Econometrica Supplementary Material

SUPPLEMENT TO "NETWORKS IN CONFLICT: THEORY AND EVIDENCE FROM THE GREAT WAR OF AFRICA"

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A. THEORY APPENDIX

A.1. *Proof of Proposition 1*

WE FIRST ESTABLISH THE EXISTENCE of a Nash equilibrium in which all groups participate in the contest (an interior equilibrium). Let $\mathbf{x}^* = (x_1^*, \ldots, x_n^*)^\top \in \mathbb{R}^n$ denote the candidate equilibrium effort vector that satisfies the FOCs; let $\mathbf{x}_{-i}^* \in \mathbb{R}^{n-1}$ denote the same vector without the ith component. Let $\pi_i(G; x_i, \mathbf{x}_{-i}^*) = \varphi_i(G; x_i, \mathbf{x}_{-i}^*)/\sum_{j=1}^n \varphi_j(G; x_i, \mathbf{x}_{-i}^*) - x_i$ denote the payoff function of a deviation from the equilibrium effort, in the range where $\varphi_i \geq 0$.

The FOCs of the profit maximization problem yield

$$0 = \frac{\partial \pi_i(G; x_i^*, \mathbf{x}_{-i}^*)}{\partial x_i} = \frac{\sum_{j=1}^n \varphi_j^* - \varphi_i^*(1 + \beta d_i^+ - \gamma d_i^-)}{\left(\sum_{j=1}^n \varphi_j^*\right)^2} - 1. \tag{22}$$

Here we have used the fact that $\partial \varphi_j^*(G; x_i, \mathbf{x}_{-i}^*)/\partial x_i = \delta_{ij} + \beta a_{ij}^+ - \gamma a_{ij}^-$ (where $\delta_{ij} = 0$ if $i \neq j$ and $\delta_{ii} = 1$); consequently, $\sum_{j=1}^n \partial \varphi_j^*/\partial x_i = 1 + \beta d_i^+ - \gamma d_i^-$. Standard algebra yields

$$\varphi_i^* = \frac{1}{1 + \beta d_i^+ - \gamma d_i^-}\left(1 - \sum_{j=1}^n \varphi_j^*\right)\sum_{j=1}^n \varphi_j^*. \tag{23}$$

Next, define $\Gamma_i^{\beta,\gamma}(G) \equiv (1 + \beta d_i^+ - \gamma d_i^-)^{-1} > 0$ and $\Lambda^{\beta,\gamma}(G) \equiv 1 - (\sum_{i=1}^n \Gamma_i^{\beta,\gamma}(G))^{-1}$, where the inequality follows from (3). Summing over i's in equation (23) implies that

$$\varphi_i^* = \Lambda^{\beta,\gamma}(G)\left(1 - \Lambda^{\beta,\gamma}(G)\right)\Gamma_i^{\beta,\gamma}(G) > 0. \tag{24}$$

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The inequality hinges on establishing that $\Lambda^{\beta,\gamma}(G) > 0$, or equivalently, $\sum_{i=1}^{n} \Gamma_i^{\beta,\gamma}(G) > 1$. Observe that $\sum_{i=1}^{n} \Gamma_i^{\beta,\gamma}(G) = \sum_{i=1}^{n} \frac{1}{1+\beta d_i^+ - \gamma d_i^-} \geq \sum_{i=1}^{n} \frac{1}{1+\beta d_i^+} \geq \frac{n}{1+\beta d_{\max}^+} > 1$. The last inequality holds true if and only if $\beta < \frac{n-1}{d_{\max}^+}$, which is, in turn, necessarily true if $\beta < 1$. This, in turn, follows from the assumption that $\beta + \gamma < 1/\max\{\lambda_{\max}(G^+), d_{\max}^-\}$ which implies that $\beta + \gamma < 1/\max\{\lambda_{\max}(G^+), \lambda_{\max}(G^-)\}$, since $\lambda_{\max}(G^-) < d_{\max}^-$ (Cvetkovic, Doon, and Sachs (1995)). Moreover, for any non-empty graph G, $\lambda_{\max}(G) \geq 1$, because for any graph G, $\lambda_{\max}(G) \geq \max_{i=1,\dots,n} \sqrt{d_i}$ (Cvetkovic, Doon, and Sachs (1995)), and $\max_{i=1,\dots,n} d_i \geq 1$ when G is not empty. Thus, $\beta \leq \beta + \gamma < 1$. This establishes that $\varphi_i^* \geq 0$ for all $i = 1, \dots, n$.

Next, we compute $\mathbf{x}^*$. Combining (2) with (24) yields

$$x_i^* + \beta \sum_{j=1}^{n} a_{ij}^+ x_j^* - \gamma \sum_{j=1}^{n} a_{ij}^- x_j^* = \Lambda^{n,\beta}(G)\big(1 - \Lambda^{n,\beta}(G)\big)\Gamma_i^{\beta,\gamma}(G). \tag{25}$$

Denoting $\mathbf{\Gamma}^{\beta,\gamma}(G) \equiv (\Gamma_1^{\beta,\gamma}(G), \dots, \Gamma_n^{\beta,\gamma}(G))^\top$, we can write this system in matrix form as

$$\big(\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-\big)\mathbf{x}^* = \Lambda^{\beta,\gamma}(G)\big(1 - \Lambda^{\beta,\gamma}(G)\big)\mathbf{\Gamma}^{\beta,\gamma}(G). \tag{26}$$

The fact that $\beta + \gamma < 1/\max\{\lambda_{\max}(G^+), \lambda_{\max}(G^-)\}$ also ensures that the matrix $\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-$ is invertible.[38] Then, (26) yields the effort levels:

$$\mathbf{x}^* = \Lambda^{\beta,\gamma}(G)\big(1 - \Lambda^{\beta,\gamma}(G)\big)\mathbf{c}^{\beta,\gamma}(G), \tag{27}$$

where $\mathbf{c}^{\beta,\gamma}(G)$ is the centrality measure defined by equation (8). Equation (27) is the matrix-form version of equation (7) in the proposition. Evaluating $\pi_i(G, \mathbf{x})$ at $\mathbf{x} = \mathbf{x}^*$ yields equation (9) in the proposition.

Thus far, we have established that $\mathbf{x}^*$ and $\boldsymbol{\varphi}^*$ satisfy the FOCs. In order to prove that the FOCs pin down a Nash equilibrium, we must establish that, for all $i = 1, 2, \dots, n$, x_i^* is a global maximum of $\pi_i(G; x_i, \mathbf{x}_{-i}^*)$ for all $x_i \in \mathbb{R}$. To prove the result, we split the horizontal line at the cutoff value $\hat{x}_i$, uniquely defined by the condition $\varphi_i(G; \hat{x}_i, \mathbf{x}_{-i}^*) = 0$. For $x_i < \hat{x}_i$, $\pi_i(G; x_i, \mathbf{x}_{-i}^*) = -D$. For $x_i \geq \hat{x}_i$, standard algebra establishes that $(\partial^2 \pi_i / \partial x_i^2)(G; x_i, \mathbf{x}_{-i}^*) = -2/(\Gamma_i^{\beta,\gamma}(G) \times \Lambda^{\beta,\gamma}(G)) < 0$, where the inequality follows from the facts, established above, that $\Gamma_i^{\beta,\gamma}(G) > 0$ and $\Lambda^{\beta,\gamma}(G) > 0$. Thus, $\pi_i(G; x_i, \mathbf{x}_{-i}^*)$ is strictly concave in x_i in the subdomain $x_i \geq \hat{x}_i$. Moreover, equation (4) establishes that $\varphi_i^* > 0 = \varphi_i(G; \hat{x}_i, \mathbf{x}_{-i}^*)$. This, together with the fact that φ_i is increasing in x_i, establishes that $x_i^* > \hat{x}_i$. The facts that (i) $\pi_i(G; x_i, \mathbf{x}_{-i}^*)$ is strictly concave in x_i, and (ii) $x_i^* > \hat{x}_i$ jointly imply that $\pi_i(G; x_i^*, \mathbf{x}_{-i}^*)$ is a global maximum of the π_i function in the subdomain $x_i \geq \hat{x}_i$. It is immediate that $\pi_i(G; x_i^*, \mathbf{x}_{-i}^*) < \infty$. Define $\underline{D} = \max_i -\pi_i(G; x_i^*, \mathbf{x}_{-i}^*)$. Then, for all $D > \underline{D}$, we have that $\pi_i(G; x_i^*, \mathbf{x}_{-i}^*) > -D$, namely, defeat is not a profitable deviation. This completes the proof of existence of an interior Nash equilibrium.

