

SUPPLEMENT TO “HOUSEHOLD LEVERAGE AND THE RECESSION”
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APPENDIX A: BASELINE ESTIMATES

THE PRIORS AND POSTERIOR estimates in the baseline estimation are given in Table **SI**. Kernel density plots are shown in the Not-for-Publication Appendix.

Priors. For the persistence and standard deviation of the AR(1) shocks, we use the same priors as [Smets and Wouters \(2007\)](#) used in their aggregate estimation. The persistence parameters are centered around 1/2. We use wide priors on the standard deviations of the shocks. For the wage and price Calvo stickiness parameters, we use a more diffuse prior than [Smets and Wouters \(2007\)](#) but centered around the same mode of $\theta_p = \theta_w = 1/2$. This is because we use a more recent sample and wider priors are consistent with a flattening of the aggregate Phillips curve. For the degree of idiosyncratic uncertainty α , we use a wide prior, centered around a value of 2.5. This prior is wide enough to allow the data to find strong or very weak effects of credit shocks, as we discuss in the paper.

Posteriors. We discard half of the draws in each chain as a burn-in. The convergence of the posterior distributions for each parameter is analyzed in the Not-for-Publication Appendix.

APPENDIX B: ROBUSTNESS EXERCISES

In this section, we present a number of robustness checks. In the Not-for-Publication Appendix, we report the full parameter estimates for these specifications. Table **SII** reports the various models’ implications for the contribution of state-level credit shocks to the relative changes of state-level employment and consumption. Table **SIII** reports the models’ aggregate implications.

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An expanded Not-for-Publication Appendix containing additional results is available at <https://callumjones.github.io>. The views expressed are those of the authors and not necessarily those of the Federal Reserve Board or the Federal Reserve System.

TABLE SI
ESTIMATED STRUCTURAL PARAMETERS, BASELINE SPECIFICATION.

Parameter	Prior				Posterior			
	Dist	Median	10%	90%	Mode	Mean	10%	90%
A. Structural Parameters								
α	N	2.5	1.5	3.8	3.23	3.44	3.05	3.93
θ_p	B	0.5	0.2	0.8	0.97	0.96	0.94	0.98
θ_w	B	0.5	0.2	0.8	0.86	0.86	0.82	0.89
B. Regional Shock Processes								
ρ_z	B	0.5	0.2	0.8	0.77	0.66	0.39	0.87
ρ_m	B	0.5	0.2	0.8	0.70	0.61	0.41	0.79
ρ_h	B	0.5	0.2	0.8	0.89	0.87	0.80	0.93
ρ_n	B	0.5	0.2	0.8	0.65	0.59	0.38	0.76
ρ_b	B	0.5	0.2	0.8	0.87	0.80	0.65	0.92
$100 \times \sigma_z$	IG	0.6	0.3	1.9	0.43	0.55	0.34	0.79
σ_m	IG	0.6	0.3	1.9	0.85	0.83	0.54	1.18
σ_h	IG	0.6	0.3	1.9	0.56	0.71	0.35	1.20
σ_n	IG	0.6	0.3	1.9	1.20	2.17	0.79	4.38
$1000 \times \sigma_b$	IG	0.6	0.3	1.9	0.67	1.56	0.49	3.08
C. Aggregate Shock Processes								
ρ_z	B	0.5	0.2	0.8	0.97	0.96	0.94	0.98
ρ_m	B	0.5	0.2	0.8	0.49	0.50	0.38	0.62
ρ_h	B	0.5	0.2	0.8	0.96	0.96	0.94	0.97
ρ_n	B	0.5	0.2	0.8	0.07	0.10	0.04	0.17
ρ_b	B	0.5	0.2	0.8	0.93	0.93	0.91	0.94
ρ_p	B	0.5	0.2	0.8	0.68	0.65	0.49	0.77
$100 \times \sigma_z$	IG	0.6	0.3	1.9	0.65	0.65	0.58	0.73
σ_m	IG	0.6	0.3	1.9	0.90	0.94	0.83	1.05
σ_h	IG	0.6	0.3	1.9	0.18	0.19	0.13	0.26
$\frac{1}{100} \times \sigma_n$	IG	0.6	0.3	1.9	0.14	0.16	0.10	0.23
$1000 \times \sigma_b$	IG	0.6	0.3	1.9	1.20	1.22	1.03	1.42
$1000 \times \sigma_p$	IG	0.6	0.3	1.9	0.68	0.76	0.50	1.08
$100 \times \sigma_r$	IG	0.6	0.3	1.9	0.78	0.80	0.70	0.90

No Population Weighting. In this robustness exercise, we estimate the model's parameters without population-weighting the contribution of individual states to the likelihood function. As Tables SII and SIII show, our results are unaffected.

Remove 5 Largest States. One concern is that shocks to large states may have important aggregate consequences, invalidating our approach of assuming independent state-level and aggregate shocks. To address this concern, we examined the robustness of our results to removing the five largest states from the estimation. Specifically, we re-estimated the model without data on California, Texas, New York, Florida, and Illinois. As Tables SII and SIII report, the estimated model's implications are similar to those in our baseline.

State Data Only. Here we estimate the model's structural parameters using regional data alone. We then fix these parameters and use the aggregate data to only estimate the parameters of the aggregate shock processes. As we show in Table SIV, using state-level data only, we estimate a much lower degree of wage and price stickiness. Intuitively, as Beraja, Hurst, and Ospina (2019) pointed out, state-level wages are quite volatile. In

TABLE SII
CONTRIBUTION OF STATE CREDIT SHOCKS TO STATE-LEVEL VARIABLES.

	Boom, 2002 to 2007		Bust, 2007 to 2010	
	Employment	Consumption	Employment	Consumption
Baseline Estimates	0.52	0.30	0.56	0.50
No Population Weighting	0.51	0.29	0.54	0.48
Remove 5 Largest States	0.63	0.36	0.60	0.55
State Data Only	0.25	0.23	0.39	0.40
Low Uncertainty ($\alpha = 5$)	0.10	0.05	0.09	0.08
High Uncertainty ($\alpha = 2$)	1.68	0.96	2.17	2.01
Lower Debt Duration ($\gamma = 0.965$)	0.63	0.35	0.64	0.59
One-period Debt ($\gamma = 0$)	0.38	0.21	0.30	0.28
Lower Labor Elasticity ($\psi = 5$)	0.57	0.33	0.63	0.56
Construction Sector	0.34	0.19	0.35	0.31
Heterogeneous Housing Elasticities	0.20	0.22	0.36	0.40
Government Spending	0.77	0.45	0.89	0.78
Option to Default	0.29	0.15	0.24	0.23
Estimated Taylor Rule	0.53	0.30	0.56	0.50

contrast, aggregate inflation has changed little during the Great Recession. Reproducing the aggregate data thus requires a greater degree of nominal stickiness, while reproducing the regional data requires a lower one. Our baseline estimates that use both aggregate and regional data thus fall somewhere in between. These results are consistent with the idea that prices respond more to large shocks than to small shocks, as in economies with menu costs (Alvarez, Le Bihan, and Lippi, 2016) and rational inattention (Mackowiak and Wiederholt, 2009). We emphasize regional differences, but similar evidence exists at the sectoral level (Boivin, Giannoni, and Mihov, 2009).

Table SII shows that the model now predicts a smaller role for credit shocks in generating fluctuations in employment and consumption across U.S. states. For example, credit shocks account for 0.25 of the variability of employment during the boom and 0.39 during the recession. A similar pattern emerges for consumption. Table SIII shows that the contribution of credit shocks to aggregate employment is also smaller.

