Econometrica Supplementary Material

SUPPLEMENT TO "MICRO TO MACRO: OPTIMAL TRADE POLICY WITH FIRM HETEROGENEITY"

(*Econometrica*, Vol. 88, No. 6, November 2020, 2739–2776)

ARNAUD COSTINOT
Department of Economics, MIT

ANDRÉS RODRÍGUEZ-CLARE
Department of Economics, UC Berkeley

IVÁN WERNING
Department of Economics, MIT

This supplement characterizes the structure of optimal taxes in environments with general technologies, general preferences, multiple countries, free entry, two-part tariffs, uniform taxes, and strategic foreign governments.

APPENDIX S.A: TECHNOLOGY

Assumptions. THERE ARE MULTIPLE FACTORS OF PRODUCTION indexed by n. $L_i \equiv \{L_{i,n}\} \geq 0$ now denotes the exogenous vector of factor endowments in country $i = H, F$, whereas $w_i \equiv \{w_{i,n}\}$ denotes the vector of factor prices in country i. For each origin country i and destination country j, a firm with blueprint φ that uses $l \equiv \{l_n\}$ units of the different factors can produce

$$q_{ij}(l, \varphi) = \left(\frac{\max\{0, g_{ij}(l, \varphi) - f_{ij}(\varphi)\}}{a_{ij}(\varphi)}\right)^{1/(1+\gamma_{ij})},$$

where $g_{ij}(\cdot, \varphi)$ is homogeneous of degree 1, strictly quasiconcave, and $\gamma_{ij} > -1/\sigma$. Firms choose their mix of factors to minimize their costs. This leads to the following demand for factor n by a firm with blueprint φ from country i selling q units in country j:

$$l_{ij,n}(q, w_i, \varphi) = \begin{cases} z_{ij,n}(w_i, \varphi)\left[a_{ij}(\varphi)q^{1+\gamma_{ij}} + f_{ij}(\varphi)\right], & \text{if } q > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $z_{ij,n}(w_i, \varphi)$ denotes the solution to $\min_l\{w_i \cdot l | g_{ij}(l, \varphi) \geq 1\}$. Without loss of generality, we normalize $g_{ij}(l, \varphi)$ so that at the equilibrium vector of factor prices, $\|z_{ij}(w_i, \varphi)\| = 1$ for all φ. For future reference, note that the cost function of a firm with blueprint φ from country i selling q units in country j is

$$c_{ij}(q, w_i, \varphi) = \sum_n w_{i,n} z_{ij,n}(w_i, \varphi)\left[a_{ij}(\varphi)q^{1+\gamma_{ij}} + f_{ij}(\varphi)\right].$$

Below, we let $c'_{ij}(q, w_i, \varphi)$ and $c''_{ij}(q, w_i, \varphi)$ denote the first and second derivatives of the cost function with respect to q.

Arnaud Costinot: costinot@mit.edu
Andrés Rodríguez-Clare: arc@berkeley.edu
Iván Werning: iwerning@mit.edu

 https://doi.org/10.3982/ECTA14763

Micro and Macro Problems. Under the previous assumptions, our micro and macro problems generalize as follows. Home's production possibility frontier is given by

$$Q_{HH}^{1/\mu}(Q_{HF}) \equiv \max_{\mathbf{l}_{HH},\mathbf{l}_{HF}} \int_{\Phi} N_H \big(q_{HH}\big(\{l_{HH,n}(\varphi)\},\varphi\big)\big)^{1/\mu}\, dG_H(\varphi),$$

$$N_H \sum_{j=H,F} \int_{\Phi} l_{Hj,n}(\varphi)\, dG_H(\varphi) \le L_{H,n}, \quad \text{for all } n,$$

$$\int_{\Phi} N_H \big(q_{HF}\big(\{l_{HF,n}(\varphi)\},\varphi\big)\big)^{1/\mu}\, dG_H(\varphi) \ge Q_{HF}^{1/\mu},$$

with $\mathbf{l}_{Hj} \equiv \{l_{Hj,n}(\varphi)\}$ for $j = H, F$. Foreign's offer curve is given by

$$Q_{FH}^{1/\mu}(Q_{HF}) \equiv \max_{\mathbf{q}_{FH},Q_{FF},w_F} \int_{\Phi} N_F q_{FH}^{1/\mu}(\varphi)\, dG_F(\varphi),$$

$$N_F \int \big[\mu c'_{FH}\big(q_{FH}(\varphi), w_F, \varphi\big) q_{FH}(\varphi)\big]\, dG_F(\varphi) = P_{FF}(Q_{FF})\, \mathrm{MRS}_F(Q_{HF}, Q_{FF}) Q_{HF},$$

$$L_{F,n} = L_{FF,n}(Q_{FF}, w_F) + N_F \int_{\Phi} l_{FH,n}\big(q_{FH}(\varphi), w_F, \varphi\big)\, dG_F(\varphi), \quad \text{for all } n,$$

$$\mu c'_{FH}\big(q_{FH}(\varphi), w_F, \varphi\big) q_{FH}(\varphi) \ge c_{FH}\big(q_{FH}(\varphi), w_F, \varphi\big),$$

where

$$L_{FF,n}(Q_{FF}, w_F) \equiv N_F \left[\int_{\Phi} l_{FF,n}\big(q_{FF}(\varphi|Q_{FF}, w_F), w_F, \varphi\big)\, dG_F(\varphi)\right],$$

with

$$q_{FF}(\varphi|Q_{FF}, w_F) = \begin{cases} \bar{q}_{FF}(\varphi|Q_{FF}, w_F), & \text{if } \mu c'_{FF}\big(\bar{q}_{FF}(\varphi), w_F, \varphi\big)\bar{q}_{FF}(\varphi|Q_{FF}, w_F) \\ & \qquad \ge c_{FF}\big(\bar{q}_{FF}(\varphi|Q_{FF}, w_F), w_F, \varphi\big), \\ 0, & \text{otherwise;} \end{cases}$$

$$p_{FF}(\varphi|Q_{FF}, w_F) = \begin{cases} \mu c'_{FF}\big(q_{FF}(\varphi), w_F, \varphi\big), & \text{if } \mu c'_{FF}\big(q_{FF}(\varphi), w_F, \varphi\big) q_{FF}(\varphi|Q_{FF}, w_F) \\ & \qquad \ge c_{FF}\big(q_{FF}(\varphi|Q_{FF}, w_F), w_F, \varphi\big), \\ \infty, & \text{otherwise;} \end{cases}$$

$$\bar{q}_{FF}(\varphi|Q_{FF}, w_F) = \big[\mu c'_{FF}\big(\bar{q}_{FF}(\varphi), w_F, \varphi\big)/P_{FF}(Q_{FF}, w_F)\big]^{-\sigma} Q_{FF},$$

$$P_{FF}(Q_{FF}, w_F) = \left(\int_{\Phi} N_F \big(p_{FF}(\varphi|Q_{FF}, w_F)\big)^{1-\sigma}\, dG_F(\varphi)\right)^{1/(1-\sigma)}.$$

Home's macro problem is given by

$$\max_{Q_{HH},Q_{FH},Q_{HF}} U_H(Q_{HH}, Q_{FH}),$$

$$Q_{FH} \le Q_{FH}(Q_{HF}),$$

$$Q_{HH} = Q_{HH}(Q_{HF}).$$

Optimal Taxes. Solving the first micro problem as we did in our baseline analysis, one can check that conditional on Q_{HF}, the optimal allocation coincides with the allocation in the decentralized equilibrium. This reflects the fact that, conditional on the size of an industry, the decentralized equilibrium under monopolistic competition with CES preferences is efficient, just like in our baseline analysis. It follows that the first part of Proposition 1 generalizes without qualification to environments with general technologies: (i) domestic taxes are uniform across all domestic producers and (ii) export taxes are uniform across all exporters.

Solving the second micro problem as we did in our baseline analysis, one can check that the first-order condition that characterizes the output level of an unconstrained firm that produces a nonzero amount is now given by

$$(1/\mu)\big[q^u_{FH}(\varphi)\big]^{1/\mu-1} = \lambda_T \mu c'_{FH}\big(q^u_{FH}(\varphi), w_F, \varphi\big)\left(1 + \frac{c''_{FH}\big(q^u_{FH}(\varphi), w_F, \varphi\big) q^u_{FH}(\varphi)}{c'_{FH}\big(q^u_{FH}(\varphi), w_F, \varphi\big)}\right) + \sum_n \lambda_{L,n} l'_{FH,n}\big(q^u_{FH}(\varphi), w_F, \varphi\big).$$

Given our restrictions on technology, this implies

$$(1/\mu)\big[q^u_{FH}(\varphi)\big]^{1/\mu-1} = c'_{FH}\big(q^u_{FH}(\varphi), w_F, \varphi\big)\left[\lambda_T \mu(1+\gamma_{FH}) + \frac{\sum_n \lambda_{L,n} z_{FH,n}(w_F, \varphi)}{\sum_n w_{F,n} z_{FH,n}(w_F, \varphi)}\right].$$

For all unconstrained firms with the same factor intensity z, that is, such that $z_{FH}(w_F, \varphi) = z$, optimal import tariffs must therefore be constant.