Next, we prove uniqueness. We assume that, contrary to the statement of the proposition, for all $D < \infty$, there exists an equilibrium where $n - \hat{n} > 0$ groups take the defeat option. Then, we show that this induces a contradiction. Since we have proved that when

[38] This follows from Weyl's theorem. The determinant of a matrix of the form $\mathbf{I}_n - \sum_{j=1}^{p} \alpha_j \mathbf{W}_j$ is strictly positive if $\sum_{j=1}^{p} |\alpha_j| < 1/\max_{j=1,\dots,p} \|\mathbf{W}_j\|$, where $\|\mathbf{W}_j\|$ is any matrix norm, including the spectral norm, which corresponds to the largest eigenvalue of $\mathbf{W}_j$.

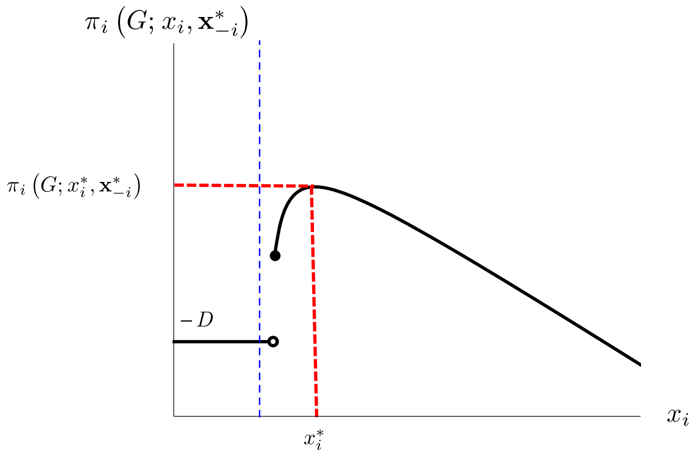


FIGURE A.1.—The figure shows the function $\pi_i(G; x_i, \mathbf{x}^*_{-i})$ for different values of x_i.

all n groups participate in the contest there exists a unique equilibrium, this establishes global uniqueness.

The condition that $\Gamma_i^{0,\gamma} > 0$, $\forall i = 1, 2, \dots, n$, ensures that, in a candidate equilibrium in which only $\hat{n} < n$ groups participate in the contest, all such $\hat{n}$ groups choose a finite effort level (this follows immediately from the analysis of the case where all n groups participate). The effort level of participants is $\mathbf{x}^*_{\hat{n}} = \Lambda^{\beta,\gamma}(G_{\hat{n}})(1 - \Lambda^{\beta,\gamma}(G_{\hat{n}}))\mathbf{c}^{\beta,\gamma}(G_{\hat{n}})$, where the graph $G_{\hat{n}}$ only includes the participating groups. Consider a non-participating group ν. For this group, in the assumed equilibrium, $\pi_\nu = -D$. Suppose group ν deviates and chooses, instead, $x_\nu = x^0_\nu$, where x^0_ν is the unique threshold such that $\varphi_\nu(G_{\hat{n}+1}; x^0_\nu, \hat{\mathbf{x}}^*_{-\nu}) = 0$. The payoff of this deviation is $\pi_\nu(G_{\hat{n}+1}; x^0_\nu, \hat{\mathbf{x}}^*_{-\nu}) = -x^0_\nu > -\infty$. Thus, for any $D > x^0_\nu$, this deviation is profitable. Repeating the argument for all partitions establishes that there exists $\underline{\underline{D}} < \infty$ such that, for all $D > \underline{\underline{D}}$, any candidate equilibrium where $n - \hat{n} > 0$ groups take the defeat option is susceptible to a profitable deviation (hence, it is not an equilibrium). Thus, the only equilibrium is interior, completing the proof.

REMARK: Figure A.1 (referred to in the text) shows the payoff function $\pi_i(G; x_i, \mathbf{x}^*_{-i})$ at the equilibrium strategy profile. Group i's payoff function is constant ($\pi_i = -D$) for all x_i below the threshold that guarantees the non-negativity of φ_i. At the threshold, the function is discontinuous, capturing the fact that when $\varphi_i \geq 0$, no defeat cost is due. To the right of the threshold, condition (3) ensures that $\pi_i(G, ; x_i, \mathbf{x}^*_{-i})$ is strictly concave in x_i. Moreover, the payoff function is hump shaped and reaches a maximum at $\varphi^*_i > 0$.

A.2. *The Case of Small Externalities*

The centrality measure $c_i^{\beta,\gamma}(G)$ in equation (8) depends, in general, on the entire network structure. However, it is instructive to consider networks in which the spillover parameters β and γ are small. In this case, our centrality measure can be approximated by the the sum of (i) the Katz–Bonacich centrality related to the network of enmities, G^-, (ii) the (negative-parameter) Katz–Bonacich centrality related to the network of alliances, G^+, and (iii) the local hostility vector, $\boldsymbol{\Gamma}^{\beta,\gamma}(G)$.[39]

[39]See Webpage Appendix C for a definition of the Katz–Bonacich centrality and for proofs of Lemmas 1–2.

LEMMA 1: *Assume that the conditions of Proposition* 1 *are satisfied. Then, as* $\beta \to 0$ *and* $\gamma \to 0$, *the centrality measure defined in equation* (8) *can be written as*

$$\mathbf{c}^{\beta,\gamma}(G) = \mathbf{b}_{\cdot}(\gamma, G^{-}) + \mathbf{b}_{\cdot}(-\beta, G^{+}) - \mathbf{\Gamma}^{\beta,\gamma}(G) + O(\beta\gamma),$$

where $O(\beta\gamma)$ *involves second and higher order terms, and the (weighted) Katz–Bonacich centrality with parameter* α *is defined as* $\mathbf{b}_{\cdot}(\alpha, G) \equiv \mathbf{b}_{\mathbf{\Gamma}^{\beta,\gamma}(G)}(\alpha, G) = (\mathbf{I}_n - \alpha\mathbf{A})^{-1}\mathbf{\Gamma}^{\beta,\gamma}(G)$ *assuming that* α *is smaller than the inverse of the largest eigenvalue of* $\mathbf{A}$.

Lemma 1 states that the centrality $\mathbf{c}^{\beta,\gamma}(G)$ can be expressed as a linear combination of the weighted Katz–Bonacich centralities $\mathbf{b}_{\cdot}(\gamma, G^{-})$, $\mathbf{b}_{\cdot}(-\beta, G^{+})$ and the vector $\mathbf{\Gamma}^{\beta,\gamma}(G)$. Each Katz–Bonacich centrality gauges the network multiplier effect attached to the system of enmities and alliances, respectively. In the case of weak network externalities (i.e. when $\beta \to 0$ and $\gamma \to 0$), the following approximation holds true:

$$\begin{aligned} b_{\cdot,i}(\gamma, G^{-}) &= \Gamma_i^{\beta,\gamma}(G) + \gamma \sum_{j=1}^{n} a_{ij}^{-} \Gamma_j^{\beta,\gamma}(G) \\ &\quad + \gamma^2 \sum_{j=1}^{n} a_{ij}^{-} \sum_{k=1}^{n} a_{jk}^{-} \Gamma_k^{\beta,\gamma}(G) + O(\gamma^3), \end{aligned} \tag{28}$$

$$\begin{aligned} b_{\cdot,i}(-\beta, G^{+}) &= \Gamma_i^{\beta,\gamma}(G) + (-\beta) \sum_{j=1}^{n} a_{ij}^{+} \Gamma_j^{\beta,\gamma}(G) \\ &\quad + (-\beta)^2 \sum_{j=1}^{n} a_{ij}^{+} \sum_{k=1}^{n} a_{jk}^{+} \Gamma_k^{\beta,\gamma}(G) + O(\beta^3). \end{aligned} \tag{29}$$

Thus, Lemma 1 suggests that, when higher order terms can be neglected, our centrality measure is increasing in γ and in the number of first- and second-degree enmities, whereas it is decreasing in β and in the number of first-degree alliances. Second-degree alliances have instead a positive effect on the centrality measure. An illustration of this result in the case of a path graph is provided in Appendix C.