Aggregate Data Only. We also estimated the model with aggregate data only. Our estimate of α was lower (3.1) as shown in Table SIV. As Table SIII shows, ignoring regional data increases the importance of credit shocks for aggregate employment.

High and Low Uncertainty. Here we illustrate the role played by the volatility of idiosyncratic taste shocks. We first reduce the volatility of taste shocks by increasing α to 5 and re-estimating all other parameters. As Table SII shows, credit shocks now produce much smaller relative movements in employment and consumption across states. When idiosyncratic uncertainty is low, agents in a state subject to a credit tightening consume out of their liquid assets, so consumption and employment change little. Similarly, as Table SIII shows, credit shocks alone generate virtually no employment decline in the aggregate.

We next increase the volatility of taste shocks by lowering α to 2. The model now attributes a significantly larger role to credit shocks in explaining state-level movements in real variables. As Table SII shows, credit shocks generate twice more volatile series for employment and consumption during the bust compared to the data. The model also

TABLE SIII
CONTRIBUTION OF AGGREGATE CREDIT SHOCKS TO AGGREGATE EMPLOYMENT.

	2007Q1 to 2010Q1		2007Q1 to 2012Q4	
	No ZLB	No FG	No ZLB	No FG
Baseline Estimates	-0.47	-1.72	-1.34	-4.76
No Population Weighting	-0.47	-1.67	-1.35	-4.62
Remove 5 Largest States	-0.62	-2.03	-1.77	-5.07
State Data Only	-0.31	-1.44	-0.63	-4.49
Aggregate Data Only	-0.33	-2.61	-1.81	-7.00
Low Uncertainty ($\alpha = 5$)	-0.08	-0.23	-0.20	-0.69
High Uncertainty ($\alpha = 2$)	-1.55	-4.93	-5.83	-8.02
Lower Debt Duration ($\gamma = 0.965$)	-1.10	-3.62	-3.33	-6.60
One-period Debt ($\gamma = 0$)	-1.50	-1.38	-4.25	-7.59
Lower Labor Elasticity ($\psi = 5$)	-0.52	-1.92	-1.52	-5.20
Construction Sector	0.32	-1.26	-0.14	-3.30
Government Spending	-0.38	-1.95	-1.73	-3.07
Option to Default	-0.60	-0.45	-1.38	-3.00
Estimated Taylor Rule	-0.25	-1.00	-0.79	-3.28

ascribes a much more important role to credit shocks in explaining the aggregate employment decline. Even in the absence of the ZLB, the model predicts an almost 6 percent decline in employment from 2007 to 2012.

Lower Debt Duration and One-Period Debt. So far we have imposed a value of γ , the parameter determining the decay rate of coupon payments in the mortgage contract, equal to 0.985, consistent with the maturity of mortgage contracts in the data. One could argue, however, that the effective duration of mortgages in the data is lower, due to households' ability to refinance their mortgages or take on home equity loans. Here we reduce γ to 0.965, implying a duration of mortgages about half that in our baseline (6 years versus 13 years) and re-estimate the model. Table SII shows that credit shocks now generate somewhat larger relative movements in employment and consumption across U.S. states. Similarly, Table SIII shows that credit shocks now account for a larger drop in aggregate employment during the recovery. For example, credit shocks alone predict a 3.3% drop in employment from 2007 to 2012 even in the absence of the ZLB and a 6.6% drop at the ZLB absent forward guidance.

We have also considered a version of the model with one-period debt, by setting $\gamma = 0$. To help match the slow-moving debt and house prices, we now assume that the shocks to credit and preference for housing are themselves AR(1) processes. As Table SII reports, the model's implications for the role of credit shocks in the cross-section are very similar to those in our Benchmark parameterization. Mechanically, the persistence and volatility of credit shocks adjusts when we change γ so as to match the comovement of credit and real variables in the data, with little consequence for the behavior of other variables. At the aggregate level, as Table SIII shows, the implications of credit shocks over the 2007 to 2010 period are unchanged compared to our baseline estimates. However, with one-period debt, credit shocks are more important over the 2007 to 2012 period.

We note that these results indicate a non-monotonic relationship between the duration of mortgage contracts and the contribution of credit shocks to employment and consumption. The reason for the non-monotonicity is that, regardless of the value of γ we use, we always re-estimate the model and extract the series for shocks to match the same observed

TABLE SIV
ESTIMATED STRUCTURAL PARAMETERS: ROBUSTNESS.

Parameter	Mean	10%	90%	Mean	10%	90%
	A. Baseline Estimates			B. No Population Weighting		
α	3.44	3.05	3.93	3.45	3.01	3.90
θ_p	0.96	0.94	0.98	0.97	0.94	0.98
θ_w	0.86	0.82	0.89	0.86	0.83	0.89
	C. State Data Only			D. $\alpha = 5$		
α	3.62	3.23	4.08	-	-	-
θ_p	0.93	0.89	0.96	0.97	0.94	0.98
θ_w	0.57	0.47	0.66	0.86	0.83	0.89
	E. $\alpha = 2$			F. $\gamma = 0.965$		
α	-	-	-	2.99	2.67	3.41
θ_p	0.96	0.93	0.98	0.96	0.94	0.98
θ_w	0.87	0.84	0.91	0.85	0.82	0.88
	G. $\gamma = 0$			H. $\psi = 5$		
α	2.65	2.41	2.87	3.34	2.98	3.78
θ_p	0.98	0.96	0.99	0.96	0.94	0.98
θ_w	0.87	0.84	0.89	0.92	0.91	0.94
	I. Construction Sector			J. Heterogeneous Housing Elasticities		
α	3.62	2.96	4.41	3.57	3.10	4.12
θ_p	0.96	0.94	0.98	0.92	0.88	0.96
θ_w	0.86	0.83	0.88	0.56	0.44	0.67
	K. Government Spending			L. Option to Default		
α	3.18	2.88	3.40	2.87	2.65	3.01
θ_p	0.96	0.95	0.98	0.96	0.93	0.98
θ_w	0.84	0.82	0.87	0.86	0.82	0.89
	M. Estimated Taylor Rule			N. Remove 5 Largest States		
α	3.43	3.17	3.85	3.28	2.94	3.62
θ_p	0.97	0.94	0.98	0.96	0.94	0.98
θ_w	0.85	0.82	0.88	0.88	0.85	0.90

debt series. Since all of the parameters of the model adjust as we change γ , the model does not predict a one-for-one relationship between γ and the relative contribution of credit shocks.

Lower Labor Elasticity. In our baseline estimation, we assigned a value of this elasticity, ψ , equal to 21, following [Christiano, Eichenbaum, and Evans \(2005\)](#). This parameter acts like a real rigidity, in that it prevents reset wages from responding too much to a given shock. Here we reduce this parameter to 5, and re-estimate all other parameters of the model. As we report in [Table SIV](#), the estimation now favors an even greater degree of nominal wage and price stickiness to compensate for the removal of the real rigidity. As [Tables SII](#) and [SIII](#) show, the model's implications for the importance of credit shocks in both the cross-section and the aggregate are virtually unchanged.

Construction Sector. In our baseline model, we assumed that the housing stock is in fixed supply, and for consistency have removed construction employment from the state

and aggregate data. Here we introduce a construction sector and add construction employment as an additional observable in the estimation. We assume that the housing stock evolves according to

$$h_{t+1}(s) = (1 - \delta_h)h_t(s) + y_t^H(s),$$

where δ_h is the rate at which housing depreciates and $y_t^H(s)$ is housing investment, produced with a decreasing returns technology using construction employment, $n_t^H(s)$,

$$y_t^H(s) = z_t^H(s)(n_t^H(s))^x,$$

where $z_t^H(s)$ is the productivity of the construction sector on island s . The problem of a firm in the construction sector is to maximize profits, given by

$$e_t(s)z_t^H(s)(n_t^H(s))^x - w_t(s)n_t^H(s) - w_t(s)\frac{\xi}{2}(n_t^H(s) - \bar{n}^H)^2,$$

where the last term is an adjustment cost that captures frictions that prevent the movement of labor across sectors.