Turning to constrained firms, the one-dimensional subproblem of finding the amount of foreign imports of variety φ is now

$$\max_q q^{1/\mu} - \lambda_T \mu c'_{FH}(q, w_F, \varphi) q - \sum_n \lambda_{L,n} l_{FH,n}(q, w_F, \varphi),$$

$$\mu c'_{FH}(q, w_F, \varphi) q \geq c_{FH}(q, w_F, \varphi).$$

Focusing again on a subset of firms with the same factor intensity z, appealing to our restrictions on technology and changing variables to $\tilde{q} = q^{1+\gamma_{FH}}$, $\tilde{\mu}_F = (1+\gamma_{FH})\mu$, $\tilde{\lambda}_L(z) = \sum_n \lambda_{L,n} z_{FH,n}(w_F, \varphi)$, and $\tilde{\lambda}_T(z) = \lambda_T \sum_n w_{F,n} z_{FH,n}(w_F, \varphi)$, this can be restated as

$$\max_{\tilde{q}} \tilde{q}^{1/\tilde{\mu}_F} - \tilde{\lambda}_T(z)\tilde{\mu}_F a_{FH}(\varphi)\tilde{q} - \tilde{\lambda}_L(z)\big[a_{FH}(\varphi)\tilde{q} + f_{FH}(\varphi)\big],$$

$$\tilde{\mu}_F \tilde{q} a_{FH}(\varphi) \geq \tilde{q} a_{FH}(\varphi) + f_{FH}(\varphi).$$

This is the same problem as in our baseline analysis. For firms with the same factor intensity z, we therefore again have the implication that tariffs should be lower for a non-empty set of less profitable firms.

APPENDIX S.B: PREFERENCES

Assumptions. The utility function of the representative agent in each country is given by

$$U_j = U_j\big(\{Q^k_{Hj}, Q^k_{Fj}\}\big),$$

$$Q^k_{ij} = \left[\int_\Phi N^k_i \big(q^k_{ij}(\varphi)\big)^{1/\mu^k}\, dG^k_i(\varphi)\right]^{\mu^k},$$

with $\mu^k \equiv \sigma^k/(\sigma^k - 1)$ and $\sigma^k > 1$ for all k.

Micro and Macro Problems. Under the previous assumptions, our micro and macro problems generalize as follows. Home's production possibility frontier is given by

$$L_H\big(\{Q^k_{HH}, Q^k_{HF}\}\big) \equiv \min_{\{\mathbf{q}^k_{Hj}\}} \sum_{j,k} N^k_H \int_\Phi l_{Hj}\big(q^k_{Hj}(\varphi), \varphi\big)\, dG^k_H(\varphi),$$

$$N^k_H \int_\Phi \big(q^k_{Hj}(\varphi)\big)^{1/\mu^k}\, dG^k_H(\varphi) \geq \big(Q^k_{Hj}\big)^{1/\mu^k}, \quad \text{for all } j, k.$$

Fix a benchmark group k_0. Foreign's offer curve can be expressed as

$$\big(Q^{k_0}_{FH}\big(\{Q^k_{FH}\}_{k\neq k_0}, \{Q^k_{HF}\}\big)\big)^{1/\mu^{k_0}} \equiv \max_{\{\mathbf{q}^k_{FH}\},\{Q^k_{FF}\}} \int_\Phi N^{k_0}_F \big(q^{k_0}_{FH}(\varphi)\big)^{1/\mu^{k_0}}\, dG^{k_0}_F(\varphi),$$

$$\sum_k N^k_F \int \big[\mu a_{FH}(\varphi) q^k_{FH}(\varphi)\big]\, dG^k_F(\varphi) = \sum_k P^k_{FF}\big(Q^k_{FF}\big)\, \mathrm{MRS}^k_{HF}\big(\{Q^{k'}_{HF}, Q^{k'}_{FF}\}\big) Q^k_{HF},$$

$$\mathrm{MRS}^{kk_0}_F\big(\{Q^{k'}_{HF}, Q^{k'}_{FF}\}\big) = P^k_{FF}\big(Q^k_{FF}\big)/P^{k_0}_{FF}\big(Q^{k_0}_{FF}\big), \quad \text{for all } k \neq k_0,$$

$$L_F = \sum_k \left[L^k_{FF}\big(Q^k_{FF}\big) + N^k_F \int_\Phi l_{FH}\big(q^k_{FH}(\varphi), \varphi\big)\, dG^k_F(\varphi)\right],$$

$$\int_\Phi N^k_F \big(q_{FH}(\varphi)\big)^{1/\mu^k}\, dG^k_F(\varphi) \geq \big(Q^k_{FH}\big)^{1/\mu^k}, \quad \text{for all } k \neq k_0,$$

$$\mu a_{FH}\big(q^k_{FH}(\varphi), \varphi\big) q^k_{FH}(\varphi) \geq l_{FH}\big(q^k_{FH}(\varphi), \varphi\big),$$

where $\mathrm{MRS}^k_{HF}(\{Q^{k'}_{HF}, Q^{k'}_{FF}\}) \equiv (\partial U_F/\partial Q^k_{HF})/(\partial U_F/\partial Q^k_{FF})$ is the marginal rate of substitution between Home's goods and Foreign's goods within group k in Foreign and $\mathrm{MRS}^{kk_0}_F(\{Q^{k'}_F\}) \equiv (\partial U_F/\partial Q^k_{FF})/(\partial U_F/\partial Q^{k_0}_{FF})$ is the marginal rate of substitution between Foreign's goods in groups k and k_0 in Foreign. Home's macro problem is given by

$$\max_{\{Q^k_{HH}\},\{Q^k_{FH}\},\{Q^k_{HF}\}} U_H\big(\{Q^k_{HH}, Q^k_{FH}\}\big),$$

$$Q^{k_0}_{FH} \leq Q^{k_0}_{FH}\big(\{Q^k_{FH}\}_{k\neq k_0}, \{Q^k_{HF}\}\big),$$

$$L_H\big(\{Q^k_{HH}, Q^k_{HF}\}\big) = L_H.$$

Optimal Taxes. Using the same arguments as in the main text, group by group, one can show that our qualitative results about the optimal structure of micro-level taxes continue to hold. Specifically, within each group, domestic taxes are uniform across all domestic producers, export taxes are uniform across all exporters with each group, and import taxes are lower on the least profitable exporters from Foreign. Like in our baseline analysis, the set of foreign firms for which there is positive discrimination is non-empty if and only if the Lagrange multiplier on the trade balance condition, λ_T, is strictly positive. Compared to our baseline analysis, as well as the previous and subsequent extensions, the only difference is that we can no longer rule out situations where λ_T is weakly negative.

APPENDIX S.C: NUMBER OF COUNTRIES

Assumptions. Consider a strict generalization of our baseline environment with arbitrarily many countries. Home remains the only strategic country, whereas all countries $i \neq H$ are passive.

