

SUPPLEMENT TO “WHAT’S NEWS IN BUSINESS CYCLES”
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THIS APPENDIX gathers Supplemental Material to Schmitt-Grohé and Uribe (2012).

A.1. TRUE IMPULSE RESPONSES OF THE OBSERVABLES IN THE
 EXAMPLE ECONOMY

See Figure A.1 for impulse responses.

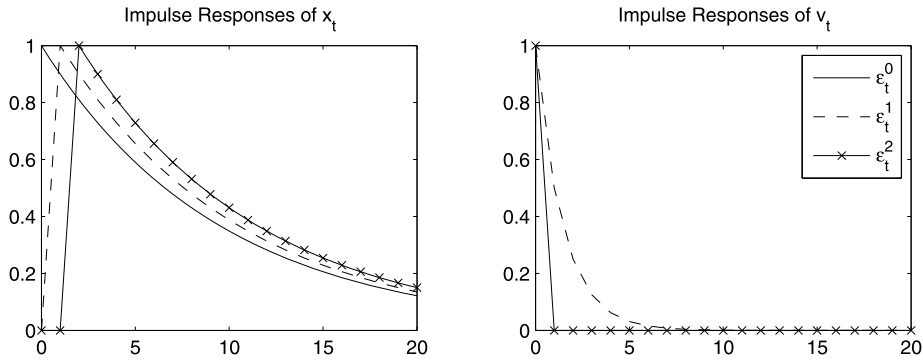


FIGURE A.1.—True impulse responses of the observables in the example economy.

A.2. IDENTIFIABILITY IN THE EXAMPLE ECONOMY: THE ISKREV TEST

The example economy of Section 2 in Schmitt-Grohé and Uribe (2012) can be written in vector form as

$$\begin{aligned}
 X_{t+1} &= h_x X_t + \eta v_{t+1}, \\
 Y_t &= g_x X_t,
 \end{aligned}$$

where $X_t = [y_{t-1} \ x_t \ \varepsilon_t^1 \ \varepsilon_t^2 \ \varepsilon_{t-1}^2]'$, $Y_t = [x_t \ v_t]'$, v_t is a 3×1 vector of i.i.d. shocks with variance/covariance matrix equal to the identity, and

$$h_x = \begin{bmatrix} \rho_y & 0 & 1 & 0 & 0 \\ 0 & \rho_x & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$g_x = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ \rho_y & 0 & 1 & 1 & 0 \end{bmatrix},$$

and

$$\eta = \begin{bmatrix} 0 & 0 & 0 \\ \sigma_0 & 0 & 0 \\ 0 & \sigma_1 & 0 \\ 0 & 0 & \sigma_2 \\ 0 & 0 & 0 \end{bmatrix}.$$

Iskrev's (2010) test consists in checking whether the derivative of the autocovariogram of Y_t with respect to the vector of estimated parameters, which we denote by $\theta \equiv [\sigma_0^2 \ \sigma_1^2 \ \sigma_2^2]$, has full column rank. It turns out that, in our example economy, it suffices to examine the derivative of the autocovariogram of orders 0, 1, and 2. This derivative is given by

$$\begin{bmatrix} \frac{\partial \text{vech} EY_t Y_t'}{\partial \theta} \\ \frac{\partial \text{vec} EY_t Y_{t+1}'}{\partial \theta} \\ \frac{\partial \text{vec} EY_t Y_{t+2}'}{\partial \theta} \end{bmatrix} = \begin{bmatrix} -1/(\rho_x^2 - 1) & -1/(\rho_x^2 - 1) & -1/(\rho_x^2 - 1) \\ 0 & -\rho_y/(\rho_x \rho_y \rho_y - 1) & 0 \\ 0 & 1 - \rho_y^2/(\rho_y^2 - 1) & 1 \\ -\rho_x/(\rho_x^2 - 1) & -\rho_x/(\rho_x^2 - 1) & -\rho_x/(\rho_x^2 - 1) \\ 0 & 1 - (\rho_x \rho_y \rho_y)/(\rho_x \rho_y \rho_y - 1) & 0 \\ 0 & -\rho_y^2/(\rho_x \rho_y \rho_y - 1) & 0 \\ 0 & \rho_y - \rho_y^3/(\rho_y^2 - 1) & 0 \\ -\rho_x^2/(\rho_x^2 - 1) & -\rho_x^2/(\rho_x^2 - 1) & -\rho_x^2/(\rho_x^2 - 1) \\ 0 & \rho_x - (\rho_x^2 \rho_y \rho_y)/(\rho_x \rho_y \rho_y - 1) & 1 \\ 0 & -\rho_y^3/(\rho_x \rho_y \rho_y - 1) & 0 \\ 0 & \rho_y^2 - \rho_y^4/(\rho_y^2 - 1) & 0 \end{bmatrix}.$$

To see that this matrix has full column rank, consider first the case $\rho_y = 0$. In this case, rows 1, 3, and 9 of this matrix form a square matrix of order 3 whose determinant equals $1/(1 + \rho_x)$, which is always different from zero. Next, consider the case $\rho_y \neq 0$. In this case, rows 1, 2, and 3 form a square matrix of order 3 whose determinant equals $\rho_y/[(\rho_x^2 - 1)(\rho_x \rho_y - 1)]$, which is always nonzero.