We set $\chi = 0.37$ as in [Garriga and Hedlund \(2020\)](#) and $\delta_h = 0.012$ to match the 4.9% share of construction employment in total employment. We estimate the process for $z_t^H(s)$ and the other parameters of the model using state and aggregate data on construction employment, in addition to the original variables. As we show in [Table SIV](#), our posterior estimates of the structural parameters are very similar to those in the baseline model. As [Table SII](#) shows, credit shocks now explain a smaller fraction, approximately one-third, of the variation in consumption and non-construction employment across states. As [Table SIII](#) shows, the contribution of credit shocks to the drop in aggregate non-construction employment declines as well.

Heterogeneous Housing Elasticities. Motivated by the evidence in [Mian and Sufi \(2014\)](#) that individual states respond differentially to aggregate credit shocks due to heterogeneity in housing supply elasticities, we allow for such heterogeneity in our model. Owing to the computational complexity of integrating state- and aggregate-level data in computing the likelihood function, here we are only able to use state-level data to conduct inference. We separate states in the U.S. into three equally-sized groups depending on how closely household debt in a given state comoves with household debt in the aggregate. We then calculate changes in all state-level variables relative to the average within each group and allow the housing supply elasticity ξ to vary across groups in an attempt to isolate the state-specific shocks from heterogeneity in elasticities. [Table SII](#) shows that the model produces smaller fluctuations in employment in response to state-specific credit shocks during the boom, but its implications for consumption and the relative importance of credit shocks in explaining the drop in employment and consumption during the bust are similar to our baseline model with ex ante identical states.

Government Spending. A crucial component of the government's response to the Great Recession was fiscal policy, which we have so far abstracted from. Here we allow for changes in government spending both at the state level and in the aggregate and argue that our results are robust to this modification. Specifically, we now assume that the final good in each state is used for both consumption and government spending, with

government spending following an AR(1) process subject to exogenous state-specific and aggregate shocks:

$$g_t = \rho_g g_{t-1} + \sigma_g \varepsilon_{g,t}.$$

The resource constraint in our model is now simply

$$c_t + g_t = y_t.$$

Lump-sum taxes finance g_t which is assumed to be zero in steady state. We augment the estimation with state- and aggregate-level data on government spending. Specifically, we use spending by the government and government enterprises in the cross-section from BEA table SAGDP2N and treat the state-level data in the same way as the other state-level observables. At the aggregate level, we use real government spending from the FRED (GCEC1), demeaned over 1984Q1 to 2015Q1, our sample period.

As Table [SII](#) shows, credit shocks imply larger movements in employment and consumption across states compared to our baseline estimates. In contrast, as Table [SIII](#) shows, the model's aggregate implications are similar to our baseline.

We present variance decompositions of this model in the Not-for-Publication Appendix. Compared to our baseline variance decompositions, the contribution of fiscal policy shocks largely comes at the expense of a smaller contribution from discount factor shocks, which suggests that the discount factor shocks in our baseline specification capture much of the cross-sectional and aggregate variation caused by perturbations in government spending.

Option to Default. We have also considered an alternative model in which households have the option to default on their mortgages and fluctuations in household credit are driven by credit supply shocks as opposed to changes in LTV limits. We assume that mortgages are one-period contracts here, to avoid the multiplicity of equilibria that arise in versions of this model with long-term debt.

Specifically, we follow [Landvoigt \(2017\)](#) and [Faria-e-Castro \(2018\)](#) in assuming that in addition to idiosyncratic preference shocks, household members experience shocks to the quality of the houses they own, ω_{it} . Each member has housing wealth $\omega_{it} e_t h_t$ and is responsible for an equal share of the family's debt b_t . The member has the option to default on its debt and does so whenever the value of its home is less than the amount owed, $\omega_{it} e_t h_t < b_t$. We also assume that financial intermediaries face an ad valorem transaction cost τ_t of issuing new loans. Letting Δ_t denote the spread between the discount rate and the rate of time preference, perfect competition between financial intermediaries drives their expected profits to zero, which gives the following debt-elastic schedule for the price of debt:

$$q_t = \frac{1 + \Delta_t}{1 + (1 + \Delta_t)\tau_t} \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left(1 - G(\hat{\omega}_{t+1}) + \frac{\theta}{\hat{\omega}_{t+1}} \int_0^{\hat{\omega}_{t+1}} \omega dG(\omega) \right),$$

where q_t is the price of one unit of mortgage debt, $G(\omega) = (\omega/\bar{\omega})^\alpha$ is the Beta distribution of housing quality shocks, which we assume are i.i.d., θ is the fraction of the housing stock that the lender can recover upon default, and

$$\hat{\omega}_t = \frac{b_t}{e_t h_t}$$

is the LTV ratio which determines the cutoff quality below which the agent defaults.

Though the household does not face an explicit borrowing limit, it recognizes that borrowing more entails a larger spread between the interest rate on mortgages and the return on liquid assets. Its optimal choice of debt therefore satisfies

$$q_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[1 - \left(1 - \chi \left(1 - \frac{\theta}{\bar{\omega}} \right) \frac{1 + \Delta_t}{1 + (1 + \Delta_t)\tau_t} \right) \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}} \right)^{\chi} \right].$$

This expression implies that household debt decreases with the transaction cost τ_t which generates a spread between mortgage rates and the liquid rate of return even in the absence of default. We refer to changes in τ_t as *credit supply* shocks.

We estimate this model using the same approach as earlier. Though we do not explicitly use data on default rates in our estimation, we show in the Not-for-Publication Appendix that the model matches the time-series of default rates during the boom and the bust cycle well. As Table SII shows, shocks to τ_t and housing preferences generate smaller movements in employment and consumption in the cross-section compared to our baseline, with credit shocks contributing about 30% and 24% of the relative variation in employment during the boom and bust, respectively. As Table SIII shows, the model's aggregate implications are similar to those of our baseline model.

Estimated Taylor Rule. In our baseline estimates, we used the parameters of the Taylor rule estimated by Justiniano, Primiceri, and Tambalotti (2011) using pre-Great Recession data. We have also estimated these policy parameters ourselves using a longer sample inclusive of the 2009 to 2015 period. We find a higher estimate of the response to the output gap α_y , and to the growth rate in the output gap α_x but otherwise similar estimates to those of Justiniano, Primiceri, and Tambalotti (2011). This is likely due to the fact that our estimation is conducted over the period of the ZLB, which through the lens of the model was a period with a significant negative output gap. Because our estimates of the Taylor Rule are similar to the ones that we calibrated in our baseline estimations, the contribution of credit shocks at the regional and aggregate level is also very similar to contribution found in our baseline.

APPENDIX C: MARGINAL PROPENSITIES TO CONSUME

We show here that our model implies relatively low marginal propensities to consume out of a transitory income change, in line with the predictions of the frictionless model. Table SV traces out the effect of varying α for several measures of the severity of liquidity constraints in our baseline model. As we vary α , we recalibrate the discount factor β to ensure that the steady-state equilibrium interest rate stays constant at 2% (annualized).

As the table reports, reducing α all the way to 1.5 increases the fraction of household members whose liquidity constraint binds to 3.6%. More consequential is the impact of

TABLE SV
MARGINAL PROPENSITY TO CONSUME.