Micro and Macro Problems. Under the previous assumptions, our micro and macro problems generalize as follows. Home's production possibility frontier is given by

$$L_H(\{Q_{Hj}\}) \equiv \min_{\{\mathbf{q}_{Hj}\}} N_H\left[\sum_j \int_\Phi l_{Hj}(q_{Hj}(\varphi), \varphi)\, dG_H(\varphi)\right],$$

$$N_H \int_\Phi (q_{Hj}(\varphi))^{1/\mu}\, dG_H(\varphi) \geq (Q_{Hj})^{1/\mu}, \quad \text{for all } j.$$

Fix a benchmark foreign country $i_0 \neq H$. We use labor in country i_0 as our numeraire, $w_{i_0} = 1$. The offer curve from the rest of the world can be expressed as

$$\left(Q_{i_0 H}(\{Q_{Hj}\}_{j\neq H}, \{Q_{iH}\}_{i\neq i_0, H})\right)^{1/\mu} \equiv \max_{\{\mathbf{q}_{iH}\}, \{Q_{ij}\}_{i\neq H, j\neq H}, \{w_i\}_{i\neq i_0, H}} \int_\Phi N_{i_0}(q_{i_0 H}(\varphi))^{1/\mu}\, dG_{i_0}(\varphi),$$

$$N_i \int [\mu w_i a_{iH}(\varphi) q_{iH}(\varphi)]\, dG_i(\varphi) + \sum_{j\neq H} P_{ij}(Q_{ij}, w_i) Q_{ij}$$

$$= P_{ii}(Q_{ii}, w_i)\, \mathrm{MRS}_{Hi}(Q_{Hi}, Q_{ii}) Q_{Hi} + \sum_{j\neq H} P_{ji}(Q_{ji}, w_j) Q_{ji}, \quad \text{for all } i \neq H,$$

$$\mathrm{MRS}_{ji}(Q_{ji}, Q_{ii}) = P_{ji}(Q_{ji}, w_j)/P_{ii}(Q_{ii}, w_i), \quad \text{for all } i \neq H \text{ and } j \neq H,$$

$$L_i = \sum_{j\neq H} L_{ij}(Q_{ij}, w_i) + N_i \int_\Phi l_{iH}(q_{iH}(\varphi), \varphi)\, dG_i(\varphi), \quad \text{for all } i \neq H,$$

$$\int_\Phi N_i(q_{iH}(\varphi))^{1/\mu}\, dG_i(\varphi)) \geq Q_{iH}^{1/\mu}, \quad \text{for all } i \neq i_0, H,$$

$$\mu a_{iH}(\varphi) q_{iH}(\varphi) \geq l_{iH}(q_{iH}(\varphi), \varphi), \quad \text{for all } i \neq H,$$

where $\mathrm{MRS}_{ji}(Q_{ji}, Q_{ii}) \equiv (Q_{ji}/Q_{ii})^{-1/\sigma}$ is the country i's marginal rate of substitution between goods from country j and its own goods, and for any $i \neq H$ and $j \neq H$, the price indices, $P_{ij}(Q_{ij}, w_i)$, and employment levels, $L_{ij}(Q_{ij}, w_i)$, associated with the sales from i

to j are such that

$$P_{ij}(Q_{ij}, w_i) = \left(\int_{\Phi} N_i\big(p_{ij}(\varphi|Q_{ij}, w_i)\big)^{1-\sigma} \, dG_i(\varphi)\right)^{1/(1-\sigma)},$$

$$L_{ij}(Q_{ij}, w_i) \equiv N_i\left[\int_{\Phi} l_{ij}\big(q_{ij}(\varphi|Q_{ij}, w_i), \varphi\big) \, dG_i(\varphi)\right],$$

with

$$q_{ij}(\varphi|Q_{ij}, w_i) = \begin{cases} \bar{q}_{ij}(\varphi|Q_{ij}, w_i), & \text{if } \mu a_{ij}(\varphi)\bar{q}_{ij}(\varphi|Q_{ij}, w_i) \geq l_{ij}\big(\bar{q}_{ij}(\varphi|Q_{ij}, w_i), \varphi\big), \\ 0, & \text{otherwise;} \end{cases}$$

$$p_{ij}(\varphi|Q_{ij}, w_i) = \begin{cases} \mu w_i a_{ij}(\varphi), & \text{if } \mu a_{ij}(\varphi) q_{ij}(\varphi|Q_{ij}, w_i) \geq l_{ij}\big(\bar{q}_{ij}(\varphi|Q_{ij}, w_i), \varphi\big), \\ \infty, & \text{otherwise;} \end{cases}$$

$$\bar{q}_{ij}(\varphi|Q_{FF}, w_F) = \big[\mu w_i a_{ij}(\varphi)/P_{ij}(Q_{ij}, w_i)\big]^{-\sigma} Q_{ij}.$$

Home's macro problem is given by

$$\max_{\{Q_{Hi}, Q_{iH}\}} U_H\big(\{Q_{iH}\}\big),$$

$$Q_{i_0 H} \leq Q_{i_0 H}\big(\{Q_{Hi}\}_{i \neq i_0}, \{Q_{iH}\}_{i \neq i_0}\big),$$

$$L_H\big(\{Q_{Hi}\}\big) = L_H.$$

Optimal Taxes. Using the same arguments as in the main text, country by country, one can show that our qualitative results about the optimal structure of micro-level taxes are unchanged: taxes should be uniform across Home firms selling to the same destination country, whereas import taxes should be lower for a non-empty set of the least profitable exporters from any given origin country.

APPENDIX S.D: FREE ENTRY

Assumptions. Producing any variety in country i requires an overhead fixed entry cost in terms of domestic labor. Firms are heterogeneous in their fixed entry costs. $N_i(f^e)$ denotes the measure of firms with entry costs below f^e and $f_i^e(\cdot)$ denotes the inverse of $N_i(\cdot)$. Once firms have paid the overhead fixed cost, they randomly draw a blueprint $\varphi \in \Phi$ from the same distribution G_i. A decentralized equilibrium with taxes is composed of schedules of output, $\mathbf{q}_{ij} \equiv \{q_{ij}(\varphi)\}$, schedules of prices, $\mathbf{p}_{ij} \equiv \{p_{ij}(\varphi)\}$, aggregate output levels, Q_{ij}, aggregate price indices, P_{ij}, wages, w_i, and measures of entrants, N_i, such that

$$q_{ij}(\varphi) = \begin{cases} \bar{q}_{ij}(\varphi), & \text{if } \mu a_{ij}(\varphi)\bar{q}_{ij}(\varphi) \geq l_{ij}\big(\bar{q}_{ij}(\varphi), \varphi\big), \\ 0, & \text{otherwise;} \end{cases} \tag{S.D.1}$$

$$p_{ij}(\varphi) = \begin{cases} \bar{p}_{ij}(\varphi), & \text{if } \mu a_{ij}(\varphi) q_{ij}(\varphi) \geq l_{ij}\big(q_{ij}(\varphi), \varphi\big), \\ \infty, & \text{otherwise;} \end{cases} \tag{S.D.2}$$

$$Q_{Hj}, Q_{Fj} \in \arg\max_{\tilde{Q}_{Hj}, \tilde{Q}_{Fj}} \left\{ U_j(\tilde{Q}_{Hj}, \tilde{Q}_{Fj}) \Big| \sum_{i=H,F} P_{ij}\tilde{Q}_{ij} = w_j L_j + \Pi_j \right.$$

$$- \int_0^{f_j^e(N_j)} w_j f^e dN_j(f^e) + T_j \Big\}, \tag{S.D.3}$$

$$P_{ij}^{1-\sigma} = \int_\Phi N_i \big[\big(1 + t_{ij}(\varphi)\big) p_{ij}(\varphi)\big]^{1-\sigma} dG_i(\varphi), \tag{S.D.4}$$

$$f_i^e(N_i) = \sum_{j=H,F} \int_\Phi \big[\mu a_{ij}(\varphi) q_{ij}(\varphi) - l_{ij}\big(q_{ij}(\varphi), \varphi\big)\big] dG_i(\varphi), \tag{S.D.5}$$

$$\Pi_i = N_i \sum_{j=H,F} \int_\Phi \big[\mu w_i a_{ij}(\varphi) q_{ij}(\varphi) - w_i l_{ij}(q_{ij}(\varphi), \varphi)\big] dG_i(\varphi), \tag{S.D.6}$$

$$L_i = N_i \Bigg[\sum_{j=H,F} \int_\Phi l_{ij}\big(q_{ij}(\varphi), \varphi\big) dG_i(\varphi)\Bigg] + \int_0^{f_i^e(N_i)} f^e \, dN_i\big(f^e\big), \tag{S.D.7}$$

$$T_i = \sum_{j=H,F} \Bigg[\int_\Phi N_j t_{ji}(\varphi) p_{ji}(\varphi) q_{ji}(\varphi) \, dG_j(\varphi)$$

$$- \int_\Phi N_i s_{ij}(\varphi) p_{ij}(\varphi) q_{ij}(\varphi) \, dG_i(\varphi)\Bigg]. \tag{S.D.8}$$

Micro and Macro Problems. Under the previous assumptions, our micro and macro problems generalize as follows. Home's production possibility frontier is given by

$$L_H(Q_{HH}, Q_{HF}) \equiv \min_{\mathbf{q}_{HH}, \mathbf{q}_{HF}, N_H} N_H \Bigg[\sum_{j=H,F} \int_\Phi l_{Hj}\big(q_{Hj}(\varphi), \varphi\big) \, dG_H(\varphi)\Bigg] + \int_0^{f_H^e(N_H)} f^e \, dN_H\big(f^e\big),$$

$$\int_\Phi N_H\big(q_{Hj}(\varphi)\big)^{1/\mu} \, dG_H(\varphi) \geq Q_{Hj}^{1/\mu}, \quad \text{for } j = H, F.$$