A.3. TECHNICAL NOTES ON APPLYING ISKREV'S TEST TO THE BASELINE DSGE MODEL

Implementing Iskrev's test consists in checking whether the derivative of the predicted autocovariogram of the vector of observables with respect to the vector of estimated parameters has a rank equal to the length of the vector of estimated parameters. Formally, let

$$m(t) \equiv \frac{\partial \text{vec} E(d_t d_0')}{\partial \theta},$$

for $t = 0, \dots, T - 1$, where d_t is the theoretical counterpart of the vector of observables used to estimate the model, θ is a vector of model parameters whose identifiability the test establishes, and T is the sample size. Let

$$M \equiv \begin{bmatrix} m(0) \\ \vdots \\ m(T - 1) \end{bmatrix}.$$

Then the estimated parameter θ is identifiable if M has full column rank.¹

Using the notation in [Schmitt-Grohé and Uribe \(2004\)](#), we can write the solution of the DSGE model up to first order as

$$y_t = g_x x_t$$

and

$$x_{t+1} = h_x x_t + \eta \varepsilon_{t+1},$$

where y_t is a vector of endogenous controls, x_t is a vector of endogenous and exogenous states, and ε_{t+1} is a white noise vector with identity variance/covariance matrix. The elements of the vector of observables are a subset of the elements of the vector of endogenous controls. The two vectors are related by an expression of the form

$$d_t = D y_t,$$

where D is a selection matrix with one unit element per row and at most one unit element per column and the remaining elements equal to zero. This relation implies that

$$\text{vec}(E d_t d_0') = (D \otimes D) \text{vec}(E y_t y_0'),$$

¹The test is performed by our Matlab code `iskrev_test.m`.

and therefore

$$m(t) = (D \otimes D) \frac{\partial \text{vec } E(y_t, y'_0)}{\partial \theta}.$$

Given the structure of the solution of the linearized DSGE model, we can write

$$E(y_t, y'_0) = g_x h'_x \Sigma_x g'_x,$$

where $\Sigma_x \equiv E x_t x'_t$ and satisfies

$$\Sigma_x = h_x \Sigma_x h'_x + \eta \eta'.$$

Taking the derivative of $\text{vec } E(y_t, y'_0)$ with respect to θ , we obtain

$$\begin{aligned} \frac{\partial \text{vec}(g_x h'_x \Sigma_x g'_x)}{\partial \theta} &= (I_y \otimes g_x h'_x \Sigma_x) dg'_x + (g_x \otimes g_x h'_x) d\Sigma_x \\ &\quad + (g_x \Sigma_x \otimes g_x) d(h'_x) + (g_x \Sigma_x h'_x \otimes I_y) dg_x. \end{aligned}$$

In this expression, the object dg_x denotes $\partial \text{vec}(g_x) / \partial \theta$, and is a matrix of order $n_y n_x \times n_\theta$, where n_y , n_x , and n_θ are the lengths of y_t , x_t , and θ , respectively. Similar notation applies to other objects.

A.3.1. Deriving dg_x and dh_x

Up to first order, the reduced form of the DSGE model can be written as

$$[f_{y'} \quad f_{x'}] E_t \begin{bmatrix} y_{t+1} \\ x_{t+1} \end{bmatrix} = -[f_y \quad f_x] \begin{bmatrix} y_t \\ x_t \end{bmatrix}.$$

Using the solution to the linearized model in the linearized equilibrium conditions, we obtain

$$[f_{y'} g_x h_x \quad f_{x'} h_x] \begin{bmatrix} x_t \\ x_t \end{bmatrix} = -[f_y g_x \quad f_x] \begin{bmatrix} x_t \\ x_t \end{bmatrix},$$

which implies that

$$f_{y'} g_x h_x + f_{x'} h_x = -f_y g_x - f_x.$$

Taking derivative with respect to θ , we obtain

$$\begin{aligned} (I_x \otimes f_{y'} g_x) dh_x + (h'_x \otimes f_{y'}) dg_x + (h'_x g'_x \otimes I_n) df_{y'} \\ + (I_x \otimes f_{x'}) dh_x + (h'_x \otimes I_n) df_{x'} \\ = -(I_x \otimes f_y) dg_x - (g'_x \otimes I_n) df_y - df_x. \end{aligned}$$

Let

$$A \equiv (h'_x \otimes f_y) + (I_x \otimes f_y),$$

$$B \equiv (I_x \otimes f_y g_x) + (I_x \otimes f_{x'}),$$

and

$$C \equiv -(h'_x g'_x \otimes I_n) df_y - (h'_x \otimes I_n) df_{x'} - (g'_x \otimes I_n) df_y - df_x.$$

Then, we can write

$$\begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} dg_x \\ dh_x \end{bmatrix} = C,$$

which can be solved to obtain

$$\begin{bmatrix} dg_x \\ dh_x \end{bmatrix} = \begin{bmatrix} A & B \end{bmatrix}^{-1} C.$$