	$\alpha = 1.5$	$\alpha = 2.5$	$\alpha = 3.5$	$\alpha = 4.5$	$\alpha = 5.5$
Fraction constrained, %	3.60	0.97	0.26	0.07	0.02
Rate of time preference, $1/\beta - 1$, annual	0.309	0.046	0.024	0.021	0.020
Marginal propensity to consume	0.127	0.061	0.032	0.017	0.010

these constraints on the rate of time preference, $1/\beta - 1$, required to match a 2% interest rate. This rate increases to 31%, a sizable amount, reflecting the households' strong precautionary savings motive. Even with such an extreme parameterization, the marginal propensity to consume out of a transitory income shock is only equal to 12.7%. As we increase α to more empirically plausible values, the fraction of constrained household members falls, as do the rate of time preference and the MPC. For example, when $\alpha = 3.5$, the mean estimate in our baseline model, the fraction of constrained household members falls to 0.26%, the discount rate falls to 2.4%, only a bit higher than the interest rate of 2%, and the MPC falls to 3.2%. As we further increase α , the fraction of constrained household members falls to nearly zero, as does the gap between the interest rate and the rate of time preference and the MPC.

We thus conclude that our results do not rely on implausibly large marginal propensities to consume. Indeed, our model's predictions along this dimension are similar to those of the frictionless consumption-savings model.

APPENDIX D: LIKELIHOOD FUNCTION

We use Bayesian likelihood methods to estimate the parameters of the island economy and the shocks. We use a panel data set across states together with aggregate data and the ZLB. We formulate the state-space of the model so as to separate our estimation into a regional component and an aggregate component and make it computationally feasible.

We discuss first the likelihood function of the state/regional component and then the likelihood function of the aggregate component. In the paper, we show how we arrive at the state-space representations that we use below to form the likelihood function.

D.1. Likelihood of the Relative State Component

We use Bayesian methods. We first log-linearize the model. The log-linearized model for the relative regional-level variables has the state-space representation

$$\hat{\mathbf{x}}_t(s) = \mathbf{Q}\hat{\mathbf{x}}_{t-1}(s) + \mathbf{G}\epsilon_t(s), \quad (\text{S1})$$

$$\hat{\mathbf{z}}_t(s) = \mathbf{H}\hat{\mathbf{x}}_t(s). \quad (\text{S2})$$

The state vector is $\hat{\mathbf{x}}_t(s)$. The error is distributed $\epsilon_t(s) \sim N(0, \Omega)$, where Ω is the covariance matrix of $\epsilon_t(s)$. We assume no observation error of the data $\hat{\mathbf{z}}_t(s)$.

Denote by ϑ the vector of parameters to be estimated. Denote by $\mathcal{Z}(s) = \{\hat{\mathbf{z}}_t(s)\}_{t=1}^T$ the sequence of $N_z \times 1$ vectors of observable variables. By Bayes's law, letting $\mathcal{P}(\vartheta)$ denote the prior of ϑ , the posterior $\mathcal{P}(\vartheta | \mathcal{Z}(s))$ satisfies

$$\mathcal{P}(\vartheta | \mathcal{Z}(s)) \propto L(s) \times \mathcal{P}(\vartheta).$$

The likelihood function $L(s)$ is computed using the sequence of structural matrices and the Kalman filter and is, for any individual state,

$$\log \mathcal{L}(s) = -\left(\frac{\bar{N}\bar{T}}{2}\right) \log 2\pi - \frac{1}{2} \sum_{t=1}^{\bar{T}} \log \det \mathbf{S}_t - \frac{1}{2} \sum_{t=1}^{\bar{T}} \tilde{\mathbf{x}}_t'(s) \mathbf{S}_t^{-1} \tilde{\mathbf{x}}_t(s),$$

where $\tilde{\mathbf{x}}_t(s)$ is the vector of forecast errors and \mathbf{S}_t is its associated covariance matrix.

Given the state-space representation of the regional component, the Kalman filtering and smoothing equations are standard. We provide those in full in the Not-for-Publication Appendix.

Block Structure. The regional component of the model has a block structure. For example, consider two states so that the log-linearized state-space representation of the state variables relative to the aggregate is

$$\begin{bmatrix} x_t(1) \\ x_t(2) \end{bmatrix} = \begin{bmatrix} \mathbf{Q} & 0 \\ 0 & \mathbf{Q} \end{bmatrix} \begin{bmatrix} x_{t-1}(1) \\ x_{t-1}(2) \end{bmatrix} + \begin{bmatrix} \mathbf{G} & 0 \\ 0 & \mathbf{G} \end{bmatrix} \begin{bmatrix} \epsilon_t(1) \\ \epsilon_t(2) \end{bmatrix}.$$

Under this block structure, the forecast error covariance matrix $\mathbf{P}_{t|t-1}$ also has a block structure.

The log-likelihood becomes a weighted sum of state-by-state log-likelihood functions. To show this: because $\mathbf{P}_{t|t-1}$ has a block structure, so does \mathbf{S}_t . And because \mathbf{S}_t has a block structure:

$$\log \det \mathbf{S}_t = \log \prod_j \det \mathbf{S}_t^j = \sum_j \log \det \mathbf{S}_t^j.$$

Finally, because \mathbf{S}_t has a block structure, so does its inverse, so that the last term in the log-likelihood can also be separated by state. The log-likelihood of the state-level components is

$$\log \mathcal{L} = \sum_s \log \mathcal{L}(s).$$

Weighting. We weight the contributions of the state likelihoods to account for differences in the size of states. Weights can, in principle, depend on the sample and the model's parameters. [Agostinelli and Greco \(2012\)](#) discussed the asymptotic properties of the weighting function which are needed for the weighted likelihood to share the same asymptotic properties as the genuine likelihood function. Using population weights for state subsamples which are constant over time is a simple weighting function which satisfies these properties.

D.2. Likelihood of the Aggregate Component

Given a sequence of ZLB durations, the state-space of the model is

$$\mathbf{x}_t = \mathbf{J}_t + \mathbf{Q}_t \mathbf{x}_{t-1} + \mathbf{G}_t \epsilon_t,$$

$$\mathbf{z}_t = \mathbf{H}_t \mathbf{x}_t.$$

The observation equation and matrix \mathbf{H}_t are time-varying because the nominal interest rate becomes unobserved when it is at the ZLB.

Denote by ϑ the vector of parameters to be estimated and by \mathbf{T} the vector of ZLB durations that are observed each period. Denote by $\mathcal{Z} = \{z_t\}_{t=1}^T$ the sequence of vectors of observable variables. With Gaussian errors, the likelihood function $\mathcal{L}^a(\mathcal{Z}, \mathbf{T} | \vartheta)$ for the aggregate component is computed using the sequence of structural matrices associated with the sequence of ZLB durations, and the Kalman filter:

$$\log \mathcal{L}^a(\mathcal{Z}, \mathbf{T} | \vartheta) = -\left(\frac{N_z T}{2}\right) \log 2\pi - \frac{1}{2} \sum_{t=1}^T \log \det \mathbf{H}_t \mathbf{S}_t \mathbf{H}_t^\top - \frac{1}{2} \sum_{t=1}^T \tilde{\mathbf{x}}_t^\top (\mathbf{H}_t \mathbf{S}_t \mathbf{H}_t^\top)^{-1} \tilde{\mathbf{x}}_t,$$

where $\tilde{\mathbf{x}}_t$ is the vector of forecast errors and \mathbf{S}_t is its associated covariance matrix.

As for the regional component of the likelihood, given the state-space representation of the aggregate variables, the Kalman filtering and smoothing equations are standard. We provide those in full in the Not-for-Publication Appendix.

