SUPPLEMENT TO "MEASURING INEQUITY AVERSION IN A HETEROGENEOUS POPULATION USING EXPERIMENTAL DECISIONS AND SUBJECTIVE PROBABILITIES": APPENDIX

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THIS APPENDIX presents tables of results and supplementary material.

S1. EXPERIMENTAL DESIGN

TABLE S.I

EXPERIMENTAL DESIGN AND NUMBER OF OBSERVATIONS IN EACH TREATMENT[a]

$N = 1213$	Proposer		Responder	Total
Ultimatum game	377		335 (322 without inconsistent behavior)	712
	Belief framing			
	Acceptance	Rejection		
	195	182		
Dictator game	260		241	511

[a]We had to drop 50 observations because of missing information in their background characteristics. 147 persons declined to participate in the experiment.

S2. INCONSISTENT RESPONDERS' CHOICES

TABLE S.II

OBSERVED CHOICE SEQUENCES FOR INCONSISTENT RESPONDERS

(Inconsistent behavior; $N = 13$)[a]								
0	150	300	450	550	700	850	1000	N
0	1	0	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1
1	0	1	1	0	1	0	0	1
0	1	0	1	1	1	0	0	1
1	1	0	1	1	1	0	0	1
1	0	0	1	1	1	1	1	1
0	0	0	1	1	0	0	1	1
0	0	0	0	1	0	0	1	1
0	1	1	1	1	1	0	1	1
1	0	0	0	1	0	0	0	1
1	1	1	1	0	0	0	0	1
1	1	1	1	1	0	0	0	1
1	0	0	0	0	0	0	0	1

[a]Table columns present the acceptance decision (coded as 1 if accepted) for all 8 possible offers. N denotes the number of observations.

S3. ECONOMETRIC MODEL

This section presents some more details on the econometric model discussed in Section 3 of the main text and gives the likelihood contributions. To keep the appendix self-contained, part of it repeats in the main text.

S3.1. *Preferences*

We assume that subjects have preferences with possibly nonlinear asymmetric inequity aversion. The utility of subject i from payoffs y_{self} to him- or herself and y_{other} to the other player is given by

$$v_i = y_{\text{self}} - \alpha_{1i}\max\{y_{\text{other}} - y_{\text{self}}, 0\} - \alpha_{2i}\max\{y_{\text{other}} - y_{\text{self}}, 0\}^2 - \beta_{1i}\max\{y_{\text{self}} - y_{\text{other}}, 0\} - \beta_{2i}\max\{y_{\text{self}} - y_{\text{other}}, 0\}^2. \tag{S1}$$

We use the specifications

$$\begin{aligned}
\alpha_{1i} &= \exp(\mathbf{x}_i'\boldsymbol{\alpha}_1 + u_i^{\alpha}),\\
\beta_{1i} &= \exp(\mathbf{x}_i'\boldsymbol{\beta}_1 + u_i^{\beta}),\\
\alpha_{2i} &= \widetilde{\mathbf{x}}_i'\alpha_2,\\
\beta_{2i} &= \widetilde{\mathbf{x}}_i'\beta_2
\end{aligned}$$

where $\widetilde{\mathbf{x}}_i = [1, \text{Responder}_i]'$ is a vector consisting of the intercept and a dummy Responder_i taking a value of 1 for responders and 0 for proposers. This vector is combined with a person's observable characteristics in the vector $\mathbf{x}_i = [\widetilde{\mathbf{x}}_i', \overline{\mathbf{x}}_i']'$. The terms u_i^{α} and u_i^{β} reflect unobserved heterogeneity, assumed to be independent of error terms and of $\mathbf{x}_i$ with a bivariate normal distribution with means zero and arbitrary covariance matrix.