We now explain how to obtain the objects df_x , $df_{x'}$, df_y , and $df_{y'}$. We explain in detail how to obtain df_x ; the other derivations are identical. We view f_x as a function of the parameter vector θ and of the vector $z(\theta) \equiv [y(\theta)' \ x(\theta)']'$, which is the steady state of the vector $[y'_t \ x'_t]'$. Thus, we write $f_x(\theta, z(\theta))$. Then, we have²

$$df_x = \frac{\partial f_x}{\partial \theta} + \frac{\partial f_x}{\partial z} \frac{\partial z(\theta)}{\partial \theta}.$$

Now, we explain how to obtain $\frac{\partial z(\theta)}{\partial \theta}$. We can write the steady state of the model as $f(\theta, z) = 0$, which implicitly defines $z(\theta)$. Differentiating, we get

$$\frac{\partial f(\theta, z(\theta))}{\partial \theta} + \frac{\partial f(\theta, z(\theta))}{\partial z} \frac{\partial z(\theta)}{\partial \theta} = 0,$$

which can be solved to obtain³

$$\frac{\partial z(\theta)}{\partial \theta} = - \left[\frac{\partial f(\theta, z(\theta))}{\partial z} \right]^{-1} \frac{\partial f(\theta, z(\theta))}{\partial \theta}.$$

²The objects $\frac{\partial f_x}{\partial \theta}$ and $\frac{\partial f_x}{\partial z}$ are produced analytically by our Matlab code `iskrev_anal_deriv.m`. To facilitate the numerical evaluation of these symbolic expressions, the code writes these derivatives to a Matlab script file called `filename_iskrev_anal_deriv.m`, where the prefix `filename` is an input of `iskrev_anal_deriv.m` chosen by the user.

³The code `iskrev_anal_deriv.m` writes this formula into the Matlab script `filename_iskrev_anal_deriv.m`.

Deriving dg'_x and dh'_x

Let R_h be a matrix such that

$$\text{vec}(h'_x) = R_h \text{vec}(h_x).$$

The matrix R_h is a permutation matrix of order n_x^2 . Its unitary elements are located in row i column $\text{fix}((i-1)/n_x) + 1 + \text{rem}(i-1, n_x)n_x$, for $i = 1, \dots, n_x^2$. Then we have that

$$dh'_x = R_h dh_x.$$

Similarly, we can deduce that

$$dg'_x = R_g dg_x,$$

where the matrix R_g is a permutation matrix (i.e., a square matrix with only one element equal to unity per row and per column and all remaining elements equal to zero) of order $n_x n_y$. Its unitary elements are located in row i column $\text{fix}((i-1)/n_x) + 1 + \text{rem}(i-1, n_x)n_y$, for $i = 1, \dots, n_x n_y$.

A.3.2. Deriving $d\Sigma_x$

Using the expression for Σ_x obtained above, we can write its derivative with respect to θ as

$$d\Sigma_x = (h_x \otimes h_x) d\Sigma_x + (h_x \Sigma_x \otimes I_x) dh_x + (I_x \otimes h_x \Sigma_x) dh'_x + d(\eta\eta').$$

Solving for $d\Sigma_x$, we obtain⁴

$$\begin{aligned} d\Sigma_x &= [I_{n_x^2} - (h_x \otimes h_x)]^{-1} \\ &\quad \times [(h_x \Sigma_x \otimes I_x) dh_x + (I_x \otimes h_x \Sigma_x) dh'_x + d(\eta\eta')]. \end{aligned}$$

A.3.3. Deriving dh_x^t

For $t = 1$, it is dh_x , which we already derived. For $t \geq 2$, we proceed iteratively, noticing that $h_x^t = h_x^{t-1} h_x$, whose derivative is given by

$$dh_x^t = (I_x \otimes h_x^{t-1}) dh_x + (h_x^{t-1} \otimes I_x) dh_x^{t-1}.$$

⁴The object $d(\eta\eta')$ is produced symbolically by `iskrev_anal_deriv.m` and then written to the script file `filename_iskrev_anal_deriv.m`.

A.3.4. What if M Is Not Full Column Rank

Suppose M is less than full column rank at a parameter value θ_0 . Then, we conclude that, with the selected observables and sample size, the parameter θ is not identifiable in the vicinity of θ_0 . This essentially means that, in this case, there will be an infinite number of parameter vectors θ that will give rise to the same autocovariogram as θ_0 . When θ is not identifiable, we can establish what linear combinations of the elements of θ will deliver the same autocovariogram as θ_0 .

