The Power of Open-Mouth Policies*

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Abstract

Central banks' announcements about future monetary policy make economic agents to react before the announced policy takes place. We evaluate the anticipation effects of such announcements in a realistically calibrated prototypical central banking model. We consider temporary and permanent anticipated changes in policy rules including inflation target and Taylor-rule coefficients, as well as anticipated switches from inflation targeting to price-level targeting and average-inflation targeting, and we find economically significant effects. Our methodological contribution is to develop a novel perturbation-based *extended function path (EFP)* framework for constructing nonstationary solutions (time-dependent decision functions) to economic models with anticipated non-Markov news shocks.

JEL classification: C61, C63, C68, E31, E52

Key Words: news shocks; turnpike theorem; time-dependent models; nonstationary models; unbalanced growth; regime switches; monetary policies; price-level targeting; average inflation targeting

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1 Introduction

Central banks increasingly rely on communication to implement their monetary policy. Through their communication to the public, the monetary authorities indicate their future intentions as well as their views of the future states of the economy. For example, a central bank may promise to fix its interest rate for a certain number of periods before normalizing its policy (forward guidance) or it may announce a future change in the inflation target. Understanding the effects of communication is essential from the policy evaluation perspective.

In this paper, we assess the effects of central bank communication by analyzing several monetary policies that are conjectured in the literature to be welfare improving. The studied policies include one-time and gradual anticipated changes in economic policies, as well as more complex scenarios in which various monetary policies happen with some probabilities. A distinctive feature of our model with communication is that the agents react to the news of a policy change even when the new policy will not take effect until a later date. We find that such anticipation effects can be very large.

Our analysis is carried out in a realistically calibrated prototypical central banking model, a smaller replica of the Terms of Trade Model (ToTEM) used by the Bank of Canada for projection and policy analysis. Importantly, our "baby" ToTEM follows the full size ToTEM model as close as possible and generates very similar impulse response functions; see Dorich et al. (2013), and Lepetyuk, Maliar and Maliar (2020, henceforth, LMM) for a description of the ToTEM and "baby" ToTEM models, respectively.

The policy experiments we consider include: (1) a gradual change in the inflation target that happens in the future either with certainty or with some probability; (2) normalization of monetary policy regarding future nominal interest rates, when the economy is initially at a zero lower bound (ZLB) on nominal interest rates; (3) a switch to a more aggressive Taylor rule; (4) a switch to price-level targeting instead of inflation targeting; (5) a switch to average inflation targeting instead of inflation targeting.

Our findings are as follows: (1) A one-percent increase in inflation rate raises the output about 0.2% and postponing an increase in the inflation target by one year produces an additional 0.1% increase in output over the transition to a new steady state. Even if the announced policy is implemented with some probability, there are still substantial anticipation effects both before and after uncertainty is resolved. (2) When an economy is at ZLB on nominal interest rates, the central bank uses policy announcements (forward guidance) about its future return to the standard interest rate rule to direct the economy's transition out of ZLB. The more it postpones such a return to the standard rule, the larger is output expansion over the transition; an initial jump in output is however invariant to the horizon of this forward guidance policy. Therefore, this experiment informs policy makers on optimal horizons of monetary policy normalizations after ZLB periods. (3) Nevertheless, a more aggressive (but realistic) behavior of the central bank toward targeting inflation and output is not translated into important anticipatory effects on the side of economic agents. (4) Switching from inflation-level targeting to price-level targeting has smaller impacts with larger implementation lags. Price-level targeting was argued in the literature to be welfare improving. Therefore, a central bank that waits to implement the new policy in practice loses time, and the economy does not get earlier benefits from higher output. (5) Finally, a switch to average inflation targeting also has modest anticipation effects; this is because average inflation targeting is in a middle ground between inflation targeting and price-level targeting. In sum, our analysis shows that the model's implications about the importance of anticipation effects depend on a specific experiment considered: there are substantial policy anticipation effects present in our experiments (1)–(2), but such effects are relatively small in experiments (3)–(5).

On the methodological side, we argue that announcements about future economic conditions and policies can be modeled as anticipated non-Markov news shocks. Such news shocks lead to nonstationary solutions (time-dependent optimal decision decision) which cannot be analyzed using conventional numerical methods for constructing one stationary (time-invariant decision function). *In the context of perturbation analysis, we modify the conventional framework to facilitate the construction of time-dependent decision functions. We refer to our method as an extended function path (EFP) because of it's similarity to an* *extended path (EP) method of Fair and Taylor (1983); the difference is that EP constructs a path for variables (time series) whereas EFP constructing a path for decision functions, see Maliar, Maliar, Taylor and Tsener (2020, henceforth, MMTT) for a discussion and review of related literature. The EFP analysis is applicable to a broad and empirically relevant class of economies in which finite-horizon trajectories converge to infinite-horizon trajectories as the time horizon increases (the property is known as turnpike theorem). The EFP perturbation method developed here is comparable in accuracy to a global projection method developed in MMTT (2020), but it is tractable in problems with much higher dimensionality, such as large-scale central-banking models. Our ubiquitous software is written using the popular Dynare platform combined with user-friendly MATLAB interface; and it can be easily adapted to other applications the reader might be interested in.*

There are other methods in the literature for analyzing changes in economic environment but they rely on the assumption of Markov shocks, in particular, models with Markov regime switching (e.g., Davig and Leeper, 2007) and models with Markov news shocks (e.g., Schmitt-Grohé and Uribe, 2012). There is an important difference between that literature and our EFP analysis: the Markov literature view shocks as recurrent random draws from a stationary Markov distribution whereas we consider shocks which are given by a sequence of historical events happening at given dates. Consequently, the Markov literature constructs jsut one time-invariant decision rule that correspondis to a given Markov process whereas our EFP method approximates an infinite sequence of optimal decision rules that correspond to a given sequence of historical events. We view the two approaches as complementary: the stationary Markov literature focuses on unanticipated recurrent events like business cycles, whereas the EFP method analyzes anticipated non-recurrent histrocial events like Brexit.

In special cases, nonstationary solutions produced by EFP can be similar to stationary Markov solutions. For example, a one time non-Markov parameter shift is similar to a regime-switching Markov model with an absorbing state and also, it is similar to a news-shock Markov model with highly persistent (random walk) news. However, more complex EFP scenarios (composed of non-recurrent periods of growth, transitions, shifts, drifts) can't be adequately modeled by using a single stationary Markov process. As an illustration, we assessed the difference between the EFP solution and the news-shock method of Schmitt-Grohé and Uribe (2012) in the context of bToTEM model and we found that such difference is economically significant. Possibly, complex non-Markov EFP scenarios can be approximated by tying up together a sequence of stationary Markov solutions, however, it is not unknown if such constructions are tractable.

The rest of the paper is organized as follows: Section 2 illustrate the EFP methodology at a glance using an example of a neoclassical growth model; Section 3 describes a large-scale central banking bToTEM model; Section 4 presents five policy experiments; Section 5 compares the solutions under Markov and non-Markov news shocks; and, finally, Section 6 concludes.

2 Anticipated non-Markov events and the EFP method at a glance

We use an example of a stylized neoclassical growth model to illustrate the perturbation-based EFP methodology at a glance and to show how anticipated non-Markov events affect the macroeconomy.

2.1 What are anticipated events?

An anticipated event is something that an individual or a group expect to happen in the future. The term 'anticipated events" is used in various contexts, including psychology, event planning, and clinical trials. In psychology, anticipation involves emotions linked to the expectation of an event, which can include pleasure or anxiety. For example, the arrival of a highly anticipated movie or the outcome of a major sports event can evoke strong emotions among fans and stakeholders. Anticipatory emotions play a crucial role in how individuals prepare for and react to future situations, influencing their behavior and mental state.

We adapt this term to economics: The economic agents become aware of some future changes including the economy fundamentals (preferences, technology, economic environment, equations, variables, parameters, expectations) and policies (legislative, monetary, fiscal). The future events can be deterministic or stochastic. The dates of the future events can also be either fixed or uncertain. Importantly, the events we consider are non-Markov; they are nonrecurrent and time-contingent (i.e., happen at given dates); in contrast, the mainstream of the literature studies Markov events which are recurrent and state-contingent.

An example of anticipated non-Markov event is Brexit, which took place on 31 January 2020 following a referendum held in the UK on 23 June 2016. After Brexit waa announced, rational agents adjust their behavior to optimally respond to the new environment prior to the date when Brexit actually happen. In turn, an examples of recurrent unanticipated Markov events in the literature are business cycles which happen at random dates with some state-contingent probabilities. Rational agents adjust they behavior to business cycle by adapting their decision rule to the presence of uncertainty, e.g., precautionary savings. Below, we illustrate the difference between unanticipated Markov and anticipated non-Markov events using the example of a stylized growth model.

2.2 A growth model with Markov and non-Markov shocks

We consider a stylized stochastic growth model:

$$\max_{\{c_t,k_{t+1}\}_{t=0}^{T}} E_0\left[\sum_{t=0}^{T}\beta^t u\left(c_t\right)\right] \tag{1}$$

$$\text{s.t. } c_t + k_{t+1} = (1-\delta)\,k_t + z_t f\left(k_t\right), \tag{2}$$

where c_t and k_t are consumption and capital, respectively; initial condition (k_0, z_0) is given; the utility and production functions $u : \mathbb{R}_+ \to \mathbb{R}$ and $f : \mathbb{R}_+ \to \mathbb{R}_+$ are twice continuously differentiable, strictly increasing, strictly quasi-concave and satisfy the Inada conditions; z_t is exogenous productivity level; $\beta \in (0,1)$ is a discount factor; $\delta \in (0,1]$ is a depreciation rate; $E_t[\cdot]$ is an operator of expectation, conditional on t-period information set $\mathcal{F}_t$; and lifetime utility (1) is bounded.

We assume that the time horizon is infinite $T=\infty$. A feasible program is a pair of random processes $\{c_t, k_{t+1}\}_{t=0}^{\infty}$, such that given information sets $(\mathcal{F}_0, \mathcal{F}_1 ...)$ they satisfy $c_t \geq 0$, $k_{t+1} \geq 0$ and (2) for $t = 1, 2....$ We assume that the set of feasible programs is not empty.

An optimal program is a feasible program that maximizes (1). We focus on the case when the optimal program of (1)-(2) is interior and satisfies the Euler equation:

$$u'(c_t) = \beta E_t\left[u'(c_{t+1})(1-\delta + z_{t+1}f'\left(k_{t+1}\right))\right]. \tag{3}$$

We assume that the interior optimal program exists and is unique.

2.3 A nonstationary model with anticipated events.

For the sake of illustration, we assume that z_t in (2) follows the usual first-order autoregressive process with an additional term A_{t+1} that represent anticipated events:

$$\ln z_{t+1} = A_{t+1} + \rho \ln z_t + \sigma\epsilon_{t+1}, \tag{4}$$

where $\sigma > 0$ and $|\rho| \leq 1$. Here $\epsilon_{t+1} \sim \mathcal{N}(0,1)$ is a Markov shock defined by a probability disctributio $\epsilon_{t+1} \sim \mathcal{N}(0,1)$; and A_t is a non-Markov shock which is defined by a time-path $(A_0, A_1, ...)$. The impact of anticipated events depends on what information about the sequence $(A_0, A_1, ...)$ is provided to the agent and when.

In the absence of anticipated events, i.e., $A_t = 0$ for all t, the model (1), (2), (4) is a business cycle model whose solution given by a time-invariant (stationary) Markov decision function $k_{t+1} = K^*(k_t, z_t)$ satisfying a fixed-point property: if we substitute $t+1$–period function $K_{t+1}(k_{t+1}, z_{t+1})$ in the right side of (3) and compute the t–period function $K_t(k_t, z_t)$ in the left side of (3), we obtain the same (fixed point) function $K_{t+1} = K_t = K^*$.[1]

However, in the presence of non-Markov anticipated shocks, the decision rules are generally nonstationary (time-dependent). Intuitively, if at $t = 0$ the agent is informed of a shock $A_t = 1$ at $t \geq 1$, she reacts immediately by adjusting her decision rules $K_0^*, ..., K_t^*$ to take advantage of a future productivity change. Her decision functions change over time; they depend on how many periods is left till the shock actually happen, i.e., $K_0^* \neq ... \neq K_t^*$.

We can think of a variety of non-Markov scenaria designed to represent the path of actual economies. In particular, the economy may face multiple anticipated events, let's say, in periods $t = 10$, $t = 17$ and $t = 23$; the events may happen with some probabilities, let's say 50%, 30% and 10%; anticipated shocks may affect multiple equations and parameters and may be correlated with or conditional on one another; the information can be revealed to the agent at once or gradually, etc.

For stationary models, we need to construct one fixed-point decision rule $K_0^* = K_1^* = ... = K^*$ that corresponds to a given Markov process - this rule works for all time periods. Conventional global nonlinear solution methods iterate on Euler equations backward until a fixed point decision rule K^* is found; and conventional perturbation methods that construct Taylor expansion by restricting the decision rules to be the same at t and $t + 1$, etc. But these methods do not work for nonstatinary models we study because fixed-point decision rules either do not exist or are not optimal in the presence of non-Markov shocks. For our nonstationary model, we need to construct an infinite sequence of optimal decision functions $K_0^*, K_1^*, ...$.

2.4 Turnipike theorem

In the present paper, we develop a pertubation solution framework for approximating a sequence of decision functions $K_0^*, K_1^*, ...$ in the presence of anticipated non-Markov shocks for a broad and empirically relevant class of nonstationary models that satisfy the turnpike theorem. Turnpike theorem states that the trajectory of the finite-horizon economy converges to that of the infinite-horizon economy as the time horizon increases.

Let us illustrate this theorem for the model (1), (2), (4) under the parameterization that lead to a closed-form solution, specifically, $u(c) = \ln(c)$, $f(k) = k^\alpha$, $A_t = \gamma_A^t$, where $\gamma_A \geq 1$ is a growth rate (we set at $\delta = 1$, $\beta = 0.99$, $\alpha = 0.36$, $\rho = 0.95$, $\sigma = 0.01$ and $\gamma_A = 1.01$).

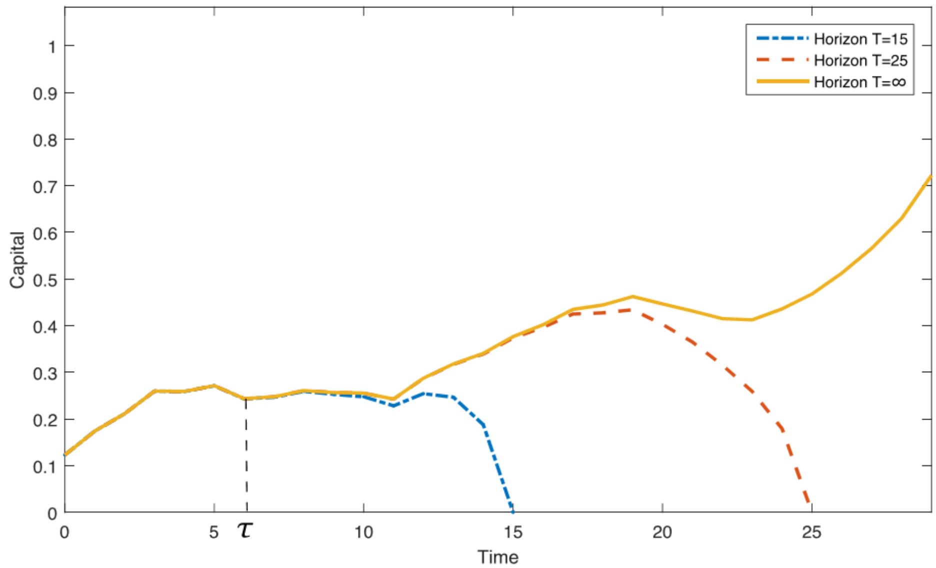
For a moment from, let us abstract from the issue of anticipated shocks and let us compare the trajectories of the finite- and infinite-horizon economies. In a finite-horizon model in which the life ends at $T + 1$, we have $k_{T+1} = 0$ and the decision rules for the previous periods $T, T-1, ...$ are given by

$$k_T = \frac{\alpha\beta}{1+\alpha\beta} z_{T-1} k_{T-1}^\alpha A_{T-1}, \quad k_{T-1} = \frac{\alpha\beta(1+\alpha\beta)}{1+\alpha\beta(1+\alpha\beta)} z_{T-2} k_{T-2}^\alpha A_{T-2}, \quad \text{etc.} \tag{5}$$

By extending the time horizon T to infinity, we obtain the solution for the infinite horizon economy. In Figure 1, we plot the capital series trajectories of the economies with finite horizons of $T = 15$ and $T = 25$

[1] Markov switching model (e.g., Davig and Leeper, 2007) and recurrent news shock model (e.g., Schmitt-Grohé and Uribe, 2012) belog to the class of stationary models.

under identical sequences of shocks as well as of the one of the economy with infinite horizon $T = \infty$.



From the figure, we observe the following fact: if all three economies start with the same initial capital stock, they follow a virtually identical path for some time and diverge only in a close proximity to the terminal date. Thus, we can accurately approximate an infinite-horizon solution during some initial number of periods τ by a finite-horizon solution – this is precisely what turnpike theorem means; see Maliar, Maliar, Taylor and Tsener (MMTT, 2019) for a formal discussion. We now have a foundation for our nonstationary analysis: Since a finite horizon model is solved by backward iteration from a given terminal condition K_T^* toproduces a sequence of time varying optimal decision rules $K_{T-1}^*, K_{T-2}^*, ...K_0^*$, we can introduce anticipated non-Markov shocks essentially at no additional cost.

2.5 Extended function path (EFP) method

On the basis of turnpike theorem, MMTT (2019) introduce a numerical method - *expended function path* (*EFP*) - for approximating a sequence of optimal decision functions in an infinite-horizon economy $(K_0^*, K_1^*, ...)$ by a finite horizon solution $(K_0, ..., K_\tau)$ during the initial τ periods.

Algorithm 1: Extended function path (EFP).
Step 1: Terminal condition. Choose $T \gg \tau$ and construct terminal condition K_T.
Step 2*:* **Backward iteration***.* Given the terminal condition K_T, iterate backward on the Euler equation to construct a path of $(K_{T-1}, ..., K_0)$.
Step 3: Turnpike property*.* Verify that the initial τ functions $(K_0, ..., K_\tau)$ are not sensitive to the choice of time horizon T and terminal condition K_T.
Use $(K_0, ..., K_\tau)$ as an approximate solution and discard the remaining $(K_{\tau+1}, ..., K_T)$.

MMTT (2019) use the EFP method to analyze several challenging applications with time-dependent decision rules including unbalanced stochastic growth models, the entry into and exit from a monetary union, information news, anticipated policy regime switches, deterministic seasonals, among others. A shortcoming of the proposed EFP method is that it relies on global projection style techniques with a high computational expense. To reduce the expense, MMTT (2019) use Smolyak (sparse) grids but the method still is costly: it takes several minutes to implement even for the simplest growth model.

2.6 Perturbation EFP method

In the present paper, we aim to analyze time-dependent scenarios in large-scale new Keynesian models of central banking. The global EFP method of MMTT (2019) will be too expensive or even intractable for so large models. Also, writing a code for global solutions is demanding for large-scale models. We therefore introduce a different version of the EFP method by using relatively inexpensive and ubiquitous perturbation techniques. To make our analysis useful to a larger community, we write our code using dynare - a popular automated software for constructing perturbation solutions in economics, see https://www.dynare.org.