D.3. Posterior Sampler

This section describes the sampler used to obtain the posterior distribution. We compute the likelihood function at the state level, the likelihood at the aggregate level, and the prior. The posterior of our full model $\mathcal{P}(\vartheta | \mathbf{T}, \mathcal{Z})$ satisfies

$$\mathcal{P}(\vartheta | \mathbf{T}, \mathcal{Z}) \propto L(\mathcal{Z}, \mathbf{T} | \vartheta) \times \mathcal{P}(\vartheta).$$

We discuss the specification of our priors below.

We use a Markov chain Monte Carlo procedure to sample from the posterior. It has a single block, corresponding to the parameters ϑ . The sampler at step j is initialized with the last accepted draw of the structural parameters ϑ_{j-1} .

The block is a standard Metropolis–Hastings random walk. First start by selecting which parameters to propose new values. For those parameters, draw a new proposal ϑ_j from a thick-tailed proposal density centered at ϑ_{j-1} to ensure sufficient coverage of the parameter space and an acceptance rate of roughly 20% to 25%. The proposal ϑ_j is accepted with probability $\frac{\mathcal{P}(\vartheta_j | \mathbf{T}, \mathcal{Z})}{\mathcal{P}(\vartheta_{j-1} | \mathbf{T}, \mathcal{Z})}$. If ϑ_j is accepted, then set $\vartheta_{j-1} = \vartheta_j$.

APPENDIX E: DATA

E.1. State

We use state-level data on employment, consumption spending, compensation, government spending, debt-to-income, and house prices. The observed state data are annual. The model is quarterly. So we used a mixed-frequency estimation procedure. The data are only used to compute forecast errors in the first quarter of the year.

To construct the data, we first take each state’s series relative to its 1999 value, compute the deviation of each state’s observation from the state mean, regress that series on time dummies, weighted by the state’s relative population, and work with the residuals. We then take out a linear trend from the resulting series. As discussed in the text, this formulation allows us to separate the state-based and aggregate components of the state-space model. The resulting series used in our baseline specification for each state from 1999 to 2015 are plotted in the Not-for-Publication Appendix.

Here, we provide more details on each series:

- Consumption: We use state-level data on Total Personal Consumption Expenditures by State from the BEA, net of housing. The data are available for download at the [BEA website](#).
- Employment: We use state-level data on Total Employment net of employment in the construction sector from the [BEA annual table SA4](#). In our empirical analysis, we scale this measure of employment by each state’s population.
- Population: We use state-level data on Population from the [BEA annual table SA1-3](#).
- Labor Compensation: We use state-level data on Compensation of Employees by Industry from the [BEA annual table SA6N](#), net of construction compensation
- Wages: We divide total labor compensation by the number of employed individuals using the two series described above.
- Income: We use state-level data on Personal Income from the [BEA annual table SA4](#).

- **Household Debt:** We use data from the FRBNY Consumer Credit Panel [Q4 State statistics by year](#). Our measures of debt include auto loans, credit card debt, mortgage debt, and student loans. This database also provides information on the number of individuals with credit scores in each state, which we use to express the debt data in per-capita terms. We then construct a debt-to-income series by dividing this measure of per-capita debt by per-capita income using the data described above on income and population from the BEA.
- **House Prices:** We used data on the Not Seasonally Adjusted House Price Index available on the [FHFA website](#).
- **Government Spending:** We use data from the BEA Table SAGDP2N gross domestic product (GDP) by state: Government and government enterprises (Millions of current dollars).

E.2. *Aggregate*

At the aggregate level, we use data on inflation, employment, output, household debt, house prices, wages, government spending, the Fed Funds rate, and ZLB durations from NY Federal Reserve Survey Data. The codes for the raw data series are as follows:

- **Gross Domestic Product:** Implicit Price Deflator (GDPDEF).
- **Personal Consumption Expenditures** (BEA Table 2.4.5U). Current, \$. We subtract housing from consumption.
- **Cumulated nonfarm business section compensation** (PRS85006062) minus employment growth (PRS85006012) and deflated by the GDP deflator.
- **Total employment net of construction**, over the civilian noninstitutional population.
- **Household Debt from FRED** (code CMDEBT) deflated by PCE deflator, and expressed relative to income (from the BEA Table 2.1). U.S. household debt-to-income ratio exhibits a trend, starting from about 0.5 in 1975 to about 1 in the last decade. Since we do not allow for trends in our model, we de-trend the data by subtracting a linear trend. We smooth this series to eliminate high-frequency noise, by projecting it on a cubic spline of order 15—the smoothed series is reported with dotted lines in the figure.
- **House Prices** from Case-Logic.
- **Government Spending:** Real government spending (GCEC1).
- **Fed Funds rate:** The interest rate is the Federal Funds Rate, taken from the Federal Reserve Economic Database.
- **ZLB Durations:** We follow the approach of Kulish, Morley, and Ronbinson (2017) and use the ZLB durations extracted from the New York Federal Reserve Survey of Primary Dealers, conducted eight times a year, from 2011Q1 onwards.¹ We take the mode of the distribution implied by these surveys. Before 2011, we use responses from the Blue Chip Financial Forecasts survey.

We plot the aggregate data used in our baseline specification in the Not-for-Publication Appendix.

¹See the website [here](#). For example, in 2011, the survey conducted on January 18, one of the questions asked was: “Of the possible outcomes below, please indicate the percent chance you attach to the timing of the first federal funds target rate increase.” (Question 2b). Responses were given in terms of a probability distribution across future quarters.

TABLE SVI
VARIANCE DECOMPOSITION, 2 QUARTER HORIZON.

Variable	Shock						
	Collateral	Housing	Productivity	Leisure	Discount	Policy	Markup
	A. State level						
Spending	13.0	0.1	0.0	3.8	83.0	–	–
Employment	6.7	0.1	45.5	3.7	44.0	–	–
Wages	0.0	0.0	0.0	99.9	0.0	–	–
Debt-to-income	99.8	0.2	0.0	0.0	0.0	–	–
House prices	0.9	98.9	0.0	0.2	0.0	–	–
	B. Aggregate level						
Consumption	0.3	0.0	30.8	0.1	42.3	20.5	6.0
Employment	0.4	0.0	3.6	0.2	58.9	28.5	8.4
Wages	0.0	0.0	0.0	92.9	0.0	0.0	7.0
Debt-to-income	40.9	0.0	0.0	1.5	56.6	0.4	0.6
House prices	0.0	80.5	3.4	0.0	13.1	2.3	0.7
Fed Funds rate	0.6	0.0	7.1	0.1	87.8	3.1	1.2
Inflation	0.0	0.0	1.0	3.6	0.3	0.0	95.1

APPENDIX F: THEORETICAL VARIANCE DECOMPOSITIONS

The theoretical forecast error variance decompositions at the 2Q horizon are shown in Table SVI. Variance decompositions at other durations are provided in the Not-for-Publication Appendix. Credit shocks account for a nontrivial fraction of the differential changes in employment and consumption at the state level at all frequencies. The aggregate component of credit shocks has a small role in accounting for aggregate consumption or employment.

Unconditionally, monetary policy shocks account for 11 percent of the variation in aggregate consumption and 13 percent of the variation in employment. To provide a point of comparison, we compare our variance decompositions with those obtained using the Smets and Wouters (2007) model, presented in the Not-for-Publication Appendix. When a similar degree of price and wage stickiness is imposed in the Smets and Wouters model and all the remaining parameters reestimated, we find that about 14 percent of the variation in output is explained by policy shocks. We find wage markup shocks account for the bulk of the variation in wages, similar to our leisure preference shocks, and that inflation is almost entirely accounted for by price markup shocks, as in our model at the aggregate level.