Let $V(\theta, T)$ be the vectorized covariogram of the vector of observables, d_t , of order T . That is,

$$V(\theta) = \begin{bmatrix} \text{vech}(Ed_0d_0') \\ \vdots \\ \text{vec}(Ed_{T-1}d_0') \end{bmatrix}.$$

Then, Taylor-expanding around θ_0 up to first order, we obtain

$$V(\theta) \approx V(\theta_0) + M(\theta_0)(\theta - \theta_0).$$

If $M(\theta_0)$ has full column rank, then $V(\theta) = V(\theta_0)$ if and only if $\theta = \theta_0$ in the neighborhood of θ_0 . If, on the other hand, $M(\theta_0)$ is rank deficient, then there exists an infinite number of vectors θ in the vicinity of θ_0 satisfying $V(\theta) = V(\theta_0)$. To obtain these vectors, perform a singular value decomposition of $M(\theta_0)'$. That is, find matrices U , S , and V such that

$$M(\theta)U = VS',$$

where U and V are unitary (i.e., $UU' = I$ and $VV' = I$) and S is diagonal with its diagonal elements nonnegative and decreasing. The matrix S has as many rows as $M(\theta)$ and as many columns as the length of θ . Now partition the matrix U as $[U^1 \ U^2]$, where U^2 has as many columns as S has zero diagonal elements. Then, we have that any vector θ of the form

$$\theta = \theta_0 + u^2\alpha$$

delivers the same autocovariogram as θ_0 for any (small) scalar α and any vector u^2 taken from the columns of U^2 .

A.4. AUTOREGRESSIVE REPRESENTATION OF ANTICIPATED SHOCKS

The law of motion of the exogenous process x_t can be written recursively as a first-order linear stochastic difference equation of the form

$$\tilde{x}_{t+1} = M\tilde{x}_t + \eta\nu_{x,t+1},$$

where $\tilde{x}_t = [\ln(x_t/x) \ \varepsilon_{x,t}^4 \ \varepsilon_{x,t-1}^4 \ \varepsilon_{x,t-2}^4 \ \varepsilon_{x,t-3}^4 \ \varepsilon_{x,t}^8 \ \varepsilon_{x,t-1}^8 \ \varepsilon_{x,t-2}^8 \ \varepsilon_{x,t-3}^8 \ \varepsilon_{x,t-4}^8 \ \varepsilon_{x,t-5}^8 \ \varepsilon_{x,t-6}^8 \ \varepsilon_{x,t-7}^8]$,

$$M = \begin{bmatrix} \rho & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix};$$

$$\eta = \begin{bmatrix} \sigma_x^0 & 0 & 0 \\ 0 & \sigma_x^4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_x^8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \text{and} \quad \nu_{x,t} = \begin{bmatrix} \nu_{x,t}^0 \\ \nu_{x,t}^4 \\ \nu_{x,t}^8 \end{bmatrix}.$$

The vector of innovations $\nu_{x,t}$ is normal i.i.d. with mean zero and variance-covariance matrix equal to the identity matrix. An alternative, but equivalent, specification, which was suggested to us by an anonymous referee, is given by

$$\begin{aligned} \ln(x_t/x) &= \rho_x \ln(x_{t-1}/x) + \nu_{x,t}^0 + \varepsilon_{t-1}^1, \\ \varepsilon_t^1 &= \varepsilon_{t-1}^2, \\ \varepsilon_t^2 &= \varepsilon_{t-1}^3, \\ \varepsilon_t^3 &= \varepsilon_{t-1}^4, \\ \varepsilon_t^4 &= \varepsilon_{t-1}^5 + \nu_{x,t}^4, \\ \varepsilon_t^5 &= \varepsilon_{t-1}^6, \end{aligned}$$

$$\begin{aligned}\varepsilon_t^6 &= \varepsilon_{t-1}^7, \\ \varepsilon_t^7 &= \varepsilon_{t-1}^8, \\ \varepsilon_t^8 &= \nu_{x,t}^8.\end{aligned}$$

An advantage of this recursive representation is that it involves only as many state variables as the longest anticipation horizon, which in our case is eight. A further advantage is that, if one were to consider, in addition to shocks anticipated 4 and 8 quarters, shocks anticipated 1, 2, 3, 5, 6, and 7 quarters, then the number of state variables would not change. One would simply add innovations $\nu_{x,t}^1$, $\nu_{x,t}^2$, $\nu_{x,t}^3$, $\nu_{x,t}^5$, $\nu_{x,t}^6$, and $\nu_{x,t}^7$, respectively.

A.5. ESTIMATED SOURCES OF UNCERTAINTY

Table A.I addresses a standard question in business-cycle analysis, namely, what is the contribution of the different sources of uncertainty considered in this study to explaining business-cycle fluctuations? We group the sources of uncertainty into three categories: technology shocks, aggregate demand shocks, and wage-markup shocks. Technology shocks consist of stationary neu-

TABLE A.I
SHARE OF UNCONDITIONAL VARIANCE EXPLAINED BY TECHNOLOGY, DEMAND, AND WAGE-MARKUP SHOCKS^a

	Posterior Shares				Prior Shares			
	<i>Y</i>	<i>C</i>	<i>I</i>	<i>h</i>	<i>Y</i>	<i>C</i>	<i>I</i>	<i>h</i>
Technology Shocks								
Mean	58	17	86	25	63	46	84	60
Median	58	17	86	25	68	44	93	67
5th percentile	52	11	83	19	14	5	38	8
95th percentile	64	24	90	30	97	93	100	97
Demand Shocks								
Mean	25	65	2	7	36	53	16	29
Median	25	65	2	7	30	55	7	20
5th percentile	21	56	1	6	2	6	0	1
95th percentile	29	73	3	9	86	95	61	85
Wage-Markup Shocks								
Mean	18	18	12	68	1	1	0	10
Median	17	18	12	68	0	0	0	4
5th percentile	14	13	9	62	0	0	0	0
95th percentile	22	24	15	75	5	4	1	44