The built-in dynare perturbation software is designed to construct a stationary solution to a Markov model. It uses the same deterministic steady state to expand the decision rules in periods t and $t+1$; and it imposes a fixed-point restriction on the coefficients of the decision rules to be the same in periods t and $t+1$. However, both of these restrictions are invalid for our nonstationary analysis so that the dynare's built-in solver cannot be used.

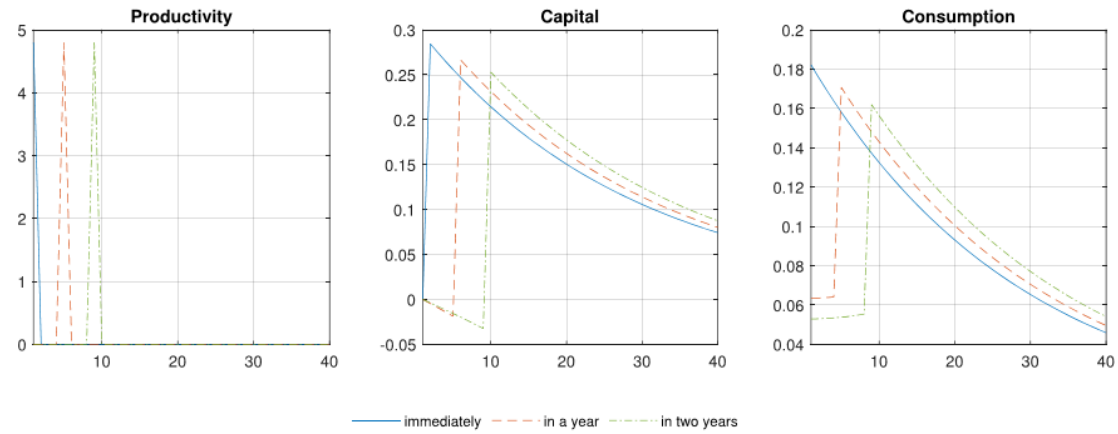
We design an EFP perturbation software that overwrites the built-in dynare routines to address three points: *i*) we solve for a time dependent sequence (path) of states around which the perturbation solutions are constructed - these points replace deterministic steady state; *ii*) we allow the decision rules to differ in consecutive periods - this construction replaces the standard fixed-point solution; and *iii*) we modify the solver to construct the optimal sequence of decision rules instead of finding a fixed point limiting solution. In the reminder of the section, we illustrating the solutions constructed by the EFP perturbation method.

In some nonstationary models, it is straightforward to identify points around which point(s) the Taylor expansion should be constructed. For example, our policy experiments with bToTEM model involve a shift in parameters that leads to a change in steady state. In those experiments, we construct perturbation solutions around the old and new steady states, and we assume that the economy switches between the solutions when the parameters change. However, this approach is infeasible in models with unbalanced growth; for such models, we first construct a deterministic growth path which the economy would follow in the absence of business-cycle shocks, and we then use the growth path as a set of points around which the Taylor expansion is constructed, see Appendix A for an elaborated example.[2]

Illustration 1: The announcement of anticipated event. In the first experiment, an unanticipated productivity increase at $t = 0$ (blue line) is compared with anticipated productivity increases that are

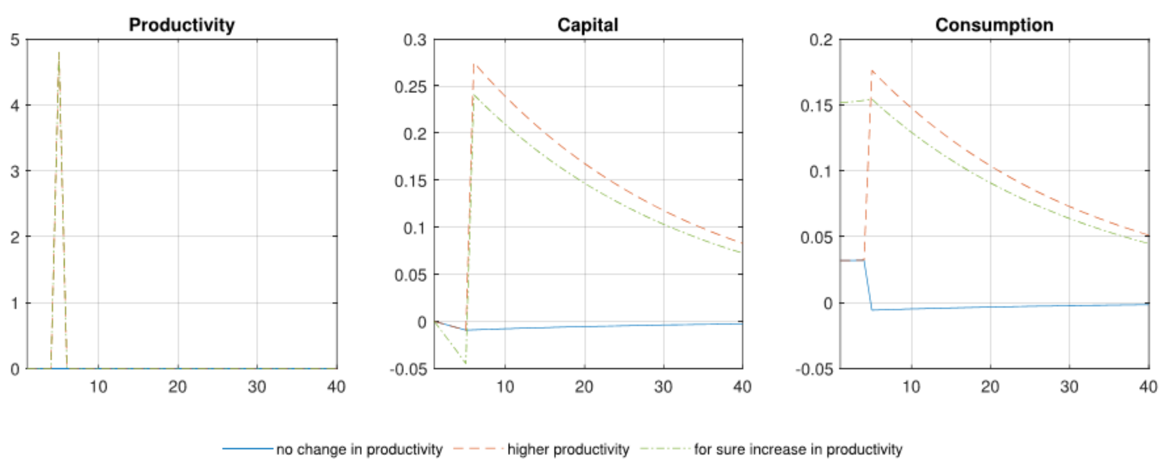
[2] *Some nonstationary models admit variable scaling to render a scaled model with a deterministic steady state. A well known example is a growth model with labor augmenting progress and homothetic preferences and technology which follows a balanced growth path and thus, can be converted into stationary; see King et al. (1988). We do not focus on those exceptional cases but on a genericlly nonstationary case.*

announced at $t = 0$ but occur in one year (period $t = 4$, red line) or in two years ($t = 8$, green line):



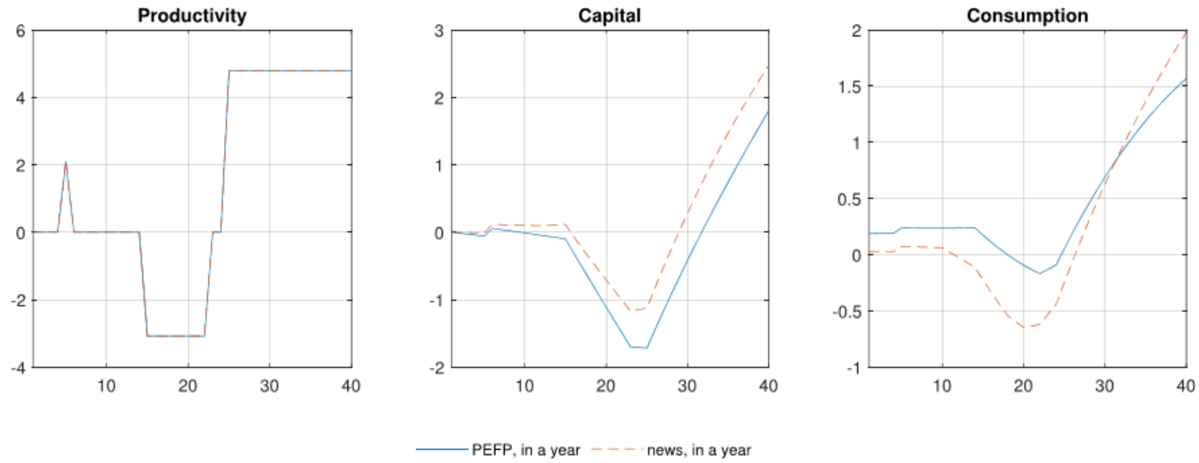
We observe well pronounced anticipatory effects on the red and green lines. Right after a productivity growth is announced at t, the agent takes advantage of the future productivity increase: she decreases (eats up) her capital to increases consumption. An instantaneous increase in consumption is larger for the red line than for the green line because in the later case, the additional resources are spread over a larger period.

Illustration 2: Anticipated events with an uncertain outcome In the second experiment, we consider the case when the agent is informed at $t = 0$ that a productivity increase will occur in one year (period $t = 4$). We compare the case when the productivity increase is certain (green line) with the case when it occurs with probability 50% (the red and blue lines show the paths when the increase actually occur and did not occur, respectively).



We observe that an anticipated growth in consumption is much larger for productivity growth that occurs with certainty (green line) than for the uncertain ones (red and blue lines). When the productivity is realized and uncertainty is resolved, we observe an additional boost in consumption for the economy where the productivity grows, and we observe a sharp consumption decline when the productivity does not grow.

Illustration 3: Modeling a historical path of actual economy. In the last experiment, we consider the economy that faces a sequence of anticipated events composed of a temporary positive shock, a prolonged but temporary negative regime shift and a permanent positive regime shift.



This example illustrates how the EFP method works for a historical path of the economy. Specifically, our goal is to construct a mixture of certain and uncertain events to reflect the historical path of an actual economy.

The EFP framework can be also used to model how economic agents learn about the importance of future events. Learning was not an addressed in our analysis so far. Our goal was to study short-term boosts in economic activity following the announcements of economic changes. However, certain announcements can serve another purpose, which is to make sure that households, firms, and markets have sufficient time to understand and prepare for the future changes. This is particularly true for important changes in policy frameworks, such as the adoption of price level targeting or average inflation targeting studied in Section 4.

There are different ways to model how the agents learn about the importance of future events. One possibility is to assume that the agent underestimates the shock initially but gradually perceives it's true size, for example, in period $t = 0$, the authority annouces that at $t = 10$, there will be a shock of a unit size $A_{10} = 1$, however, the agent believes that the size of the shock will be smaller, specifically, in periods $t = 0, 1, 2, ...10$ the agent believes that A_{10} will be $0, \frac{1}{10}, \frac{2}{10}, ..., 1$, respectively, understanding the correct size of the shock only at the end. Another possibility is to model learning in terms of a probability of shock by assuming that the agent underestimates the probability of shock initially but improves her forecast over time, for example, in periods $t = 0, 1, 2, ...10$, the agent believes that that $A_{10} = 1$ will happen with probabilities $0, \frac{1}{10}, \frac{2}{10}, ..., 1$, understanding the correct probability only at the end. The EFP allows us to model these and other learning schemes in order to represent the historical process taking place in actual economies.

3 Large-scale central banking models

Nowadays, the central banks, leading international organizations and government agencies, use large-scale macroeconomic models for projection and policy analysis. A prominent example is the Terms of Trade Economic Model (ToTEM) of the Bank of Canada. That model includes several types of utility-maximizing consumers, several profit-maximizing production sectors, monetary and fiscal authorities, as well as a foreign sector. ToTEM is huge: it contains 356 equations and unknowns, including 215 state variables; see Dorich et al. (2013).

In the paper, we consider a scaled-down version of ToTEM developed by LMM (2020). Like the full-scale model, the "baby" ToTEM (in short, bToTEM) is a small open-economy model. It features multiple new-Keynesian Phillips curves – one due to sticky prices in domestic production, one due to sticky wages and one due to sticky import prices. We incorporate the rule-of-thumb price settlers in line with Galí and

Gertler (1999). We assume quadratic adjustment costs of investment and convex costs of capital utilization to generate more realistic model's performance, in particular, with respect to monetary-policy transmission. The international trade consists of exporting domestic consumption goods and commodities and importing foreign goods for domestic production. Even though bToTEM is much smaller than ToTEM (it has only 47 equations and unknowns, including 21 state variables) it generates realistic impulse-responses of the Canadian economy to shocks, which are very similar to those produced by the full scale ToTEM model; see LMM (2020) for comparison results. In this section, we present a bToTEM central banking model and the implementation of the EFP analysis.

Final-good production. Final consumption goods are produced in two stages. In the first stage, intermediate goods are produced competitively using labor, capital, commodities and imports. In the second stage, final goods are aggregated from differentiated goods that are each produced by a monopolistically competitive firm from the intermediate goods and from the final goods. The final goods can be consumed by households. They can also be transformed using linear technologies into other types of goods, namely, investment goods and noncommodity exports goods.

In the first production stage, a representative perfectly competitive firm produces an intermediate good by solving the following profit maximization problem:

$$\max_{\left\{Z_t^g, Z_t^n, L_t, K_t, I_t, COM_t^d, M_t, u_t, d_t\right\}} E_0 \sum_{t=0}^{\infty} \mathcal{R}_{0,t}\left(P_t^z Z_t^n - W_t L_t - P_t^i I_t - P_t^{com} COM_t^d - P_t^m M_t\right)$$

$$\text{s.t. } Z_t^g = \left[\delta_l \left(A_t L_t\right)^{\frac{\sigma-1}{\sigma}} + \delta_k \left(u_t K_{t-1}\right)^{\frac{\sigma-1}{\sigma}} + \delta_{com}\left(COM_t^d\right)^{\frac{\sigma-1}{\sigma}} + \delta_m \left(M_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{6}$$

$$\log A_t = \varphi_a \log A_{t-1} + (1-\varphi_a)\log \bar{A} + \xi_t^a, \tag{7}$$

$$K_t = (1-d_t) K_{t-1} + I_t, \tag{8}$$

$$d_t = d_0 + \bar{d} e^{\rho(u_t - 1)}, \tag{9}$$

$$Z_t^n = Z_t^g - \frac{\chi_i}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 I_t, \tag{10}$$

where Z_t^g and Z_t^n are gross production and net (of adjustment costs) production of final goods; L_t, K_t, I_t COM_t^d, M_t, u_t and d_t are labor, capital, investment, commodity inputs, imports, capital utilization and depreciation rate, respectively; A_t is the level of labor-augmenting technology; ξ_t^a is a normally distributed variable, and φ_a is an autocorrelation coefficient. The firm discounts nominal payoffs according to household's stochastic discount factor $\mathcal{R}_{t,t+j} = \beta^j (\lambda_{t+j}/\lambda_t)(P_t/P_{t+j})$, where λ_t is the household's marginal utility of consumption, and P_t is the final good price. Investment goods and noncommodity exports are assumed to be produced from the final goods according to linear technology, $P_t^i = \iota_i P_t$ and $P_t^{nc} = \iota_x P_t$, where P_t^i and P_t^{nc} are the price of investment goods and noncommodity exports goods, respectively.

In the second stage of production, monopolistically competitive firms produce a continuum of differentiated good. Then, these differentiated goods are aggregated into the final good by an aggregating firm that solves the following cost minimization problem

$$\min_{\{Z_{it}\}} \int_0^1 P_{it} Z_{it} di$$

$$\text{s.t. } Z_t = \left(\int_0^1 Z_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where Z_t and P_{it} are given; Z_{it} is a differentiated good i, The cost minimization implies the following demand function for the differentiated good i:

$$Z_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Z_t, \quad \text{with } P_t \equiv \left(\int_0^1 P_{it}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$

Each differentiated good is produced from the intermediate goods and from the final goods using technology featuring perfect complementarity,

$$Z_{it} = \min\left(\frac{Z_{it}^n}{1-s_m}, \frac{Z_{it}^{mi}}{s_m}\right), \tag{11}$$

where Z_{it}^n is an intermediate good and Z_{it}^{mi} is a final good input, and s_m is a Leontief parameter.

There are two types of the monopolistically competitive firms producing differentiated goods: rule-of-thumb firms of measure ω and forward-looking firms of measure $1-\omega$. Both rule-of-thumb firms and forward-looking firms index their price to the inflation target $\bar{\pi}_t$ with probability θ as $P_{it} = \bar{\pi}_t P_{i,t-1}$. The rule-of-thumb firms partially index their price to lagged inflation and target inflation with probability $1-\theta$,

$$P_{it} = (\pi_{t-1})^{\gamma} (\bar{\pi}_t)^{1-\gamma} P_{i,t-1}. \tag{12}$$

Forward-looking firms choose their prices P_t^* with probability $1-\theta$ to maximize profits generated when the price remains effective

$$\max_{P_t^*} E_t \sum_{j=0}^{\infty} \theta^j \mathcal{R}_{t,t+j} \left(\prod_{k=1}^{j} \bar{\pi}_{t+k} P_t^* Z_{i,t+j} - (1-s_m) P_{t+j}^z Z_{i,t+j} - s_m P_{t+j} Z_{i,t+j}\right) \tag{13}$$

$$\text{s.t. } Z_{i,t+j} = \left(\frac{\prod_{k=1}^{j} \bar{\pi}_{t+k} P_t^*}{P_{t+j}}\right)^{-\varepsilon} Z_{t+j}.$$

The production in the first stage Z_t^n and that in the second stages Z_t are related via price dispersion Δ_t,

$$Z_t^n = \int_0^1 Z_{it}^n di = (1-s_m) \int_0^1 Z_{it} di = (1-s_m) \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Z_t di = (1-s_m) \Delta_t Z_t, \tag{14}$$

where $\Delta_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di$.

Production of commodities. Commodities are produced by a domestic firm using final goods and land as inputs. They are sold domestically or exported to the rest of the world. The domestic firm solves

$$\max_{Z_t^{com}, COM_t} \{P_t^{com} COM_t - P_t Z_t^{com}\}$$

$$\text{s.t. } COM_t = (Z_t^{com})^{s_z} (A_t F)^{1-s_z} - \frac{\chi_{com}}{2} \left(\frac{Z_t^{com}}{Z_{t-1}^{com}} - 1\right)^2 Z_t^{com}, \tag{15}$$

where Z_t^{com} is the final good input, and F is a fixed production factor, which may be considered as land. Similar to production of final goods, the commodity producers incur quadratic adjustment costs when they adjust the level of final good input. The commodities are sold domestically or exported to the rest of the world, $COM_t = COM_t^d + X_t^{com}$. They are sold at the world price adjusted by the nominal exchange rate, $P_t^{com} = e_t P_t^{comf}$, where e_t is the nominal exchange rate (i.e., domestic price of a unit of foreign currency), and P_t^{comf} is the world commodity price; in real terms, the latter price is given by $p_t^{com} = s_t p_t^{comf}$, where $p_t^{com} \equiv P_t^{com}/P_t$ and $p_t^{comf} \equiv P_t^{comf}/P_t^f$ are domestic and foreign relative prices of commodities, respectively, P_t^f is the foreign consumption price level, and $s_t = e_t P_t^f / P_t$ is the real exchange rate.

Production of imports. The representative perfectly competitive firm produces the final imported good M_t from a continuum of intermediate imported goods M_{it} and solves the following cost-minimization problem,

$$\min_{\{M_{it}\}} \int_0^1 P_{it}^m M_{it} di$$
$$\text{s.t. } M_t = \left(\int_0^1 M_{it}^{\frac{\varepsilon_m - 1}{\varepsilon_m}} di \right)^{\frac{\varepsilon_m}{\varepsilon_m - 1}},$$

where M_{it} is an intermediate imported good i. The demand for an intermediate imported good i is given by

$$M_{it} = \left(\frac{P_{it}^m}{P_t^m} \right)^{-\varepsilon_m} M_t, \quad \text{with } P_t^m \equiv \left(\int_0^1 \left(P_{it}^m\right)^{1-\varepsilon_m} di \right)^{\frac{1}{1-\varepsilon_m}}.$$

Prices of the intermediate imported goods are sticky in a similar way as the prices of the differentiated final goods. A measure ω_m of the importers follows the rule-of-thumb pricing, and the others are forward looking. The optimizing forward-looking importers choose the price P_t^{m*} in order to maximize profits generated when the price remains effective

$$\max_{P_t^{m*}} E_t \sum_{j=0}^{\infty} (\theta_m)^j \mathcal{R}_{t,t+j} \left(\prod_{k=1}^{j} \bar{\pi}_{t+k} P_t^{m*} M_{i,t+j} - e_{t+j} P_{t+j}^{mf} M_{i,t+j} \right)$$
$$M_{i,t+j} = \left(\frac{\prod_{k=1}^{j} \bar{\pi}_{t+k} P_t^{m*}}{P_{t+j}^m} \right)^{-\varepsilon_m} M_{t+j},$$

where P_t^{mf} is the price of imports in the foreign currency. All importers face the same marginal cost given by the foreign price of imports.