We can also obtain a simple approximate expression for the equilibrium efforts and the payoffs in Proposition 1.[40]

LEMMA 2: *As* $\beta \to 0$ *and* $\gamma \to 0$, *the equilibrium effort and payoff of group* i *in network* G *can be written as*

$$x_i^*(G) = A_1^{\beta,\gamma}(G) - B_1(\beta d_i^{+} - \gamma d_i^{-}) + O(\beta\gamma),$$
$$\pi_i^*(G) = A_2^{\beta,\gamma}(G) + B_2(\beta d_i^{+} - \gamma d_i^{-}) + O(\beta\gamma),$$

where $A_1^{\beta,\gamma}(G)$, B_1, $A_2^{\beta,\gamma}(G)$, *and* B_2 *are positive constants with* $A_1^{\beta,\gamma}(G)$ *and* $A_2^{\beta,\gamma}(G)$ *being of the order of* $O(\beta) + O(\gamma)$.

[40]See the proof of Lemma 2 for the explicit expressions for the constants $A^{\beta,\gamma}(G)$, B, $C^{\beta,\gamma}(G)$, and D. It is useful to note that, when $\beta = \gamma = 0$, then $\Lambda^{\beta,\gamma}(G) = 1 - \frac{1}{n}$, and $c_i^{\beta,\gamma}(G) = 1$. Then, the equilibrium expressions in Proposition 1 simplify to $x_i^*(G) = (n-1)/n^2$ and $\pi_i^*(G) = 1/n^2$ which are the standard solutions in the Tullock CSF.

Lemma 2 shows that, when network externalities are small, a group's fighting effort increases in the weighted difference between the number of enmities (weighted by γ) and alliances (weighted by β), that is, the net local externalities $d_i^- \gamma - d_i^+ \beta$. The opposite is true for the equilibrium payoff, which is increasing in $d_i^+ \beta - d_i^- \gamma$. Thus, ceteris paribus, an increase in the spillover from alliances (enmities), parameterized by β (γ), and an increase in the number of allies (enemies) decreases (increases) group i's fighting effort and increases (reduces) its payoff. Intuitively, a group with many enemies tends to fight harder and to appropriate a smaller share of the prize, whereas a group with many friends tends to fight less and to appropriate a large size of the prize. One must remember, however, that this simple result hinges on β and γ being small; in general, higher-degree links have sizeable effects.

A.3. *Appendix to Section 2.4 (Heterogeneity)*

In the following, we provide a complete equilibrium characterization of the extension of our model that we have introduced in Section 2.4. When the fighting strength φ_i of group i depends on an idiosyncratic shifter $\tilde{\varphi}_i$ as in equation (11), then the following proposition characterizes the corresponding Nash equilibrium.

PROPOSITION 2: *Let* $\Gamma_i^{\beta,\gamma}(G)$ *and* $\Lambda^{\beta,\gamma}(G)$ *be defined as in equation* (6), *and let*

$$\mathbf{c}_{\boldsymbol{\mu}}^{\beta,\gamma}(G) \equiv \left(\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-\right)^{-1} \boldsymbol{\mu} \tag{30}$$

be a centrality vector, whose generic element $c_{\boldsymbol{\mu},i}^{\beta,\gamma}(G)$ *describes the centrality of group i in the network for some vector* $\boldsymbol{\mu} \in \mathbb{R}^n$, *and assume that the same parameter restrictions on* β *and* γ *hold as in Proposition* 1. *Then for the cost of defeat*, D, *large enough*, *there exists a unique Nash equilibrium of the n-player simultaneous move game with payoffs given by equation* (1), *groups' OPs in equation* (11), *and strategy space* $\mathbb{R}^n$, *where the equilibrium effort levels are given by*

$$x_i^*(G) = \Lambda^{\beta,\gamma}(G)\bigl(1 - \Lambda^{\beta,\gamma}(G)\bigr) c_{\Gamma^{\beta,\gamma}(G),i}^{\beta,\gamma}(G) - c_{\tilde{\varphi},i}^{\beta,\gamma}(G), \tag{31}$$

for all $i = 1, \dots, n$. *Moreover, the aggregate and individual equilibrium OPs are*, *respectively*,

$$\sum_{i=1}^{n} \varphi_i = \Lambda^{\beta,\gamma}(G), \tag{32}$$

$$\varphi_i = \Lambda^{\beta,\gamma}(G)\bigl(1 - \Lambda^{\beta,\gamma}(G)\bigr)\Gamma_i^{\beta,\gamma}(G), \tag{33}$$

and the equilibrium payoffs are given by

$$\pi_i^*(G) \equiv \pi_i\bigl(G, \mathbf{x}^*\bigr) = \bigl(1 - \Lambda^{\beta,\gamma}(G)\bigr)\bigl(\Gamma_i^{\beta,\gamma}(G) - \Lambda^{\beta,\gamma}(G) c_{\Gamma^{\beta,\gamma}(G),i}^{\beta,\gamma}(G)\bigr) + c_{\tilde{\varphi},i}^{\beta,\gamma}(G). \tag{34}$$

PROOF: We start by considering an equilibrium in which all groups participate in the contest (i.e., an interior equilibrium). With equation (11), we can write group i's payoff as follows:

$$\pi_i(G,\mathbf{x}) = \frac{\varphi_i}{\sum_{j=1}^{n} \varphi_j} - x_i$$
$$= \frac{x_i + \beta \sum_{j=1}^{n} a_{ij}^{+} x_j - \gamma \sum_{j=1}^{n} a_{ij}^{-} x_j + \tilde{\varphi}_i}{\sum_{j=1}^{n} \left(x_j + \beta \sum_{k=1}^{n} a_{jk}^{+} x_k - \gamma \sum_{k=1}^{n} a_{jk}^{-} x_k + \tilde{\varphi}_j \right)} - x_i. \tag{35}$$

The partial derivatives are given by

$$\frac{\partial \pi_i}{\partial x_i} = \frac{\frac{\partial \varphi_i}{\partial x_i} \sum_{j=1}^{n} \varphi_j - \varphi_i \sum_{j=1}^{n} \frac{\partial \varphi_j}{\partial x_i}}{\left(\sum_{j=1}^{n} \varphi_j \right)^2} - 1$$
$$= \frac{\sum_{j=1}^{n} \varphi_j - \varphi_i \left(1 + \beta d_i^{+} - \gamma d_i^{-}\right)}{\left(\sum_{j=1}^{n} \varphi_j \right)^2} - 1, \tag{36}$$

where we have used the fact that $\frac{\partial \varphi_j}{\partial x_i} = \delta_{ij} + \beta a_{ij}^{+} - \gamma a_{ij}^{-}$ and consequently $\sum_{j=1}^{n} \frac{\partial \varphi_j}{\partial x_i} = 1 + \beta d_i^{+} - \gamma d_i^{-}$. The FOCs are then given by $\frac{\partial \pi_i}{\partial x_i} = 0$. From the partial derivative in equation (36), the FOC for group i can be written as follows:

$$\frac{\partial \pi_i}{\partial x_i} = \frac{\sum_{j=1}^{n} \varphi_j - \varphi_i \left(1 + \beta d_i^{+} - \gamma d_i^{-}\right)}{\left(\sum_{j=1}^{n} \varphi_j \right)^2} - 1 = 0,$$

from which we get

$$\varphi_i = \frac{1}{1 + \beta d_i^{+} - \gamma d_i^{-}} \left(1 - \sum_{j=1}^{n} \varphi_j \right) \sum_{j=1}^{n} \varphi_j.$$