^a*Y*, *C*, *I*, and *h* refer, respectively, to the growth rates of output, consumption, investment, and hours. Estimates are based on 500,000 draws from the posterior distribution. Technology shocks are e_t^i , for $i = 0, 4, 8$ and $j = z, x, z^I, a$. Demand shocks are e_{jt}^i , for $i = 0, 4, 8$ and $j = g, \zeta$. Wage-markup shocks are $e_{\mu,t}^i$, for $i = 0, 4, 8$.

tral productivity shocks, z_t , permanent neutral productivity shocks, X_t , stationary investment-specific productivity shocks, z_t^I , and permanent investment-specific productivity shocks, A_t . Aggregate demand shocks consist of government spending shocks, g_t , and preference shocks, ζ_t . Table A.I presents the share of the overall predicted variance of the variables of interest attributed to each of the three categories of shocks. It shows that the majority of the variances of output and investment are accounted for by technology shocks. Consumption is explained mostly by aggregate demand shocks, and hours are driven to a large extent by wage-markup shocks. These findings are in line with variance decompositions reported in related studies. For instance, Justiniano, Primiceri, and Tambalotti (2008, Table 4) reported that technology shocks account for 71 percent of variations in output and 92 percent of variations in investment. At the same time, these authors found that the majority of fluctuations in consumption and hours are accounted for by, respectively, aggregate demand shocks (57 percent), and markup shocks (70 percent).

Among the anticipated sources of uncertainty, the most relevant is ε_μ^4 , the four-quarter anticipated innovation in wage markups (see Table III in the main paper). This disturbance may reflect the macroeconomic effects of anticipated news regarding protracted wage negotiations of major labor unions. The reason this shock is favored by our data sample is that it helps account for the observed regularity that output and the main components of aggregate demand (consumption and investment spending) all lead employment. Figure A.2 displays the correlations of output, consumption, and investment with current and future values of hours. All of these cross correlations are positive, indicating that employment lags the other macroeconomic indicators. The figure also shows the predictions of our estimated DSGE model for these cross correlations.⁵ In addition, the figure displays the predicted cross correlations when the variance of the shock ε_μ^4 is set to zero. Clearly, the anticipated wage-markup shock contributes to making output, consumption, and investment leading indicators of employment.

Figure A.3 provides a flavor for why this is the case and for why the unanticipated component of wage markups does not play this role. The figure displays the impulse responses of output, consumption, investment, and hours to a four-quarter anticipated increase in the wage markup. In response to this anticipated negative cost-push shock, firms immediately cut investment spending and capital utilization, and, likewise, households cut consumption spending. Hours, however, are relatively little changed after the announcement and prior to the materialization of the shock. This is because the estimated wealth effect of labor supply (captured by the parameter γ) is virtually nil. Hours fall significantly but only four quarters after the announcement. As a result, the reactions of output, consumption, and investment all precede that of employment.

⁵The DSGE model is parameterized at the posterior median of the parameter estimate.