Households. Households maximize the lifetime utility by choosing holdings of domestic and foreign-currency denominated bonds, labor and consumption, and they are subject to habits in consumption. Each household supplies a variety of differentiated labor service to the labor market, which is monopolistically competitive. The representative household of type h solves the following utility-maximization problem:

$$\max_{C_t, L_{ht}, B_t, B_t^f} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\mu}{\mu - 1} \left(C_t - \xi \bar{C}_{t-1} \right)^{\frac{\mu-1}{\mu}} \exp\left(\frac{\eta (1-\mu)}{\mu (1+\eta)} \int_0^1 (L_{ht})^{\frac{\eta+1}{\eta}} dh \right) \eta_t^c \right\},$$
$$\text{s.t. } P_t C_t + \frac{B_t}{R_t} + \frac{e_t B_t^f}{R_t^f \left(1 + \kappa_t^f\right)} = B_{t-1} + e_t B_{t-1}^f + \int_0^1 W_{ht} L_{ht} dh + \Pi_t, \tag{16}$$
$$\log \eta_t^c = \varphi_c \log \eta_{t-1}^c + \xi_t^c, \tag{17}$$

where C_t, L_{ht}, B_t, B_t^f are consumption of final goods, labor service of type h, holdings of domestic and foreign-currency denominated bonds, respectively; $\bar{C}_t$ is the aggregate consumption, taken by the household as given; $\beta \in (0,1)$ is a subjective discount factor; μ and η are the utility-function parameters; η_t^c is a consumption demand shock, ξ_t^c is a normally distributed variable, and φ_c is an autocorrelation coefficient; R_t and R_t^f are domestic and foreign nominal interest rate, respectively; κ_t^f is the risk premium on the foreign interest rate; W_{ht} is the nominal wage of labor of type h; Π_t is profits paid by the firms. The representative household supplies a variety of differentiated labor service to the labor market, which is monopolistically competitive.

Labor packer. A labor packer aggregates differentiated labor services by solving

$$\min_{\{L_{ht}\}} \int_0^1 W_{ht} L_{ht} dh$$
$$\text{s.t. } L_t = \left(\int_0^1 L_{ht}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dh\right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}},$$

where L_t is aggregated labor demanded by firms in the first stage of production. Cost minimization of the labor packer implies the following demand for individual labor:

$$L_{ht} = \left(\frac{W_{ht}}{W_t}\right)^{-\varepsilon_w} L_t, \quad \text{with } W_t \equiv \left(\int_0^1 W_{ht}^{1-\varepsilon_w} dh\right)^{\frac{1}{1-\varepsilon_w}}. \tag{18}$$

Labor unions. Labor unions set wages. There are two types of labor unions: rule-of-thumb unions of measure ω_w and forward-looking unions of measure $1 - \omega_w$. Within each type, with probability θ_w the labor unions index their wage to the inflation target $\bar{\pi}_t$ as follows $W_{it} = \bar{\pi} W_{i,t-1}$. The rule-of-thumb unions that do not index their wage in the current period follow the rule

$$W_{it} = \left(\pi_{t-1}^w\right)^{\gamma_w} \left(\bar{\pi}_t\right)^{1-\gamma_w} W_{i,t-1}. \tag{19}$$

A forward-looking unions that do not index its wage solves

$$\max_{W_t^*} E_t \sum_{j=0}^{\infty} (\beta\theta_w)^j \left\{ \frac{\mu}{\mu - 1} \left(C_{t+j} - \xi \bar{C}_{t+j-1}\right)^{\frac{\mu-1}{\mu}} \exp\left(\frac{\eta(1-\mu)}{\mu(1+\eta)} \int_0^1 \left(L_{ht+j}\right)^{\frac{\eta+1}{\eta}} dh\right) \eta_{t+j}^c \right\} \tag{20}$$

$$\text{s.t. } L_{h,t+j} = \left(\frac{\prod_{k=1}^j \bar{\pi}_{t+k} W_t^*}{W_{t+j}}\right)^{-\varepsilon_w} L_{t+j}, \tag{21}$$

$$P_{t+j} C_{t+j} = \prod_{k=1}^{j} \bar{\pi}_{t+k} W_t^* L_{h,t+j} dh + \Psi_{t+j}, \tag{22}$$

where Ψ_{t+j} includes terms in budget constraints (16) other than C_{t+j} and $L_{h,t+j}$.

Monetary authority. The central bank uses a Taylor rule to set the short-term nominal interest rate,

$$R_t = \rho_r R_{t-1} + (1 - \rho_r) \left[\bar{R} + \rho_\pi (\pi_t - \bar{\pi}_t) + \rho_Y \left(\log Y_t - \log \bar{Y}_t\right)\right] + \eta_t^r, \tag{23}$$

where ρ_r measures the degree of smoothing of the interest rate; $\bar{R}$ is the nominal neutral interest rate; ρ_π measures a response to the inflation gap; $\bar{\pi}_t$ is the inflation target; ρ_Y measures a response to the output gap; $\bar{Y}_t$ is potential output; η_t^r is an interest rate shock following a process

$$\eta_t^r = \varphi_r \eta_{t-1}^r + \xi_t^r,$$

where ξ_t^r is a normally distributed variable, and φ_r is an autocorrelation coefficient. Potential output changes with productivity according to

$$\log \bar{Y}_t = \varphi_z \log \bar{Y}_{t-1} + (1 - \varphi_z) \log\left(\frac{A_t \bar{Y}}{\bar{A}}\right).$$

If an effective lower bound R_t^{elb} is imposed on the nominal interest rate, the interest rate is determined as a maximum of (23) and R_t^{elb}:

$$R_t = \max\left\{R_t^{elb}, \Phi_t\right\}.$$

The ELB restriction was never binding in our experiments. However, our earlier LMM (2020) paper studied a stationary version of the bToTEM model with an occasionally binding ELB constraint using IRIS perturbation software (see https://iris.igpmn.org), and Occbin perturbation toolbox of Guerrieri and Iacoviello (2015); these methods can be also used in the context of our nonstationary analysis.

Foreign demand for noncommodity exports. By analogy with the demand for imports, the foreign demand function for noncommodity exports is assumed to be

$$X_t^{nc} = \gamma^f \left(\frac{P_t^{nc}}{e_t P_t^f} \right)^{-\phi} Z_t^f, \tag{24}$$

where P_t^{nc} is a domestic price of noncommodity exports; γ^f is the demand-function parameter. In real terms, we have

$$X_t^{nc} = \gamma^f \left(\frac{s_t}{p_t^{nc}} \right)^{\phi} Z_t^f. \tag{25}$$

Balance of payments. The balance of payments in nominal terms is given by

$$\frac{e_t B_t^f}{R_t^f \left(1 + \kappa_t^f\right)} - e_t B_{t-1}^f = P_t^{nc} X_t^{nc} + P_t^{com} X_t^{com} - P_t^m M_t, \tag{26}$$

where B_t^f is domestic holdings of foreign-currency denominated bonds, and R_t^f is the nominal interest rate on the bonds. By normalizing the bonds holdings as $b_t^f \equiv \frac{e_t B_t^f}{\pi_{t+1}^f P_t \bar{Y}}$, the balance of payments in real terms becomes

$$\frac{b_t^f}{r_t^f \left(1 + \kappa_t^f\right)} - b_{t-1}^f \frac{s_t}{s_{t-1}} = \frac{1}{\bar{Y}} \left(p_t^{nc} X_t^{nc} + p_t^{com} X_t^{com} - p_t^m M_t \right), \tag{27}$$

where r_t^f is the real interest rate on the foreign-currency denominated bonds.

Rest-of-the-world economy. The rest of the world is specified by three exogenous processes that, respectively, describe the evolution of foreign output Z_t^f, the foreign real interest rate r_t^f, and the foreign commodity price p_t^{comf},

$$\log Z_t^f = \varphi_{zf} \log Z_{t-1}^f + \left(1 - \varphi_{zf}\right) \log \bar{Z}^f + \xi_t^{zf}, \tag{28}$$

$$\log r_t^f = \varphi_{rf} \log r_{t-1}^f + \left(1 - \varphi_{rf}\right) \log \bar{r} + \xi_t^{rf}, \tag{29}$$

$$\log p_t^{comf} = \varphi_{comf} \log p_{t-1}^{comf} + \left(1 - \varphi_{comf}\right) \log \bar{p}^{comf} + \xi_t^{comf}, \tag{30}$$

where ξ_t^{zf}, ξ_t^{rf} and ξ_t^{comf} are normally distributed random variables, and φ_{Zf}, φ_{rf} and φ_{comf} are autocorrelation coefficients.

Uncovered interest rate parity. We impose an augmented uncovered interest rate parity condition

$$e_t = E_t \left[(e_{t-1})^{\varkappa} \left(e_{t+1} \frac{R_t^f \left(1 + \kappa_t^f\right)}{R_t} \right)^{1-\varkappa} \right]. \tag{31}$$

Stationarity condition for the open-economy model. The risk premium κ_t^f is a decreasing function of foreign assets

$$\kappa_t^f = \varsigma\left(\bar{b}^f - b_t^f\right), \tag{32}$$

where $\bar{b}^f$ is the steady state level of the normalized bond holdings. This assumption ensures a decreasing rate of return to foreign assets.

Summary of the model's variables. For each period t, there are the following four types of variables in this model: 47 endogenous (or non-predetermined) variables,

$$y_t \equiv \left\{ \begin{array}{c} F_{1t}, F_{2t}, F_{1t}^w, F_{2t}^w, F_{1t}^m, F_{2t}^m, q_t, \lambda_t, s_t, \\ L_t, K_t, I_t, COM_t^d, M_t, u_t, d_t, Z_t^g, Z_t^n, Z_t, C_t, Y_t, \pi_t, rmc_t, \Delta_t, \pi_t^m, \bar{\pi}_t, p_t^m, R_t, p_t^z, w_t, \\ MPK_t, R_t^k, p_t^i, \kappa_t^f, b_t^f, X_t^{nc}, X_t^{com}, COM_t, Z_t^{com}, \pi_t^w, w_t^*, \Delta_t^w, \bar{Y}_t, p_t^{com}, p_t^{nc}, p_t^{mf}, p_t^y, \end{array} \right\},$$

where $\{F_{1t}, F_{2t}\}$, $\{F_{1t}^w, F_{2t}^w\}$, $\{F_{1t}^m, F_{2t}^m\}$ are supplementary variables in Phillips curves for prices, wages and imports, respectively; q_t is Tobin's q; rmc_t and MPK_t are real marginal cost and marginal productivity of capital, respectively; p_t^{nc} and p_t^y are prices of noncommodity goods and output, respectively. We have 15 endogenous state variables:

$$\left\{C_{t-1}, R_{t-1}, s_{t-1}, \pi_{t-1}, \Delta_{t-1}, w_{t-1}, \pi_{t-1}^w, \Delta_{t-1}^w, p_{t-1}^m, \pi_{t-1}^m, I_{t-1}, Z_{t-1}^{com}, b_{t-1}^f, \bar{Y}_{t-1}, K_{t-1}\right\},$$

where π_{t-1}^w, Δ_{t-1}^w, p_{t-1}^m, π_{t-1}^m are wage inflation, wage dispersion, and price and inflation of imports; we have 19 endogenous forward variables:

$$\left\{ \begin{array}{c} F_{1t+1}, F_{2t+1}, F_{1t+1}^w, F_{2t+1}^w, F_{1t+1}^m, F_{2t+1}^m, \lambda_{t+1}, q_{t+1}, u_{t+1}, I_{t+1}, \\ Z_{t+1}^{com}, \pi_{t+1}, \bar{\pi}_{t+1}, \pi_{t+1}^m, \pi_{t+1}^w, p_{t+1}^{com}, p_{t+1}^z, MPK_{t+1}, s_{t+1} \end{array} \right\};$$

and we have 6 exogenous state variables:

$$z_t \equiv \left\{A_t, \eta_t^R, \eta_t^c, p_t^{comf}, r_t^f, Z_t^f\right\}.$$

Our calibration procedure for bToTEM is described in Appendix C and it closely follows the calibration of the full scale ToTEM model.

The implementation of the perturbation EFP method for the bToTEM model LMM (2020) construct conventional time-invariant solutions to a stationary bToTEM model.[3] Here, we construct the novel time-dependent (nonstationary) solutions to the model. We model announcements about economic policies that will be implemented at some future dates as anticipated non-Markov news shocks, and we analyze a reaction of economic agents to such announcements. Assuming that we are at $t = 0$ and that a given policy will be implemented at $T > 0$, we construct a sequence of optimal decision functions for periods $t = 0, 1, ..., T$ that characterize the anticipatory effects (obviously, the optimal decision functions will depend on how far the economy is from the moment the policy is introduced). Below, we outline our perturbation-based framework for analyzing economies with non-Markov news shocks.

We represent bToTEM as an infinite-horizon nonstationary equilibrium problem in which a solution is characterized by a set of equilibrium conditions for $t = 0, 1, \ldots,$

$$E_t\left[G_t\left(y_{t-1}, y_t, y_{t+1}, z_t, z_{t+1}\right)\right] = 0, \tag{33}$$

$$z_{t+1} = Z_t\left(z_t, \epsilon_{t+1}\right), \tag{34}$$

[3] LLM (2020) compare perturbation solutions with more accurate global projection solutions constructed using deep learning analysis. That paper finds that high order perturbation solutions are sufficiently accurate in the bToTEM model. Since our nonstationary analysis is more costly, we limit attention to perturbation solutions only.

where (z_0, y_{-1}) is given; E_t denotes the expectations operator conditional on information available at t; $z_t \in \mathbb{R}^{d_z}$ is a vector of exogenous state variables at t; Z_t is a time-dependent law of motion for z_t; $y_t \in \mathbb{R}^{d_y}$ is a vector of endogenous variables; $\epsilon_{t+1} \in \mathbb{R}^{d_\epsilon}$ is a vector of shocks; G_t is a continuously differentiable vector function. Note that the latter function is time-dependent because the model is nonstationary (due to, for example, time-dependent parameters in policy rules, production function, utility function). A solution is given by a set of time-dependent equilibrium functions $y_t = Y_t(z_t, y_{t-1})$ that satisfy (33), (34) in the relevant area of the state space.

Our perturbation analysis proceeds in the following two steps:

Step I: solving a T-period stationary economy.

Assume that in a very remote period T, the economy becomes stationary, i.e., $G_t(\cdot) = G(\cdot)$ and $Z_t(\cdot) = Z(\cdot)$ for all $t \geq T$. Therefore, the system (33), (34) becomes

$$E_t\left[G\left(y_{t-1}, y_t, y_{t+1}, z_t, z_{t+1}\right)\right] = 0, \tag{35}$$

$$z_{t+1} = Z\left(z_t, \epsilon_{t+1}\right). \tag{36}$$

Solving (35), (36) allows us to find the solution $y_T = Y_T(z_T, y_{T-1})$.

Step II: constructing a function path.

Using a T-period solution $y_T = Y_T(z_T, y_{T-1})$ as a terminal condition, iterate backward for $T-1, ..., 1$ on the corresponding equilibrium conditions to construct a sequence (path) of time-dependent value and decision functions $\{Y_{T-1}(\cdot), ..., Y_1(\cdot)\}$. For example, for period t, the system on which we iterate backward is

$$E_t\left[G_t\left(y_{t-1}, y_t, Y_{t+1}\left(z_{t+1}, y_t\right), z_t, z_{t+1}\right)\right] = 0,$$

$$z_{t+1} = Z_t\left(z_t, \epsilon_{t+1}\right),$$

Here, we solve for today's endogenous variables y_t, given tomorrow's functions $z_{t+1} = Z_t(z_t, \epsilon_{t+1})$ and $y_{t+1} = Y_{t+1}(z_{t+1}, y_t)$.

In both steps, we use perturbation to find numerical approximations of the decision functions. In Step I, a Taylor expansion of the policy functions in a stationary model is found around the deterministic steady state $\bar{v}$ of the model. In Step II, we consider two alternative options. The first option is to find solutions for v_{t+1} and v_t around v_t and v_{t-1}, respectively, such that $v_t = v_{t-1} \equiv \bar{v}_t$; in dynare, it can be implemented by coding v_t and v_{t+1} using the same variable names. The other option is to consider v_{t+1} and v_t perturbed around $\bar{v}_t$ and $\bar{v}_{t-1}$, respectively, such that $\bar{v}_t = \bar{v}_{t-1}\gamma_{v,t-1}$, where $\gamma_{v,t-1}$ is a time-dependent growth rate; in dynare, it can be implemented by coding v_t and v_{t+1} with different variable names. For bToTEM model, we limit our attention to the second option but in Appendix A, we compare both options using the example of a neoclassical growth model.

4 Analyzing anticipated news shocks

In this section, we show a series of policy experiments in which we consider anticipated changes in one or several model's parameters. In all the figures, the variables are shown in percentage deviations from the initial risky steady state, except for the interest rate and the inflation rate, which are both shown in percentage point deviations from the risky steady state and expressed in annualized terms.[4]

4.1 A change in the inflation target

The inflation target in central banking models, particularly within the New Keynesian framework, significantly influences the conduct and effectiveness of monetary policy. If the central bank raises its inflation target, it becomes more tolerant of higher inflation. In the short run, this can stimulate output by lowering

[4] By risky steady state, we mean a state to which a stochastic economy converges in the absence of exogenous shocks.

the real interest rate and encouraging spending. However, in the long run, this can lead to higher actual inflation and may require the central bank to raise nominal interest rates to bring inflation back under control, potentially causing economic contraction. The level of the inflation target affects the frequency and severity of ZLB episodes. A higher target provides a larger buffer against the ZLB, allowing for greater monetary stimulus during downturns. The optimal inflation target in the New Keynesian model depends on the central bank's preferences and the specific economic conditions. In general, a moderate inflation target allows the central bank to balance its goals of price stability and output stabilization. In turn, a very high target may raise concerns about credibility and unintended consequences.

The optimal level of the inflation target is a subject of ongoing research and debate. It depends on various factors, including the structure of the economy, the nature of shocks, and the effectiveness of unconventional monetary policy tools. Recent discussions have focused on the potential benefits and costs of raising the inflation target above the commonly observed 2% level, particularly in light of the persistent low-interest-rate environment and the increased risk of hitting the ZLB. However, any change to the inflation target must be carefully considered, weighing the potential benefits against the potential costs and risks to central bank credibility.

