels for the five goods were

$$\begin{aligned} p_L &= (2.3925,\ 2.0100,\ 1.4364,\ 1.3571,\ 1.4793) \\ p_U &= (2.9559,\ 2.1499,\ 1.4364,\ 1.3571,\ 1.4793). \end{aligned}$$

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- The level of unemployment support in the economy may be defined as the income ratio of the unemployed to the employed. Here it is

$$
\begin{aligned}
UNSL &= \frac{.0353 * 2.0562}{(1 - .0353) * 2.1799} \\
&= 3.45\%.
\end{aligned}
$$

Productivity and Employment

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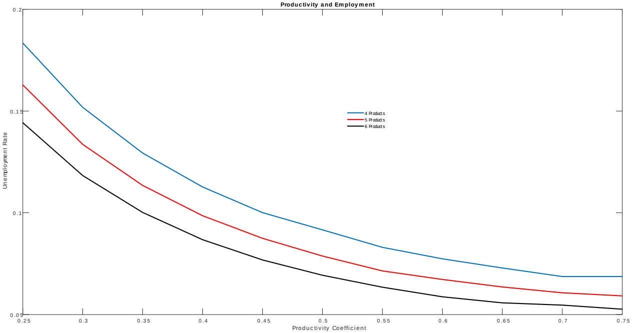
- The graph shows that increased productivity coupled with an expansion in the number of productive activities may help maintain employment levels.


Productivity and Employment
Unemployment Rate
Productivity Coefficient
4 Products
5 Products
6 Products
0.2
0.15
0.1
0.05
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75

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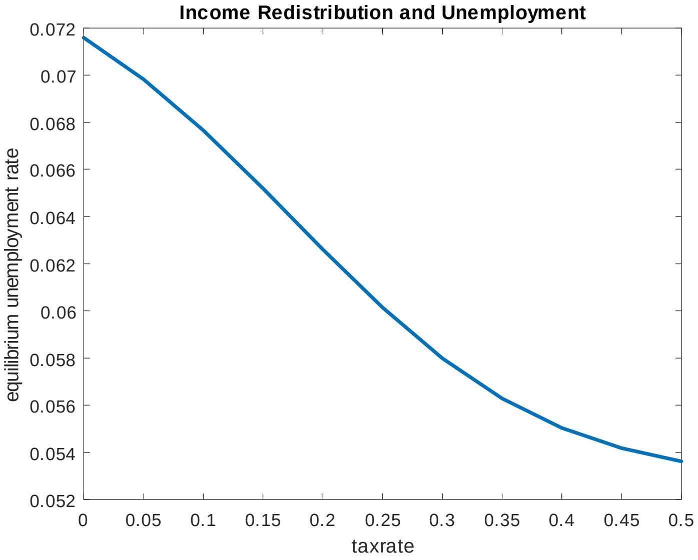
- The figure shows the effects on unemployment of income redistribution through taxation.


Income Redistribution and Unemployment
equilibrium unemployment rate
taxrate
0.072
0.07
0.068
0.066
0.064
0.062
0.06
0.058
0.056
0.054
0.052
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

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- There is a case on equilibrium grounds for enhancing unemployment support in the face of a productivity shock.

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- There is then one output, Y and two types of labor that we denote by L_1 and L_2.
- For a random supply of labor we introduce a utility function for the output and leisure for the two labor groups.
- Consider a Cobb-Douglas utility function with maximal labor supplies of A_1, A_2 for the two groups and utility function

$$u(Y, L_1, L_2) = Y^{\beta}(A_1 - L_1)^{\theta_1}(A_2 - L_2)^{\theta_2}.$$

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- There is a resulting random supply of output.

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$$\begin{aligned} w_{L1} L_{S1} &= \frac{\beta + \theta_2}{\beta + \theta_1 + \theta_2} w_{L1} A_1 - \frac{\theta_1}{\beta + \theta_1 + \theta_2} \left(w_{L2} A_2 + \pi \right) \\ w_{L2} L_{S2} &= \frac{\beta + \theta_1}{\beta + \theta_1 + \theta_2} w_{L2} A_2 - \frac{\theta_2}{\beta + \theta_1 + \theta_2} \left(w_{L1} A_1 + \pi \right) \\ Y_D &= \frac{\beta}{\beta + \theta_1 + \theta_2} \frac{w_{L1} A_1 + w_{L2} A_2 + \pi}{p_U}. \end{aligned}$$

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- The stress levels in all cases were set at 0.2.

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- The solution gave working wages for the two labor types in units of output consumption with deflator p_U, of $w_{U1}/p_U = 0.3981$ and $w_{U2}/p_U = 0.2235$ or a 78% premium for the skilled group.

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- The average labor supplied was 0.5696 and 0.4951 for groups one and two respectively.

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Conclusion

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- We go on to introduce random demands and supplies in the labor market.
- Equilibria then reflect equilibrium levels of unemployment, unemployment insurance, and unemployment supports.
- Productivity shocks and income redistribution are related to lower unemployment rates and greater levels of equilibrium unemployment support.