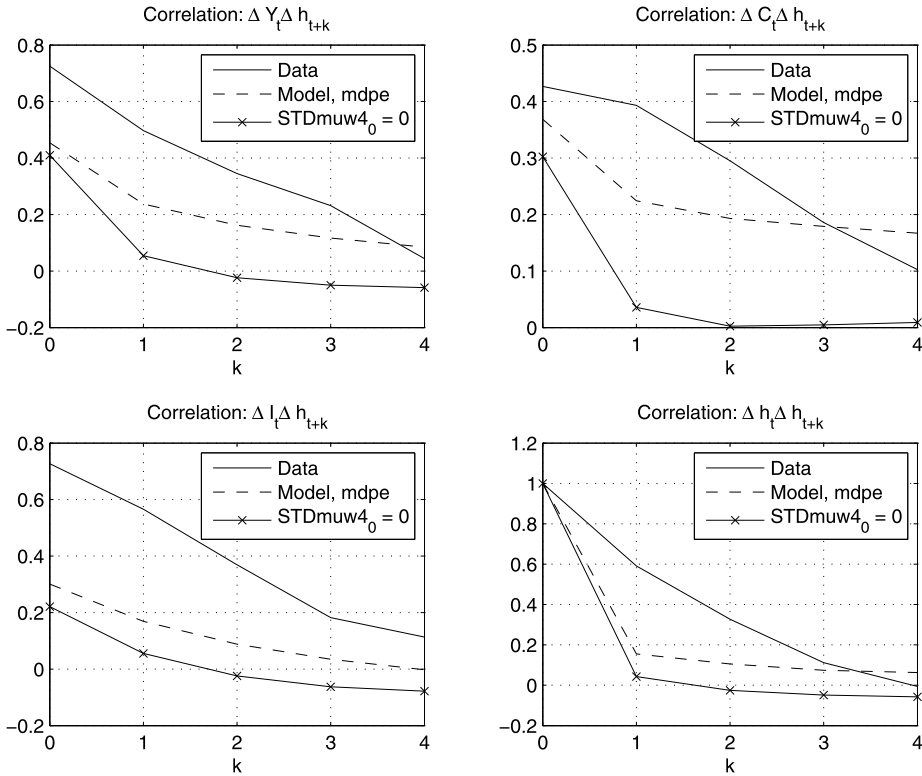


FIGURE A.2.—The cross correlogram of hours.

We note in addition that, as is clear from Figure A.2, the four-quarter anticipated wage-markup shock helps explain the observed positive autocorrelation in employment growth. When we shut the four-quarter anticipated wage-markup shock, the model predicts virtually no autocorrelation in employment growth.

A.6. ANTICIPATED SHOCKS IN THE FREQUENCY DOMAIN

In Schmitt-Grohé and Uribe (2012), we analyzed the role of anticipated shocks using the time domain. Here, we conduct a brief exploration of the significance of anticipated shocks from the perspective of the frequency domain. Figure A.4 displays the population spectra of the anticipated component (solid lines) and the unanticipated components (broken lines) of output growth, hours growth, consumption growth, and investment growth. The population spectra were computed at the posterior mean of the vector of estimated parameters. Business-cycle frequencies, defined as 8 to 32 quarters, are marked by two dotted vertical lines. The fact that the spectra associated with the antic-

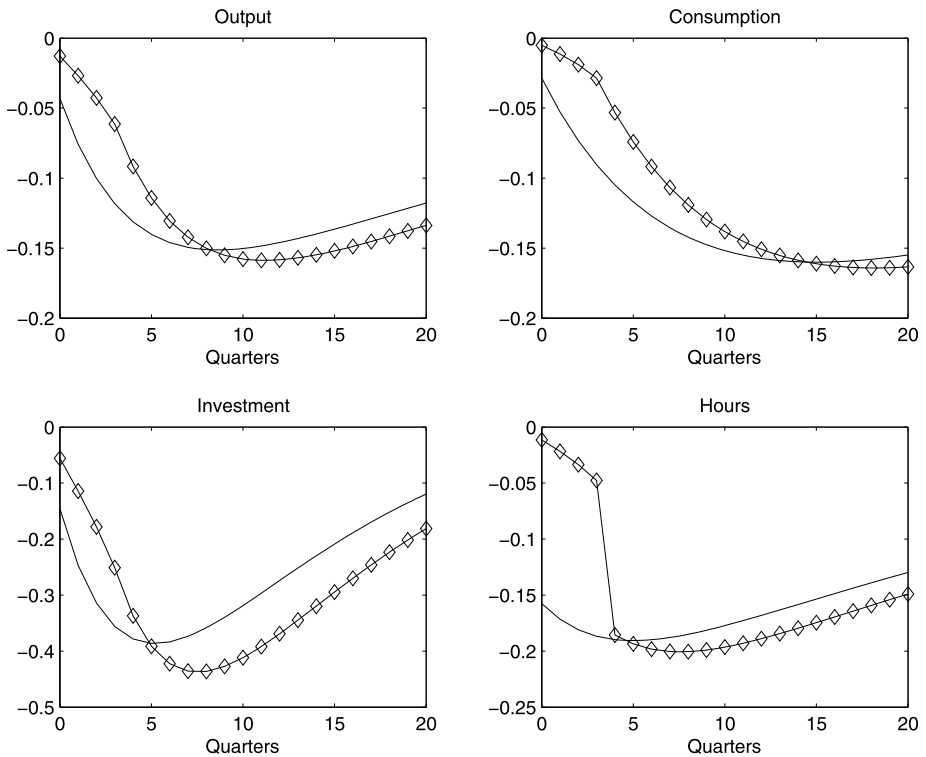


FIGURE A.3.—Impulse responses to an increase in the wage markup. —: Unanticipated. —◇—◇: Four-quarter anticipated.

ipated and the unanticipated components have different shapes suggests that these two components play distinct roles in explaining business cycles. For instance, for hours worked, the spectrum of the anticipated component is downward sloping, whereas the spectrum of the unanticipated component is upward sloping. This means that anticipated shocks are estimated to be relatively more important at the lower range of business-cycle frequencies. In the case of output and investment, even though the spectra of both the anticipated and the unanticipated components are downward sloping, the one associated with the anticipated component has more density around the lower end of the business-cycle spectrum, indicating again that anticipated shocks are relatively more important in explaining lower frequency business-cycle movements. Finally, the consumption spectrum shows that movements in consumption at business-cycle frequencies are accounted for by anticipated and unanticipated shocks in equal parts.