In our first bToTEM experiment, we consider a change in the inflation target that appears in the Taylor rule (23). In particular, we assume that the central bank announces in advance that it will increase the inflation target $\bar{\pi}_t$ and that everyone considers the announcement to be fully credible. Why is it a relevant policy experiment? During the Great Recession of 2007–2009, central bank's nominal policy rates across a number of countries fell to a ZLB on nominal interest rates. There is ample literature arguing that the inflation target is a good policy instrument for dealing with ZLB episodes. For example, Summers (1991) and Fischer (1996) suggest to keep an inflation target as high as 2 or 3 percent if the economy hits ZLB. Krugman (1998) proposes to use a 4 percent inflation target in the Japanese economy to deal with persisting deflation. Furthermore, Blanchard, Dell'Arriccia and Mauro (2010), Williams (2009) and Ball (2013) argue that a higher inflation target would have prevented the interest rate from falling to the ZLB.

In Canada, inflation-targeting framework was adopted in 1991, and since 1995, the inflation target was maintained at the level of 2 percent. The inflation target is reviewed and renewed every five years. In particular, the last review was in October of 2016, when the Bank of Canada decided to keep the target at the same level; this renewal covers the period from January 1st, 2017 to December 31st, 2021. There are two types of possible anticipation effects here. First, we would have had a policy implementation lag leading to anticipation effects if the Bank of Canada decided to change the target in October 2016. Second, in spite of the fact that the inflation target was not changed in 2016, anticipation effects were still present as there were some chances that it would be changed given that Canada was close to the ZLB at that time and policymakers were seriously discussing this possibility.

Figure 1 displays dynamics of the main model's variables. We present the results for the method that finds a perturbation solution obtained around a deterministic steady state (labeled as Method 2 in Appendix B; our sensitivity results for other methods predict similar patterns of behavior).

We consider two cases: first, at $t = 1$, the central bank makes an announcement that starting from $t = 1$, it will gradually increase the inflation target $\bar{\pi}_t$ from 2 percent to 3 percent during a period of 8 quarters, and second, the same change takes place but starting from $t = 5$ (i.e., in one year); the inflation target remains at the new (higher) level after it is reached.

When the inflation-target change begins at $t = 1$, inflation follows the same pattern as the target. What is the reason for such behavior of inflation? In our experiment, we assume full credibility of the inflation-target policy. Inflation repeats the pattern of the inflation target because agents determining the behavior of inflation are mainly non-optimizers who index their price by inflation target. As a result, the nominal interest rate gradually increases over the first fifteen periods by 1 percent, and it stays at the new level forever (see Figure 1; note that the real neutral rate is the same as before). Following the announcement, output, investment and commodity exports jump up, and over the transition, the economy experiences an investment- and export-driven growth with the peak increase of output of 0.2 percent. Output begins to descend toward its original level after one year. Consequently, there is only a temporary expansionary

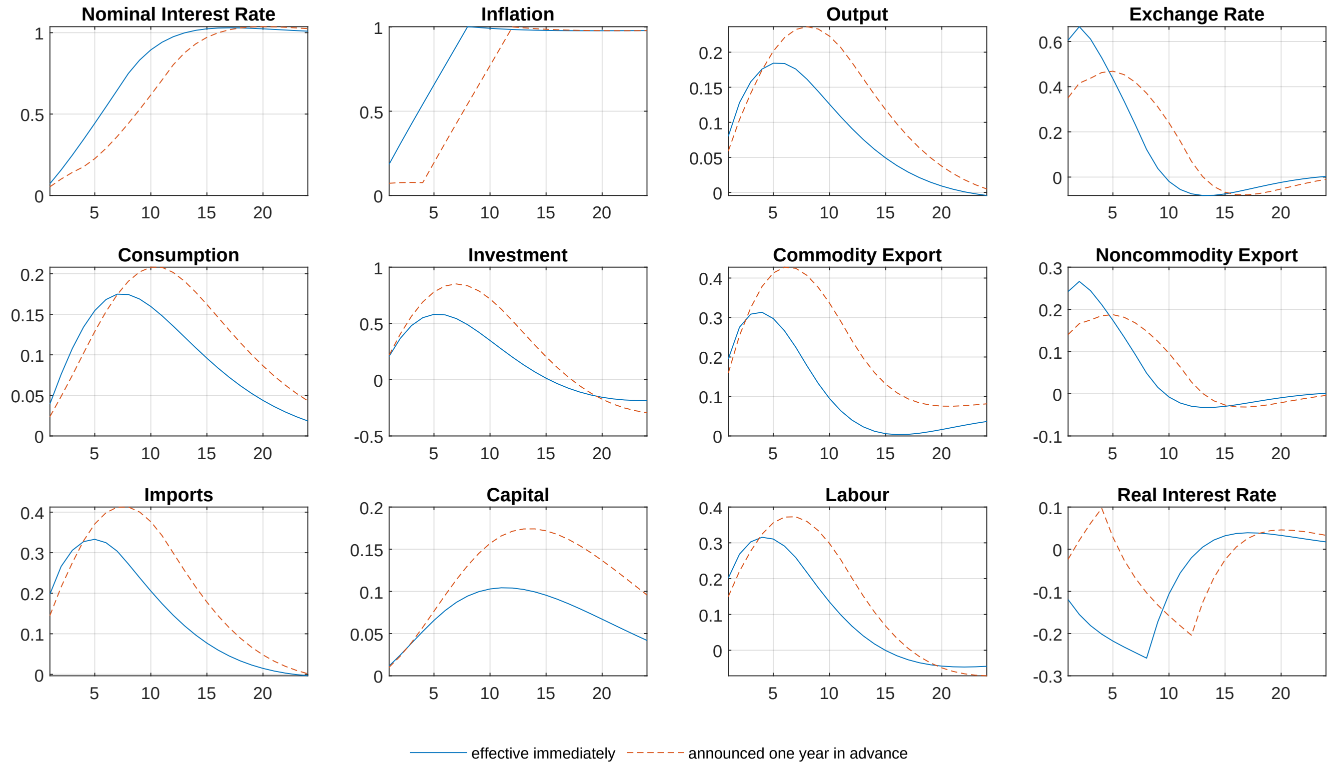


Figure 1: A gradual increase in the inflation target

effect on the economy due to a higher inflation target.[5]

When the inflation-target change is delayed for one year, the variables behave qualitatively similar. One visible difference from the previous case is that most variables in the figure experience larger increases at the peak, in particular, the output increases to about 0.3 percent (the exchange rate and noncommodity export are exceptions). Therefore, it pays for the central bank to announce this type of policy in advance as output increases more during the transition. The larger jumps in such variables as output, consumption, investment, capital are entirely due to anticipatory effects. That is, agents expect the real interest rate to be lower in the near future, and they accumulate more capital in advance of the more favorable environment which has positive effects on the economy's output today.

Table 1 contains the mean and maximum residuals in the model's equations used for computing the corresponding variables in the table. As we can see, the maximum residuals range between $10^{-3.13}$ and $10^{-5.84}$, i.e., between .07 percent and .0001 percent, which are very low.[6] For the remaining experiments, the residuals in equations are of similar size so our solutions are very accurate (to save on space, these residuals are not reported).

In the second experiment, we model a probabilistic setting in which agents rationally expect that the inflation target might change to two possible levels with some probabilities. Specifically, we assume that there is a 50-percent chance that starting from $t = 5$ the inflation target $\bar{\pi}_t$ gradually increases from 2 to 3 percent during 8 quarters; otherwise, the inflation target remains the same. Our computational method is easy to adapt to modeling more sophisticated anticipation scenarios like one considered in that experiment.

[5] *Garin et al. (2018) analyses the impact of the inflation-target shocks on output in a canonical three-equation New Keynesian model. They find that the output response depends considerably on persistance of such shock, see their Figure 2. For sufficiently persistent shocks, the effect of the inflation target on output is quantitatively similar to the one in our bToTEM model. (However, that paper does not analyze how postponing the inflation target shock affects the output, which is a distintive feature of our analysis).*

[6] In LMM (2020), we reported larger residuals when the economy was hit by a large negative demand shock and the ELB was reached.

	R_t	π_t	Y_t	C_t	I_t	X_t^{nc}	X_t^{com}	M_t	L_t	K_t
Average	-4.66	-5.25	-4.09	-4.47	-5.11	-4.10	-3.17	-5.10	-4.42	-5.91
Maximum	-4.30	-5.17	-4.03	-4.38	-5.03	-4.08	-3.13	-4.98	-4.37	-5.84

Table 1: Residuals in the model's equations on the simulated path, $\log_{10}$ units. R_t, π_t, Y_t, C_t, I_t, X_t^{nc}, X_t^{com}, M_t, L_t, K_t are the nominal interest rate, inflation, output, consumption, investment, noncommodity export, commodity export, imports, labor and capital, respectively.

When computing policies in period $t = 4$, we explicitly use the Dynare macro language to set the period 4 expectation functions to be equal to the weighted sums of expectations over the two possible realizations in period $t = 5$.

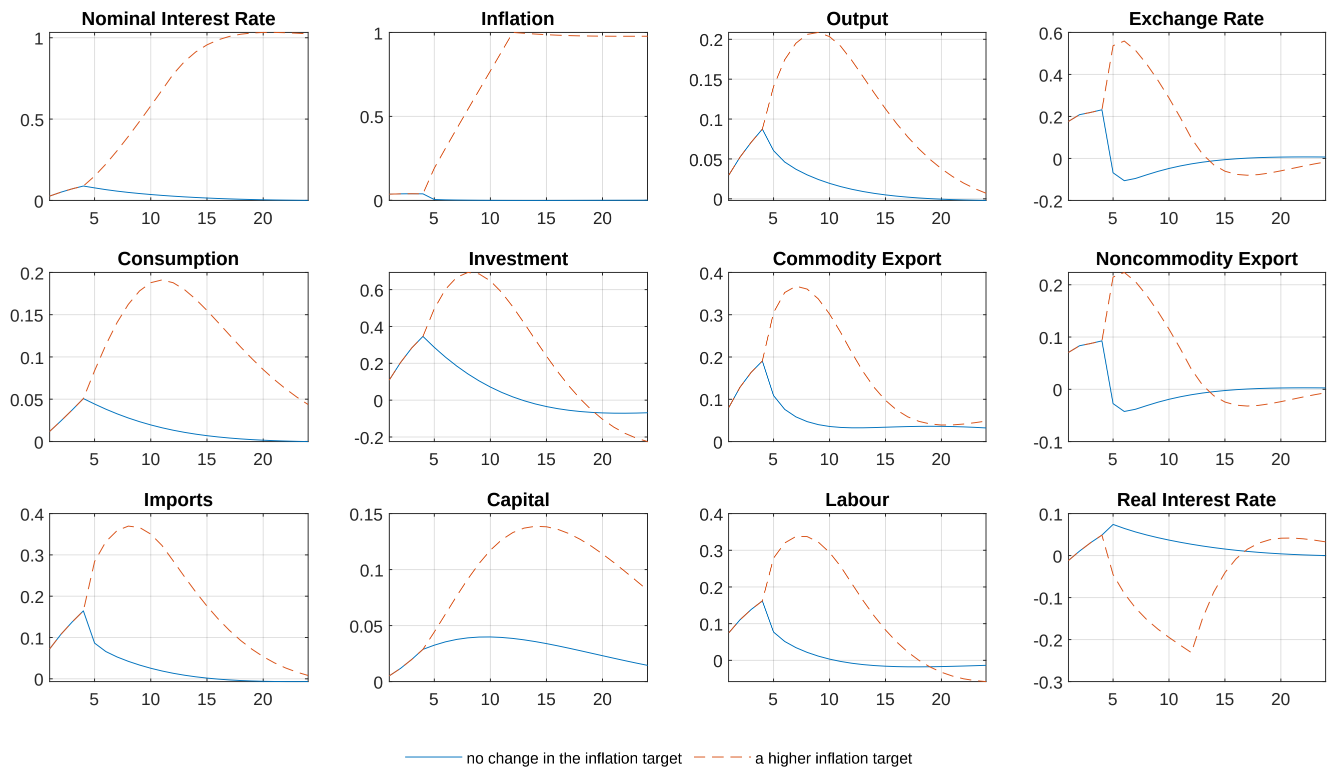


Figure 2: A gradual increase in the inflation target (50% probability)

This experiment is plotted in Figure 2. There are two alternative transition paths differing from period 5 onwards, one per each scenario, i.e., with and without an increase in $\bar{\pi}_t$. Similar to the previous experiment, inflation mimics the behavior of the inflation target: it gradually rises to the new steady state level. Starting from the risky steady state at $t = 1$, all the variables experience mild increases, which are due to anticipatory effects on the side of economic agents. Once it becomes known whether the target will go up or not, all the variables quickly return to the original steady state if the target does not increase, and they experience a more pronounced hump-shape behavior and return to a new steady state if the target increases. In Appendix B, we extend the latter experiment to vary the probability of switching to a higher inflation target at $t = 5$, namely, it is either 25 percent or 75 percent (instead of 50 percent). As those figures show, in case of no inflation-target change, the transition back to the old steady state is significantly faster for the 25-percent case than for the 75-percent case.

4.2 Monetary policy normalization

Monetary policy normalization refers to the central bank's actions to raise nominal interest rates back to a standard rule after a period of being at the zero lower bound (ZLB) and having a loose monetary policy. It has several effects within the New Keynesian model. First, it leads to a decrease in output and a decrease in inflation in the short run because higher interest rates dampen aggregate demand by making borrowing more expensive and saving more attractive. Second, it can affect financial markets by leading to higher bond yields and potentially lower equity prices because higher interest rates make bonds more attractive relative to other assets. Overall, the impact of monetary policy normalization on the New Keynesian model depends on various factors, including the initial state of the economy, the speed and communication of the normalization process, and the credibility of the central bank. The effectiveness of monetary policy normalization can be limited if the economy is close to ZLB. In this case, the central bank may need to rely on unconventional monetary policy tools, such as forward guidance and quantitative easing, to stimulate the economy. By announcing their intentions to eventually raise rates, central banks can guide expectations and influence economic behavior even before the actual policy change occurs.

Now, let us consider the Canadian economy. During the Great Recession of 2007–2009, the nominal interest rate hit the ZLB. As a result, central banks could not rely on Taylor rules to conduct their monetary policy and resorted to forward guidance – an unconventional monetary policy consisting in announcing future interest-rate changes. As emphasized by the literature, central bank's communication of the policy-rate's future path is the main channel through which forward guidance policy affects the economy. Eggertsson and Woodford (2003) demonstrate that a central bank's promises to keep low interest rates for longer periods helps alleviate negative consequences of binding ZLB. As agents expect future interest rates to be lower than in the absence of forward guidance, they increase today's investment and consumption, which stimulates today's economy. Campbell et al. (2012) name this form of forward guidance Odyssean. Another form is Delphic forward guidance: a central bank may have better information about the state of the shocks that hit the economy, and it communicates a forecasted path of policy rates.[7] In the Odyssean case, future intentions are known, while in the Delphic case, forward guidance is implied – agents do not know its exact duration.

In this paper, we assume that the central bank uses forward guidance to convey a policy change when lifting off from an effective lower bound (ELB) on the bank's policy rates. In particular, we assume that initially the economy is at ELB and at $t = 1$ the central bank announces that it will keep the interest rate at that level for T periods and afterwards it will return to the standard Taylor rule (23). To model the central bank's policy at the ELB periods, we assume that the nominal interest rate is given by

$$R_t = R^{elb},$$

where R^{elb} is the ELB. When the interest-rate policy is normalized after T periods, the Taylor rule's coefficients return back to normal values, and the policy is described by the rule (23).

Figure 3 presents the results for this experiment when the solutions are approximated around a deterministic steady state. The change in the interest rate rule announced at $t = 1$ is anticipated by agents. We compare three cases, depending on whether the interest-rate policy returns to normal (i) in one quarter ($T = 1$), (ii) in one year ($T = 4$), (iii) in two years ($T = 8$). In all the cases, the initial interest rate is below its risky steady state, however, it eventually returns to the steady state.

When the policy is announced, the exchange rate, inflation, and the real variables jump up above the steady state. Local currency depreciation makes domestic exports more competitive, which leads to an increase in exports of both commodities and noncommodity goods. Domestic firms benefit from increased sales, which leads to immediate increases in output, labor, investment and capital. On the other hand, as households work more, they demand more of imported goods, so that imports go up as well. Evidently, an output increase is the largest when the announced policy is kept for the longest horizon of eight periods.

[7] Marinkov (2020) argues that during the ZLB period, agents may misjudge a central bank's reaction function and bias their expectations. In this case, the central bank may want to use forward guidance as a guiding tool to correct agents' beliefs.

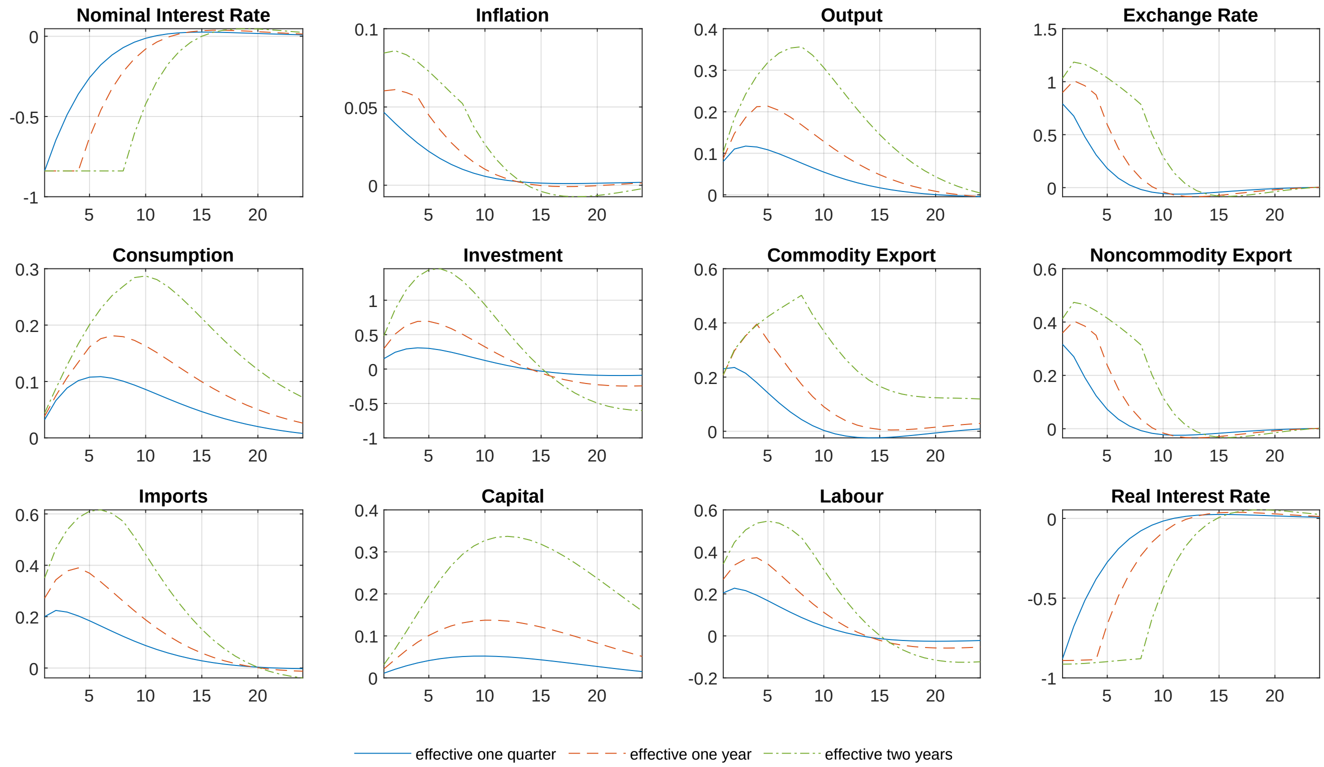


Figure 3: Monetary policy normalization

The peak increase in output is 70 percent higher than the one for the forward guidance horizon of four quarters. The differences in output dynamics across the considered cases are only present over the transition but not in the initial period – in all three cases, an initial output jump is of equal size. The dependence of the initial reaction in output on the horizon of the forward guidance policy is known in the literature as a forward guidance puzzle. Even though there is no such dependence in the figure, we do still see that the policy horizon matters for the total effect on output: it reacts more if the policy change is postponed further away in the future.

