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SUPPLEMENT TO CORRIGENDUM TO "COMMUNICATION AND EQUILIBRIUM IN DISCONTINUOUS GAMES OF INCOMPLETE INFORMATION"

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THIS IS A SUPPLEMENT that contains some omitted details from a "Corrigendum to 'Communication and Equilibrium in Discontinuous Games of Incomplete Information'."

We thank Andreas Blume for pointing out an error in a claim that we made in Example 1, and Eric Balder for pointing out that the proof of a lemma contains an assertion that is far from obvious and that two steps of the main proof contain typos and misstatements that make the steps difficult to follow. We offer corrections here.

In Example 1 we claim that no equilibrium exists for any type-independent tie-breaking rule. Andreas Blume has pointed out the following counterexample to that claim. Consider a tie-breaking rule where all ties are broken in favor of bidder 1.[1] Bidder 2 bids 3.5 for all types. Bidder 1 bids 3.5 if his type is .5 or higher, and bids $3.5 - \varepsilon t_1$ when of type $t_1 < .5$. For small $\varepsilon > 0$, this is an equilibrium. The error in our proof of the nonexistence claim comes in the assertion that if there exists an equilibrium, then there exists one in weakly increasing strategies. To justify that assertion, we appealed to Proposition 1 of Maskin and Riley (2002), to which the above is also a counterexample.[2]

A slight modification of our Example 1 resurrects the claim that there does not exist an equilibrium for any type-independent tie-breaking rule. Consider exactly the same setting except for the following: with probability $1 - \varepsilon$ there is one item available that is awarded to the high bidder at the high price; and with probability $0 < \varepsilon < 1/200$ there are two items available, in which case the auctioneer randomly draws a number x from a uniform distribution on $[0, 6]$ and gives an item to each bidder whose bid exceeds x at a price of 0.[3] With this modification, whenever there exists an equilibrium there also exists one in weakly increasing strategies. This follows from the observation that if one type of a bidder weakly prefers a high bid to a low one, then a higher type of the bidder will strictly prefer the higher bid to the lower one, as now any increase in bids increases the probability of obtaining an object. The remaining proof shows that in any equilibrium the bottoms of the supports of the bidding strategies must be identical and must be bid by both bidders with positive probability. A contradiction is then reached by showing that for any choice of this lowest bid that results in a positive probability of ties, at least one bidder would deviate.

A detailed proof is as follows: With this perturbation, monotonicity is easy to see: Consider any pair of bids b', b'' with $b'' > b'$, and consider player 1. Because types are

[1] The claim that there are no equilibria is true if the tie-breaking rule gives both bidders a positive probability of winning an object.

[2] The statement of Proposition 1 in earlier working paper versions of Maskin and Riley (2002) asserts monotonicity only when there is a strictly positive probability of winning, and is correct.

[3] This has some flavor of an Amsterdam auction (see Goeree and Offerman (2004)) in that its effect is to make bidders care about their bids even in the presence of asymmetries (here, in the tie-breaking).

independent, and because we are considering a tie-breaking rule that does not depend on player types, the set of events (on the types and bids of the other player, the outcome of any randomizations by the seller in the event of ties, and the randomization over whether a second object is available) in which player 1 wins with b'' but loses with b', and the probabilities over those events are unaffected by t_i. But then, if b'' does at least as well as b' for $t_1 = t'$, then it does strictly better for $t_1 = t''$ for any $t'' > t'$. This follows since, given the perturbation, b'' has a strictly higher probability than b' of winning, and own type only enters expected utility calculations multiplied by probability of winning, while the rest of the expected utility calculation is independent of own type.

Thus, there is no loss in assuming the bidding strategies b_1, b_2 are monotone, in the sense that if $t_i' > t_i$, then every bid in the support of $b_i(t_i')$ is at least as large as every bid in the support of $b_i(t_i)$. It follows immediately that there is an at most countable set of signals t_i for which the support of $b_i(t_i)$ is not a singleton. For such t_i, replace b_i by the infimum of the support of $b_i(t_i)$. The modified bid functions b_1, b_2 again constitute an equilibrium. Thus we have an equilibrium in monotone, pure behavioral strategies. Altering bids at signals $0, 1$ if necessary, there is no loss in assuming that b_1, b_2 are continuous at $0, 1$.

Note first then, that $b_1(0) = b_2(0) \equiv \underline{b}$. For instance, if $b_1(0) < b_2(0)$, then a small increase in bid never hurts, and helps if there happens to be a second object (the value of which is always positive).

Imagine that $\underline{b} \geq 4\frac{3}{4}$. The average value of the object, even if allocated optimally to the player with larger t, is $5 + 2/3 - 4(1/3) = 4\frac{2}{3}$, since $2/3$ and $1/3$ are the expected higher and lower values of two draws from the uniform distribution. So, conditional on there being one object, at least $1/12$ is lost. On the other hand, when there are two objects, each player earns at most 6 (the maximum possible value of the object), for an expectation of 12δ. Since $\delta < 1/200$, someone is thus losing money on average, and would be better off to bid 0 always. Hence, $\underline{b} < 4\frac{3}{4}$.