Foreign's offer curve is given by

$$Q_{FH}^{1/\mu}(Q_{HF}) \equiv \max_{\mathbf{q}_{FH}, Q_{FF}, N_F} \int_\Phi N_F q_{FH}^{1/\mu}(\varphi) \, dG_F(\varphi),$$

$$N_F \int \mu a_{FH}(\varphi) q_{FH}(\varphi) \, dG_F(\varphi) = P_{FF}(Q_{FF}, N_F) \, \mathrm{MRS}_F(Q_{HF}, Q_{FF}) Q_{HF}$$

$$f_F^e(N_F) = \pi_{FF}(Q_{FF}, N_F) + \int \big[\mu a_{FH}(\varphi) q_{FH}(\varphi) - l_{FH}\big(q_{FH}(\varphi), \varphi\big)\big] \, dG_F(\varphi),$$

$$L_F = \int_0^{f_F^e(N_F)} f^e \, dN_F\big(f^e\big) + L_{FF}(Q_{FF}, N_F) + N_F \int_\Phi l_{FH}\big(q_{FH}(\varphi), \varphi\big) \, dG_F(\varphi),$$

$$\mu a_{FH}(\varphi) q_{FH}(\varphi) \geq l_{FH}\big(q_{FH}(\varphi), \varphi\big),$$

where expected profits and total employment associated with the local sales of foreign firms, $\pi_{FF}(Q_{FF}, N_F)$ and $L_{FF}(Q_{FF}, N_F)$, are given by

$$\pi_{FF}(Q_{FF}, N_F) \equiv \int_\Phi \mu a_{FF}(\varphi) q_{FF}(\varphi | Q_{FF}, N_F) \, dG_F(\varphi)$$

$$- \int_{\Phi} l_{FF}\big(q_{FF}(\varphi|Q_{FF}, N_F), \varphi\big)\, dG_F(\varphi), \tag{S.D.9}$$

$$L_{FF}(Q_{FF}, N_F) \equiv N_F\bigg[\int_{\Phi} l_{FF}\big(q_{FF}(\varphi|Q_{FF}, N_F), \varphi\big)\, dG_F(\varphi)\bigg], \tag{S.D.10}$$

with

$$q_{FF}(\varphi|Q_{FF}, N_F) = \begin{cases} \bar{q}_{FF}(\varphi|Q_{FF}, N_F), & \text{if } \mu a_{FF}(\varphi)\bar{q}_{FF}(\varphi|Q_{FF}, N_F) \\ & \quad \geq l_{FF}\big(\bar{q}_{FF}(\varphi|Q_{FF}, N_F), \varphi\big), \\ 0, & \text{otherwise;} \end{cases} \tag{S.D.11}$$

$$p_{FF}(\varphi|Q_{FF}, N_F) = \begin{cases} \mu a_{FF}(\varphi), & \text{if } \mu a_{FF}(\varphi) q_{FF}(\varphi|Q_{FF}, N_F) \\ & \quad \geq l_{Fj}\big(q_{FF}(\varphi|Q_{FF}, N_F), \varphi\big), \\ \infty, & \text{otherwise;} \end{cases} \tag{S.D.12}$$

$$\bar{q}_{FF}(\varphi|Q_{FF}, N_F) = \big[\mu a_{FF}(\varphi)/P_{FF}(Q_{FF}, N_F)\big]^{-\sigma} Q_{FF}, \tag{S.D.13}$$

$$P_{FF}(Q_{FF}, N_F) = \bigg(\int_{\Phi} N_F\big(p_{FF}(\varphi|Q_{FF}, N_F)\big)^{1-\sigma}\, dG_F(\varphi)\bigg)^{1/(1-\sigma)}. \tag{S.D.14}$$

Home's macro problem is given by

$$\max_{Q_{HH}, Q_{FH}, Q_{HF}} U_H(Q_{HH}, Q_{FH}),$$
$$Q_{FH} \leq Q_{FH}(Q_{HF}),$$
$$L_H(Q_{HH}, Q_{HF}) = L_H.$$

Optimal Taxes. Solving the first micro problem as we did in our baseline analysis, one can check that conditional on Q_{HH}, Q_{HF}, and N_H, the optimal allocation again coincides with the allocation in the decentralized equilibrium:

$$q_{Hj}(\varphi|Q_{HH}, Q_{HF}, N_H) = \begin{cases} \big(\mu a_{Hj}(\varphi)/\lambda_{Hj}\big)^{-\sigma}, & \text{if } \varphi \in \Phi_{Hj}, \\ 0, & \text{otherwise,} \end{cases} \tag{S.D.15}$$

with $\Phi_{Hj} \equiv \{\varphi : (\mu - 1) a_{Hj}(\varphi)(\mu a_{Hj}(\varphi)/\lambda_{Hj})^{-\sigma} \geq f_{Hj}(\varphi)\}$ the set of domestically produced varieties sold in country j. The only difference compared to our baseline analysis is the outer problem that minimizes over N_H. At an interior solution, the derivative of the value function associated with the inner problem should be equal to zero. By the Envelope theorem, this condition simplifies into

$$f_H^e(N_H) = \sum_{j=H,F} \int_{\Phi} \big(\lambda_{Hj}\big(q_{Hj}(\varphi|Q_{HH}, Q_{HF}, N_H)\big)^{1/\mu}$$
$$- l_{Hj}\big(q_{Hj}(\varphi|Q_{HH}, Q_{HF}, N_H), \varphi\big)\big)\, dG_H(\varphi). \tag{S.D.16}$$

This determines the optimal measure of domestic entrants, $N_H(Q_{HH}, Q_{HF})$. The optimal micro quantities are then given by $q_{Hj}(\varphi|Q_{HH}, Q_{HF}) \equiv q_{Hj}(\varphi|Q_{HH}, Q_{HF}, N_H(Q_{HH}, Q_{HF}))$.

It follows that the first part of Proposition 1 also generalizes without qualification to environments with free entry: (i) domestic taxes are uniform across all domestic producers and (ii) export taxes are uniform across all exporters.

To solve the second micro problem, we proceed as we did in our baseline analysis to express optimal imports conditional on Q_{HF}, Q_{FF} and N_F as

$$q_{FH}(\varphi|Q_{HF}, Q_{FF}, N_F) = \begin{cases} \big(\mu\chi^E_{FH}a_{FH}(\varphi)\big)^{-\sigma}, & \text{if } \varphi \in \Phi^u_{FH}, \\ f_{FH}(\varphi)/\big((\mu-1)a_{FH}(\varphi)\big), & \text{if } \varphi \in \Phi^c_{FH}, \\ 0, & \text{otherwise}, \end{cases} \tag{S.D.17}$$

with $\chi^E_{FH} \equiv \lambda_L + \lambda_T + (\mu - 1)(\lambda_T + \lambda_E/N_F) > 0$, where λ_E is the Lagrange multiplier associated with the free entry condition in Foreign, and the two sets of imported varieties defined by

$$\Phi^u_{FH} \equiv \big\{\varphi : \theta_{FH}(\varphi) \in \big[\big(\max\big\{(\lambda_L - \lambda_E/N_F)/\chi^E_{FH}, 1\big\}\big)^{1/\sigma}\chi^E_{FH}, \infty\big)\big\},$$
$$\Phi^c_{FH} \equiv \big\{\varphi : \theta_{FH}(\varphi) \in \big[\lambda_L + \lambda_T, \chi^E_{FH}\big)\big\}.$$

Since $\chi^E_{FH} > 0$, the constrained set Φ^c_{FH} is non-empty if and only if $\lambda_T + \lambda_E/N_F > 0$. To show that import taxes remain lower for a non-empty set of the least profitable exporters from Foreign under free entry, it is therefore sufficient to establish that $\lambda_T + \lambda_E/N_F > 0$.