Decisions of Proposers in the Ultimatum Game

Each proposer had eight choices ($j = 1, \dots, 8$), involving own payoffs $y_{\text{self}}(1), \dots, y_{\text{self}}(8)$. We assume expected utility maximization, where proposer i uses the own subjective probability Q_{ij} that offer j will be accepted. Since utility is zero if the offer is rejected, the expected utility of offer j is given by $Q_{ij}v_{ij}$, where v_{ij} denotes person i's utility of payoffs $(y_{\text{self}}(j), 1000 - y_{\text{self}}(j))$ (cf. equation (S1)). The subjective expected utility of making an offer $y_{\text{self}}(j)$ is therefore given by

$$\begin{aligned}
Q_{ij}v_{ij} = Q_{ij}\big[&y_{\text{self}}(j) - \alpha_{1i}\max\{1000 - 2y_{\text{self}}(j), 0\}\\
&- \alpha_{2i}\max\{1000 - 2y_{\text{self}}(j), 0\}^2\\
&- \beta_{1i}\max\{2y_{\text{self}}(j) - 1000, 0\}\\
&- \beta_{2i}\max\{2y_{\text{self}}(j) - 1000, 0\}^2\big].
\end{aligned}$$

To allow for suboptimal behavior, we add idiosyncratic error terms ϵ_{ij} multiplied with a noise-to-signal ratio parameter λ_i. We assume that errors ϵ_{ij} are independent of each other and of other variables in the model (i.e. (u_i^α, u_i^β) and $\mathbf{x}_i$), and that the difference of any two ϵ_{ij} across options follows a logistic distribution. We assume that proposer i chooses the option j that maximizes $Q_{ij}v_{ij} + \lambda_i\epsilon_{ij}$.

Decisions of Responders in the Ultimatum Game

Responder i has to trade off the utility of accepting or rejecting each offer. The utility of rejecting is zero, and the responder utility v_{ij} of accepting offer j immediately follows from equation (S1):

$$\begin{aligned} v_{ij} = y_{\text{self}}(j) &- \alpha_{1i}\max\{1000 - 2y_{\text{self}}(j), 0\} \\ &- \alpha_{2i}\max\{1000 - 2y_{\text{self}}(j), 0\}^2 \\ &- \beta_{1i}\max\{2y_{\text{self}}(j) - 1000, 0\} - \beta_{2i}\max\{2y_{\text{self}}(j) - 1000, 0\}^2. \end{aligned}$$

A perfectly utility maximizing responder thus accepts offer j if and only if $v_{ij} > 0$.

Again, we assume the responder accepts offer j if $v_{ij} + \lambda_i\epsilon_{ij} > 0$, where $\lambda_i\epsilon_{ij}$ denote idiosyncratic error terms which are assumed to follow a logistic distribution.

The size of the noise parameter λ_i drives the likelihood of suboptimal choice. We will allow the noise parameter λ_i to vary with background characteristics and the responder's role by assuming that $\lambda_i = \exp(\mathbf{x}_i'\boldsymbol{\lambda})$.

Decisions of Proposers in the Dictator Game

To test the performance of our model, we generate out-of-sample predictions for the subsample of dictators, assuming their preferences and noise levels are the same as those of proposers in the ultimatum game. Thus we use the same specification as for proposers in the ultimatum game except that Q_{ij} is replaced by 1 for all j.

S3.2. *Expectations of Proposers in the Ultimatum Game*

The observed stated probabilities are assumed to be generated by the process

$$P_{ij}^* = \bar{\mathbf{x}}_i'\delta + \gamma_j + \phi_j F_i + u_i^P + \epsilon_{ij}^P,$$

$$P_{ij} = \begin{cases} 0, & \text{if } P_{ij}^* \leq 0, \\ P_{ij}^*, & \text{if } 0 < P_{ij}^* < 1, \\ 1, & \text{if } P_{ij}^* \geq 1. \end{cases}$$

The correct process generating proposer expectations is assumed to be

$$Q_{ij}^* = \overline{\mathbf{x}}_i'\delta + \gamma_j + u_i^P,$$
$$Q_{ij} = \begin{cases} 0, & \text{if } Q_{ij}^* \leq 0, \\ Q_{ij}^*, & \text{if } 0 < Q_{ij}^* < 1, \\ 1, & \text{if } Q_{ij}^* \geq 1. \end{cases}$$

We assume that the triplet $(u_i^\alpha, u_i^\beta, u_i^P)$ is distributed as a trivariate normal distribution with arbitrary covariance matrix, independent of background variables and other error terms in the model.