Summation over i gives

$$\sum_{i=1}^{n} \varphi_i = \left(1 - \frac{1}{\sum_{i=1}^{n} \frac{1}{1 + \beta d_i^{+} - \gamma d_i^{-}}} \right).$$

With $\Gamma_i^{\beta,\gamma}(G)$ and $\Lambda^{\beta,\gamma}(G)$ as in equation (6), we can write the aggregate operational performance as

$$\sum_{i=1}^{n} \varphi_i = \Lambda^{\beta,\gamma}(G),$$

which is equivalent to equation (32). The individual operational performance can be written as

$$\varphi_i(G, \mathbf{x}) = \Lambda^{\beta,\gamma}(G)\big(1 - \Lambda^{\beta,\gamma}(G)\big)\Gamma_i^{\beta,\gamma}(G), \tag{37}$$

which is equivalent to equation (32). We then get

$$\varphi_i(G, \mathbf{x}) = x_i + \beta \sum_{j=1}^{n} a_{ij}^{+} x_j - \gamma \sum_{j=1}^{n} a_{ij}^{-} x_j + \tilde{\varphi}_i = \Lambda^{n,\beta}(G)\big(1 - \Lambda^{n,\beta}(G)\big)\Gamma_i^{\beta,\gamma}(G). \tag{38}$$

We can write

$$x_i + \beta \sum_{j=1}^{n} a_{ij}^{+} x_j - \gamma \sum_{j=1}^{n} a_{ij}^{-} x_j = \varphi_i - \tilde{\varphi}_i = \Lambda^{n,\beta}(G)\big(1 - \Lambda^{n,\beta}(G)\big)\Gamma_i^{\beta,\gamma}(G) - \tilde{\varphi}_i.$$

Denoting $\Gamma^{\beta,\gamma}(G) \equiv (\Gamma_1^{\beta,\gamma}(G), \dots, \Gamma_n^{\beta,\gamma}(G))^{\top}$, we can write this in vector-matrix form as

$$\big(\mathbf{I}_n + \beta \mathbf{A}^{+} - \gamma \mathbf{A}^{-}\big)\mathbf{x} = \Lambda^{\beta,\gamma}(G)\big(1 - \Lambda^{\beta,\gamma}(G)\big)\boldsymbol{\Gamma}^{\beta,\gamma}(G) - \tilde{\boldsymbol{\varphi}}.$$

When the matrix $I_n + \beta A^{+} - \gamma A^{-}$ is invertible, we obtain a unique solution given by

$$\mathbf{x} = \Lambda^{\beta,\gamma}(G)\big(1 - \Lambda^{\beta,\gamma}(G)\big)\big(\mathbf{I}_n + \beta \mathbf{A}^{+} - \gamma \mathbf{A}^{-}\big)^{-1}\boldsymbol{\Gamma}^{\beta,\gamma}(G) - \big(\mathbf{I}_n + \beta \mathbf{A}^{+} - \gamma \mathbf{A}^{-}\big)^{-1}\tilde{\boldsymbol{\varphi}}. \tag{39}$$

With the definition of the centrality in equation (30), we then can write equation (39) in the form of equation (31) in the proposition. Moreover, using the fact that equilibrium payoffs are given by $\pi_i(G, x^*) = \frac{\varphi_i^*(G)}{\sum_{j=1}^{n} \varphi_j^*(G)} - x_i^*$, we obtain equation (34) in the proposition.

Next, note that from equation (36) we find that

$$\frac{\partial^2 \pi_i}{\partial x_i^2} = -\frac{2}{\Lambda^{\beta,\gamma}(G)\Gamma_i^{\beta,\gamma}(G)} = -2\frac{\sum_{j=1}^{n} \Gamma_j^{\beta,\gamma}(G)}{\sum_{j=1}^{n} \Gamma_j^{\beta,\gamma}(G) - 1}\big(1 + \beta d_i^{+} - \gamma d_i^{-}\big), \tag{40}$$

which is negative under the same parameter restrictions on β and γ as in Proposition 1. Further, as in the proof of Proposition 1, one can then proceed to establish that there exists a unique interior Nash equilibrium when the cost of defeat, D, is large enough. *Q.E.D.*

B. EMPIRICAL APPENDIX

In this section, we discuss technical details related to Sections 4 (ambiguous links) and 6 (random utility model). Then, we include the tables and figures discussed in the main text and in this appendix. Additional tables and figures can be found in the webpage Appendix C.

B.1. *Appendix to Section 4.2.2*

While the large majority of bilateral links are incontrovertible, there are some that may potentially be ambiguous, that is, for which experts do not all reach the same clear-cut assessments or where the battleground observations in ACLED are not fully reflected in the expert accounts. As mentioned earlier, Table B.V checks the robustness of our results with respect to ambiguous group links. The columns 1–5 of the table have already been discussed in the main text. Here, we discuss columns 6–9. In column 6, we exclude the CNDD since this group appears to have switched its relation with the Mayi-Mayi militia. Next, we classify Uganda and the RCD-G as enemies (column 7). While this violates our coding rule (that classifies them as neutral), it is more consistent with the narrative. Next, we code all member states of the Southern African Development Community (SADC) as allies of each other and of the FARDC (column 8). Finally, we define as "governments allied to the FARDC" all governments allied to the FARDC in the baseline treatment plus all SADC member states. Finally, we let all "governments allied to the FARDC" be (i) allied among themselves, (ii) allied to the FARDC, and (iii) enemies to Rwanda-I and Uganda (column 9). The results are in all cases similar to the baseline table.

In Table B.VI, we investigate potentially ambiguous links on the basis of accounts in Autesserre (2008), Prunier (2011), Sanchez de la Sierra (2016), Stearns (2011). We start by focusing in column 1 on the FARDC links with ALIR. In the baseline regression, they are coded as enemies under Laurent Kabila and as neutral under Joseph Kabila. The earlier link is potentially ambiguous, since the ALIR was part of the Hutu exodus that clashed with Rwanda and Uganda (see, e.g., Prunier (2011, p. 234)). This might suggest an alliance between the FARDC and ALIR. However, this is in contradiction with the ACLED data where these two groups are observed fighting against each other on fourteen occasions between 1998 and 2001, and never on the same side as brothers in arms. Column 1 shows that the results are not sensitive if we code this dyad as an alliance or as an enmity.

In the same vein, columns 2 and 3 run the same regressions as in columns 4 and 5 of Table B.V where we investigated the ambiguous links between the FARDC and FDLR, but assuming now in addition the same links for the FARDC-Interahamwe pair as for the FARDC-FDLR pair. In particular, in our baseline estimates the FARDC-FDLR are first allies under Laurent Kabila, and then neutral under Joseph Kabila, whereas the FARDC-Interahamwe are allies throughout the whole period. Here, we assume that FARDC-Interahamwe and FARDC-FDLR are in both cases allies under Laurent Kabila and enemies under Joseph Kabila (column 2), or neutral throughout the entire period (column 3).

In column 4, we scrutinize the relationship between the FARDC-LK and ADF. In the baseline, we code them as allied, based on an expert source (see International Crisis Group (1998)). However, Prunier (2011, p. 177) mentioned one incident where the 10th division of the FARDC attacks ADF rebels. While this appears to be an isolated episode, in column 4 we code FARDC-LK and ADF as enemies.