Our above experiment adds to the discussion on central bank's communication strategy. After the Great Recession of 2007–2009, when the economic conditions improved, an important policy question was how and when to normalize the monetary policy, where normalizing means switching back to some Taylor rule; see Yellen (2015). In particular, the following questions arose after the end of the crisis: (1) Should the central bank normalize policy now or later? (2) Should the central bank do it gradually or all at once? (3) Should the regime shift be announced in advance? (4) Should the policy normalization be time or state dependent? All these questions are hard to address in the context of conventional stationary new Keynesian framework because by definition a monetary policy normalization is a nonstationary change. Nevertheless, the technique developed in this paper enables us to study these questions easily. In our above experiment, we compare the economy's behavior under policies that differ in horizon of return to normal values, which corresponds to questions (1) and (3). We conclude that there are gains from announcing a future lift-off in advance and we quantify these gains for different durations of forward guidance. Similarly, questions (2) and (4) can be answered using our techniques; we leave them for future research.

An effective commitment to keep the interest rate at the ELB implies that the rate should be kept at this low level longer than a Taylor rule would imply. In particular, during the COVID-19 pandemic, the interest rates reached ZLB across a number of developed economics. In the U.S., the Fed has already announced that it expects to keep its benchmark interest rate pinned near zero through 2023.[8] Taylor

[8] See https://apnews.com/9b9a335a1ce05d69fc97a1d6197371ab

(2021) argues that the Taylor rules considered by the Fed in the February 2021 Monetary Policy Report imply that the federal fund rate should be higher than the actual zero level and that the Fed "should now engage in a strategy or rule in which people and markets understand that it would raise the policy interest rate if economic growth increases and inflation rises as they are now forecast to do." Our above analysis, however, plays up the importance of commitment to the announced policy on which hinges the desired monetary expansion.

4.3 Switching to a more aggressive Taylor rule

A more aggressive Taylor rule means that the central bank responds more strongly to changes in inflation and output. It has several important implications for central banking models: First, a more aggressive Taylor rule (with a coefficient on inflation greater than one) is often crucial for achieving a unique and stable equilibrium. It helps anchor inflation expectations and prevents the economy from spiraling into inflationary or deflationary episodes. Second, while a more aggressive Taylor rule promotes stability, it can also lead to increased volatility in output and interest rates in the short run because the central bank responds more forcefully to economic fluctuations, which can amplify the initial shocks. Third, a credible commitment to a more aggressive Taylor rule can enhance the central bank's ability to manage inflation expectations, leading to a lower and more stable inflation in the long run, even with a less volatile policy. Fourth, a more aggressive Taylor rule can pose challenges for identification because the central bank's strong response to inflation can make it difficult to disentangle the effects of the policy rule from other factors influencing inflation. Overall, a more aggressive Taylor rule can be a powerful tool for central banks to achieve their objectives of price stability and output stabilization. However, it also requires careful consideration of the potential trade-offs and challenges associated with its implementation.

In our bToTEM experiment, we consider a one-time change in the sensitivity of the policy rate to inflation and the output gap in the Taylor rule (23), as measured by ρ_π and ρ_Y, respectively.[9] It differs from previous experiments, in which the anticipated changes in the model's parameters are gradual. Figure 4 plots the economy's responses to two-time increases in either ρ_π or ρ_Y or both, relative to benchmark parameterization. Note that this change in the coefficient values is quite large relative to what a central bank would typically consider. Switching to more aggressive Taylor rules is anticipated at $t = 1$ but occurs at $t = 2$, so that there are immediate anticipatory effects in all the model's variables.

As we can see, both policies – a higher ρ_π and a higher ρ_Y – are inflationary. However, a double increase in the sensitivity to inflation ρ_π is more effective in expanding the economy: output, consumption, investment, capital, labor are visibly higher both at peak and in the new steady state than in the old steady state; commodity production slightly drops, which is related a lower commodity exports. A double increase in the sensitivity to the output gap has more modest effects however. When there is a stronger response to both inflation and the output gap, the quantitative expressions of the effects are roughly in between the other two cases. That is, given that there is a trade off between inflation and the output gap in the policy rule, responding stronger to the output gap undoes the effects of stronger responses to inflation. W*e also observe that the risky steady state of interest rate is lower for when the Taylor rule has a stronger response to inflation gap, and it is higher when the Taylor rule has a stronger response to inflation gap. The intuition behind this result is as follows: When the response to the inflation gap becomes stronger, we need a smaller increase in the interest rate to achieve an equivalent inflation stabilization and similarly, when the response to the output gap becomes stronger, we need a larger increase the interest rate to achieve an equivalent inflation stabilization.* Overall, total effects are not quantitatively important in our experiment: switching to a significantly more aggressive Taylor rule has only minor effects on the economy's behavior when the economy is not hit by any shocks.

[9] For example, Taylor (1999) argues that the Taylor rule with $\rho_\pi = 0.5$ and $\rho_Y = 1$ is more reasonable than the one advocated in Taylor (1993) when $\rho_\pi = 0.5$ and $\rho_Y = 0.5$.

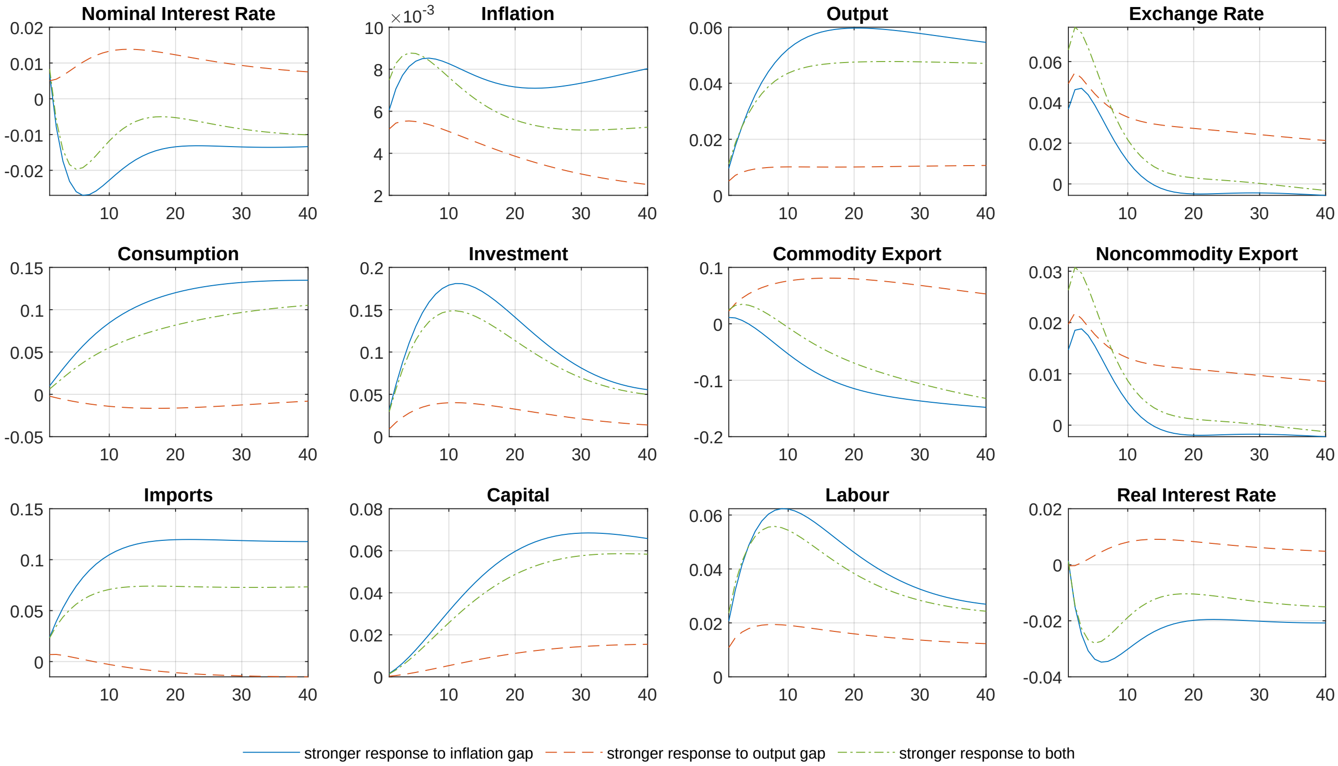


Figure 4: A switch to more aggressive Taylor rules

4.4 Switching from inflation targeting to price-level targeting

Switching from inflation targeting to price-level targeting has several implications for central banking models: First, price-level targeting has the potential to reduce the volatility of output and inflation through automatic, self-correcting changes in inflation expectations which could be particularly beneficial during periods of economic stress. Second, price-level targeting can be especially useful at the ZLB, as it allows for a catch-up period of above-target inflation following a period of below-target inflation which can help stimulate the economy when conventional monetary policy tools are constrained. However, price-inflation targeting requires a strong focus on managing public expectations, as its effectiveness hinges on people's forward-looking behavior and their understanding of the policy, which may necessitate greater transparency and communication from central banks. Furthermore, the success of price-level targeting depends heavily on its credibility. If people doubt the central bank's commitment to the price-level path, the policy may not be as effective in stabilizing the economy. Overall, the effects of switching from inflation targeting to price-level targeting are complex and depend on various factors and transitioning to PLT may be challenging because of the need to build credibility and public understanding of the new framework.

Concerning the literature, the seminal paper of Svensson (1999) argues that price-level targeting is a "free lunch" in a sense that it positively affects a short-run trade off between inflation and output variability (namely, it reduces inflation variability without an increase in output variability); see also Hatcher and Minford (2016) and Ambler (2009) for surveys. Bernanke (2017) proposes to use a temporary price-level target when short-term interest rates are at (or near) ZLB. When ZLB prevents policymakers from providing adequate stimulus, inflation is below target. Price-level-targeting policymakers compensate for periods of low inflation below target by following a temporary surge in inflation The Bank of Canada has seriously considered the use of price-level targeting; see Kahn (2009) and Bank of Canada (2011).

We first consider a bToTEM scenario, in which central bank switches from the standard Taylor rule

(23) targeting inflation to the one targeting a price-level gap,

$$R_t = \rho_r R_{t-1} + (1-\rho_r)\left\{\bar{R} + \rho_\pi\left(\log P_t - \log \bar{P}_t\right) + \rho_Y\left(\log Y_t - \log \bar{Y}_t\right)\right\} + \eta_t^r, \tag{37}$$

where P_t is the actual price level, and $\bar{P}_t$ is the target price level that grows at the rate of inflation target $\bar{P}_t = \bar{P}_{t-1}\bar{\pi}_t$. Therefore, price-level targeting does not suggest that policymakers pursue a constant price level but set a target for the price level that rises over time.

An inflation-targeting central bank does not pay attention to temporary changes in inflation as long as inflation comes back to target after some time ("lets bygones to be bygones"). In contrast, price-level-targeting central bank aims at reversing temporary deviations of inflation from target each time it misses it (e.g., a central bank increases inflation when inflation falls below target). As a result, under inflation targeting, an inflation shock permanently shifts price path to a different level, while under price-level-targeting, any movement in inflation above target is matched with an equal and opposite movement in inflation below target, so that the economy goes along a predetermined price path. Consequentially, with inflation targeting, agents will face a considerable amount of uncertainty about the future price level (the central bank treats past target misses as bygones and returns inflation to the target level gradually, without taking into account any impact on the price level), while with price-level targeting, agents will be much more confident on where the prices will be in the future, even with a positive average inflation.

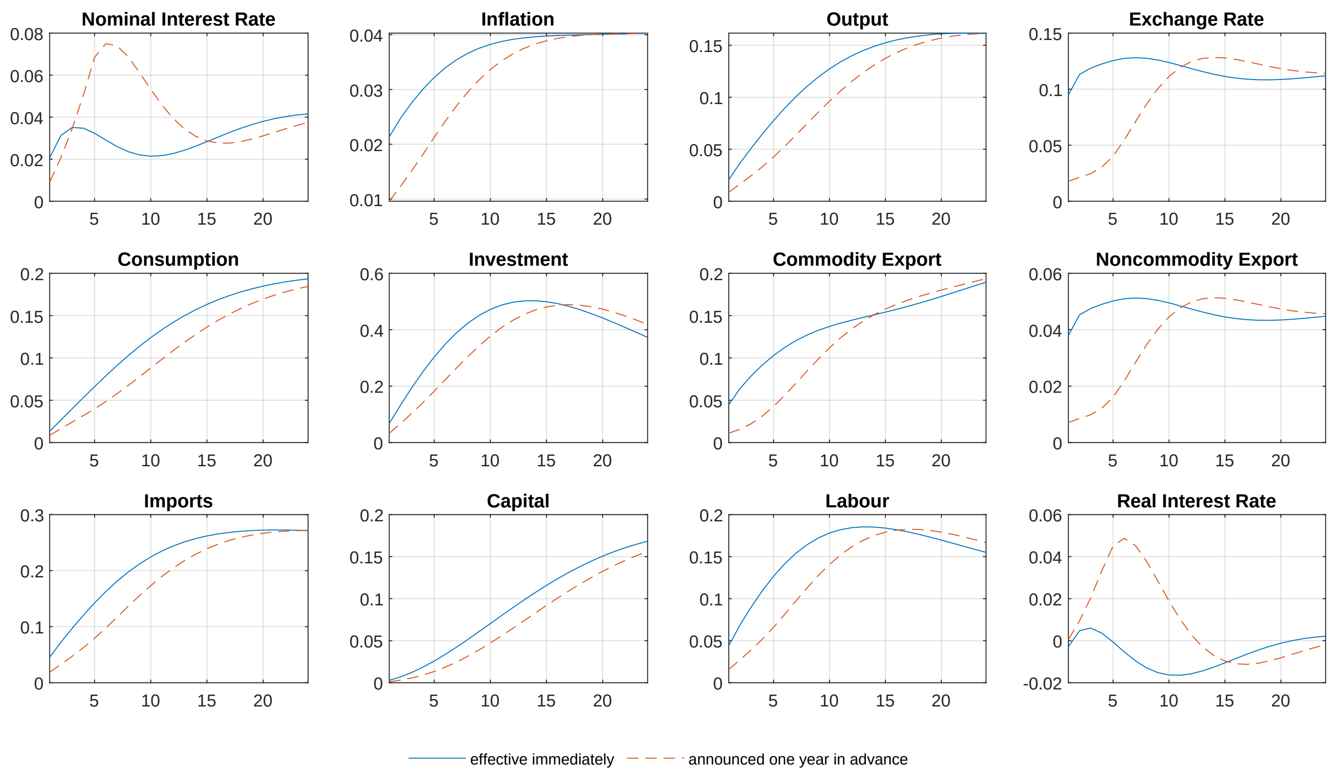


Figure 5: A switch to price-level targeting

In Figure 5, we present the results for two policy experiments in which the policy change becomes effective either immediately (at $t = 1$) or in one year after being announced (at $t = 5$). The new interest-rate rule is associated with higher steady state levels for all the model's variables in the figure. Therefore, switching to price-level targeting has expansionary effects on the economy. Moreover, for all of the variables (except of the nominal interest rate), the immediately implemented policy gives larger benefits than the policy announced one year in advance. That is, if the central bank postpones to implement the switch, the economy reaches the new steady state almost at the same time as the immediate policy, but over the transition the effects are smaller.

In the next experiment, we shock the economy, so that there appears a large output gap. In particular, we consider a permanent negative demand shock – a decrease in foreign demand, modeled as a negative innovation in the random-walk process for this shock. (A version of this experiment with a permanent decrease in productivity is presented in Appendix B.) In response, the central bank can either continue using a policy rule with inflation targeting or can switch to price-level targeting, which, as we saw, leads to higher steady state output. As a result, switching to price-level targeting can be viewed as an attempt to revive the economy.

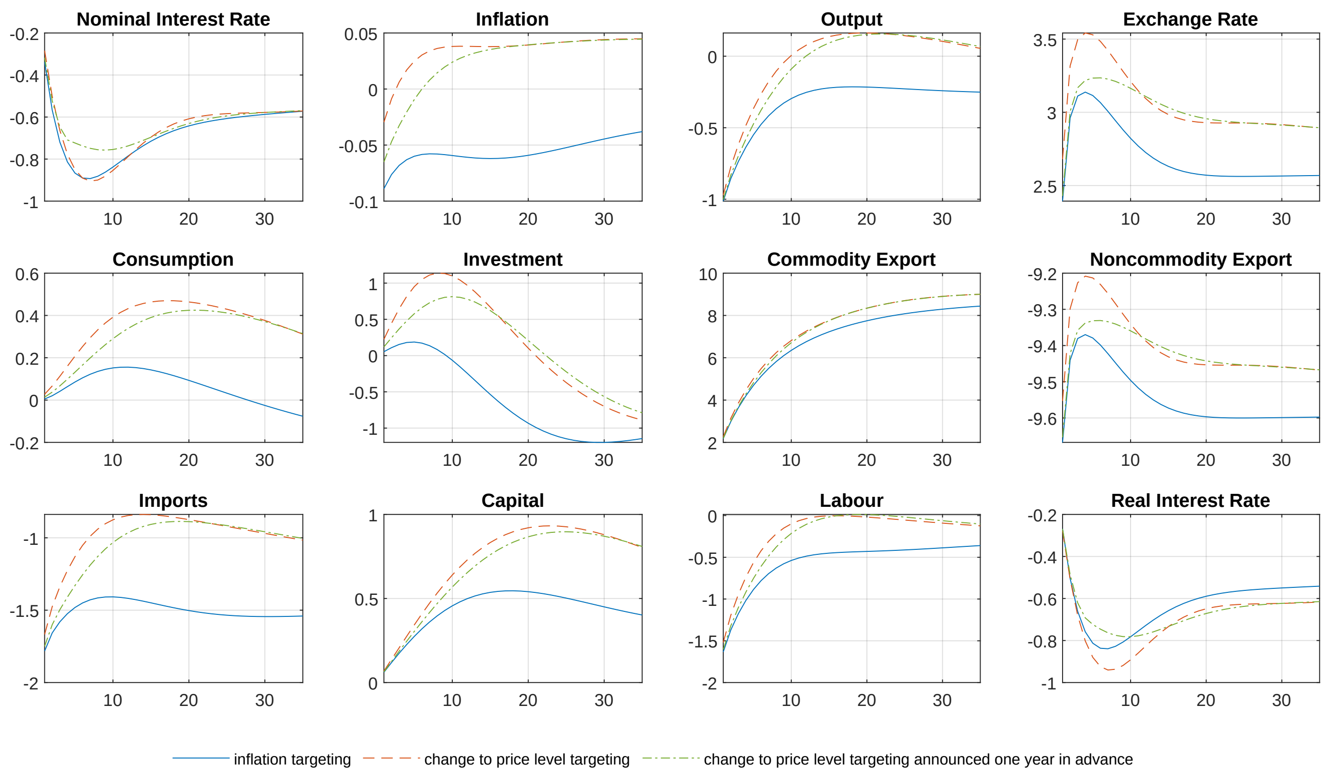


Figure 6: A negative foreign demand shock and a switch to price-level targeting

Figure 6 presents the results of this experiment when the switch is either implemented immediately or is delayed for one year (but it is still announced today). As we see, there is barely any difference between the immediate and delayed changes in monetary policy for such variables as commodities and commodity export. For all other variables, the immediate policy change has larger impacts than the delayed policy change, which is in line with the previous experiment in Figure 5. Therefore, the anticipation effects work in the direction of softening the effects of the negative demand shock, with impulse responses lying between the cases of no-change and immediate change. In particular, the dynamics of the nominal interest rate is smoother in case of anticipated policy, which leads to smoother behavior of the remaining variables. The initial impact on the economy is significant, e.g., output and labor fall by 1 and 1.5 percent in the three scenarios considered. With no switch in monetary policy, output recovers a bit but its new steady state is still below old steady state. With the policy switch, output is nearly the same as before the shock, and consumption is even higher. Note that each considered policy implies that the central bank tightens monetary policy, even in the economic downturn.