S3.3. *Likelihood Function*

Proposers in the Ultimatum Game

From the assumptions of logistic errors, the probability that proposer i chooses offer j, conditional on $(\mathbf{x}_i, u_i^\alpha, u_i^\beta)$, is given by

$$C_{ij}(\mathbf{x}_i, u_i^\alpha, u_i^\beta) = \frac{\exp(Q_{ij}v_{ij}/\lambda_i)}{\sum_{k=1}^{8}\exp(Q_{ij}v_{ik}/\lambda_i)}.$$

For each proposer i, we observe a sequence of subjective beliefs $\{P_{ij} : j = 1, \ldots, 8\}$, where b_{ij} denotes the probability placed on an offer j being accepted. Let $\mathbf{1}[A]$ denote the indicator function which takes a value of 1 when event A is true and 0 otherwise. The likelihood of the subjective beliefs of proposer i, conditional on $\mathbf{x}_i$ and u_i^P, is given by

$$B_i(\overline{\mathbf{x}}_i, F_i, u_i^P) = \prod_{j=1}^{8} \Pr(P_{ij} = 0|\overline{\mathbf{x}}_i, F_i, u_i^P)^{\mathbf{1}[P_{ij}=0]} h(P_{ij}|\overline{\mathbf{x}}_i, F_i, u_i^P)^{\mathbf{1}[0<P_{ij}<1]}$$
$$\times \Pr(P_{ij} = 1|\overline{\mathbf{x}}_i, F_i, u_i^P)^{\mathbf{1}[P_{ij}=1]},$$

where $h(\cdot)$ denotes the normal density over noncensored beliefs. The individual likelihood of proposer i who chooses offer j and who has a sequence of beliefs $\{P_{ij} : j = 1, \ldots, 8\}$, conditional on $\mathbf{x}_i$, is given by

$$L_i^P(\mathbf{x}_i) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} [C_{ij}(\mathbf{x}_i, u_i^\alpha, u_i^\beta) \cdot B_i(\overline{\mathbf{x}}_i, F_i, u_i^P)] \times f(u_i^\alpha, u_i^\beta, u_i^P)\, du_i^\alpha\, du_i^\beta\, du_i^P, \tag{S2}$$

where $f(u_i^\alpha, u_i^\beta, u_i^P)$ denotes the trivariate normal density with mean vector $\mathbf{0}$ and covariance matrix $\boldsymbol{\Omega}$.

Responders in the Ultimatum Game

From the assumption of logistic errors, the probability that responder i accepts offer j, conditional on $(\mathbf{x}_i, u_i^\alpha, u_i^\beta)$, is given by

$$R_{ij}(\mathbf{x}_i, u_i^\alpha, u_i^\beta) = \frac{1}{1+\exp(-v_{ij}/\lambda_i)}.$$

For responder i, we observe a sequence of eight decisions $\{d_{ij} : j = 1, \dots, 8\}$, where $d_{ij} = 1$ if responder i accepts offer j and 0 otherwise. The likelihood of the strategy sequence of responder i, conditional on $\mathbf{x}_i$, is given by

$$\text{(S3)} \qquad L_i^R(\mathbf{x}_i) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left[\prod_{j=1}^{8} [R_{ij}(\mathbf{x}_i, u_i^\alpha, u_i^\beta)]^{d_{ij}} [1 - R_{ij}(\mathbf{x}_i, u_i^\alpha, u_i^\beta)]^{1-d_{ij}} \right] \times g(u_i^\alpha, u_i^\beta)\, du_i^\alpha\, du_i^\beta,$$

where $g(\cdot, \cdot)$ denotes the bivariate normal distribution.