In column 5, we investigate the link MLC-Hema ethnic militia. In the baseline estimations, we code them as enemy, based on ACLED reporting them to fight five times against each other and never as brothers in arms. Stearns (2011, p. 230) referred to one incident where MLC troops cooperated with the Hema ethnic militia to attack locals. Thus, in the robustness check of column 5, we code them as allies.

Finally, the link FDLR-MayiMayi is considered in column 6. In the baseline estimations, we code two MayiMayi fractions as allied to FDLR (based on ACLED incidences), namely, the Yakutumba and Pareco fractions. In contrast, the relationship FDLR-MayiMayi is coded as neutral for all other MayiMayi fractions (again based on ACLED). Prompted by Sanchez de la Sierra's statement that "Mayi-Mayi groups and the FDLR often conducted joint operations during the second Congo war" (2016, p. 84), we code all MayiMayi fractions as being allied to the FDLR.

In all columns, the baseline results are virtually unchanged. The results are also robust to changing all links simultaneously (except for column 3, since it would be incompatible with column 2).

B.2. *Appendix to Section 6 (Random Utility Model)*

B.2.1. *Multinomial Logit Estimation*

Table B.XII displays the results of the multinomial logit estimation. Table B.XIII displays the results of the Random Utility Model with time-varying network. Figure B.1 reports the cross-dyad distribution of predicted probabilities for enmities (left panel) and alliances (right panel). In the left panel, the dark sample represents the distribution for observed enmities (alliances) and the light sample represents the distribution for the other observed links. Figure B.2 displays the observed and predicted distributions of six network statistics that play an important role in our baseline model: degree one enemies (panel *a*), degree one allies (panel *b*), number of degree one links (panel *c*), common enemies (panel *d*), common allies (panel *e*), and conflicting neighbors (panel *f*). For each statistic, the panel compares the data with the average distribution over 1000 simulated networks as predicted by the conditional logit model (plus/minus one standard deviation interval). The figure shows that all these important moments of the data are predicted accurately. More precisely, each Monte Carlo draw consists in drawing a random utility shock $\tilde{u}_{ij}(a)$ for each dyad ij and alternative a from a type I extreme value distribution with mean $\gamma \approx 0.577$ (the Euler–Mascheroni constant), and variance, $\sqrt{\pi}/6$. Using equation (21), we compute each simulated alternative-dependent joint surplus $U_{ij}^{\mathrm{sim}}(a)$ and select the predicted link for dyad ij as $a_{ij}^{\mathrm{pre}} = \arg\max U_{ij}^{\mathrm{sim}}(a)$. This leads to a simulated network for which we can compute the six distributions of network statistics. This procedure is iterated 1000 times. Each panel reports the average distribution (± 1 SD) across the Monte Carlo draws.

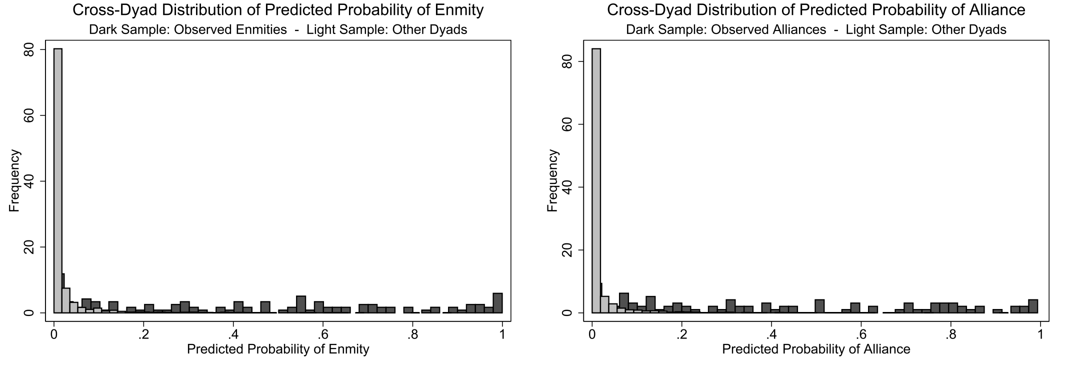
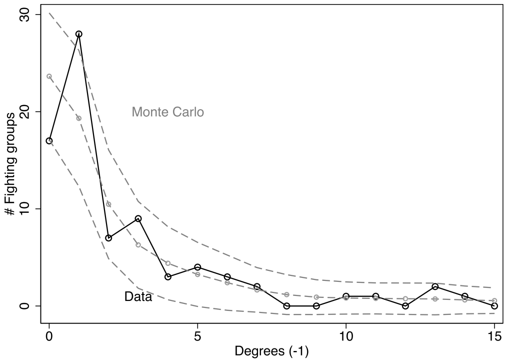
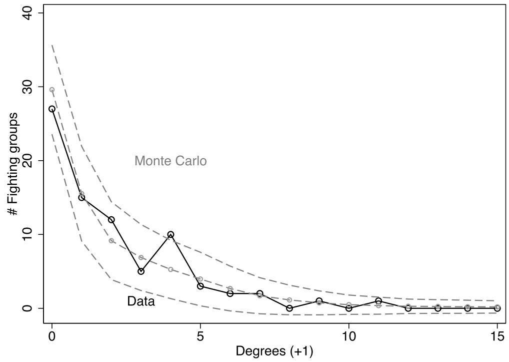


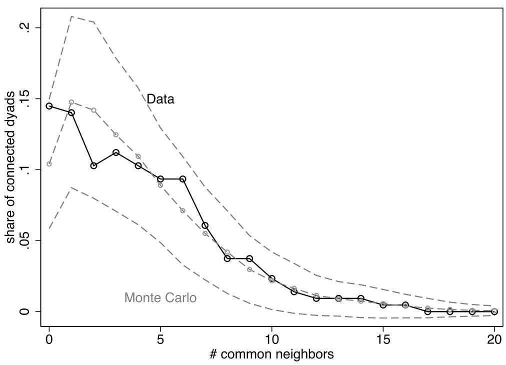
FIGURE B.1.—Predicted probabilities of enmities and alliances.



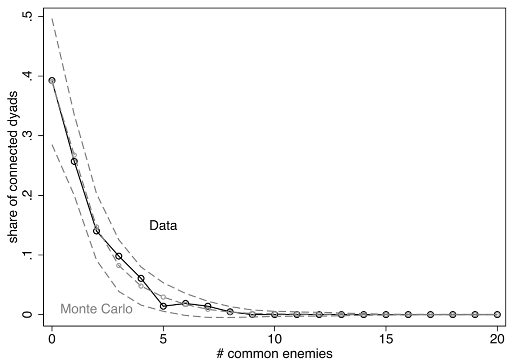
(a) Degree one enemies.



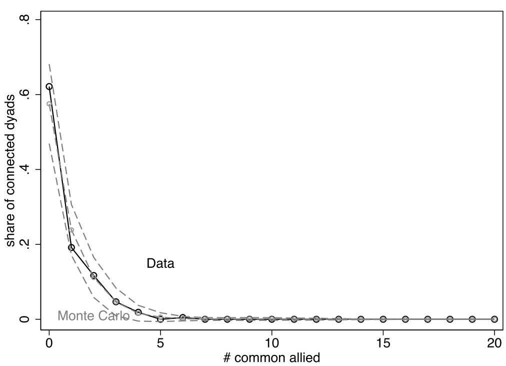
(b) Degree one allies.



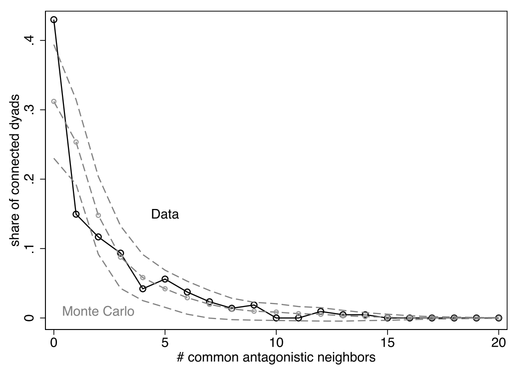
(c) Number of degree one links.