APPENDIX G: IDENTIFICATION OF α

G.1. Local Projections

In the paper, we discussed what identifies the parameter α by examining what the model predicts for the comovement between employment, consumption, and household debt for different values of α . Here, we examine those predictions for longer leads and lags. Figure S1 plots the impulse response of employment and consumption to a change in debt computed using the local projections method of Jordà (2005). The first two panels show the impulse responses computed using the data. Following a change in household debt, employment increases on average by 0.05 percent and consumption increases by almost

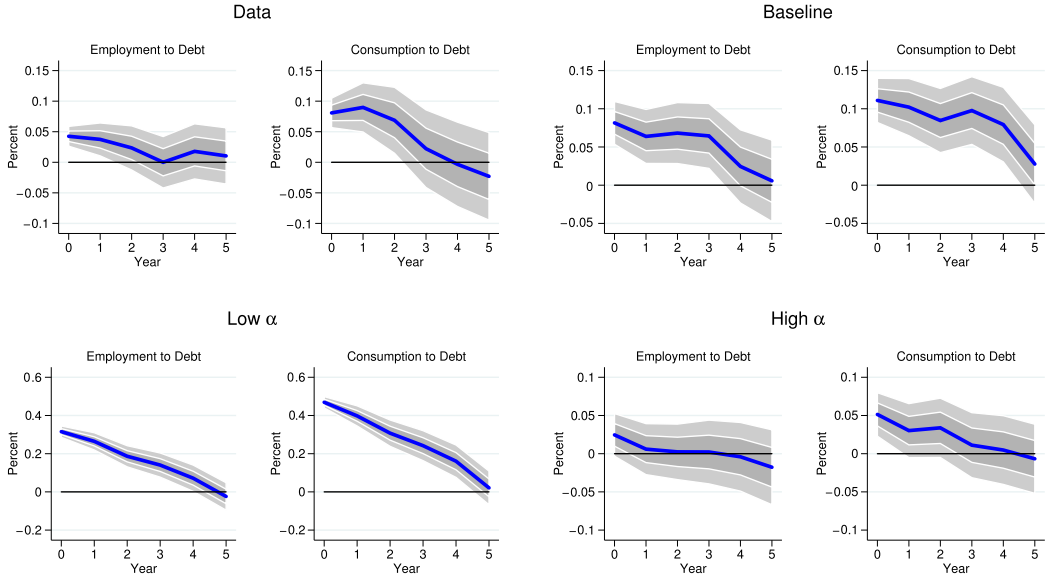


FIGURE S1.—Local projections of effect of debt shock, state level.

0.1 percent, and mean revert after approximately 3 to 4 years. We find that model simulated series show very similar patterns. However, a panel simulated using $\alpha = 2$ greatly overstates the response of employment and consumption to changes in credit, while a panel simulated using $\alpha = 10$ understates the responses.

G.2. GMM

We also consider an alternative, perhaps more transparent limited-information approach to estimating the model's parameters. Specifically, we choose the model's parameters by minimizing the distance between moments computed using state-level data and those implied by the model. The moments we target are the standard deviation of the variables, the contemporaneous second moments, and the persistence in the data. Hence, denote

$$M_t \equiv \begin{bmatrix} \text{vech}(\mathbf{z}_t \mathbf{z}_t') \\ \text{diag}(\mathbf{z}_t \mathbf{z}_t') \\ \text{diag}(\mathbf{z}_t \mathbf{z}_{t-1}') \end{bmatrix}, \quad (\text{S3})$$

where \mathbf{z}_t denotes the five state-level time series we described above at an annual frequency, the $\text{vech}(\bullet)$ operator selects the lower triangular elements of a matrix and orders them in a vector, and the $\text{diag}(\bullet)$ operator selects the diagonal elements of a matrix. Let Θ denote the vector of structural parameters that we wish to estimate; then the GMM estimator is given by

$$\hat{\Theta}_{GMM} = \arg \min \left(\frac{1}{T} \sum_{t=1}^T M_t - \mathbb{E}[\mathbf{M}(\Theta)] \right)' \mathbf{W} \left(\frac{1}{T} \sum_{t=1}^T M_t - \mathbb{E}[\mathbf{M}(\Theta)] \right), \quad (\text{S4})$$

where $\mathbb{E}[\mathbf{M}(\Theta)]$ denotes the model-implied moments that are counterparts to M_t when taking a first-order approximation to the model conditions and evaluates them at Θ . \mathbf{W} is

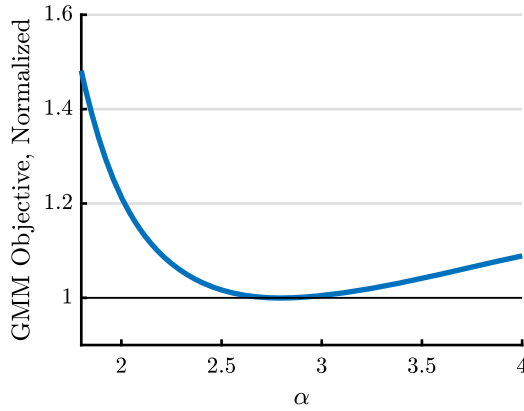


FIGURE S2.—GMM objective across parameters.

a positive definite weighting matrix, which is positive definite. We use a conventional two-step approach. First, we use an identity matrix for \mathbf{W} to obtain an initial estimate of the parameters denoted by Θ_0 . Then, we use the inverse of the variance-covariance matrix of $(\frac{1}{T} \sum_{t=1}^T M_t - \mathbb{E}[\mathbf{M}(\Theta_0)])$ as the weighting matrix, which is obtained with a Newey–West estimator with 1 lag (since we are using annual data).

This approach, which we apply to state-level data only, yields an estimate of α of 2.9, very close to the maximum likelihood estimate that uses state-level data only (see Table SIV). Figure S2 shows how the GMM objective function varies with α and that this parameter is indeed well identified by state-level data.

APPENDIX H: OTHER MODEL IMPLICATIONS

We next study our model’s ability to reproduce several additional variables that we have not directly used in estimation. We also look at the predictions for default rates from the model with default, which was not used in estimation.

H.1. *Tradables and Nontradables*

Consider first Figure S3 which shows how, in our model, state-level tradable and nontradable employment comove with consumption during the Great Recession. As Mian and Sufi reported, most of the decline in employment at the state level was due to a decline in nontradable employment. Our model reproduces this fact well: the elasticity of nontradable employment to consumption is equal to 0.75 in the model (0.55 in the data, as reported by Kehoe, Midrigan, and Pastorino (2019)). Similarly, tradable employment comoves little with state-level consumption, both in the model (the elasticity of tradable employment to consumption is -0.21) and in the data (an elasticity of -0.03).