Suppose that one player, say player 2, bids $\underline{b}$ with probability 0. Let $b' \in (\underline{b}, 4\frac{3}{4})$ be such that $b_2 \leq b' \Rightarrow t_2 < 1/32$. When there are two objects, b' is a better bid than any lower bid. Let $P' > 0$ be the probability that player 2 bids less than b'. When there is a single object and bidding b' wins, the object is worth at least $5 + 0 - 4(1/32) = 4\frac{7}{8}$ to player 1, and so profits conditional on there being a single object are at least $P'/8 > 0$. But, for types near 0, 1's equilibrium bid is arbitrarily close to $\underline{b}$ (and so is less than b') and wins with probability arbitrarily close to 0, since 2 bids in some small neighborhood of $\underline{b}$ with arbitrarily small probability. So, an interval of 1's types has a profitable deviation, a contradiction.

Thus, both players bid $\underline{b}$ with positive probability. For each i, let $\tau_i = \text{supp}\{t \mid b_i(t) \leq \underline{b}\} > 0$.

Assume that ties at $\underline{b}$ are broken with probability $p \in (0, 1)$ in favor of player 1. Let t' and t'', $t' < t''$, be two values of t for which $b_1(t) = \underline{b}$. It follows that $5 + t' - 4E(t_2 | b_2(t_2) = \underline{b}) \geq \underline{b}$, else 1 would be better to bid $\underline{b} - \varepsilon$ with t' (note that by taking ε small enough, the cost of lowering the bid in the event there are two objects can be made arbitrarily small). But then, $5 + t'' - 4E(t_2 | b_2(t_2) = \underline{b}) > \underline{b}$, and so 1 should deviate to $\underline{b} + \varepsilon$ with t''. This is a contradiction.

So, all ties at $\underline{b}$ are broken in favor of one player, say player 1. Thus, with $t = \tau_2 - \varepsilon$, player 2 never wins when there is a single object, while by bidding ε more, he can also win when $t_1 \in [0, \tau_1)$. For this not to be a profitable deviation, it must be that $\underline{b} \geq 5 + \tau_2 - 4(Et_1 | t_1 \leq \tau_1) = 5 + \tau_2 - 2\tau_1$. Note that this implies that $\tau_1 > 1/8$, since $\underline{b} < 4\frac{3}{4}$.

Suppose that $\tau_2 < 1$. Pick $t = \tau_2 + \varepsilon$, and consider replacing $b_2(t)$ (which is by definition greater than $\underline{b}$) by any bid b' in $(\underline{b}, b_2(t))$. When this changes a win into a loss, $t_1 \geq \tau_1$, and hence, v_2 is at best $5 + \tau_2 + \varepsilon - 4\tau_1 < \underline{b}$, and so the change is profitable. At least $(1-\delta)/8$ of the time, there was a single object and $t_1 < \tau_1$. In this event, b' still wins, and pays $b_2(t) - b'$ less. Finally, lowering the bid costs at most 6 (the maximal value of the object) at most $\delta(b_2(t) - b')/6$ of the time (as this is the probability that there is a second object and the "reserve" is between b' and $b_2(t)$). As $\delta < 1/200$, $(1-\delta)/8 > \delta$, and so this is a profitable deviation, a contradiction. Thus, $\tau_2 = 1$.

Since $\tau_2 = 1$, $E(t_2|b_2 = \underline{b}) = 1/2$. Hence, $\underline{b} \leq 3 = 5 + 0 - 4(1/2)$; otherwise 1 is better to bid $\underline{b} - \varepsilon$ with t_1 near 0. But then, player 2 can profitably bid $\underline{b} + \varepsilon$ when he has t above 0, a contradiction. Thus, there is no equilibrium to this game.

There are other modified versions of the auction that also have no equilibria with type-independent tie-breaking rules, but the details of formulating such examples are very delicate.

In the proof of Lemma 2 part (i) we state that "we may implicitly define a unique element $\mu(E) = \Omega$ by requiring that $\phi \cdot \mu(E) = \lambda_\phi(E)$ for every $\phi \in \mathcal{E}^*$." As it is not obvious why this is true, we provide a fuller proof of part (i) of Lemma 2 here.

PROOF OF LEMMA 2(i): To show that $M(X, \Omega)$ is weak-* compact, we must show that every net in $M(X, \Omega)$ has a weak-* convergent subnet. To this end, let $\{\mu_\alpha\} \subset M(X, \Omega)$ be a net. For each integer $k \geq 1$ and each k-tuple $\Phi = (\varphi_1, \ldots, \varphi_k) \in (\mathcal{E}^*)^k$, consider the net $\{\Phi \cdot \mu_\alpha\}$ of $\mathbb{R}^k$-valued measures (equivalently, of k-tuples of scalar measures). Since the range of each μ_α lies in Ω, the range of each $\Phi \cdot \mu_\alpha$ lies in $\Phi(\Omega)$, which is a compact convex subset of $\mathbb{R}^k$. Hence, we can find a subnet $\{\mu_\beta\}$ of $\{\mu_\alpha\}$ such that, for each Φ, the net $\{\Phi \cdot \mu_\beta\}$ converges weak-* to some $\mathbb{R}^k$-valued measure λ_Φ. Because each of the measures $\Phi \cdot \mu_\beta$ has its range in $\Phi(\Omega)$, which is a compact convex set, so does the limit measure λ_Φ. That is, for each Borel set $E \subset X$, we have $\lambda_\Phi(E) \in \Phi(\Omega)$. Hence, for each Borel set $E \subset X$, the set