(d) Common enemies.



(e) Common allies.



(f) Conflicting neighbors.

FIGURE B.2.—Goodness-of-fit statistics.

B.2.2. *Effect of Removing Armed Groups (With Network Rewiring)*

Table B.XIV summarizes the effect of removing one or more groups from the conflict when endogenous network adjustments are allowed. We focus, for comparability, on the top 15 groups in Table IV. All but three groups remain in the top 15 even after allowing for network recomposition.[41] The UPC is especially interesting. This is a medium-large group whose activity accounts for 2.2% of the total violence. Its removal in Table IV yields a reduction in violence of the order of 3%, with a sizeable multiplier of 1.4. However, the recomposition of the network after its removal offsets two thirds of the gains.

TABLE B.I

SUMMARY STATISTICS[a]

Variable	Obs.	Mean	Std. Dev.	Min	Max
Total Fight.	1040	5.929	25.046	0	300
Total Fight. Enemies (TFE)	1040	69.237	109.95	0	682
Total Fight. Allies (TFA)	1040	48.603	85.75	0	563
Total Fight. Neutrals (TFN)	1040	350.539	241.616	1	1042
d^- (# enemies)	1040	2.95	4.306	0	26
d^+ (# allies)	1040	2.4	3.45	0	21
Rainfall $(t-1)$	1040	125.839	26.164	59.639	195.56

[a]The sample comprises the 80 fighting groups that are involved in at least one fighting event in ACLED during the period 1998–2010.

[41]The three groups that drop out of the top 15 are the UPC, Mutiny of FARC, and FRPI. The groups entering the top 15 are two branches of the RCD (the first collects all events involving "unspecified" RCD; the second is the group labeled RCD-National; both are likely to suffer with large measurement error) and the National Army for the Liberation of Uganda.

TABLE B.II

BENCHMARK SECOND STAGE WITH LARGE_6[a]

	Dependent Variable: Total Fighting							
	OLS	Reduced IV	Full IV	Neutrals	Battles	$d^- \geq 1$ & $d^+ \geq 1$	GED Coord.	GED Union
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Total Fight. Enemies (TFE)	0.066***	0.130**	0.100***	0.119***	0.121***	0.118***	0.123***	0.161***
	(0.016)	(0.057)	(0.033)	(0.035)	(0.035)	(0.035)	(0.036)	(0.045)
Total Fight. Allies (TFA)	0.001	−0.218**	−0.149***	−0.139***	−0.153***	−0.161***	−0.138***	−0.152***
	(0.017)	(0.086)	(0.051)	(0.047)	(0.051)	(0.062)	(0.047)	(0.050)
Total Fight. Neutrals (TFN)				0.005	0.004	0.009	0.005	0.006
				(0.006)	(0.006)	(0.014)	(0.006)	(0.005)
Additional controls	Reduced	Reduced	Full	Full	Full	Full	Full	Full
Estimator	OLS	IV	IV	IV	IV	IV	IV	IV
Set of Instrument Variables	n.a.	Restricted	Full	Full	Full	Full	Full	Full
Kleibergen–Paap F-stat	n.a.	10.6	15.6	15.1	17.4	14.9	15.1	17.3
Hansen J (p-value)	n.a.	0.16	0.75	0.83	0.79	0.70	0.81	0.79
Observations	1040	1040	1040	1040	988	598	1040	1781
R-squared	0.510	0.265	0.474	0.459	0.446	0.462	0.455	0.389

[a]All regressions include group fixed effects and control for rainfall in the own group's territory. Columns 1–3 include time fixed effects. Robust standard errors corrected for Spatial HAC in parentheses. Significance levels are indicated by $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

TABLE B.III
ROBUSTNESS[a]

	Dependent Variable: Total Fighting									
	Only Deg. 2 (1)	Sample Split (2)	Only Violent (3)	Batt., Riots, Viol. (4)	Excl. Bilateral (5)	Lags (6)	1 Evt. Enemy (7)	2 Evts. Ally (8)	RUM (9)	3d Poly. (10)
Total Fight. Enemies (TFE)	0.074***	0.147**	0.083***	0.083***	0.063***	0.090***	0.119***	0.089***	0.106***	0.082***
	(0.027)	(0.062)	(0.019)	(0.019)	(0.023)	(0.026)	(0.029)	(0.019)	(0.033)	(0.019)
Total Fight. Allies (TFA)	−0.207**	0.002	−0.114***	−0.114***	−0.110***	−0.104***	−0.170***	−0.107***	−0.112**	−0.115***
	(0.083)	(0.079)	(0.033)	(0.034)	(0.033)	(0.040)	(0.051)	(0.033)	(0.050)	(0.034)
Total Fight. Neutrals (TFN)	0.008	0.011	0.004	0.004	0.004	0.007	0.005	0.002	0.002	0.004
	(0.006)	(0.007)	(0.004)	(0.004)	(0.004)	(0.005)	(0.006)	(0.004)	(0.006)	(0.004)
Additional controls	Full	Full	Full	Full	Full	Full	Full	Full	Full	Full
Estimator	IV	IV	IV	IV	IV	IV	IV	IV	IV	IV
Set of Instrument Variables	Full	Full	Full	Full	Full	Full	Full	Full	Full	Full
Kleibergen–Paap F-stat	13.4	37.7	22.5	24.6	20.4	20.3	11.8	25.3	4.0	28.6
Hansen J (p-value)	0.28	0.84	0.58	0.58	0.54	0.69	0.51	0.51	0.62	0.58
Observations	1040	640	1040	1027	1040	960	1040	1040	1040	1040
R-squared	0.489	0.589	0.568	0.570	0.550	0.577	0.394	0.571	0.554	0.567

[a]All regressions include group fixed effects and control for rainfall in the own group's territory. Robust standard errors corrected for Spatial HAC in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

TABLE B.IV

ROBUSTNESS: GROUP DEFINITION (FARDC, RWANDA & OTHERS)[a]

	Dependent Variable: Total Fighting			
	FARDC	MayMay	Rwanda	FARDC & RWA
Merge	(1)	(2)	(3)	(4)
Total Fight. Enemies (TFE)	0.040**	0.068***	0.074***	0.040**
	(0.019)	(0.022)	(0.022)	(0.020)
Total Fight Allies (TFA)	−0.058*	−0.084***	−0.108***	−0.066*
	(0.034)	(0.032)	(0.032)	(0.037)
Total Fight. Neutrals (TFN)	−0.002	−0.000	0.003	−0.003
	(0.004)	(0.005)	(0.005)	(0.004)
Additional controls	Full	Full	Full	Full
Estimator	IV	IV	IV	IV
Set of Instrument Variables	Full	Full	Full	Full
Kleibergen–Paap F-stat	34.6	15.4	19.7	29.8
Hansen J (p-value)	0.44	0.66	0.59	0.51
Observations	1027	962	1027	1014
R-squared	0.631	0.614	0.589	0.642

[a]All regressions include group fixed effects and control for rainfall in the group's homeland. Robust standard errors corrected for Spatial HAC in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

TABLE B.V

ROBUSTNESS: AMBIGUOUS NETWORK LINKS I[a]