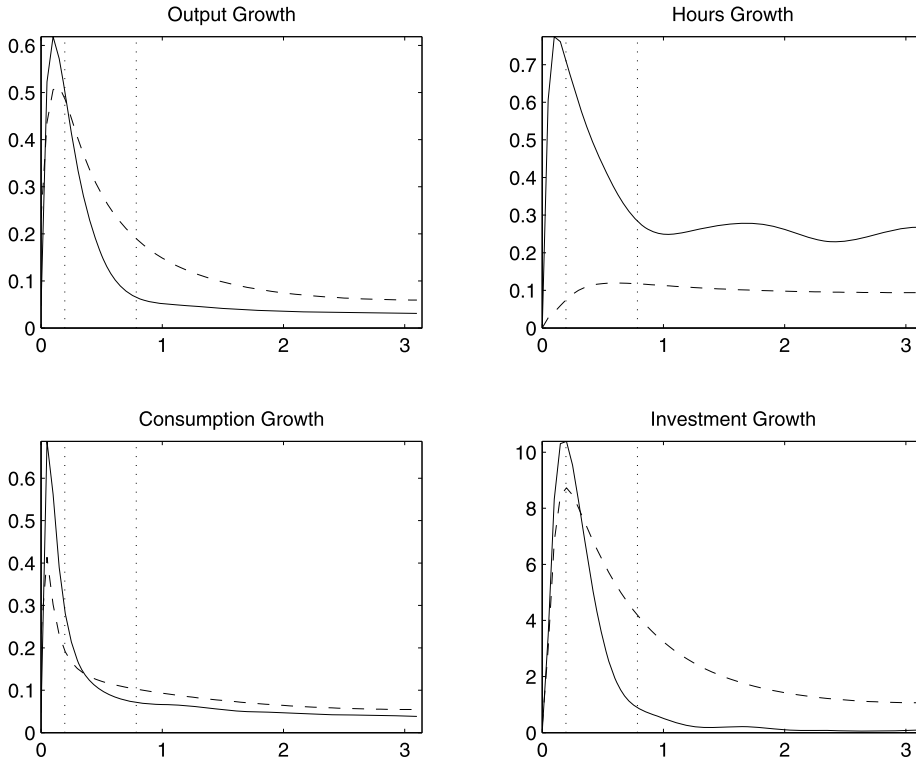


FIGURE A.4.—The population spectrum. —: Anticipated component. ---: Unanticipated component. Computed at the mean of the posterior distribution. The two vertical dotted lines mark frequencies between 8 and 32 quarters.

A.7. A PARSIMONIOUS SHOCK SPECIFICATION

In this subsection, we address two potential issues regarding the shock structure and observability assumptions maintained thus far. In regard to the shock structure of the model analyzed in previous sections, a potential concern is that it contains a number of nonstructural and ad hoc sources of uncertainty. Among these are the preference shock, ζ_t , the wage-markup shock, μ_t , and the shock shifting the law of motion of the capital stock, z_t^l . Although these shocks are customarily included in estimated medium-scale DSGE models, it is of interest to ascertain whether the importance of anticipated shocks is robust to omitting them. For this reason, in this section, we estimate a special case of our model in which we set

$$\sigma_k^i = 0,$$

for $k = z^l, \zeta, \mu$ and $i = 0, 4, 8$.

A second potential concern with our baseline estimation is the inclusion of total factor productivity as an observable variable. In particular, the construction of an empirical measure of TFP requires the use of data on the capital stock, which, as is well known, is difficult to measure accurately. Consequently, in this section, we omit TFP from the set of observables.

We estimate the resulting parsimonious version of the model using Bayesian methods. For the parameters that are estimated, we impose priors identical to those used in the baseline estimation. In line with the findings of the extensive literature devoted to fitting DSGE models to quarterly postwar data, the exclusion of the nonstructural shocks results in a weakening of the model's ability to fit the data. The central question for our purposes, however, concerns the predictions of the estimated model regarding the importance of anticipated disturbances. The estimated parsimonious model predicts that about two thirds of the variances of output, consumption, investment, and hours is accounted for by anticipated shocks. The exact shares are 0.68, 0.68, 0.69, and 0.69, respectively. It follows that our central result, namely, that anticipated shocks are important drivers of business cycles, is robust to doing away with the set of nonstructural or ad hoc shocks that are customarily used to fit medium-scale macroeconomic models to the data. The facts that the model allows for fewer sources of uncertainty and that the set of observables excludes TFP naturally result in a significant increase in the importance of TFP shocks. Indeed, more than 90 percent of the volatility of output growth is now explained by stationary and nonstationary neutral productivity shocks. Of the two types of TFP shocks, nonstationary neutral TFP shocks are the single most important source of fluctuations, explaining about 70 percent of the volatility of output growth. Further, the single most important component of nonstationary TFP shocks are eight-quarter anticipated innovations, which alone explain almost 50 percent of movements in output, consumption, investment, and hours. This result is in line with the findings of [Beaudry and Portier \(2006\)](#) obtained in the context of an empirical VAR model. In the next section, we explore further the connection between the predictions of our estimated parsimonious DSGE model and those stemming from empirical VAR models.