We proceed by contradiction. Suppose that $\lambda_T + \lambda_E/N_F \leq 0$. Consider the Lagrangian,

$$\begin{aligned}\mathcal{L}(\mathbf{q}_{FH}, Q_{HF}, Q_{FF}, N_F) = {} & \int_\Phi N_F q^{1/\mu}_{FH}(\varphi)\, dG_F(\varphi) \\ & + \lambda_T\bigg(P_{FF}(Q_{FF}, N_F)\,\mathrm{MRS}_F(Q_{HF}, Q_{FF})Q_{HF} \\ & - N_F \int \mu a_{FH}(\varphi)q_{FH}(\varphi)\, dG_F(\varphi)\bigg) \\ & + \lambda_E\bigg(f^e_F(N_F) - \pi_{FF}(Q_{FF}, N_F) \\ & - \int \big[\mu a_{FH}(\varphi)q_{FH}(\varphi) - l_{FH}\big(q_{FH}(\varphi), \varphi\big)\big]\, dG_F(\varphi)\bigg) \\ & + \lambda_L\bigg(L_F - \int_0^{f^e_F(N_F)} f^e\, dN_F\big(f^e\big) - L_{FF}(Q_{FF}, N_F) \\ & - \int_\Phi N_F l_{FH}\big(q_{FH}(\varphi), \varphi\big)\, dG_F(\varphi)\bigg).\end{aligned}$$

At an interior optimum, we must have

$$\begin{aligned}&\frac{\partial\mathcal{L}\big(\big\{q_{FH}(\varphi|Q_{HF}, Q_{FF}, N_F)\big\}, Q_{HF}, Q_{FF}, N_F\big)}{\partial N_F} \\ &\quad = \int_\Phi q^{1/\mu}_{FH}(\varphi|Q_{HF}, Q_{FF}, N_F)\, dG_F(\varphi) + \lambda_T\frac{\partial P_{FF}(Q_{FF}, N_F)}{\partial N_F}\,\mathrm{MRS}_F(Q_{HF}, Q_{FF})Q_{HF}\end{aligned}$$

$$
\begin{aligned}
&- \lambda_T \int \mu a_{FH}(\varphi) q_{FH}(\varphi|Q_{HF}, Q_{FF}, N_F)\, dG_F(\varphi) + \lambda_E\biggl(\frac{df_F^e(N_F)}{dN_F} - \frac{\partial \pi_{FF}(Q_{FF}, N_F)}{\partial N_F}\biggr) \\
&- \lambda_L\biggl(f_F^e(N_F) + \frac{\partial L_{FF}(Q_{FF}, N_F)}{\partial N_F} + \int_\Phi l_{FH}\bigl(q_{FH}(\varphi|Q_{HF}, Q_{FF}, N_F)\bigr), \varphi\biggr)\, dG_F(\varphi) \\
&= 0.
\end{aligned}
$$

Under the assumption that $\lambda_T + \lambda_E/N_F \leq 0$, Φ_{FH}^c is empty. Equation (S.D.17) therefore implies

$$
\int_\Phi q_{FH}^{1/\mu}(\varphi|Q_{HF}, Q_{FF}, N_F))\, dG_F(\varphi) = \int_\Phi \mu \chi_{FH}^E a_{FH}(\varphi) q_{FH}(\varphi|Q_{HF}, Q_{FF}, N_F)\, dG_F(\varphi).
$$

Substituting this expression into the previous first-order condition, we get

$$
\begin{aligned}
&\lambda_T\biggl(\frac{\partial P_{FF}(Q_{FF}, N_F)}{\partial N_F} \mathrm{MRS}_F(Q_{HF}, Q_{FF}) Q_{HF} \\
&\quad + (\mu - 1) \int_\Phi \mu a_{FH}(\varphi) q_{FH}(\varphi|Q_{HF}, Q_{FF}, N_F)\, dG_F(\varphi)\biggr) \\
&\quad + \frac{\lambda_E}{N_F}\biggl(N_F \frac{df_F^e(N_F)}{dN_F} - N_F \frac{\partial \pi_{FF}(Q_{FF}, N_F)}{\partial N_F} \\
&\quad + (\mu - 1) \int_\Phi \mu a_{FH}(\varphi) q_{FH}(\varphi|Q_{HF}, Q_{FF}, N_F)\, dG_F(\varphi)\biggr) \\
&\quad - \lambda_L\biggl(f_F^e(N_F) + \frac{\partial L_{FF}(Q_{FF}, N_F)}{\partial N_F} \\
&\quad - \int_\Phi \bigl[\mu a_{FH}(\varphi) q_{FH}(\varphi|Q_{HF}, Q_{FF}, N_F) - l_{FH}\bigl(q_{FH}(\varphi), \varphi|Q_{HF}, Q_{FF}, N_F\bigr)\bigr]\, dG_F(\varphi)\biggr) \\
&= 0.
\end{aligned}
$$

Since $\lambda_T + \lambda_E/N_F \leq 0$, this further implies

$$
\begin{aligned}
&\lambda_T\biggl(\frac{\partial P_{FF}(Q_{FF}, N_F)}{\partial N_F} \mathrm{MRS}_F(Q_{HF}, Q_{FF}) Q_{HF}\biggr) + \lambda_E\biggl(\frac{df_F^e(N_F)}{dN_F} - \frac{\partial \pi_{FF}(Q_{FF}, N_F)}{\partial N_F}\biggr) \\
&\quad - \lambda_L\biggl(f_F^e(N_F) + \frac{\partial L_{FF}(Q_{FF}, N_F)}{\partial N_F} \\
&\quad - \int_\Phi \bigl[\mu a_{FH}(\varphi) q_{FH}(\varphi|Q_{HF}, Q_{FF}, N_F) - l_{FH}\bigl(q_{FH}(\varphi), \varphi|Q_{HF}, Q_{FF}, N_F\bigr)\bigr]\, dG_F(\varphi)\biggr) \\
&\geq 0.
\end{aligned}
\tag{S.D.18}
$$

To simplify the previous expression, note that

$$
\begin{aligned}
L_{FF}(Q_{FF}, N_F) = \min_{\mathbf{q}_{FF}} N_F \int_\Phi l_{FF}\bigl(q_{FF}(\varphi), \varphi|Q_{FF}, N_F\bigr)\, dG_F(\varphi), \\
\int_\Phi N_F q_{FF}^{1/\mu}(\varphi)\, dG_F(\varphi) \geq Q_{FF}^{1/\mu}.
\end{aligned}
$$

The counterpart of equation (S.D.15) for the foreign country therefore implies

$$\frac{\partial L_{FF}(Q_{FF}, N_F)}{\partial N_F} = -\bigg(\int_\Phi \big(\mu a_{FF}(\varphi) q_{FF}(\varphi|Q_{FF}, N_F) - l_{FF}\big(q_{FF}(\varphi|Q_{FF}, N_F), \varphi\big)\big)\, dG_F(\varphi)\bigg), \tag{S.D.19}$$

Substituting this expression into (S.D.18) and using the free entry condition (S.D.5) for country $i = F$, we obtain

$$\lambda_E\bigg(\frac{df_F^e(N_F)}{dN_F} - \frac{\partial \pi_{FF}(Q_{FF}, N_F)}{\partial N_F}\bigg) \geq -\lambda_T \frac{\partial P_{FF}(Q_{FF}, N_F)}{\partial N_F} \mathrm{MRS}_F(Q_{HF}, Q_{FF}) Q_{HF}. \tag{S.D.20}$$

To conclude, note that: (i) $\lambda_T > 0$, (ii) $\frac{\partial P_{FF}(Q_{FF}, N_F)}{\partial N_F} \leq 0$, (iii) $\frac{\partial \pi_{FF}(Q_{FF}, N_F)}{\partial N_F} \leq 0$, and (iv) $\frac{df_F^e(N_F)}{dN_F} \geq 0$. Condition (i) derives from the same argument as in Section 3.4. Condition (ii) can be established by contradiction. If $P_{FF}(Q_{FF}, N_F)$ were strictly increasing in N_F, then by equation (S.D.14), $p_{FF}(\varphi|Q_{FF}, N_F)$ would have to be strictly increasing in N_F for some goods, which, in turn, would require $\bar{q}_{FF}(\varphi|Q_{FF}, N_F)$ to be strictly decreasing in N_F for those goods. By equation (S.D.13), this would require $P_{FF}(Q_{FF}, N_F)$ to be strictly decreasing in N_F, a contradiction. Condition (iii) derives from the fact that $\pi_{FF}(q, \varphi) \equiv \mu a_{FF}(\varphi) q - l_{FF}(q, \varphi)$ is strictly increasing in q and that since $\partial P_{FF}(Q_{FF}, N_F)/\partial N_F \leq 0$, $q_{FF}(\varphi|Q_{FF}, N_F)$ is decreasing in N_F. Finally, condition (iv) derives from the fact that $f_F^e(\cdot)$ is the inverse of a strictly increasing function.

Given conditions (i)–(iv), equation (S.D.20) implies $\lambda_E \geq 0$. Since $\lambda_T > 0$, this further implies that $\lambda_T + \lambda_E / N_F > 0$, a contradiction. This establishes that import taxes remain lower for a non-empty set of the least profitable firms under free entry.