4.5 Switching from inflation targeting to average inflation targeting

Switching from inflation targeting to average inflation targeting affects central banking models in several ways: First, it leads to an increased flexibility for policymakers: it allows the inflation rate to temporarily deviate from the target rate, as long as the average inflation rate over a certain period returns back to the *target. This provides greater flexibility to respond to economic shocks and address short-term fluctuations. Second, by committing to a long-run average inflation target, policymakers can enhance the credibility of their commitment to price stability and anchor long-run inflation expectations. This can lead to more stable and predictable inflation dynamics. Third, the higher flexibility under average inflation targeting enables policymakers to better stabilize output and employment in response to economic shocks, potentially leading to improved economic outcomes. Average inflation targeting is particularly beneficial in the context of the ZLB: by allowing for temporary overshooting of the inflation target, policimakers can mitigate the constraints imposed by the ZLB and enhance the effectiveness of monetary policy during downturns. In certain scenarios, average inflation targeting exhibits properties that are quantitatively similar to price-level targeting. This suggests that average inflation targeting can offer some of the benefits of price-level targeting, such as reduced output and inflation volatility, without the same degree of implementation challenges. However, the success of average inflation targeting also depends on effective communication of the policy framework to the public and its credibility.*

The US Fed switched from inflation targeting to average inflation targeting was announced by the Fed's Chair Jeromy Powell on August 27th; see Powell (2020). However, as was stated by Richard Clarida, during his presentation at the Hoover Economic Policy Working Group on January 13, 2021, one month prior to that, there was evidence that Fed would introduce that framework, and as a result, there were substantial anticipatory price moves in the U.S. economy.

In our bToTEM experiment, we consider a switch from the inflation-targeting Taylor rule (23) to a rule that incorporates an average of the past inflation (including the actual inflation),

$$R_t = \rho_r R_{t-1} + (1-\rho_r)\left[\bar{R} + \rho_\pi \left(\frac{1}{M+1}\sum_{j=0}^{M}\pi_{t-j} - \bar{\pi}_t\right) + \rho_Y\left(\log Y_t - \log \bar{Y}_t\right)\right] + \eta_t^r. \tag{38}$$

The policy of average inflation targeting shares many of the properties of price-level targeting. As was suggested by the previous literature, average inflation targeting is a middle ground between price-level targeting and inflation targeting; see Nessén and Vestin (2005). Under average inflation targeting, a central bank reacts to a deviation of today's inflation averaged with previous inflation from target inflation. For example, if the inflation target is 2 percent, the averaging window is 3 years, and after consistently archiving 2 percent inflation in the past, in the most recent year, inflation deviates to 3 percent, the central bank will aim to achieve policy-induced inflation of 1 percent in the next year. As a result, inflation will oscillate around average inflation target and the average inflation target is achieved on average. The price level will stay close to its trend, even though the level will sometimes deviate from the fixed trend.

Figure 7 displays the results for two cases: one is when the switch happens immediately and the other when it is implemented with a lag of one year after it was announced. Amano et al. (2020) study optimal history dependence under average inflation targeting in the context of the standard new Keynesian model accounting for the ELB, and they find that optimal M ranges from 2 to 8. We assume $M = 8$ which is the largest number of lags found by Amano et al. (2020).

It turns out that this policy change has very modest anticipation effects on the economy in the absence of any shocks. In fact, when the policy becomes effective immediately, there are larger responses in such variables as output, labor, imports, and noncommodity exports. That is, reacting to average inflation rather than inflation smooths out dynamics to a new steady state. Therefore, we would not expect the economy to experience any drastic changes in the course of transition to average inflation targeting.

5 Markov versus non-Markov news shocks

There are other methods in the literature that focus on changes in economic environment including Markov regime switching models (e.g., Davig and Leeper, 2007) and models with Markov news shocks (e.g., Schmitt-Grohé and Uribe, 2012); see also Barro and King (1984), Beaudry and Portier (2006, 2007), Jaimovich

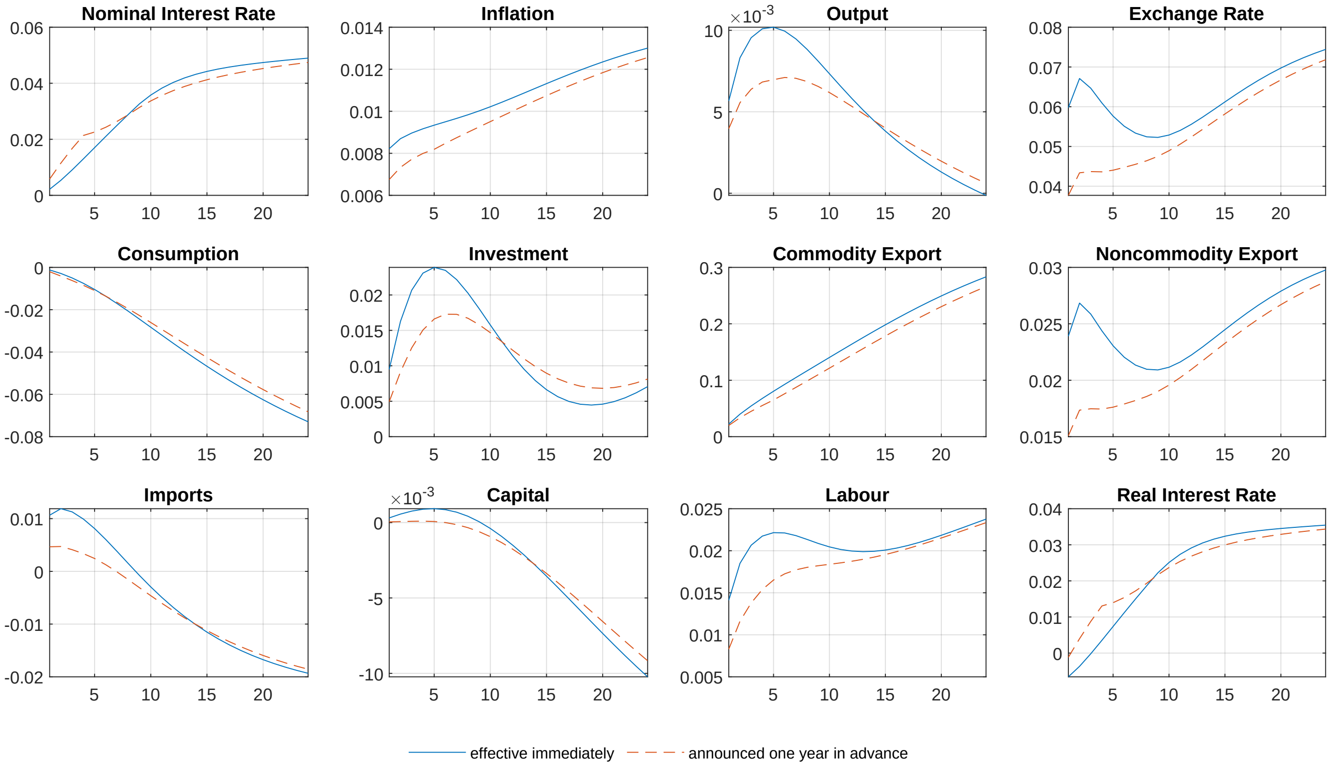


Figure 7: A switch to average inflation targeting

and Rebelo (2009) However, there is an important conceptial difference between that literature and the EFP analysis: the literature considers recurrent changes driven by a Markov process and constructs one time-invariant decision rule; whereas, we consider a sequence (path) of non-Markov events and construct a sequence (path) of time-varying decision rules. The quantitative difference between the stationary solutions produced by those Markov methods and nonstationary non-Markov analysis can be quite large since one decision rule constructed for any given Markov process cannot describe a variety of non-Markov scenarios that might occur on a historical path.

To illustrate this difference, we compare Schmitt-Grohé and Uribe's (2012) and EFP solutions in the context of an experiment of Section 4.3 in which the central bank switches to a more aggressive Taylor rule; namely, we assume that the sensitivity to inflation ρ_π in the Taylor rule (23) is doubled relative to its benchmark value: the change is announced at $t = 1$ and implemented at $t = 2$. Our framework with non-Markov shocks provides a natural way of modeling this scenario. Namely, we construct a stationary solution for period $t = 2$, and we find a solution for period $t = 1$ that matches a given terminal condition (decision rule) constructed for period $t = 2$.

Schmitt-Grohé and Uribe (2012) does not specify how their perturbation method can be used for analyzing non-Markov anticipated shocks. We tried out two ways of adapting their method to our experiment: First, we consider a unit-root process for $\rho_{\pi,t}$, i.e., $\rho_{\pi,t} = \rho_{\pi,t-1} + \varepsilon_{t-1}$, in which initially $\rho_{\pi,0} = \rho_{\bar{\pi}}$. In this specification, the shock innovation, ε_t captures news that become known at period t and that have a direct impact at $t+1$. In our experiment, $\varepsilon_1 = \rho_{\bar{\pi}}$ at period $t = 1$, and at all other periods the shock innovation is zero. It implies that $\rho_{\pi,1} = \rho_{\bar{\pi}}$ and $\rho_{\pi,t} = 2\rho_{\bar{\pi}}$ for all $t \geq 2$. Second, we consider a temporary news shock, i.e., $\rho_{\pi,t} = \rho_{\bar{\pi}} + \varepsilon_{t-1}$. In that case, we get $\rho_{\pi,2} = 2\rho_{\bar{\pi}}$ in period $t = 2$ and $\rho_{\pi,t} = \rho_{\bar{\pi}}$ in all other periods.

Figure 8 compares our second-order perturbation EFP solution with two solutions produced by Schmitt-Grohé and Uribe's (2012) method. The volatility of the news shock is assumed to be zero, so the initial risky steady state is the same for all three solutions. The following observations are in order: First, the EFP solution with non-Markov shocks is situated in between the two Markov news-shock solutions. Second, both the EFP solution and permanent Markov news-shock solution converge to new (although different) risky steady states, while the temporary Markov news-shock solution converges to the old steady state, given the temporary nature of the shock. Third, in the three cases, all the variables behave in qualitatively similar manner: a more aggressive central bank leads to an increase in inflation and a decrease in nominal and real interest rates, which raises output, investment, capital and imports. Fourth, the gap between our solution and permanent news shock solution depends on the initial condition: we observe in our sensitivity experiments (not reported) that the gap is smaller if we start below steady state. This is because the anticipation effects are mixed up with upward-sloping transition dynamics. Finally, the difference between the two solutions with Markov news shocks and our second-order perturbation solution comes from the differences in slopes of the decision rules. If we were to consider the first-order perturbation, the economy would remain at the deterministic steady state in the two Markov news-shock solutions but not in our solution.

Furthermore, we observe that the permanent Markov news-shock model predicts dramatically larger effects associated with the switch to a more aggressive Taylor rule than the EFP method (except for consumption). This is true both for the anticipation effects and for differences in steady states. For example, anticipation effects in investment are five times larger at peak for the Markov news shocks than for our perturbation solutions. We conclude that, the two approaches may lead to qualitatively different results: here, the Markov news-shock approach significantly overstates the importance of a given anticipated event because a unit-root process for $\rho_{\pi,t}$ implies that once a news shock happens, its effects will persist forever.

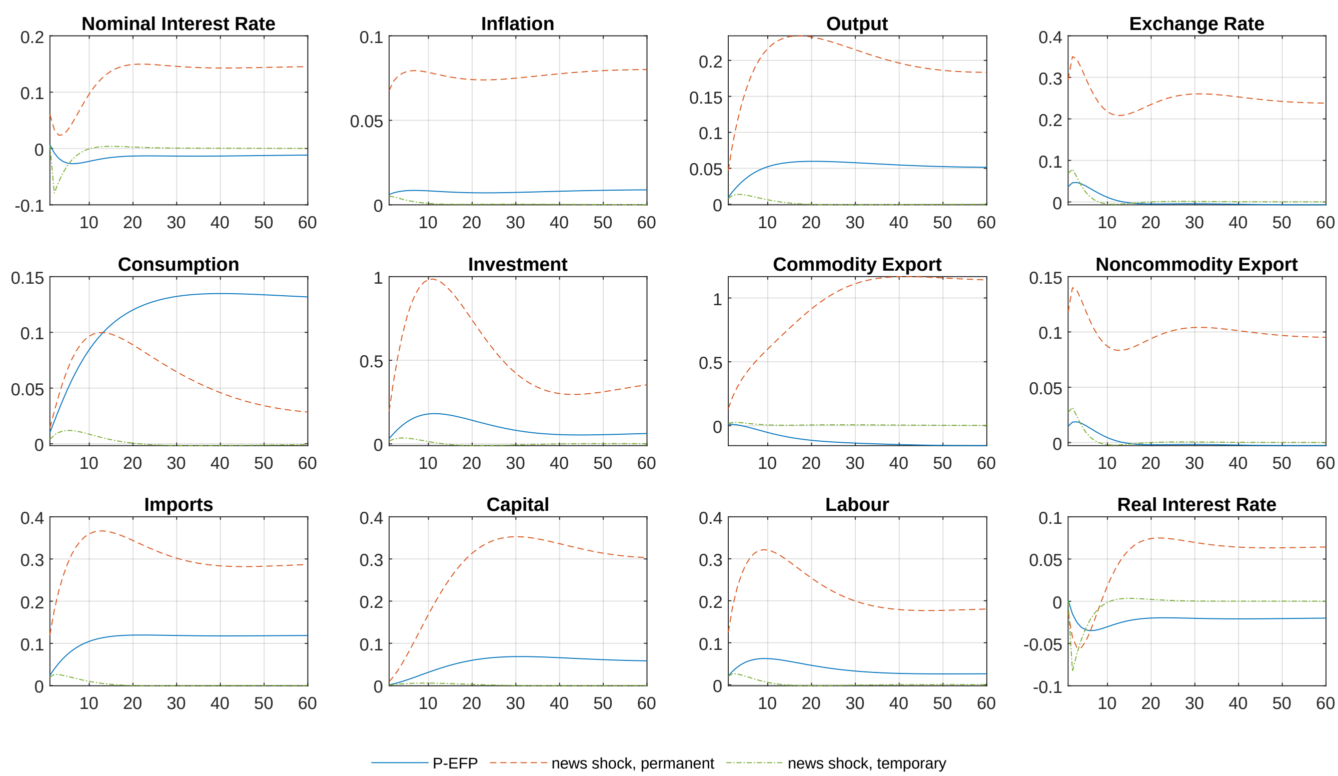


Figure 8: A switch to a more aggressive Taylor rule at $t = 2$ announced at $t = 1$.

There is a simple intuition on why our non-Markov solutions differ from those produced for Markov news shocks. In our case, the agent's decision rule is the best response to a given news shock and in Schmitt-Grohé and Uribe's (2012) framework, it is the best response to the given Markov stationary process. With Markov process, the response to news is determined not only by the news itself but also by the properties of the Markov process which was assumed for constructing the solution–this feature is absent in our case.

For the same reason, regime switching Markov models do not provide an adequite framework for analyzing non-Markov news shocks.

6 Conclusion

The literature recognizes that private sector's expectations are important for policy outcomes and central banks use the open mouth policies to anchor the expectations. However, little work has been done on evaluating the effects of the open mouth policy within a DSGE framework. This paper fills in this gap. We find that the anticipation effects are the strongest for such time dependent economic policies as a policy-rate normalization in the aftermath of the ZLB crisis, and a gradual change in the inflation target level. The other time dependent policy changes like a switch to a more aggressive policy rate rule, a switch to price-level or average inflation targeting lead to more modest anticipation effects.

The proposed EFP methodology is not limited to central-banking models. Many economic policies are announced ahead of being implemented. For instance, changes to taxes, tariffs, minimum wage, pension reforms, Social Security are frequently signed into law well before they are put in practice. Other notable examples include an announcement about a new member state's accession to the European Union (EU) or a member state's exit from the EU (i.e., Brexit), an announcement of the outcome of presidential elections before the new elected president comes to power. Our perturbation-based framework for solving, calibrating, simulating and estimating of parameters provides a simple and tractable way of analyzing non-Markov transitions associated with such policy changes. Literally, the EFP analysis makes it possible to construct a model-consistent historical path of a real-world economy.

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Appendix A

In this section, we illustrate the implementation of perturbation-based method on a toy example – a neoclassical stochastic growth model with labor augmenting technological progress. We consider a version of the model that allows for balanced growth. To solve this model, we proceed as if growth was unbalanced, and then compare our solutions to those obtained by an accurate projection method that solves detrended (stationary) model.

The growth model with labor-augmenting technological progress. We consider the following neoclassical stochastic growth model with labor augmenting technological progress:

$$\max_{\{c_t,k_{t+1}\}_{t=0}^{\infty}} E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right] \tag{39}$$

$$\text{s.t. } c_t + k_{t+1} = (1-\delta)k_t + z_t f(k_t, A_t), \tag{40}$$

$$\ln z_{t+1} = \rho \ln z_t + \sigma\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0,1), \tag{41}$$

where (k_0, z_0) is given; E_t is an operator of conditional expectation; $c_t \geq 0$ and $k_t \geq 0$ are consumption and capital, respectively; $A_t = A_0\gamma_A^t$ is labor augmenting technological progress with the rate $\gamma_A \geq 1$; u and f are utility and production functions, respectively; and $\beta \in (0,1)$; $\delta \in [0,1]$; $\rho \in (-1,1)$; $\sigma \in (0,\infty)$.