Sample Likelihood

Given N_P proposers and N_R responders in the ultimatum game, the sample log-likelihood is

$$\mathcal{L} = \sum_{i=1}^{N_P} \log(L_i^P(\mathbf{x}_i)) + \sum_{i=1}^{N_R} \log(L_i^R(\mathbf{x}_i)).$$

S3.4. *Approximation of Integrals by Simulation*

Estimation of the model parameters requires approximating the integrals in (S2) and (S3). In the case of (S2), this amounts to computing the expectation of the proposer likelihood with respect to the density function $f(u_i^\alpha, u_i^\beta, u_i^P)$, a trivariate normal distribution with vector mean $\mathbf{0}$ and covariance matrix $\boldsymbol{\Omega}$. Using a Choleski decomposition, we have that $\boldsymbol{\Omega} = \boldsymbol{\Gamma}\boldsymbol{\Gamma}'$, where $\boldsymbol{\Gamma}$ denotes a lower triangular matrix. Draws can be taken from the joint distribution $f(u_i^\alpha, u_i^\beta, u_i^P)$ using

$$\boldsymbol{\mu}_i = \boldsymbol{\Gamma}\boldsymbol{\varepsilon}_i,$$

where $\boldsymbol{\varepsilon}_i$ is a 3 by 1 vector of independent draws from a standard normal distribution and where $\boldsymbol{\mu}_i = [u_i^\alpha, u_i^\beta, u_i^P]'$. Given a sequence of draws $\{\boldsymbol{\varepsilon}_i^m : m = 1, \dots, M\}$, a sequence of draws from a trivariate normal distribution with variance–covariance matrix $\boldsymbol{\Omega}$ is obtained as $\{\boldsymbol{\mu}_i^m = \boldsymbol{\Gamma}\boldsymbol{\varepsilon}_i^m : m = 1, \dots, M\}$, where $\boldsymbol{\mu}_i^m = [u_i^{\alpha(m)}, u_i^{\beta(m)}, u_i^{P(m)}]'$. We generate a sequence of independent standard normal draws $\{\boldsymbol{\varepsilon}_i^m : m = 1, \ldots, M\}$ based on an underlying Halton sequence. These sequences are part of a list of variance reduction techniques. These techniques offer improved coverage of the domain of integration and result in lower simulation noise than traditional random number generators (see Train (2003) for details). The approximated likelihood of proposer i is given by

$$\widetilde{L}_i^P(\mathbf{x}_i) \approx \sum_{m=1}^{M} \big[C_{ij}(\mathbf{x}_i, u_i^{\alpha(m)}, u_i^{\beta(m)}) \cdot B_i(\mathbf{x}_i, u_i^{P(m)})\big].$$

In a similar way, the approximated likelihood of responder i is given by

$$\widetilde{L}_i^R(\mathbf{x}_i) \approx \sum_{m=1}^{M} \left[\prod_{j=1}^{8} \big[R_{ij}(\mathbf{x}_i, u_i^{\alpha(m)}, u_i^{\beta(m)})\big]^{d_{ij}} \times \big[1 - R_{ij}(\mathbf{x}_i, u_i^{\alpha(m)}, u_i^{\beta(m)})\big]^{1-d_{ij}} \right].$$

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REFERENCE

TRAIN, K. E. (2003): *Discrete Choice Methods With Simulation*. Cambridge, U.K.: Cambridge University Press.

TABLE S.III

MODEL FIT OF OFFER AND RESPONSE DISTRIBUTIONS USING THE MODEL WITH RATIONAL EXPECTATIONS[a]

	All		Low Age		High Age		Low Education		High Education		Dictators	
	1	2	3	4	5	6	7	8	9	10	11	12
	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted
Proposers												
0 CP	0.005	0.010	0.015	0.008	0.000	0.008	0.000	0.013	0.008	0.009	0.073	0.671
150 CP	0.005	0.022	0.015	0.123	0.000	0.007	0.000	0.015	0.000	0.024	0.015	0.000
300 CP	0.042	0.025	0.091	0.089	0.023	0.007	0.016	0.009	0.059	0.029	0.108	0.000
450 CP	0.393	0.392	0.500	0.425	0.282	0.329	0.398	0.398	0.378	0.398	0.511	0.020
550 CP	0.531	0.506	0.364	0.309	0.656	0.618	0.544	0.524	0.546	0.495	0.250	0.285
700 CP	0.013	0.026	0.000	0.025	0.023	0.018	0.041	0.022	0.000	0.025	0.019	0.015
850 CP	0.000	0.009	0.000	0.009	0.000	0.006	0.000	0.008	0.000	0.009	0.008	0.004
1000 CP	0.011	0.008	0.015	0.008	0.015	0.005	0.000	0.008	0.008	0.007	0.015	0.003
Sample size	377		66		131		123		119			
Responders												
0 CP	0.053	0.063	0.048	0.080	0.010	0.033	0.079	0.064	0.050	0.070		
150 CP	0.155	0.122	0.253	0.163	0.100	0.075	0.159	0.109	0.175	0.146		
300 CP	0.317	0.351	0.446	0.435	0.210	0.259	0.261	0.307	0.358	0.402		
450 CP	0.932	0.922	0.952	0.954	0.900	0.877	0.943	0.902	0.933	0.941		
550 CP	0.916	0.927	1.000	0.995	0.840	0.835	0.829	0.884	0.925	0.942		
700 CP	0.683	0.698	0.916	0.924	0.490	0.465	0.568	0.580	0.708	0.712		
850 CP	0.581	0.607	0.892	0.871	0.340	0.352	0.477	0.470	0.600	0.620		
1000 CP	0.549	0.571	0.867	0.847	0.310	0.308	0.443	0.421	0.567	0.585		
Sample size	322		83		100		88		120			