A.8. RELATING MODEL-BASED TO VAR-BASED ESTIMATES OF ANTICIPATED SHOCKS

Beaudry and Portier (2006; hereafter, BP) estimated the importance of anticipated permanent TFP shocks using an empirical vector error correction model (VECM). Their identification strategy was designed to uncover anticipated permanent changes in total factor productivity. Specifically, these authors imposed two conditions for an innovation in TFP growth to be an anticipated shock: First, the shock must affect TFP in the long run (we refer to this restriction as the long-run identification scheme), and second, the shock cannot affect TFP contemporaneously (we refer to this restriction as the short-run

identification scheme). The shocks that satisfy both BP identification schemes in our model are the anticipated components of the nonstationary neutral productivity shock, that is, $\varepsilon_{x,t}^4$ and $\varepsilon_{x,t}^8$. We note, however, that our DSGE model does not have a VAR representation of the type considered in BP. One reason for the lack of a BP-style VAR representation is that the number of innovations we consider is larger than the number of observables included in the VARs considered by BP. It follows that the shocks identified by the BP methodology cannot be interpreted as $\varepsilon_{x,t}^4$, or $\varepsilon_{x,t}^8$, or a combination thereof. We therefore interpret the BP empirical results as a particular filtering of the data that can be compared to a similar filtering performed on artificial data generated by our theoretical model.

Figure A.5 displays impulse responses of adjusted TFP—that is, $z_t X_t^{1-\alpha k}$ —and the value of the firm applying the Beaudry–Portier long- and short-run

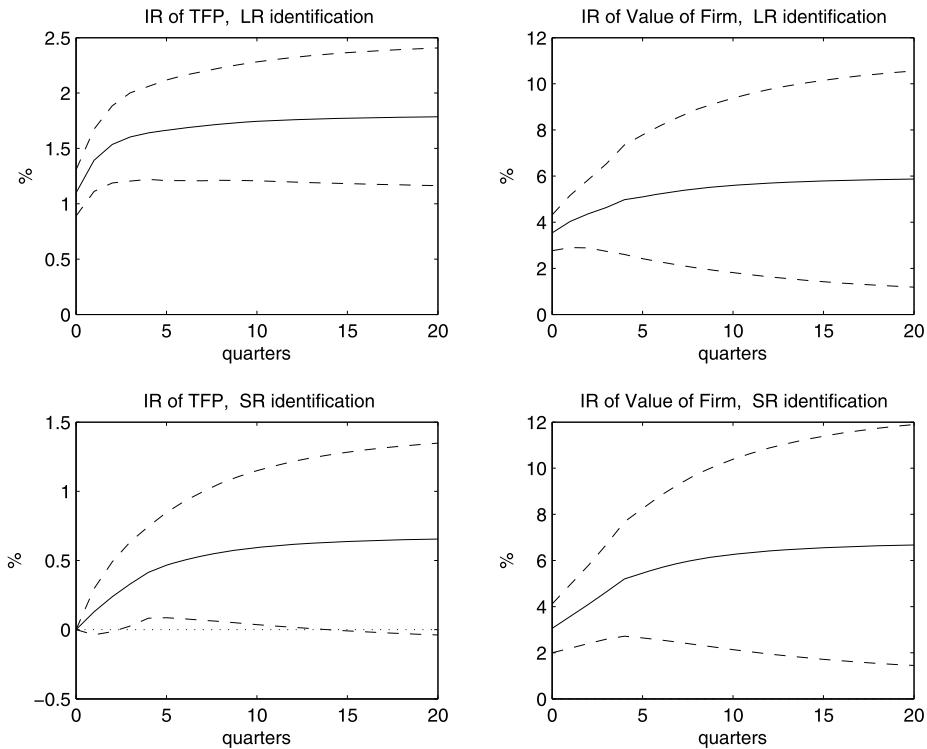


FIGURE A.5.—Beaudry–Portier-style impulse response functions model generated data. Solid lines correspond to mean point estimates and broken lines to point estimates \pm two standard-deviation bands. Impulse responses are computed from a bivariate VAR in the growth rates of TFP and the value of the firm. Artificial data are generated from the parsimonious specification of the model. The VAR is estimated 1000 times. Each time, an artificial time series of length 1212 is created, but only the last 212 observations are used in the estimation of the VAR.

identification schemes to a VAR in the growth rates of TFP and the value of the firm estimated on artificial data generated using the parsimonious specification of the model. We use this version of the model because it assigns a relatively large role to anticipated permanent TFP shocks. We generate artificial data of length 1212 quarters and discard the first 1000 elements. The remaining time series are of equal length as those used in the empirical work of BP. We repeat the estimation of the VAR 1000 times and report the mean and standard deviation of the BP-style impulse responses. The figure shows that, in response to a BP-style innovation, TFP and the value of the firm display a significant increase. In this sense, the predictions of our estimated model are consistent with the empirical results of BP. We reiterate our hesitation to interpret the shocks identified in this exercise as being anticipated TFP shocks because, as explained above, our theoretical model does not imply a bivariate VAR representation.

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