Discussion. Why cannot we solve a nonstationary model with conventional solution methods? For (39)–(41), the Euler equations is given by

$$u(c_t) = \beta E_t\left[u'(c_{t+1})(1-\delta + z_{t+1}f_k(k_{t+1}, A_{t+1}))\right].$$

The mainstream of economic literature considers stationary models. We can make our model stationary by setting at $A_t = A$ for all t. The solution to such model is characterized by time-invariant (stationary) decision functions. Conventional solution methods iterate on the Euler equation until a fixed-point decision function for consumption $c_t = C(k_t, z_t)$ is found.[10] However, if A_t grows over time, then the optimal decision function changes over time $C_t(\cdot) \neq C_{t+1}(\cdot)$, then there is no fixed-point solution $C(\cdot)$ so that the conventional methods are not applicable. Under additional restrictions on preferences and technology, the model with labor augmenting progress has balanced growth and can be converted into stationary; see King et al. (1988). We will not focus on this special case but will approximate a sequence (path) of time-dependent functions $\{C_0(\cdot), C_1(\cdot), ...\}$.

In doing this, we exploit a turnpike theorem; see Majumdar and Zilcha (1987) and Mitra and Nyarko (1991) for examples of turnpike theorems for nonstationary models. A turnpike theorem studies the convergence of finite-horizon economies to infinite horizon economies as the time horizon increases. For this model, the turnpike theorem is proven in MMTT (2020). Two important consequences of the turnpike theorem help us compute solutions in the nonstationary economy: First, the infinite- and finite-horizon solutions follow closely one another for a long time and diverge only when the economy approaches to a terminal condition. Second, two terminal conditions $k_T = k'$ and $k_T = k''$ that are close to the solution to nonstationary model make the finite-horizon path closer to the infinite-horizon path. The former allows us to approximate infinite-horizon solutions by finite-horizon solutions, while the latter tells us that it is important to select a good terminal condition, the one close to the infinite-horizon equilibrium path.

To implement the perturbation procedure described in the main text, in Step I, we construct a stationary (time-invariant) model of period T and construct the corresponding Markov decision rule for consumption $C_T(\cdot)$, and in Step II, we use C_T to iterate backward on Euler equations in order to construct a sequence (path) of time-dependent value and decision functions $\{C_{T-1}(\cdot), C_{T-2}(\cdot), ..., C_0(\cdot)\}$, respectively. As a final step, we check the turnpike theorem by verifying that the constructed finite-horizon solution for initial τ periods converges periodwise to a limiting $\{C_0^*(\cdot), C_1^*(\cdot), ..., C_\tau^*(\cdot)\}$ as time horizon T increases, where τ is the final time period in which we want the solution to be accurate. We elaborate on Steps I and II in details below.

First of all note that the model with growth has no natural steady state. To deal with this issue, we introduce time-varying growth rates of capital γ_{kt} that capture how much this state variable grows from period t to $t+1$ due to the time trend or the parameter change.

[10] Solution to the growth model could equally well be expressed by a decision function for next period capital $k_{t+1} = K(k_t, z_t)$.

Step I: Solving for terminal decision functions. In Step I, we aim to construct stationary Markov terminal condition in the form of a decision function for consumption $c_T = C_T(k_T, z_T)$ which is as close as possible to unknown decision function of the infinite horizon model. We assume balanced growth $\gamma_{kT} = \gamma_{cT} = \gamma_A$, and feed the resulting two equations to a Dynare perturbation. Assuming $c_T = C_T(k_T, z_T)$ and $c_{T+1} = C_T(k_{T+1}, z_{T+1})$, we obtain the usual stationary solution to

$$u'(c_T) = \beta E_T \left[u'(c_{T+1}\gamma_A)(1 - \delta + z_{T+1} f_k(k_{T+1}\gamma_A, A_T\gamma_A)) \right],$$
$$c_T = (1 - \delta) k_T + z_T f(k_T, A_T) - k_{T+1}\gamma_A.$$

Unless $\gamma_A = 1$, the model does not have a balanced growth and our approximation does not coincide with the infinite horizon solution at T. But the turnpike theorem implies that the specific terminal condition assumed at T does not affect significantly the solution up to τ provided that $\tau \leq T$. There are ways of constructing more accurate terminal conditions at additional costs.[11]

Step II: Finding a path of decision functions. In Step II, we start from the constructed terminal condition for T and proceed backward to compute the path of the decision functions for $t = T-1, T-2, ..., 0$ by iterating backward on

$$u'(c_t) = \beta E_t \left[u'(C_{t+1}(k_{t+1}, z_{t+1}))(1 - \delta + z_{t+1} f_k(k_{t+1}, A_{t+1})) \right], \tag{42}$$
$$k_{t+1} = (1 - \delta) k_t + z_t f(k_t, A_t) - c_t. \tag{43}$$

In particular, for period $T - 1$, given $c_T = C_T(k_T, z_T)$, Dynare produces the decision function for $c_{T-1} = C_{T-1}(k_{T-1}, z_{T-1})$, in period $T-2$, given $c_{T-1} = C_{T-1}(k_{T-1}, z_{T-1})$ we find $c_{T-2} = C_{T-2}(k_{T-2}, z_{T-2})$ and so on until the entire solution path is constructed.

Perturbation solutions we construct are obtained around a deterministic growth path. We consider five alternative methods for constructing such a path. We either assume some exogenous growth rates or precompute the growth rates endogenously by shutting down uncertainty in the model. Also, our methods differ in a way the policy functions are specified. In particular, for each deterministic growth-path specification, we have two versions of the algorithm: one in which a next-period policy function takes into account the volatility of uncertainty σ, and the other in which it does not setting $\sigma = 0$. Why might we want to handle the volatility differently? Perturbation policy functions of second and higher orders of approximation are not passing in general through a deterministic steady state of the model. Even in the balanced growth model, if true policy functions for period $t+1$ are combined with the model's equations written for period t, the deterministic steady state would not be a solution of the deterministic version of the combined system of equations. This feature can be overcome by recognizing explicitly that $C_{t+1}(\cdot)$ depends on σ and by setting σ to zero when computing the deterministic steady state.

Methods 1 and 2. Methods 1 and 2 find local approximations of today's consumption policy function $C_t(k_t, z_t)$ in period t from equations (42) and (43) given the next-period function $C_{t+1}(k_{t+1}, z_{t+1})$. The difference between the two methods lies only in the point around which the local approximation is taken and it is related to our implementation in Dynare.

In period t, Method 1 finds local approximation around a point $(k_t^*, 1)$ that solves the following system of two equations for c_t^* and k_t^*:

$$u'(c_t^*) = \beta u'(C_{t+1}(k_t^*, 1))\left[1 - \delta + f_k(k_t^*, A_{t+1})\right], \tag{44}$$
$$k_t^* = (1 - \delta) k_t^* + f(k_t^*, A_t) - c_t^*. \tag{45}$$

[11] MMTT (2020) offer an alternative way of constructing a terminal condition. Namely, they assume that the solution is stationary in periods T, $T+1$ and $T+2$ provided that it is adjusted to growth. This gives 4 equations (Euler equation and constraint) for T and $T+1$, which can be solved with respect to steady state k_T^*, c_T^* and growth rates γ_{kT} and γ_{cT}.

Here today's and tomorrow's capital are the same and equal to k_t^* because we assume that the growth rate of capital is one.

To understand Method 2, recall that the consumption decision function obtained by perturbation depends on the uncertainty parameter σ and is given by $C_{t+1}(.,.;\sigma)$ in period $t+1$; see (**??**) for a general representation.[12] In Method 2, we perturb around a point that is computed taking $C_{t+1}(.,.;\sigma)$ without the effect of uncertainty, $\sigma=0$; this approach is similar to finding a deterministic steady state first (as $\sigma=0$). In other words, the approximation is conducted around a point $\left(k_t^{\dagger},1\right)$ that solves the following system of two equations for $c_t^{\dagger}$ and $k_t^{\dagger}$

$$u'(c_t^{\dagger}) = \beta u'(C_{t+1}\left(k_t^{\dagger},1;0\right))\left[1-\delta+f_k\left(k_t^{\dagger},A_{t+1}\right)\right], \tag{46}$$

$$k_t^{\dagger} = (1-\delta)\,k_t^{\dagger} + f\left(k_t^{\dagger},A_t\right) - c_t^{\dagger}. \tag{47}$$

Evidently, the first-order perturbation solutions obtained by Method 1 and 2 are identical, as such solutions do not depend on uncertainty.[13]

Methods 3 and 4. These two methods explicitly account for time-varying growth rates $\{\gamma_{kt}\}_{t=1}^{T}$ (recall that for both Methods 1 and 2 we assume that growth rates are equal to unity). Similarly to the latter methods, our Methods 3 and 4 differ in points around which we find Taylor's expansions and parallel to Methods 1 and 2, respectively. To construct a path of growth rates $\{\gamma_{kt}\}_{t=1}^{T}$, both Methods 3 and 4 solve a deterministic version of the model. Namely, we shut down uncertainty by assuming $z_t=1$ for all t, set $\tilde{c}_{T+1}$ and $\tilde{k}_{T+1}$ equal to the steady state of the stationary model in the terminal period, and solve the following system of equations:[14]

$$u'(\tilde{c}_t) = \beta u'(\tilde{c}_{t+1})\left(1-\delta+f\left(\tilde{k}_{t+1},A_{t+1}\right)\right),$$

$$\tilde{k}_{t+1} = (1-\delta)\,\tilde{k}_t + f\left(\tilde{k}_t,A_t\right) - \tilde{c}_t.$$

Given the solution $\left\{\tilde{k}_{t+1}\right\}_{t=1}^{T}$, we compute the growth rates as $\gamma_{kt}=\tilde{k}_{t+1}/\tilde{k}_t$. Both Methods 3 and 4 take $\{\gamma_{kt}\}_{t=1}^{T}$ as given.

In period t, Method 3 perturbs the solution around a point $(k_t^*,1)$ that solves for k_t^* and c_t^* the following system of two equations:

$$u'(c_t^*) = \beta u'(C_{t+1}(\gamma_{kt}k_t^*,1))(1-\delta+f(\gamma_{kt}k_t^*,A_{t+1}), \tag{48}$$

$$\gamma_{kt}k_t^* = (1-\delta)\,k_t^* + f(k_t^*,A_t) - c_t^*. \tag{49}$$

Note that a variable k_{t+1}^* is replaced by $\gamma_{kt}k_t^*$ meaning that we take into account growth when computing the point of approximation. In turn, Method 4 finds a perturbation solution around a point $\left(k_t^{\dagger},1;0\right)$ and finds $c_t^{\dagger}$ and $k_t^{\dagger}$ by solving

$$u'(c_t^{\dagger}) = \beta u'(C_{t+1}\left(\gamma_{kt}k_t^{\dagger},1;0\right))1-\delta+f\left(\gamma_{kt}k_t^{\dagger},A_{t+1}\right), \tag{50}$$

$$\gamma_{kt}k_t^{\dagger} = (1-\delta)\,k_t^{\dagger} + f\left(k_t^{\dagger},A_t\right) - c_t^{\dagger}. \tag{51}$$

[12] Note that the dependence of $C_{t+1}(k_t^*,1)$ on σ is implicit in Method 1, i.e., we mean $C_{t+1}(k_t^*,1;\sigma)$ there.

[13] Note, however, that higher-order approximations will differ between the two methods not only because the intercepts associated with uncertainty are distinct (equal to $C_{\sigma\sigma,t+1}(k_t^*,1)\,\sigma^2$ and $C_{\sigma\sigma,t+1}\left(k_t^{\dagger},1;0\right)\sigma^2$ for Method 1 and Method 2, respectively) but also because the points around we approximate differ.

[14] To implement this step in Dynare, we just solve a system of equations backward in terms of variables $\left\{\tilde{c}_t,\tilde{k}_t\right\}$.

Method 5. Method 5 is close to Method 3, but the path for growth rates is computed iteratively. We begin by exogenously fixing the path $\{\gamma_{k,t}\}_{t=1}^{T}$ and obtaining the policy functions for a stochastic version of the model; this is similar to Method 3. As a next step, we simulate the model with the realized values of shocks which are set to zero, we compute the growth rates of capital over this simulated path, and we obtain the policy functions for a stochastic version of the model. We can repeat this step as many times as necessary. We do not offer any counterpart of Method 5 (i.e., Method 6) that corrects for volatility as it is the case of the methods above because the stochastic growth path is computed in a stochastic version of the model, in which the growth path is obtained endogenously.

Numerical results. In this section, we present the results of our numerical analysis. We assume the standard utility and production functions:

$$u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}, \quad f(k, A) = A^{1-\alpha}k^{\alpha}.$$

For all the experiments, we fix the parameters $\{\alpha, \beta, \delta, \rho\}$ at the following values:

$$\alpha = 0.36, \quad \beta = 0.99, \quad \delta = 0.025, \quad \rho = 0.95.$$

We vary the values of the remaining parameters $\{\gamma, \sigma_\varepsilon, \gamma_A, T\}$; in the benchmark case, we set them to the following values:

$$\gamma = 5, \quad \sigma_\varepsilon = 0.03, \quad \gamma_A = 1.01, \quad T = 200.$$

We simulate the model's solution for different values of the terminal date T. For all simulations, we use the same initial condition (k_0, z_0) and the same sequence of productivity shocks $\{z_t\}_{t=1}^{\mathcal{T}}$.

To see whether our perturbation-based method computes accurate solutions, we obtain an (almost) exact solution by exploiting the property of balanced growth. For this purpose, we first introduce labor-augmenting technical change into the model, then derive the first-order conditions, and finally, detrend them. The resulting stationary model is solved by a very accurate standard projection method with Smolyak grid, third-order polynomial approximation, and 10-node Gauss-Hermite quadrature (the maximum residuals in the model's equations are of order 10^{-9} in $\log_{10}$ units). We compare the simulated series generated by such a projection method with those of our perturbation method on a fixed sequence of shocks of length $\mathcal{T}$.

In Table 2, we report absolute unit-free mean and maximum differences between our approximate and balanced growth ("exact") solutions (in $\log_{10}$ units) on a simulated path $[0, \mathcal{T}]$ with $\mathcal{T} \in \{50, 100, 150, 175, 200\}$. We consider both, first- and second-order approximations.

As is evident from the table, the first-order perturbation solutions are significantly less accurate than the second-order solutions; the difference between the two can reach two orders of magnitude. However, in terms of running times (both solution and simulation), the two solutions are roughly comparable. It is not faster to obtain a first- than second-order solution because each perturbation step takes just few seconds and the largest share of time is spent on finding different decision rules for each period. For second-order approximations, the most basic method, Method 1, yields very accurate solutions: the mean difference from the exact solution is at most 1 percent across the considered simulation lengths, while the maximum difference reaches 1.5 percent. The ranking of the methods in terms of accuracy varies with time horizon $\mathcal{T}$. For example, for $\mathcal{T} = 200$, Method 1 is the least accurate method, followed by Method 2, and then by Methods 4 and 3 (we look at the maximum errors). However, the ranking between Methods 2 and 4 reverses when the other Ts in the table are considered. Methods 3 and 5 are about the same in terms of accuracy and they are the most accurate.

Figure 9 plots our first- and second-order solutions for capital of the nonstationary model (produced by Method 5), as well as the exact solution of the balanced growth model (produced by the standard projection method); the left panel displays the growing solutions, while the right panel contains the detrended

	Method 1		Method 2		Method 3		Method 4		Method 5	
First-order solution										
Errors, in $\log_{10}$ units										
Horizon		Mean	Max			Mean	Max		Mean	Max
$[0, 50]$		-1.41	-1.13			-1.50	-1.22		-1.53	-1.23
$[0, 100]$		-1.24	-0.97			-1.33	-1.11		-1.35	-1.12
$[0, 150]$		-1.14	-0.77			-1.25	-1.08		-1.27	-1.08
$[0, 175]$		-1.07	-0.58			-1.22	-0.94		-1.23	-0.97
$[0, 200]$		-1.04	-0.58			-1.19	-0.94		-1.20	-0.97
Running time, in seconds										
Solution	161.57		157.60		294.28		288.03		317.38	
Simulation	0.0387		0.0275		0.0346		0.0293		0.0271	
Second-order solution										
Errors, in $\log_{10}$ units										
Horizon	Mean	Max	Mean	Max	Mean	Max	Mean	Max	Mean	Max
$[0, 50]$	-2.28	-2.03	-2.83	-2.53	-3.48	-2.95	-2.79	-2.24	-3.51	-2.92
$[0, 100]$	-2.12	-1.90	-2.77	-2.53	-3.30	-2.93	-2.46	-2.05	-3.30	-2.91
$[0, 150]$	-2.05	-1.80	-2.75	-2.53	-3.26	-2.88	-2.33	-2.03	-3.26	-2.88
$[0, 175]$	-2.00	-1.71	-2.71	-2.15	-3.14	-2.23	-2.38	-2.03	-3.14	-2.23
$[0, 200]$	-2.04	-1.71	-2.61	-1.79	-3.07	-2.23	-2.43	-2.03	-3.09	-2.23
Running time, in seconds										
Solution	167.99		167.18		308.87		296.46		337.60	
Simulation	0.0256		0.0249		0.0326		0.0312		0.0348	

Notes: Mean and Max are, respectively, the average and maximum of absolute difference between the P-EFP and exact solutions (in $\log_{10}$ units) on a stochastic simulation of 200 observations.

Table 2: Difference of a simulated solution path from the balanced growth path in $\log_{10}$ units

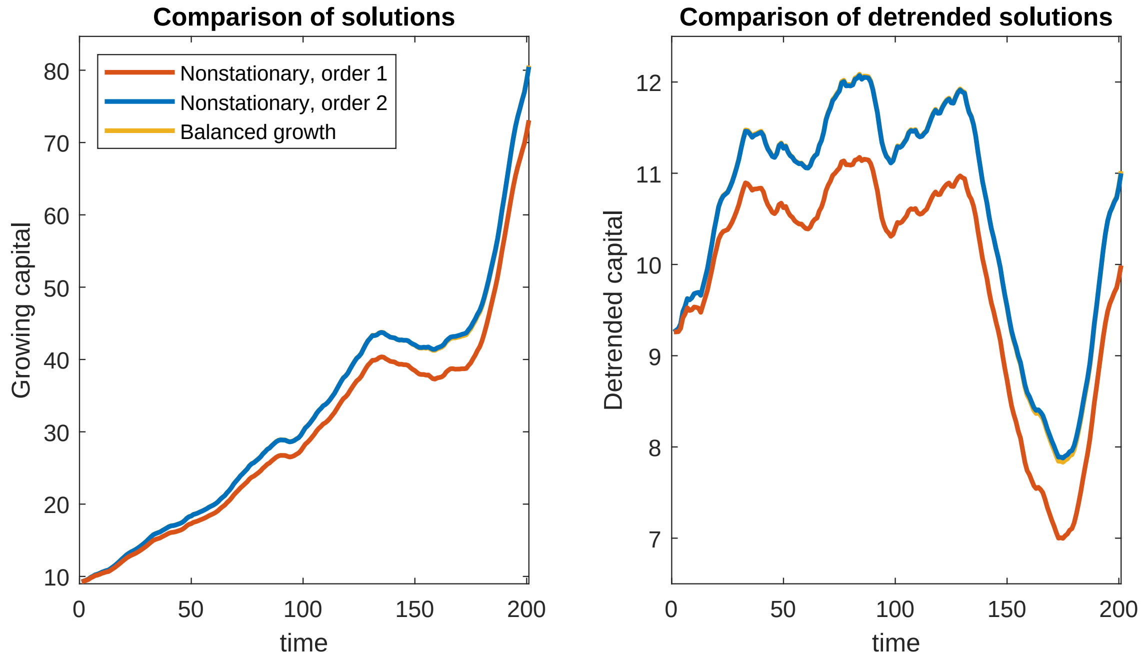

Figure 9: Comparison of the nonstationary P-EFP solutions computed by Method 5 and the balanced growth solution

solutions. One striking feature of our solutions is that its second-order approximation is virtually identical to the exact solution (blue and yellow lines coincide). In turn, the first-order solution is a visible upward shift of the other two solutions, and therefore, can imply substantial inaccuracy.