$$E_\Phi = \Phi^{-1}(\lambda_\Phi(E)) \cap \Omega = \{\omega \in \Omega : \Phi(\omega) = \lambda_\Phi(E)\}$$

is not empty. Since Φ is continuous and Ω is compact, E_Φ is also compact. Note that if $\Phi \in (\mathcal{E}^*)^k$, $\Psi \in (\mathcal{E}^*)^\ell$, then $(\Phi, \Psi) \in (\mathcal{E}^*)^{k+\ell}$ and

$$E_\Phi \cap E_\Psi = \Phi^{-1}(\lambda_\Phi(E)) \cap \Psi^{-1}(\lambda_\Psi(E)) = (\Phi, \Psi)^{-1}(\lambda_{(\Phi,\Psi)}(E)) = E_{(\Phi,\Psi)}.$$

Hence, the family of sets $\{E_\Phi : \Phi \in (\mathcal{E}^*)^k, \text{ some } k\}$ has the finite intersection property. Because the sets E_Φ are compact, there is an element of Ω that belongs to all of them. Because $\mathcal{E}$ is locally convex, continuous linear functionals separate points of $\mathcal{E}$, so there is a unique such element. Define $\mu(E)$ to be this unique element of Ω.

By construction, $\varphi \cdot \mu(E) = \lambda_\varphi(E)$ for each Borel set $E \subset X$, so weak countable additivity of μ follows immediately from weak countable additivity of each λ_ϕ. Weak-* convergence of $\{\mu_\beta\}$ to μ is immediate from the definition and construction. We have shown that every net in $M(X, \Omega)$ has a weak-* convergent subnet, so the proof is complete.

The last paragraph on page 1735 contains several typographical errors (t_i^*'s should be a_i^*'s) and a misstatement of (36). Delete from the beginning of the paragraph ("Fix an arbitrary...") through equation (37) and substitute the following.

Fix an arbitrary $(s_i^*, a_i^*) \in H$. Continuity of u_i (in outcomes and types) and continuity of Ψ on T_i^k guarantees that there is a compact neighborhood L of $h(s_i^*, a_i^*)$ in T_i^k such that if $(s_i, a_i) \in S_i \times A_i$ and $t_i, t_i' \in T_i^k$, then

$$(36) \qquad \left|Eu_i(s_i, a_i|\sigma_{-i}, t_i, \theta) - Eu_i(s_i, a_i|\sigma_{-i}, t_i', \theta)\right| < \frac{1}{4j}.$$

Continuity of h on H means that we can choose a compact neighborhood K of (s_i^*, a_i^*) in H such that $h(K) \subset L$. Applying (36), then (35), and then (36) again yields that

$$(37) \qquad Eu_i(s_i, a_i|\sigma_{-i}, t_i, \theta) > Eu_i(s_i, a_i|\sigma_{-i}, h(s_i, a_i), \theta) - \frac{1}{4j}$$

$$> Eu_i(\sigma_i|\sigma_{-i}, h(s_i, a_i), \theta) + \frac{3}{4j}$$

$$> Eu_i(\sigma_i|\sigma_{-i}, t_i, \theta) + \frac{1}{2j}.$$

The β's and B's in Step 5 are missing subscripts that make the argument hard to follow. On page 1737, delete the two sentences starting "Define B:" and "For each r, define $\beta^r \ldots$" and substitute the following

For each r and each type/action pair $(\bar{s}_i, \bar{a}_i) \in S_i^r \times A_i^r$, define $\beta^r_{(\bar{s}_i,\bar{a}_i)} : \Delta_{-i} \to \Omega$ by $\beta^r_{(\bar{s}_i,\bar{a}_i)}(s_{-i}, a_{-i}, s_{-i}) = \theta^r(\bar{s}_i, \bar{a}_i, s_{-i}, a_{-i})$ and $B_{(\bar{s}_i,\bar{a}_i)} : \Delta_{-i} \to \Omega$ by

$$B_{(\bar{s}_i,\bar{a}_i)}(s_{-i}, a_{-i}, s_{-i}) = \Theta(\bar{s}_i, \bar{a}_i, s_{-i}, a_{-i}).$$

Note that $\beta^r_{(\bar{s}_i,\bar{a}_i)}$ is a selection from $B_{(\bar{s}_i,\bar{a}_i)}$.

Correspondingly, two paragraphs later replace "$\beta^{rm} \bar{\sigma}^r_{-i} \to \xi$" with "$\beta^{rm}_{(s_i^{rm}, a_i^{rm})} \bar{\sigma}^r_{-i} \to \xi$." Finally, in the last paragraph on page 1737 replace "there is a selection β from B" with "there is a selection β from $B_{(s_i,a_i)}$."

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