APPENDIX S.E: TWO-PART TARIFFS

Assumptions. In addition to the ad valorem taxes available in our baseline model, we now assume that Home's government also has access to firm-specific fixed fees: $\{s_{Hj}^f(\varphi), t_{jH}^f(\varphi)\}_{j=H,F}$. In order to sell any amount in Home's market, a firm with blueprint φ from country j needs to pay $t_{jH}^f(\varphi)$. Conversely, any firm from Home that sells any amount in market j receives $s_{Hj}^f(\varphi)$. Foreign's government is still passive.

Micro and Macro Problems. Under the previous assumptions, our first micro problem and our macro problem are unchanged. The only difference is Foreign's offer curve. It is given by

$$Q_{FH}^{1/\mu}(Q_{HF}) \equiv \max_{\mathbf{t}_{FH}^f, \mathbf{q}_{FH}, Q_{FF}} \int_\Phi N_F q_{FH}^{1/\mu}(\varphi)\, dG_F(\varphi),$$

$$N_F \int \big[\mu a_{FH}(\varphi) q_{FH}(\varphi) - t_{FH}^f(\varphi)\big]\, dG_F(\varphi) = P_{FF}(Q_{FF}) \mathrm{MRS}_F(Q_{HF}, Q_{FF}) Q_{HF},$$

$$L_F = L_{FF}(Q_{FF}) + N_F \int_\Phi l_{FH}\big(q_{FH}(\varphi), \varphi\big)\, dG_F(\varphi),$$

$$\mu a_{FH}(\varphi) q_{FH}(\varphi) - t_{FH}^f(\varphi) \geq l_{FH}\big(q_{FH}(\varphi), \varphi\big).$$

The Lagrangian associated with the one-dimensional subproblem of finding the amount of foreign imports of variety φ is

$$\max_{q,t} q^{1/\mu} - \lambda_T\big[\mu a_{FH}(\varphi)q - t\big] - \lambda_L l_{FH}(q,\varphi) + \lambda_\pi(\varphi)\big[\mu a_{FH}(\varphi)q - t - l_{FH}(q,\varphi)\big].$$

Linearity with respect to t implies $\lambda_\pi(\varphi) = \lambda_T$. Thus, this can be rearranged as

$$\max_{q} q^{1/\mu} - (\lambda_T + \lambda_L) l_{FH}(q,\varphi).$$

Like in the case of domestic output and exports, this leads to the first-best level of q given the Lagrange multipliers λ_T and λ_L, implying that the non-negativity of profits does not affect the level of imports.

Optimal Taxes. The first micro problem is unchanged, so the first part of Proposition 1 generalizes without qualification to environments with nonlinear taxes: (i) domestic taxes are uniform across all domestic producers and (ii) export taxes are uniform across all exporters.

Since the profitability constraint does not affect the level of q, we get the same variable taxes on all foreign exporters. Fixed fees, $\{t^f_{FH}(\varphi)\}$, however, vary. The same argument as in the baseline implies that $\lambda_T > 0$ and thus $\lambda_\pi(\varphi) > 0$. Hence, by complementary slackness,

$$t^f_{FH}(\varphi) = \big[\mu a_{FH}(\varphi) q_{FH}(\varphi) - l_{FH}\big(q_{FH}(\varphi), \varphi\big)\big].$$

This implies that the fees extract all producer surplus from selling to Home, and hence there is now positive discrimination across all firms, with lower fees on the least profitable firms.

APPENDIX S.F: UNIFORM TAXES

Assumptions. Suppose that Home's government is constrained to set uniform taxes: $t_{HF}(\varphi) = \bar{t}_{HF}$, $t_{HH}(\varphi) = \bar{t}_{HH}$, $s_{HF}(\varphi) = \bar{s}_{HF}$, and $s_{HH}(\varphi) = \bar{s}_{HH}$ for all φ. Foreign's government is still passive.

Micro and Macro Problems. Compared to our baseline analysis, the only difference is that the micro problems now include an additional constraint:

$$q_{ij}\big(\varphi'\big)/q_{ij}(\varphi) = \big(a_{ij}\big(\varphi'\big)/a_{ij}(\varphi)\big)^{-\sigma} \quad \text{for any } \varphi, \varphi' \text{ such that } q_{ij}\big(\varphi'\big), q_{ij}(\varphi) > 0. \qquad \text{(S.F.1)}$$

For varieties from Home that are sold in any market, $i = H$ and $j = H, F$, constraint (S.F.1) was already satisfied by the solution to our first micro problem. So the value of $L_H(Q_{HH}, Q_{HF})$ remains unchanged. In contrast, for foreign varieties that are imported by Home, $i = F$ and $j = H$, constraint (S.F.1) will bind at the solution to our second micro problem, leading to a new offer curve in Foreign. The other equations that characterize the solution to Home's relaxed planning problem are unchanged. In particular, one can still reduce Home's macro planning problem to

$$\max_{Q_{HH}, Q_{FH}, Q_{HF}} U_H(Q_{HH}, Q_{FH}),$$

$$Q_{FH} \le Q_{FH}(Q_{HF}),$$

$$L_H(Q_{HH}, Q_{HF}) = L_H.$$

Optimal Taxes. Starting from the macro problem, one can follow the exact same steps as in our baseline analysis to show that

$$\frac{(1+\bar{t}^*_{FH})/(1+\bar{t}^*_{HH})}{(1+\bar{s}^*_{HF})/(1+\bar{s}^*_{HH})} = 1/\eta^*. \tag{S.F.2}$$

Conditional on Q_{HF}, one can check that the decentralized equilibrium abroad must be such that

$$\mathrm{MRS}_F\big(Q_{HF}, Q_{FF}(Q_{FH})\big) = P_{HF}/P_{FF}, \tag{S.F.3}$$

$$\mathrm{MRT}_F\big(Q_{FH}, Q_{FF}(Q_{FH})\big) = \tilde{P}_{FH}/P_{FF}, \tag{S.F.4}$$

$$P_{HF}Q_{HF} = \tilde{P}_{FH}Q_{FH}, \tag{S.F.5}$$

with $\tilde{P}_{FH}$ the untaxed price of Home's imports, and $Q_{FF}(Q_{FH})$ given by the implicit solution of

$$L_F(Q_{FH}, Q_{FF}) = L_F. \tag{S.F.6}$$

By equations (S.F.3) and (S.F.4), Home's terms of trade can be expressed as

$$P(Q_{FH}, Q_{HF}) = \frac{\mathrm{MRS}_F\big(Q_{HF}, Q_{FF}(Q_{FH})\big)}{\mathrm{MRT}_F\big(Q_{FH}, Q_{FF}(Q_{FH})\big)}. \tag{S.F.7}$$

Combining equation (S.F.7) with the trade balance condition (S.F.5), we can describe Foreign's offer curve implicitly as

$$P(Q_{FH}, Q_{HF})Q_{HF} = Q_{FH}. \tag{S.F.8}$$

Totally differentiating the previous expression with respect to Home's aggregate exports and imports, Q_{HF} and Q_{FH}, we obtain

$$\eta = (1+\rho_{HF})/(1-\rho_{FH}), \tag{S.F.9}$$

where Home's terms-of-trade elasticities, $\rho_{ij} \equiv \partial \ln P(Q_{FH}, Q_{HF})/\partial \ln Q_{ij}$, can be computed using equation (S.F.7),

$$\rho_{HF} = -1/\sigma, \tag{S.F.10}$$

$$\rho_{FH} = -(1/x_{FF} - 1)/\sigma - \kappa_F, \tag{S.F.11}$$

with $x_{FF} \equiv P_{FF}Q_{FF}/(L_F + \Pi_F)$ and $\kappa_F \equiv d \ln \mathrm{MRT}_F(Q_{FH}, Q_{FF}(Q_{FH}))/d \ln Q_{FH}$.

Combining equation (S.F.2) with equations (S.F.9)–(S.F.11), we obtain

$$\frac{(1+\bar{t}^*_{FH})/(1+\bar{t}^*_{HH})}{(1+\bar{s}^*_{HF})/(1+\bar{s}^*_{HH})} = 1 + \frac{1+\sigma\kappa^*_F x^*_{FF}}{(\sigma-1)x^*_{FF}},$$

where κ^*_F and x^*_{FF} are the values κ_F, and x_{FF} evaluated at those taxes.