An important question is: How does our perturbation solutions compare to the existing methods that can solve nonstationary models?

In Table 3, we make a comparison of our perturbation method to three other methods, an extended path method of Fair and Taylor (1983) method, a naive method and a global EFP method of Maliar et al. (2020). Fair and Taylor's (1983) method solves for a path of variables and not functions (as our method does). A naive method finds a different solution for each period t under the assumption that the t-period level of technology prevails in each subsequent period. For each of the methods, we use $T = 200$ in the solution procedure, and we simulate the model for $\mathcal{T} \in \{50, 100, 150, 175, 200\}$.

As is seen from the table, among the three alternative methods, the ranking of the methods is always the same: the naive method is the least accurate and the global EFP is the most accurate, with Fair and Taylor's (1983) method being in between. The latter reaches a notorious accuracy of 0.0001 percent for $\mathcal{T} = 50$; the residuals increase to 3.5% for $\mathcal{T} = 200$. The main finding in the table is that for $\mathcal{T} = 175$ our second-order method is almost as accurate as third-degree solution obtained with the global EFP method, and for $\mathcal{T} = 200$, the second-order solution overpasses the third-degree global EFP solution by a half order of magnitude. Moreover, our perturbation solution is not only more accurate for longer $\mathcal{T}$ but also much faster. This is because of perturbation used as a basis of the method.

Appendix B

In this section, we present sensitivity experiments.

A gradual increase in inflation target implemented with probability. In Figures 10 and 11, we present the supplementary experiments for Section 3.2. Namely, we consider two experiments that are parallel to the one in Figure 2, where a gradual increase in the inflation target happens with probability

Table 3: Comparison of the P-EFP to the other methods

	Fair-Taylor (1983) method	Naive method	Global EFP	P-EFP	
Type of approximation	path	path	3rd order	1st order	2nd order
Maximum errors, in $\log_{10}$ units					
$[0, 50]$	-1.29	-1.04	-6.35	-1.27	-2.24
$[0, 100]$	-1.18	-0.92	-4.76	-1.11	-2.05
$[0, 150]$	-1.14	-0.89	-3.22	-1.07	-2.03
$[0, 175]$	-1.14	-0.89	-2.47	-0.94	-2.03
$[0, 200]$	-1.14	-0.89	-1.51	-0.94	-2.03
Running time, in seconds					
Solution	1.2(+4)	28.9	199.4	317.4	337.6
Simulation	-	2.6	0.0244	0.0271	0.0348
Total	1.2(+4)	31.5	199.4	317.4	337.6

Note: Maximum errors are the maximum of the absolute difference between the given and exact solutions (in $\log_{10}$ units) on a stochastic simulation of $\mathcal{T}$ observations.

of 50 percent. In Figures 10 and 11, such a gradual change occurs with probabilities 75 and 25 percent, respectively. As is seen from the figures, a larger probability of implementing a higher inflation target leads to slightly larger expansionary effects on output, consumption, investment, and commodity exports. Although the qualitative patterns are the same, the anticipation effects (changes up to the fifth period when the actual change takes place) are visibly larger with 75 percent probability than with 25 percent probability.

A negative supply shock and a switch to price-level targeting. In Figure 6, we focus on a switch to price-level targeting after a negative foreign demand shock. Here, we present a supplementary experiment for Section 3.5. namely, we consider a negative supply shock instead.

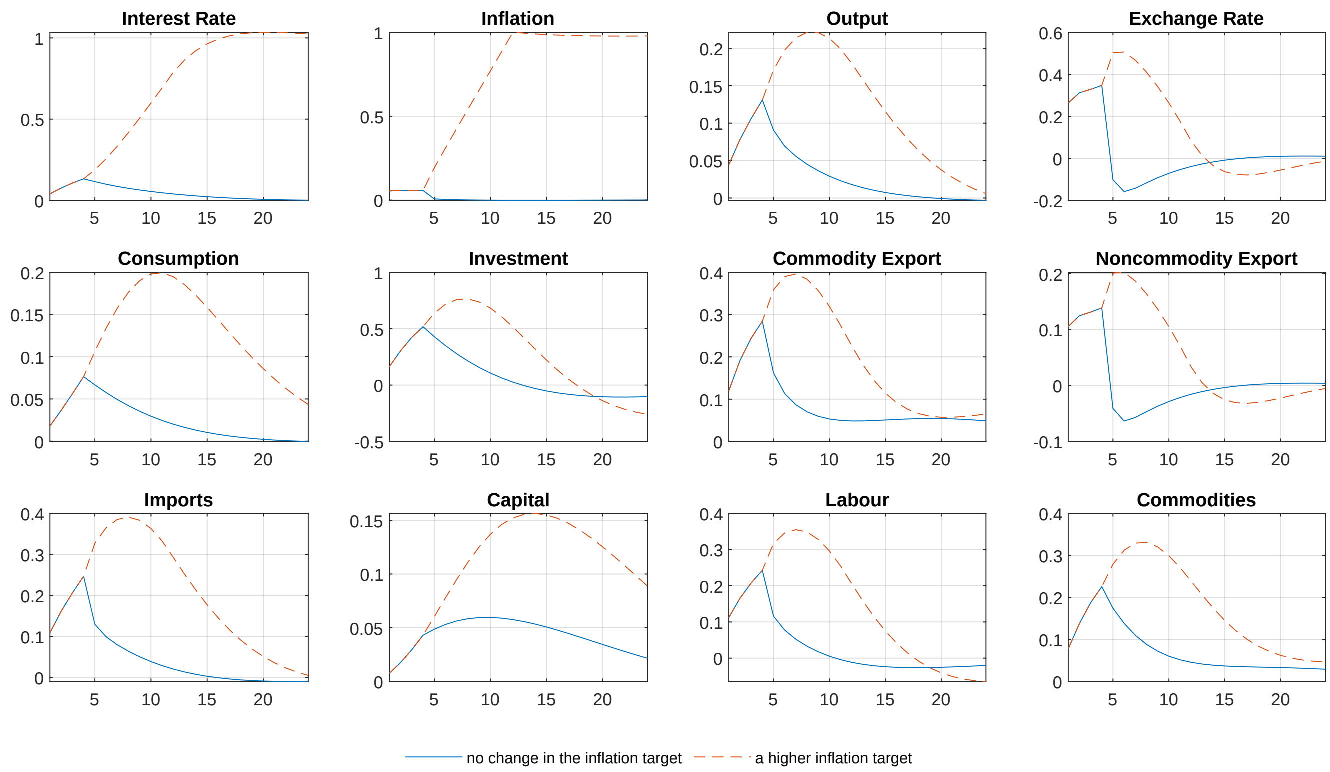

Figure 10: A gradual increase in the inflation target (75% probability)

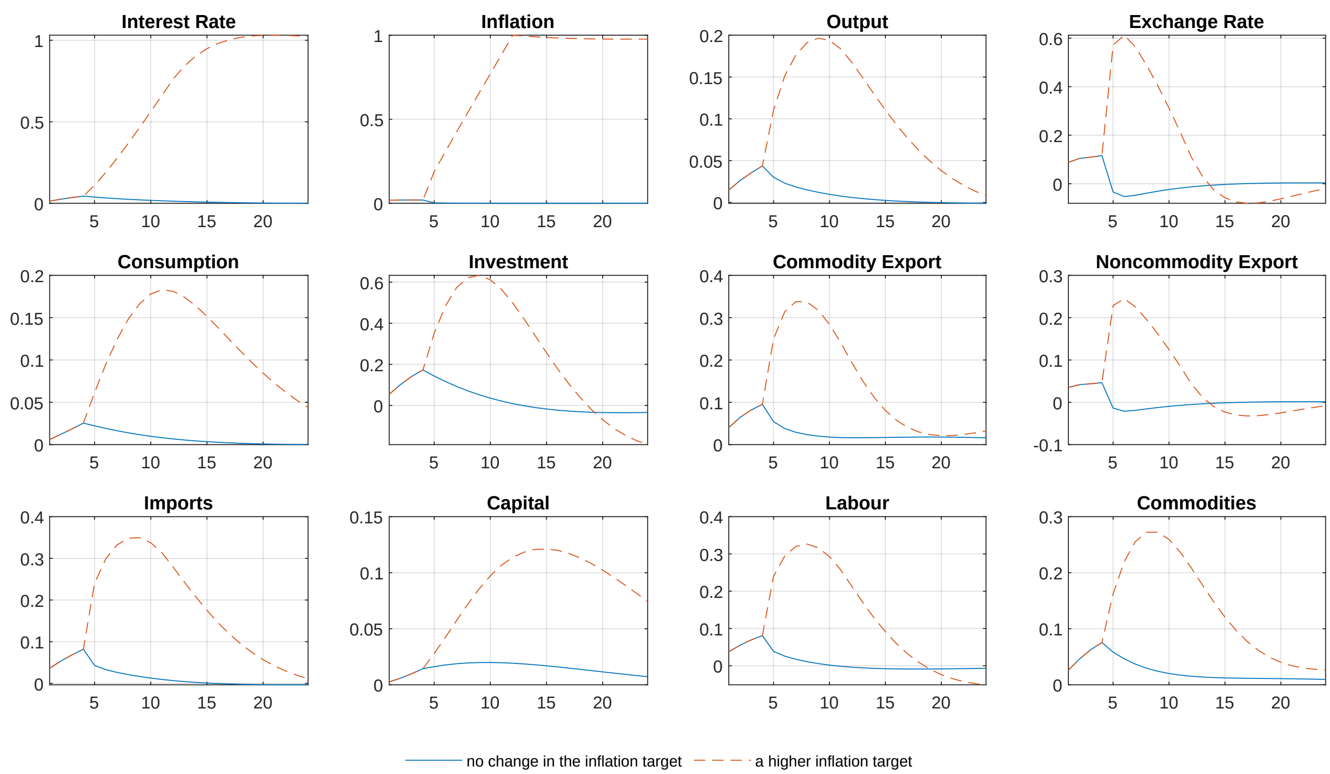

Figure 11: A gradual increase in the inflation target (25% probability)

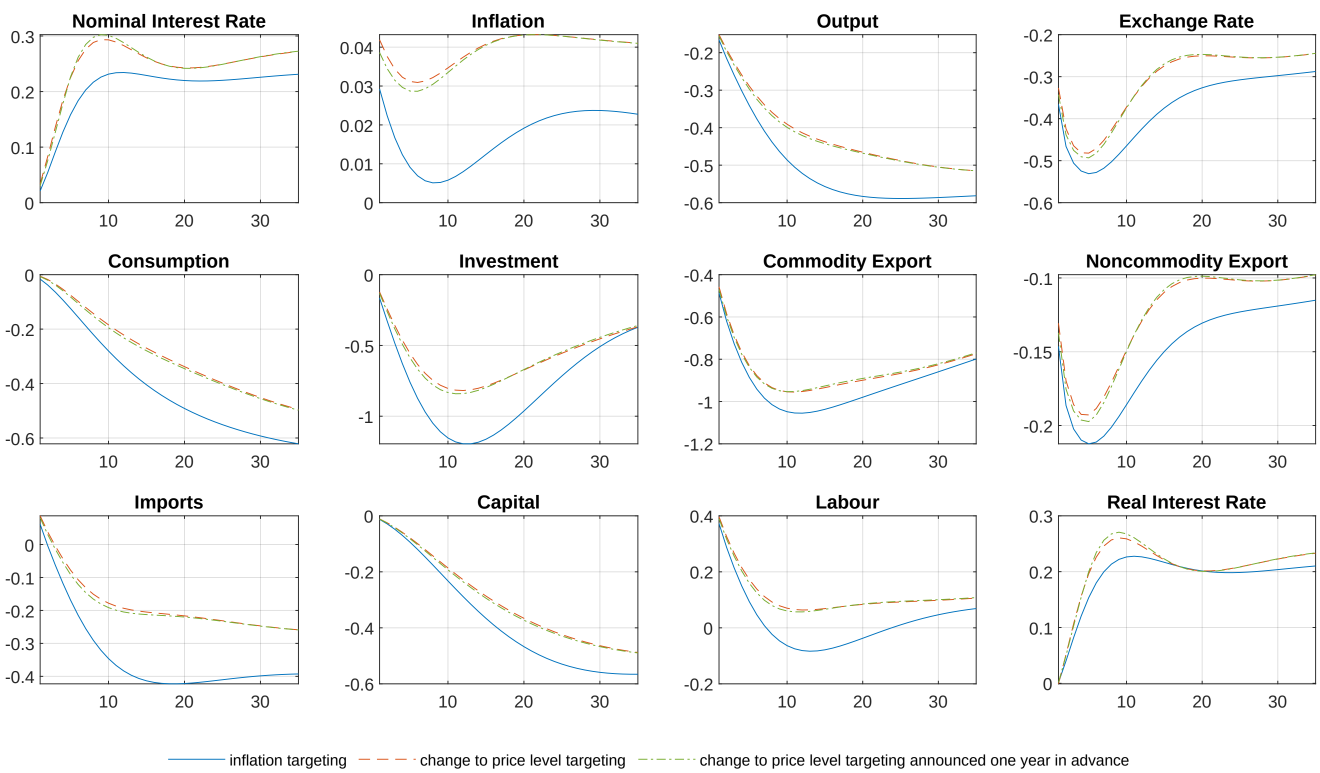

Figure 12: A negative supply shock and a switch to price-level targeting

Online appendix

For the reader's convenience, we provide a description of the calibration procedure, which is similar to LMM (2020).

6.1 Calibration

The model contains 61 parameters to be calibrated. Whenever possible, we use the same values of parameters in the scaled-down model as those in the full-scale model, and we choose the remaining parameters to reproduce a selected set of observations from the Canadian time series data. In particular, our calibration procedure targets the ratios of six nominal variables to nominal GDP $P_t^y Y_t$, namely, consumption $P_t C_t$, investment $P_t^i I_t$, noncommodity export $P_t^{nc} X_t^{nc}$, commodity export $P_t^{com} X_t^{com}$, import $P_t^m M_t$, total commodities $P_t^{com} COM_t$, and labor input $W_t L_t$. Furthermore, we calibrate the persistence of shocks so that the standard deviations of the selected bToTEM variables coincide with those of the corresponding ToTEM variables, namely, those of domestic nominal interest rate R_t, productivity A_t, foreign demand Z_t^f, foreign commodity price p_t^{comf}, and foreign interest rate r_t^f. The parameters choice is summarized in Tables 4 and 5 below.

Parameter	Symbol	Value	Source
Rates			
– real interest rate	$\bar{r}$	1.0076	ToTEM
– discount factor	β	0.9925	ToTEM
– inflation target	$\bar{\pi}$	1.005	ToTEM
– nominal interest rate	$\bar{R}$	1.0126	ToTEM
– ELB on the nominal interest rate	R^{elb}	1.0076	fixed
Output production			
– CES elasticity of substitution	σ	0.5	ToTEM
– CES labor share parameter	δ_l	0.249	calibrated
– CES capital share parameter	δ_k	0.575	calibrated
– CES commodity share parameter	δ_{com}	0.0015	calibrated
– CES import share parameter	δ_m	0.0287	calibrated
– investment adjustment cost	χ_i	20	calibrated
– fixed depreciation rate	d_0	0.0054	ToTEM
– variable depreciation rate	$\bar{d}$	0.0261	ToTEM
– depreciation semielasticity	ρ	4.0931	calibrated
– real investment price	ι_i	1.2698	ToTEM
– real noncommodity export price	ι_x	1.143	ToTEM
– labor productivity	$\bar{A}$	100	normalization
Price setting parameters for consumption			
– probability of indexation	θ	0.75	ToTEM
– RT indexation to past inflation	γ	0.0576	ToTEM
– RT share	ω	0.4819	ToTEM
– elasticity of substitution of consumption goods	ε	11	ToTEM
– Leontieff technology parameter	s_m	0.6	ToTEM
Price setting parameters for imports			
– probability of indexation	θ^m	0.8635	ToTEM
– RT indexation to past inflation	γ^m	0.7358	ToTEM
– RT share	ω^m	0.3	ToTEM
– elasticity of substitution of imports	ε^m	4.4	
Price setting parameters for wages			
– probability of indexation	θ^w	0.5901	ToTEM

– RT indexation to past inflation	γ^w	0.1087	ToTEM
– RT share	ω^w	0.6896	ToTEM
– elasticity of substitution of labor service	ε^w	1.5	ToTEM
Household utility			
– consumption habit	ξ	0.9396	ToTEM
– consumption elasticity of substitution	μ	0.8775	ToTEM
– wage elasticity of labor supply	η	0.0704	ToTEM
Monetary policy			
– interest rate persistence parameter	ρ_r	0.83	ToTEM
– interest rate response to inflation gap	ρ_π	4.12	ToTEM
– interest rate response to output gap	ρ_y	0.4	ToTEM
Other			
– capital premium	κ^k	0.0674	calibrated
– exchange rate persistence parameter	$\varkappa$	0.1585	ToTEM
– foreign commodity price	$\bar{p}^{comf}$	1.6591	ToTEM
– foreign import price	$\bar{p}^{mf}$	1.294	ToTEM
– risk premium response to debt	ς	0.0083	calibrated
– export scale factor	γ^f	18.3113	calibrated
– foreign demand elasticity	ϕ	0.4	calibrated
– elasticity in commodity production	s_z	0.8	calibrated
– land	F	0.1559	calibrated
– share of other components of output	υ_z	0.7651	calibrated
– share of other components of GDP	υ_y	0.311	calibrated
– adjustment cost in commodity production	χ_{com}	16	calibrated
– persistence of potential GDP	φ_z	0.75	calibrated

Table 4: Calibrated parameters in endogenous model's equations

Parameter	Symbol	Value	Source
Shock persistence			
– persistence of interest rate shock	φ_r	0.25	ToTEM
– persistence of productivity shock	φ_a	0.9	fixed
– persistence of consumption demand shock	φ_c	0	fixed
– persistence of foreign output shock	φ_{zf}	0.9	fixed
– persistence of foreign commodity price shock	φ_{comf}	0.87	calibrated
– persistence of foreign interest rate shock	φ_{rf}	0.88	calibrated
Shock volatility			
– standard deviation of interest rate shock	σ_r	0.0006	calibrated
– standard deviation of productivity shock	σ_a	0.0067	calibrated
– standard deviation of consumption demand shock	σ_c	0.0001	fixed
– standard deviation of foreign output shock	σ_{zf}	0.0085	calibrated
– standard deviation of foreign commodity price shock	σ_{comf}	0.0796	calibrated
– standard deviation of foreign interest rate shock	σ_{rf}	0.0020	calibrated

Table 5: Calibrated parameters in exogenous model's equations