[a]"All" refers to overall fit; "Low Age" refers to subjects with less than 35 years of age; "High Age" refers to subjects with above 54 years of age; "Low Education" refers to subjects with either a primary or general secondary education; "High Education" refers to individuals with either university training or high vocational training.

TABLE S.IV

PARAMETER ESTIMATES OF THE STRUCTURAL MODEL WITH RATIONAL EXPECTATIONS[a]

	α_1	β_1	λ	Random Components	
Constant	−0.999	0.080	4.235	$V(u^\alpha)$	0.743***
	(1.416)	(0.459)	(0.448)		(0.107)
Responders	1.094	−0.414	−0.645	$V(u^\beta)$	0.766***
	(1.414)	(0.423)	(0.683)		(0.179)
Male	0.021	0.176*	0.359**	$\text{Corr}(u^\alpha, u^\beta)$	0.243***
	(0.142)	(0.103)	(0.163)		(0.041)
Education					
Medium	0.038	−0.305**	0.229	$\alpha_2^{\text{proposers}}$	−0.379
	(0.146)	(0.111)	(0.224)		(0.246)
High	−0.231	−0.254**	0.112	$\alpha_2^{\text{responders}}$	−0.279
	(0.145)	(0.126)	(0.267)		(0.350)
Age					
Medium	0.162	0.769***	−0.196	$\beta_2^{\text{proposers}}$	−2.331
	(0.148)	(0.162)	(0.184)		(1.358)
High	0.253	1.126***	0.245	$\beta_2^{\text{responders}}$	−0.227
	(0.197)	(0.208)	(0.304)		(1.914)
Income					
Medium	−0.264	−0.128	−0.170		
	(0.217)	(0.161)	(0.258)		
High	−0.228	−0.360**	−0.229		
	(0.256)	(0.176)	(0.242)		
Paid work	0.218	0.002	0.038		
	(0.192)	(0.142)	(0.261)		
House work	−0.081	−0.363*	−0.356		
	(0.251)	(0.197)	(0.303)		
Retired	0.581**	0.385**	−0.357		
	(0.273)	(0.196)	(0.321)		

[a]Standard errors are given in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

TABLE S.V

PREDICTED AVERAGE PREFERENCE PARAMETERS FOR PROPOSERS AND RESPONDERS[a]

	α_1		α_2		β_1		β_2	
	Subj.	Obj.	Subj.	Obj.	Subj.	Obj.	Subj.	Obj.
Proposers	0.297	0.642	−0.515	−0.379	1.314	3.117	0.221	−2.331
	(0.078)	(0.179)	—	—	(0.648)	(1.673)	—	—
Responders	1.587	1.827	−0.277	−0.279	1.742	1.901	−0.289	−0.277
	(0.442)	(0.500)	—	—	(0.942)	(1.072)	—	—

[a]Predicted average preference parameters for proposers and responders using the model incorporating subjective expectations and the model which assumes that proposers have expectations which coincide with the objective acceptance probabilities. Standard deviations of the predictions are in parentheses. Predictions for α_2 and β_2 correspond to the point estimates for proposers and responders, respectively, and do not vary for a given player type.