To conclude, we show that $\kappa_F^* < 0$ whenever G_F has strictly positive density around blueprints φ with profitability such that foreign firms are indifferent between selling and not selling in at least one market $j = H, F$. By equation (S.F.4), we know that

$$\mathrm{MRT}_F\big(Q_{FH}, Q_{FF}(Q_{FH})\big) = \frac{\left(\int_{\Phi_{FH}} \big(a_{FH}(\varphi)\big)^{1-\sigma}\, dG_F(\varphi)\right)^{1/(1-\sigma)}}{\left(\int_{\Phi_{FF}} \big(a_{FF}(\varphi)\big)^{1-\sigma}\, dG_F(\varphi)\right)^{1/(1-\sigma)}},$$

with the set of foreign varieties sold in market $j = H, F$ such that

$$\Phi_{Fj} = \left\{\varphi : (\mu - 1) a_{Fj}^{1-\sigma}(\varphi)\left(N_F \int_{\Phi_{Fj}} a_{Fj}^{1-\sigma}(\varphi)\, dG_F(\varphi)\right)^{-\mu} Q_{Fj} \geq f_{Fj}(\varphi)\right\}.$$

If Q_{Fj} increases, Φ_{Fj} expands and $(\int_{\Phi_{Fj}} (a_{Fj}(\varphi))^{1-\sigma}\, dG_F(\varphi))^{1/(1-\sigma)}$ decreases. Furthermore, if G_F has strictly positive density around blueprints φ with profitability such that foreign firms are indifferent between selling and not selling in market j, $(\int_{\Phi_{Fj}} (a_{Fj}(\varphi))^{1-\sigma}\, dG_F(\varphi))^{1/(1-\sigma)}$ must strictly decrease. Since labor market clearing requires Q_{FF} to be strictly decreasing in Q_{FH}, it follows that $\mathrm{MRT}_F(Q_{FH}, Q_{FF}(Q_{FH}))$ is strictly decreasing in Q_{FH} whenever G_F has strictly positive density around blueprints φ with profitability such that foreign firms are indifferent between selling and not selling in at least one market $j = H, F$.

APPENDIX S.G: NASH EQUILIBRIUM

Assumptions. Both governments are strategic and simultaneously set their taxes, taking the taxes of the other government as given. Namely, the government of country $i = H, F$ sets ad valorem taxes, $\mathbf{t}_{Hi} \equiv \{t_{Hi}(\varphi)\}$, $\mathbf{t}_{Fi} \equiv \{t_{Fi}(\varphi)\}$, $\mathbf{s}_{iH} \equiv \{s_{iH}(\varphi)\}$, $\mathbf{s}_{iF} \equiv \{s_{iF}(\varphi)\}$, in order to maximize the utility of the representative agent in country i, taking as given the ad valorem taxes, $\mathbf{t}_{Hj} \equiv \{t_{Hj}(\varphi)\}$, $\mathbf{t}_{Fj} \equiv \{t_{Fj}(\varphi)\}$, $\mathbf{s}_{jH} \equiv \{s_{jH}(\varphi)\}$, $\mathbf{s}_{jF} \equiv \{s_{jF}(\varphi)\}$, in country $j \neq i$. This leads to the following definition of a Nash equilibrium.

DEFINITION 1: At a Nash equilibrium, the government of country $i = H, F$ solves

$$\max_{\{T_j\}_{j=H,F}, \{\mathbf{t}_{ji}, \mathbf{s}_{ij}\}_{j=H,F}, w_i, \{\mathbf{q}_{ij}, Q_{ij}, \mathbf{p}_{ij}, P_{ij}\}_{i,j=H,F}} U_i(Q_{Hi}, Q_{Fi}),$$

subject to conditions (S.D.1)–(S.D.8) taking as given $\{\mathbf{t}_{j-i}, \mathbf{s}_{-ij}\}_{j=H,F}$.

Planning Problem. The problem faced by the two countries is symmetric. Without loss of generality, we focus on Home's problem. It can be expressed as

$$\max_{\mathbf{q}_{HH}, \mathbf{q}_{FH}, \mathbf{q}_{HF}, Q_{HH}, Q_{FF}, Q_{FH}, Q_{HF}} U_H(Q_{HH}, Q_{FH}),$$

$$\begin{aligned} N_F \mu \int \frac{a_{FH}(\varphi)}{1 + s_{FH}(\varphi)} q_{FH}(\varphi)\, dG_F(\varphi) &= N_H \int \frac{\big(q_{HF}(\varphi)\big)^{1/\mu}}{1 + t_{HF}(\varphi)} P_{FF}(Q_{FF}) \\ &\quad \times \mathrm{MRS}_F(Q_{HF}, Q_{FF}) Q_{HF}^{1/\sigma}\, dG_H(\varphi), \end{aligned} \tag{S.G.1}$$

$$L_{FF}(Q_{FF}) + \int N_F l_{FH}\big(q_{FH}(\varphi), \varphi\big)\, dG_F(\varphi) = L_F, \qquad \text{(S.G.2)}$$

$$\int_\Phi N_F\big(q_{FH}(\varphi)\big)^{1/\mu}\, dG_F(\varphi) = Q_{FH}^{1/\mu}, \qquad \text{(S.G.3)}$$

$$\mu a_{FH}(\varphi) q_{FH}(\varphi) \geq l_{FH}\big(q_{FH}(\varphi), \varphi\big), \qquad \text{(S.G.4)}$$

$$\int_\Phi N_H\big(q_{Hj}(\varphi)\big)^{1/\mu}\, dG_H(\varphi) = Q_{Hj}^{1/\mu}, \quad \text{for } j = H, F, \qquad \text{(S.G.5)}$$

$$N_H\left[\sum_j \int l_{Hj}\big(q_{Hj}(\varphi), \varphi\big)\, dG_H(\varphi)\right] \leq L_H, \qquad \text{(S.G.6)}$$

with

$$P_{FF}^{1-\sigma}(Q_{FF}) = \int_{\phi:\mu a_{FF}(\varphi)\bar{q}_{FF}(\varphi|Q_{FF}) \geq l_{FF}(\bar{q}_{FF}(\varphi|Q_{FF}),\varphi)} N_F\left[\frac{1+t_{FF}(\varphi)}{1+s_{FF}(\varphi)}\mu a_{FF}(\varphi)\right]^{1-\sigma} dG_F(\varphi),$$

$$L_{FF}(Q_{FF}) = \int_{\phi:\mu a_{FF}(\phi)\bar{q}_{FF}(\varphi|Q_{FF}) \geq l_{FF}(\bar{q}_{FF}(\varphi|Q_{FF}),\varphi)} N_F l_{FF}\big(\bar{q}_{FF}(\varphi|Q_{FF}), \varphi\big)\, dG_F(\varphi),$$

$$\bar{q}_{FF}(\varphi|Q_{FF}) = \left[\frac{1+t_{FF}(\varphi)}{1+s_{FF}(\varphi)}\frac{\mu a_{FF}(\varphi)}{P_{FF}(Q_{FF})}\right]^{-\sigma} Q_{FF}.$$

Noting that $\text{MRS}_F(Q_{HF}, Q_{FF})Q_{HF}^{1/\sigma} = Q_{FF}^{1/\sigma}$ is independent of Q_{HF}, we can solve Home's problem by first solving the relaxed problem,

$$\max_{\mathbf{q}_{HH},\mathbf{q}_{FH},\mathbf{q}_{HF},Q_{HH},Q_{FF},Q_{FH}} U_H(Q_{HH}, Q_{FH}),$$

$$N_F\mu \int \frac{a_{FH}(\varphi)}{1+s_{FH}(\varphi)} q_{FH}(\varphi)\, dG_F(\varphi) = N_H \int \frac{\big(q_{HF}(\varphi)\big)^{1/\mu}}{1+t_{HF}(\varphi)} P_{FF}(Q_{FF}) Q_{FF}^{1/\sigma}\, dG_H(\varphi), \qquad \text{(S.G.7)}$$

$$L_{FF}(Q_{FF}) + \int N_F l_{FH}\big(q_{FH}(\varphi), \varphi\big)\, dG_F(\varphi) = L_F, \qquad \text{(S.G.8)}$$

$$\int_\Phi N_F\big(q_{FH}(\varphi)\big)^{1/\mu}\, dG_F(\varphi) = Q_{FH}^{1/\mu}, \qquad \text{(S.G.9)}$$

$$\mu a_{FH}(\varphi) q_{FH}(\varphi) \geq l_{FH}\big(q_{FH}(\varphi), \varphi\big), \qquad \text{(S.G.10)}$$

$$\int_\Phi N_H\big(q_{HH}(\varphi)\big)^{1/\mu}\, dG_H(\varphi) = Q_{HH}^{1/\mu}, \qquad \text{(S.G.11)}$$

$$N_H\left[\sum_j \int l_{Hj}\big(q_{Hj}(\varphi), \varphi\big)\, dG_H(\varphi)\right] \leq L_H, \qquad \text{(S.G.12)}$$

and then setting Q_{HF} such that constraint (S.G.5) holds for $j = F$. To solve for the optimal micro quantities $\mathbf{q}_{HH}$, $\mathbf{q}_{HF}$, and $\mathbf{q}_{FH}$ associated with the previous problem, we can use the same argument as in our baseline analysis and solve a series of one-dimensional Lagrangian problems.