	Dependent Variable: Total Fighting								
	Uga-Rwa Neutral	Uga-Rwa Allies	Uga-Rwa Allies Then Neutral	FARDC-FDLR Allies Then Enemies	FARDC-FDLR Neutral	Exclude CNDD	Uga-RCD-G Enemies	SADC Allies	Dyadic Closure Main Groups
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Total Fight. Enemies (TFE)	0.082***	0.081***	0.082***	0.090***	0.083***	0.083***	0.074***	0.082***	0.075***
	(0.021)	(0.021)	(0.020)	(0.020)	(0.020)	(0.020)	(0.019)	(0.020)	(0.020)
Total Fight Allies (TFA)	−0.105***	−0.109***	−0.112***	−0.116***	−0.112***	−0.111***	−0.104***	−0.116***	−0.100***
	(0.031)	(0.033)	(0.033)	(0.033)	(0.036)	(0.033)	(0.031)	(0.035)	(0.031)
Total Fight. Neutrals (TFN)	0.004	0.003	0.004	0.003	0.003	0.002	0.004	0.004	0.000
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
Additional controls	Full	Full	Full	Full	Full	Full	Full	Full	Full
Estimator	IV	IV	IV	IV	IV	IV	IV	IV	IV
Set of Instrument Variables	Full	Full	Full	Full	Full	Full	Full	Full	Full
Kleibergen–Paap F-stat	19.9	20.2	19.8	16.7	23.4	27.2	20.8	19.2	12.7
Hansen J (p-value)	0.60	0.59	0.58	0.60	0.56	0.54	0.57	0.55	0.58
Observations	1040	1040	1040	1040	1040	1027	1040	1040	1040
R-squared	0.574	0.573	0.569	0.577	0.565	0.574	0.577	0.573	0.589

[a] All regressions include group fixed effects and control for rainfall in the group's homeland. Robust standard errors corrected for Spatial HAC in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

TABLE B.VI

ROBUSTNESS: AMBIGUOUS NETWORK LINKS II[a]

	Dependent Variable: Total Fighting					
	FARDC-ALIR Ally	FARDC-FDLR/Interah. Ally Then Enemy	FARDC-FDLR/Interah. Neutral	FARDC-ADF Enemy	Hema Ethnic Militia-MLC Ally	FDLR-MayiMayi Ally
	(1)	(2)	(3)	(4)	(5)	(6)
Total Fight. Enemies (TFE)	0.087***	0.077***	0.078***	0.081***	0.085***	0.070***
	(0.020)	(0.022)	(0.019)	(0.018)	(0.019)	(0.020)
Total Fight. Allies (TFA)	−0.117***	−0.087***	−0.103***	−0.117***	−0.113***	−0.107***
	(0.035)	(0.025)	(0.034)	(0.036)	(0.035)	(0.033)
Total Fight. Neutrals (TFN)	0.004	0.002	0.003	0.004	0.004	0.003
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.006)
Additional controls	Full	Full	Full	Full	Full	Full
Estimator	IV	IV	IV	IV	IV	IV
Set of Instrument Variables	Full	Full	Full	Full	Full	Full
Observations	1040	1040	1040	1040	1040	1040
Kleibergen–Paap F-stat	21.4	10.1	25.2	25.1	20.1	28.3
Hansen J (p-value)	0.58	0.58	0.55	0.56	0.57	0.58
R-squared	0.562	0.605	0.576	0.570	0.566	0.574

[a] All regressions include group fixed effects and control for rainfall in the own group's territory. Robust standard errors corrected for Spatial HAC in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

TABLE B.VII

MEASUREMENT ERROR IN RAINFALL[a]

	Dependent Variable: GPCC Gauge Rainfall Measure							
	Linear		Log-linear		Linear		Log-linear	
Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
# ACLED conflict events	0.017	0.008	0.009	0.001	−0.069	0.016	−0.016	0.005
	(0.032)	(0.011)	(0.008)	(0.003)	(0.057)	(0.014)	(0.014)	(0.004)
TRMM satellite rainfall measure	0.639***	0.513***	0.714***	0.619***	–	–	–	–
	(0.018)	(0.012)	(0.015)	(0.013)	–	–	–	–
GPCP satellite rainfall measure	–	–	–	–	0.790***	1.073***	0.843***	1.233***
	–	–	–	–	(0.044)	(0.078)	(0.055)	(0.096)
(0.5 × 0.5) Grid Cell FE	No	Yes	No	Yes	No	Yes	No	Yes
Annual TE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	9893	9893	9893	9893	9893	9893	9893	9893
R-squared	0.578	0.446	0.604	0.490	0.555	0.494	0.541	0.503

[a]The unit of observation is a cell of resolution 0.5 × 0.5 degrees in a given year. The panel contains 761 cells covering DRC between 1998 and 2010. In columns 3, 4, 7, and 8, all rainfall variables are in log. Robust standard errors are clustered at the (0.5 × 0.5) cell level in columns 1–4 and at the (2.5 × 2.5) cell level in columns 5–8. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

TABLE B.VIII

ALTERNATIVE RAINFALL DATA[a]

Column in baseline Table I	4 (All Evts.)	5 (Battles)	4 (All Evts.)	5 (Battles)	4 (All Evts.)	5 (Battles)
Definition "large group"	$d^- \geq 10$		$d^- \geq 6$		Not Included	
	(1)	(2)	(3)	(4)	(5)	(6)
			Panel a (TRMM)			
Total Fight. Enemies (TFE)	0.025	0.084**	0.053*	0.102**	0.131**	0.197***
	(0.029)	(0.042)	(0.029)	(0.049)	(0.060)	(0.074)
Total Fight. Allies (TFA)	−0.092**	−0.146***	−0.142***	−0.194***	−0.126**	−0.187***
	(0.038)	(0.047)	(0.053)	(0.058)	(0.058)	(0.066)
Total Fight. Neutrals (TFN)	0.006	0.007	0.005	0.007	0.004	0.003
	(0.005)	(0.008)	(0.007)	(0.009)	(0.006)	(0.010)
Kleibergen–Paap F-stat	10.3	3.9	10.9	4.2	18.1	10.7
Hansen J (p-value)	0.50	0.68	0.55	0.72	0.39	0.72
Observations	1040	988	1040	988	1040	988
R-squared	0.586	0.534	0.495	0.403	0.431	0.264
			Panel b (GPCP)			
Total Fight. Enemies (TFE)	0.017	0.023	0.060	0.064	0.138**	0.155**
	(0.041)	(0.041)	(0.050)	(0.053)	(0.063)	(0.062)
Total Fight. Allies (TFA)	−0.068***	−0.097***	−0.130***	−0.186***	−0.106*	−0.144**
	(0.026)	(0.036)	(0.049)	(0.062)	(0.055)	(0.064)
Total Fight. Neutrals (TFN)	−0.009	−0.006	−0.010	−0.004	−0.002	0.001
	(0.008)	(0.009)	(0.011)	(0.013)	(0.012)	(0.013)
Kleibergen–Paap F-stat	7.2	7.4	6.4	11.3	10.1	10.0
Hansen J (p-value)	0.60	0.69	0.66	0.72	0.58	0.71
Observations	1040	988	1040	988	1040	988
R-squared	0.606	0.590	0.514	0.446	0.444	0.390
			Panel c (TRMM & GPCP)			
Total Fight. Enemies (TFE)	0.035	0.060**	0.069**	0.095***	0.128***	0.163***
	(0.026)	(0.026)	(0.029)	(0.033)	(0.049)	(0.050)
Total Fight Allies (TFA)	−0.063***	−0.099***	−0.108***	-0.157***	−0.087*	−0.129***
	(0.024)	(0.028)	(0.038)	(0.043)	(0.046)	(0.050)
Total Fight. Neutrals (TFN)	0.006	0.007	0.002	0.004	0.003	0.001
	(0.005)	(0.007)	(0.006)	(0.008)	(0.006)	(0.009)
Kleibergen–Paap F-stat	38.6	27.8	51.4	33.3	58.0	15.6
Hansen J (p-value)	0.89	0.93	0.93	0.94	0.85	0.87
Observations	1040	988	1040	988	1040	988
R-squared	0.609	0.586	0.528	0.464	0.471	0.398

[a]The dependent variable is total fighting. The unit of observation is an armed group in a given year. The panel contains 80 armed groups between 1998 and 2010. All regressions include group fixed effects and control for rainfall in the own group's territory, as well as the same additional controls and instruments as in the columns (4) and (5) of the baseline Table I. Robust standard errors corrected for Spatial HAC in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

TABLE B.IX

MONTE CARLO SIMULATIONS TESTING LINK MISMEASUREMENT[a]