H.2. *Mortgage Rate*

Consider next the model’s implications for the mortgage interest rate. Since in our model, mortgages are long-term perpetuities with decaying coupon payments, the return on such securities is not directly comparable to the interest rate on 30-year mortgages in the data. Nevertheless, we can derive the implied rate at which the flows underlying these

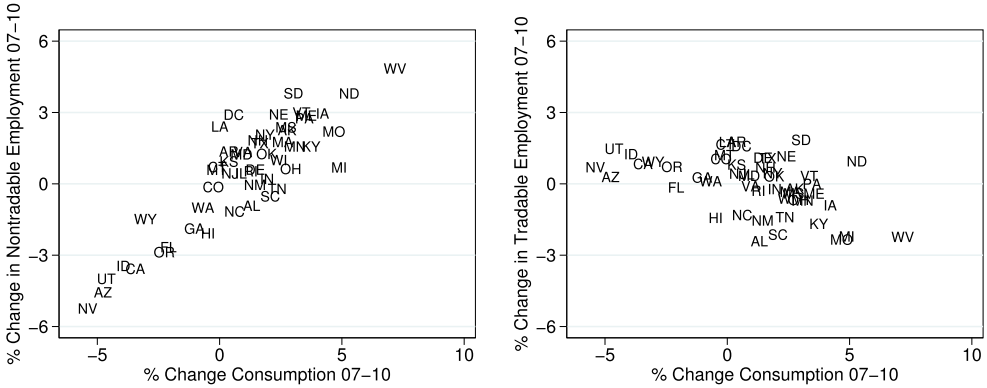


FIGURE S3.—Dynamics of sectoral employment during the recession.

securities are discounted as the rate i_t^m that rationalizes the price of the security q_t . This rate is defined by

$$q_t = \frac{1}{1 + i_t^m} \sum_{k=0}^{\infty} \left(\frac{\gamma}{1 + i_t^m} \right)^k,$$

which gives

$$1 + i_t^m = \frac{1}{q_t} + \gamma.$$

Figure S4 compares this implied long-term rate in our model with the average interest rate on 30-year mortgages in the data. Since the latter series has a trend, we de-trend both the model and the data (and add the average rate over 2001 to 2015 in both the model

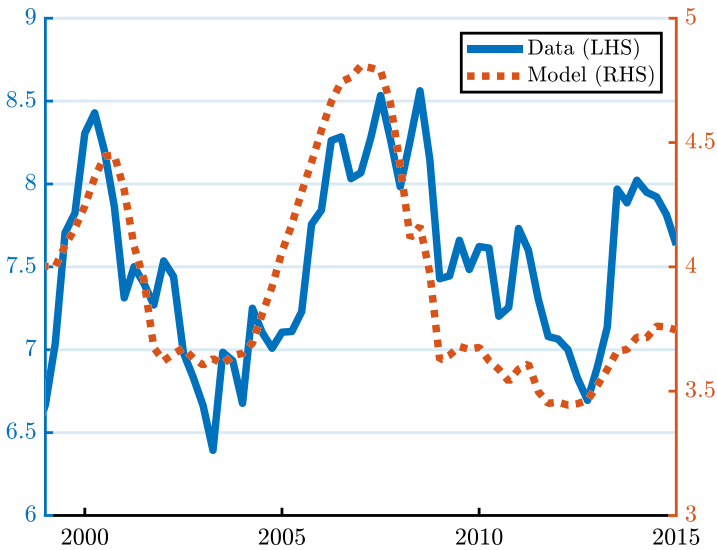


FIGURE S4.—Dynamics of the mortgage rate.

and data). The model does a reasonable job at reproducing medium-term movements in the mortgage rate in the data, though it misses the high-frequency variation. Since the model abstracts from several sources of risk embedded in mortgage rates, such as default and prepayment risk, we do not view the model's failure to match these high-frequency fluctuations as critical. Indeed, since our Kalman filter isolates the credit shocks from the dynamics of mortgage debt in the data, changes in mortgage spreads in the data are captured in a reduced-form way as shifts in the borrowing constraint. In our Robustness section below, we extend our model to explicitly model mortgage default and show that our conclusions are robust to adding a time-varying default spread between mortgage rates and the interest rate on liquid assets.

APPENDIX I: ALTERNATIVE SOURCES OF IDIOSYNCRATIC RISK

Here we discuss alternative approaches to introducing idiosyncratic risk. Though all these approaches would mimic qualitatively the approach we pursue in the paper, they are less tractable analytically. We start by deriving the liquid asset supply curve in a simple version of our baseline model with Pareto-distributed taste shocks, and then discuss the alternative approaches.

I.1. *Pareto Taste Shocks*

For transparency, we focus on a simple closed-economy, flexible price version of our model with one-period assets. We also abstract from housing and labor supply, and assume a borrowing limit $b_{t+1} \leq \bar{b}$. The problem of the representative household is to maximize its life-time utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int \log(c_{it}) di,$$

s.t.

$$\begin{aligned} x_t &= (1 + r_{t-1})(a_t - b_t) + y + b_{t+1}, \\ b_{t+1} &\leq \bar{b}, \\ c_{it} &\leq x_t, \\ a_{t+1} &= x_t - \int c_{it} di. \end{aligned}$$

The first-order optimality conditions are

$$\begin{aligned} \frac{v_{it}}{c_{it}} &= \beta(1 + r_t)\mathbb{E}_t \lambda_{t+1} + \xi_{it}, \\ \lambda_t &= \beta(1 + r_t)\mathbb{E}_t \lambda_{t+1} + \int \xi_{it} di, \\ \lambda_t &= \beta(1 + r_t)\mathbb{E}_t \lambda_{t+1} + \mu_t. \end{aligned}$$

Here λ_t is the multiplier on the budget constraint, ξ_{it} is the multiplier on the liquidity constraint, and μ_t is the multiplier on the borrowing constraint. Let

$$\hat{c}_t = \frac{1}{\beta(1+r_t)\mathbb{E}_t\lambda_{t+1}}$$

be the consumption of the agents with $v_{it} = 1$. We then have

$$c_{it} = v_{it}\hat{c}_t \quad \text{if } v_{it}\hat{c}_t \leq x_t,$$

and

$$c_{it} = x_t$$

otherwise. We next calculate the average multiplier on the liquidity constraints. We have

$$\frac{\int \xi_{it} di}{\beta(1+r_t)\mathbb{E}_t\lambda_{t+1}} = \int v_{it} \frac{\hat{c}_t}{c_{it}} di - 1.$$

Using the assumption of Pareto-distributed taste shocks, this expression simplifies to

$$\int v_{it} \frac{\hat{c}_t}{c_{it}} di - 1 = \int_1^{\frac{x_t}{\hat{c}_t}} (1-1) dF(v) + \alpha \frac{\hat{c}_t}{x_t} \int_{\frac{x_t}{\hat{c}_t}}^{\infty} v^{-\alpha} dv - \left(\frac{x_t}{\hat{c}_t}\right)^{-\alpha} = \frac{1}{\alpha-1} \left(\frac{x_t}{\hat{c}_t}\right)^{-\alpha}.$$

We can therefore write

$$\frac{\lambda_t}{\beta(1+r_t)\mathbb{E}_t\lambda_{t+1}} - 1 = \frac{1}{\alpha-1} \left(\frac{x_t}{\hat{c}_t}\right)^{-\alpha}.$$

Let Δ_t be the wedge in the aggregate Euler equation, implicitly defined as

$$\lambda_t = (1 + \Delta_t)\beta(1+r_t)\mathbb{E}_t\lambda_{t+1}.$$

Clearly,

$$\Delta_t = \frac{1}{\alpha-1} \left(\frac{x_t}{\hat{c}_t}\right)^{-\alpha}.$$

The household's total consumption expenditure is

$$\frac{c_t}{\hat{c}_t} = \alpha \int_1^{\frac{x_t}{\hat{c}_t}} v^{-\alpha} dv + \frac{x_t}{\hat{c}_t} \left(\frac{x_t}{\hat{c}_t}\right)^{-\alpha} = \frac{\alpha}{1-\alpha} \left[\left(\frac{x_t}{\hat{c}_t}\right)^{1-\alpha} - 1 \right] + \left(\frac{x_t}{\hat{c}_t}\right)^{1-\alpha},$$

or

$$\frac{c_t}{\hat{c}_t} = \frac{\alpha}{\alpha-1} - \frac{1}{\alpha-1} \left(\frac{x_t}{\hat{c}_t}\right)^{1-\alpha}.$$