Let λ_T^H, λ_{HH}^H, and $\lambda_{LH}^H > 0$ denote the Lagrange multipliers associated with constraints (S.G.7), (S.G.11), and (S.G.12), respectively.[1] Superscripts H keep track of the fact that these Lagrange multipliers all correspond to country H's planning problem. The Lagrangian associated with the one-dimensional sub-problem for domestic sales and exports can be expressed as

$$\min_q l_{Hj}(q, \varphi) - \tilde{\lambda}_{Hj}^H(\varphi) q^{1/\mu},$$

with

$$\tilde{\lambda}_{HH}^H(\varphi) = \lambda_{HH}^H / \lambda_{LH}^H,$$
$$\tilde{\lambda}_{HF}^H(\varphi) = \lambda_T^H \big[P_{FF}(Q_{FF}) Q_{FF}^{1/\sigma}\big] / \big[\lambda_{LH}^H \big(1 + t_{HF}(\varphi)\big)\big].$$

Likewise, let λ_{LF}^H and $\lambda_{FH}^H > 0$ denote the Lagrange multipliers associated with constraints (S.G.8) and (S.G.9), respectively.[2] The Lagrangian associated with the one-dimensional sub-problem for imports can be expressed as

$$\max_q q^{1/\mu} - \tilde{\lambda}_T^H(\varphi) \mu a_{FH}(\varphi) q - \tilde{\lambda}_{LF}^H l_{FH}(q, \varphi),$$
$$\mu a_{FH}(\varphi) q \geq l_{FH}(q, \varphi),$$

with

$$\tilde{\lambda}_T^H(\varphi) = \lambda_T^H / \big[\big(1 + s_{FH}(\varphi)\big) \lambda_{FH}^H\big],$$
$$\tilde{\lambda}_{LF}^H = \lambda_{LF}^H / \lambda_{FH}^H.$$

To solve the two previous micro problems, one can simply substitute $\tilde{\lambda}_{HH}^H(\varphi)$, $\tilde{\lambda}_{HF}^H(\varphi)$, $\tilde{\lambda}_T^H(\varphi)$, and $\tilde{\lambda}_{LF}^H$ for λ_{HH}, λ_{HF}, λ_T, and λ_L into the solutions derived in Sections 3.2 and 3.3.

Nash Taxes. By comparing the solution to Home's planning problem to the decentralized equilibrium with taxes, as we did in Sections 4.1 and 4.2, we can characterize Home's best-response to a given schedule of Foreign's taxes and subsidies. Fix a benchmark variety $\varphi_{HH} \in \Phi_{HH}$ that is sold domestically, with $s_{HH}^{BR} \equiv s_{HH}^{BR}(\varphi_{HH})$ and $t_{HH}^{BR} \equiv t_{HH}^{BR}(\varphi_{HH})$; a benchmark variety $\varphi_{HF} \in \Phi_{HF}$ that is exported, with $s_{HF}^{BR} \equiv s_{HF}^{BR}(\varphi_{HF})$, and a benchmark variety $\varphi_{FH} \in \Phi_{FH}^u$ that is imported, with $t_{FH}^{BR} \equiv t_{FH}^{BR}(\varphi_{FH})$. One can check that

$$\big(1 + s_{HH}^{BR}(\varphi)\big) / \big(1 + t_{HH}^{BR}(\varphi)\big) = \big(1 + s_{HH}^{BR}\big) / \big(1 + t_{HH}^{BR}\big), \quad \text{if } \varphi \in \Phi_{HH},$$

[1] To see that $\lambda_{LH}^H > 0$, suppose, by contradiction, that $\lambda_{LH}^H = 0$. By complementary slackness, inequality (S.G.12) must then be slack. If so, one can strictly increase $q_{HH}(\varphi)$ for a positive measure of blueprints φ and, in turn, strictly increase Q_{HH} and U_H, a contradiction.

[2] To see that $\lambda_{FH}^H > 0$, note that at an interior optimum of Home's planning problem, the first-order condition with respect to Q_{FH} implies

$$\frac{dU_H(Q_{HH}, Q_{FH})}{dQ_{FH}} = \lambda_{FH}^H Q_{FH}^{1/\mu - 1} / \mu > 0.$$

$$s^{BR}_{HF}(\varphi) = s^{BR}_{HF}, \quad \text{if } \varphi \in \Phi_{HF},$$

$$t^{BR}_{FH}(\varphi) = \begin{cases} (1 + t^{BR}_{FH}) \dfrac{\lambda^H_{LF}(1 + s_{FH}(\varphi)) + \mu\lambda^H_T}{\lambda^H_{LF}(1 + s_{FH}(\varphi_{FH})) + \mu\lambda^H_T} - 1, & \text{if } \varphi \in \Phi^u_{FH}, \\ (1 + t^{BR}_{FH}) \dfrac{(1 + s_{FH}(\varphi))\lambda^H_{FH}\theta_{FH}(\varphi)}{\lambda^H_{LF}(1 + s_{FH}(\varphi_{FH})) + \mu\lambda^H_T} - 1, & \text{if } \varphi \in \Phi^c_{FH}, \end{cases}$$

Analogous expressions hold for the best-response policies in Foreign.

Let $\{s^{NE}_{ij}(\varphi), t^{NE}_{ij}(\varphi)\}$ denote the Nash taxes. At a Nash equilibrium, both Home and Foreign simultaneously play best-response. For domestic taxes, the equilibrium conditions for $\varphi \in \Phi_{HH}$ and $\varphi \in \Phi_{FF}$ immediately imply that, like in our baseline analysis, Nash domestic taxes should be uniform across firms. For trade taxes, we can simultaneously solve for Nash export taxes by Foreign and import tariffs at Home by using our characterization of $s^{BR}_{FH}(\varphi)$, evaluated at $t_{FH}(\varphi) = t^{BR}_{FH}(\varphi)$, and our characterization of $t^{BR}_{FH}(\varphi)$, evaluated at $s_{FH}(\varphi) = s^{BR}_{FH}(\varphi)$. This implies that

$$s^{NE}_{FH}(\varphi) = s^{NE}_{FH}, \quad \text{if } \varphi \in \Phi_{FH},$$

$$t^{NE}_{FH}(\varphi) = (1 + t^{NE}_{FH}) \min\left\{1, \frac{(1 + s^{NE}_{FH})\lambda^H_{FH}\theta_{FH}(\varphi)}{(\lambda^H_{LF}(1 + s^{NE}_{FH}) + \mu\lambda^H_T)}\right\} - 1, \quad \text{if } \varphi \in \Phi_{FH}.$$

Analogous expressions hold for Nash import tariffs in Foreign and export subsidies at Home.

We have already argued that $\lambda^H_{FH} > 0$. We must also have

$$\lambda^H_{LF}/\lambda^H_{FH} + (\mu\lambda^H_T)/[(1 + s^{NE}_{FH})\lambda^H_{FH}] > 0.$$

Otherwise, Home's optimal imports would be infinite for the same reason as in Section 3.2. Thus, import tariffs at Home strictly increase with the profitability of foreign exporters, $\theta_{FH}(\varphi)$ for all $\varphi \in \Phi^c_{FH}$, whereas export subsidies are constant, like in our baseline analysis.

To conclude, we now demonstrate that Φ^c_{FH} is non-empty so that import tariffs must be non-discriminatory over some range at a Nash equilibrium. For the one-dimensional Lagrangian problem for exports, $\min_q l_{HF}(q, \varphi) - \tilde{\lambda}^H_{HF}(\varphi)q^{1/\mu}$, to admit a nonzero solution for at least some φ, we know that

$$\tilde{\lambda}^H_{HF}(\varphi) = \lambda^H_T[P_{FF}(Q_{FF})Q^{1/\sigma}_{FF}]/[\lambda^H_{LH}(1 + t_{HF}(\varphi))] > 0.$$

This implies $\lambda^H_T > 0$. For the same reasons as in Section 4.4, it follows that Φ^c_{FH} is non-empty.

Co-editor Fabrizio Zilibotti handled this manuscript.

Manuscript received 5 October, 2016; final version accepted 6 May, 2020; available online 12 May, 2020.