		Probability of mismeasurement											
		0		0.01		0.1		0.2		0.5		1	
		TFA	TFE	TFA	TFE	TFA	TFE	TFA	TFE	TFA	TFE	TFA	TFE
Enmity links only	Mean	0.118	0.085	0.117	0.084	0.112	0.077	0.106	0.0713	0.102	0.046	0.119	0.002
	S.D.	0.012	0.011	0.012	0.011	0.014	0.013	0.019	0.022	0.024	0.020	0.018	0.015
Alliance links only	Mean	0.119	0.085	0.117	0.0843	0.107	0.075	0.095	0.066	0.059	0.055	0.000	0.083
	S.D.	0.012	0.011	0.014	0.011	0.019	0.015	0.023	0.019	0.027	0.022	0.008	0.008
Alliance & Enmity links	Mean	0.118	0.085	0.116	0.083	0.101	0.067	0.086	0.051	0.040	0.012	0.001	-0.001
	S.D.	0.012	0.010	0.014	0.012	0.022	0.019	0.028	0.023	0.037	0.028	0.017	0.014

[a]The table reports the mean and standard deviations (S.D.) of the Monte Carlo sampling distributions (1000 draws) of the baseline 2SLS estimates (col. 4, Table I) of TFE and TFA for different probabilities of network link mismeasurement. The data generating process is based on the coefficients = 0.114 for TFA and = 0.083 for TFE.

TABLE B.X

BOOTSTRAPPING ACLED EVENTS[a]

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
TFA	Mean	0.006	−0.231	−0.121	−0.114	−0.113	−0.138	−0.111	−0.132
	S.D.	0.007	0.039	0.022	0.023	0.023	0.033	0.023	0.013
TFE	Mean	0.065	0.131	0.067	0.088	0.086	0.087	0.089	0.135
	S.D.	0.005	0.024	0.016	0.016	0.016	0.019	0.016	0.011
TFN	Mean				0.003	0.003	0.013	0.004	0.005
	S.D.				0.003	0.003	0.008	0.003	0.002

[a]The table reports the mean and standard deviations (S.D.) of the Monte Carlo sampling distributions (1000 draws) of the estimates of TFE, TFA, TFN for all specifications of the baseline Table I. In each Monte Carlo draw, the analysis is conducted on a random sample of ACLED events drawn with replacement.

TABLE B.XI

MONTE CARLO SIMULATIONS TESTING ACLED EVENT MISMEASUREMENT[a]

	Probability of mismeasurement							
	0.01		0.1		0.2		0.5	
	TFA	TFE	TFA	TFE	TFA	TFE	TFA	TFE
Mean	−0.1139	0.0831	−0.1132	0.0836	−0.1109	0.0828	−0.1054	0.0813
S.D.	0.0009	0.0008	0.0028	0.0025	0.0044	0.0011	0.0063	0.0053

[a]The table reports the mean and standard deviations (S.D.) of the Monte Carlo sampling distributions (1000 draws) of the baseline 2SLS estimates (col. 4, Table I) of TFE and TFA for different probabilities of ACLED event mismeasurement for small groups. A group is defined to be small if $(d^- + d^+) < 3$, a condition that is satisfied for 49.7 percent of the groups in the sample.

TABLE B.XII

MULTINOMIAL LOGIT[a]

	CSF Surplus	
	0.000 (0.000)	
	Enmity link	Alliance link
Common enemy	−0.387**	0.425**
	(0.183)	(0.181)
Common allied	0.097	−0.529***
	(0.194)	(0.175)
Common allied and enemy	0.454***	−0.727***
	(0.119)	(0.161)
Geodistance	−0.003***	−0.004***
	(0.001)	(0.001)
Same ethnic group	0.842	0.865
	(0.595)	(0.601)
Same Hutu Tutsi	0.831	0.550
	(0.813)	(0.787)
Different Hutu Tutsi	1.295**	−0.914
	(0.596)	(1.148)
Zero Government	−1.977***	0.849
	(0.711)	(0.763)
Zero Foreign	1.514***	0.761
	(0.523)	(0.576)
Number of observations	9480	
Log likelihood	−445.92	

[a]Conditional logit estimator. The unit of observation is a pair of fighting groups in a given year. Alternatives correspond to Enmity (column 2), Alliance (column 3), and Neutrality as reference. Alternative-dependent group fixed effects are included. Standard errors in parentheses, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

TABLE B.XIII

RUM WITH TIME-VARYING NETWORK[a]

	Dep. Var.: Total Fighting		
	(1)	(2)	(3)
Total Fight. Enemies (TFE)	0.106***	0.114***	0.110***
	(0.033)	(0.035)	(0.038)
Total Fight. Allies (TFA)	−0.112**	−0.122**	−0.137**
	(0.050)	(0.053)	(0.062)
Total Fight. Neutrals (TFN)	0.002	0.001	0.011
	(0.006)	(0.006)	(0.023)
Kleibergen–Paap F-stat	4.0	3.6	5.1
Hansen J (p-value)	0.62	0.67	0.73
Observations	1040	1040	469
R-squared	0.554	0.579	0.597

[a]An observation is a given armed group in a given year. The panel contains 80 armed groups between 1998 and 2010. All regressions include group fixed effects and control for rainfall in the group's homeland. Robust standard errors corrected for Spatial HAC in parentheses. Column (1) replicates the baseline specification of the main table (column (4), Table I) but with RUM. Column (2) replicates column (1) of Table III but with RUM, while column (3) replicates column (4) of Table III but with RUM. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

TABLE B.XIV

WELFARE EFFECT OF REMOVING ARMED GROUPS WITH NETWORK RECOMPOSITION[a]

Group	Sh. Fight. (1)	−ΔRD (Exog. Netw.) (2)	−ΔRD (End. Netw.) (3)	Multipl. (End. Netw.) (4)	MAD (5)	−ΔRD due to rewiring (6)	New Enm. at Med. (7)	New All. at Med. (8)
RCD-G	0.087	0.151	0.137	1.6	0.025	−0.014	−2	−2
RCD-K	0.060	0.094	0.077	1.3	0.027	−0.017	−1	−1
Rwanda	0.053	0.066	0.105	1.9	0.039	0.039	−3	4
LRA	0.041	0.056	0.051	1.2	0.005	−0.005	0	−1
FDLR	0.066	0.055	0.058	0.9	0.008	0.004	0	1
Mayi-Mayi	0.057	0.046	0.083	1.5	0.024	0.037	−1	2
Uganda	0.043	0.043	0.066	1.5	0.036	0.023	−3	3
CNDP	0.043	0.041	0.041	0.9	0.008	0	0	0
MLC	0.031	0.039	0.053	1.7	0.016	0.014	−1	2
UPC	0.022	0.030	0.009	0.5	0.021	−0.021	−2	−1
Lendu Ethnic Mil.	0.024	0.022	0.047	2.0	0.020	0.025	1	−2
Mutiny FARDC	0.016	0.016	0.016	1.0	0	0	0	0
Interahamwe	0.014	0.014	0.032	2.3	0.024	0.018	−1	0
ADF	0.013	0.012	0.038	2.9	0.013	0.026	−1	1
FRPI	0.009	0.010	0.010	1.1	0	0	0	0

[a]The computation of the counterfactual equilibrium is based on the baseline point estimates of column 4 in Table I. The results are based on 1,000 Monte Carlo simulations of an endogenous network recomposition. For each group, we report the observed share of total fighting involving this group (col. 1); the counterfactual reduction in rent dissipation associated with its removal (exogenous network) (col. 2); the counterfactual reduction in rent dissipation associated with its removal with network recomposition (col. 3); a multiplier defined as the ratio of col. 3 over col. 1 (col. 4); the Median Absolute Deviation in reduction in RD across Monte Carlo draws (col. 5); the difference between col. 3 and col. 2 (col. 6); post-rewiring number of new enmities and alliances at the median Monte Carlo draw (cols. 7–8).

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