Finally, savings are

$$a_{t+1} = x_t - c_t = \hat{c}_t \left(\frac{x_t}{\hat{c}_t} - \frac{c_t}{\hat{c}_t} \right),$$

and scaling by consumption (or equivalently, income), we have

$$\frac{a_{t+1}}{c_t} = \frac{x_t}{c_t} - 1 = \frac{x_t/\hat{c}_t}{c_t/\hat{c}_t} - 1 = \frac{x_t}{\hat{c}_t} \left(\frac{\alpha}{\alpha-1} - \frac{1}{\alpha-1} \left(\frac{x_t}{\hat{c}_t} \right)^{1-\alpha} \right)^{-1} - 1.$$

1.2. Gaussian Taste Shocks

All of our analysis goes through with alternative distributions of idiosyncratic shocks, but at the loss of some analytical tractability. To illustrate this, we next assume that idiosyncratic shocks are normally distributed, with $\log v_{it} \sim N(\mu_v, \sigma_v^2)$. The wedge in the Euler equation is now equal to

$$\Delta_t = \frac{\hat{c}_t}{x_t} \int_{\frac{x_t}{\hat{c}_t}}^{\infty} v \, d\Phi\left(\frac{\log v - \mu_v}{\sigma_v}\right) + \Phi\left(\frac{\log \frac{x_t}{\hat{c}_t} - \mu_v}{\sigma_v}\right) - 1,$$

where $\Phi(\cdot)$ is the cdf of the standard normal. Since

$$\int_{\frac{x_t}{\hat{c}_t}}^{\infty} v \, d\Phi\left(\frac{\log v - \mu_v}{\sigma_v}\right) = \exp\left(\mu_v + \frac{\sigma_v^2}{2}\right) \Phi\left(\frac{\mu_v + \sigma_v^2 - \ln \frac{x_t}{\hat{c}_t}}{\sigma_v}\right),$$

we have

$$\Delta_t = \left(\frac{x_t}{\hat{c}_t}\right)^{-1} \exp\left(\mu_v + \frac{\sigma_v^2}{2}\right) \Phi\left(\frac{\mu_v + \sigma_v^2 - \ln \frac{x_t}{\hat{c}_t}}{\sigma_v}\right) + \Phi\left(\frac{\log \frac{x_t}{\hat{c}_t} - \mu_v}{\sigma_v}\right) - 1.$$

Given $\frac{x_t}{\hat{c}_t}$, we can find aggregate consumption using

$$\frac{c_t}{\hat{c}_t} = \int_0^{\frac{x_t}{\hat{c}_t}} v \, d\Phi\left(\frac{\log v - \mu_v}{\sigma_v}\right) + \frac{x_t}{\hat{c}_t} \left(1 - \Phi\left(\frac{\log \frac{x_t}{\hat{c}_t} - \mu_v}{\sigma_v}\right)\right),$$

or

$$\frac{c_t}{\hat{c}_t} = \exp\left(\mu_v + \frac{\sigma_v^2}{2}\right) \Phi\left(\frac{\ln \frac{x_t}{\hat{c}_t} - \mu_v - \sigma_v^2}{\sigma_v}\right) + \frac{x_t}{\hat{c}_t} \left(1 - \Phi\left(\frac{\log \frac{x_t}{\hat{c}_t} - \mu_v}{\sigma_v}\right)\right),$$

and

$$\begin{aligned} a_{t+1} &= x_t - \hat{c}_t, \\ a_{t+1} &= x_t - c_t = \hat{c}_t \left(\frac{x_t}{\hat{c}_t} - \frac{c_t}{\hat{c}_t} \right), \end{aligned}$$

so the asset-to-income ratio is

$$\frac{a_{t+1}}{c_t} = \left(\frac{c_t}{\hat{c}_t}\right)^{-1} \left(\frac{x_t}{\hat{c}_t} - \frac{c_t}{\hat{c}_t}\right).$$

Though this liquid asset supply curve is more involved, the model with Gaussian taste shocks produces very similar responses to a tightening of credit, provided one recalibrates the volatility of taste shocks, σ_v^2 , appropriately.

I.3. Persistent Shocks

One can also allow for serially correlated taste shocks, though once again at the expense of analytical tractability. For example, suppose

$$\log v_{it} = (1 - \rho)\mu_v + \rho \log v_{it-1} + (1 - \rho^2)^{1/2} \sigma_v \varepsilon_{it},$$

where $\varepsilon_{it} \sim N(0, 1)$. Then the conditional mean is

$$\mathbb{E} \log v_{it} | \log v_{it-1} = (1 - \rho)\mu_v + \rho \log v_{it-1},$$

and the conditional variance is

$$\mathbb{V} \log v_{it} | \log v_{it-1} = (1 - \rho^2) \sigma_v^2,$$

and the formula determining the amount transferred to a consumer who had a taste v_{it-1} in the previous period is

$$\begin{aligned} 1 + \Delta_t &= \frac{\lambda_t}{\beta(1 + r_t) \mathbb{E}_t \lambda_{t+1}} = \Phi \left(\frac{\log \frac{x_t(v_{it-1})}{\hat{c}_t} - (1 - \rho)\mu_v - \rho \log v_{it-1}}{(1 - \rho^2)^{1/2} \sigma_v} \right) \\ &+ \left(\frac{x_t(v_{it-1})}{\hat{c}_t} \right)^{-1} \exp \left((1 - \rho)\mu_v + \rho \log v_{it-1} + \frac{(1 - \rho^2) \sigma_v^2}{2} \right) \\ &\times \Phi \left(\frac{(1 - \rho)\mu_v + \rho \log v_{it-1} + (1 - \rho^2) \sigma_v^2 - \ln \frac{x_t(v_{it-1})}{\hat{c}_t}}{(1 - \rho^2)^{1/2} \sigma_v} \right). \end{aligned}$$

This is more involved, since it requires solving a nonlinear equation for each v_{it-1} , but conceptually the problem is unchanged.

I.4. Income Shocks

We now assume that the idiosyncratic uncertainty takes the form of income, rather than preference shocks. In particular, we assume that income y_{it} is an i.i.d. random variable, realized after the household decides how much funds x_t to transfer to individual household members. The agent's consumption is thus limited by the sum of the transfer it receives and its idiosyncratic income realization. The representative household's problem is to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int \log(c_{it}) di,$$

subject to

$$x_t = (1 + r_{t-1})(a_t - b_t) + b_{t+1},$$

$$\begin{aligned}
b_{t+1} &\leq \bar{b}, \\
c_{it} &\leq x_t + v_{it}, \\
a_{t+1} &= x_t + \int v_{it} di - \int c_{it} di.
\end{aligned}$$

The first-order conditions are

$$\begin{aligned}
\frac{1}{c_{it}} &= \beta(1+r_t)\mathbb{E}_t\lambda_{t+1} + \xi_{it}, \\
\lambda_t &= \beta(1+r_t)\mathbb{E}_t\lambda_{t+1} + \int \xi_{it} di, \\
\lambda_t &= \beta(1+r_t)\mathbb{E}_t\lambda_{t+1} + \mu_t,
\end{aligned}$$

so letting

$$\hat{c}_t = \frac{1}{\beta(1+r_t)\mathbb{E}_t\lambda_{t+1}}$$

denote the unconstrained level of consumption, we have

$$c_{it} = \min(\hat{c}_t, x_t + v_{it}).$$

To find the multipliers, we have

$$\frac{\lambda_t}{\beta(1+r_t)\mathbb{E}_t\lambda_{t+1}} = 1 + \hat{c}_t \int \xi_{it} di = 1 + \int_0^{\hat{c}_t - x_t} \left(\frac{\hat{c}_t}{x_t + v} - 1 \right) dF(v),$$

where $F(\cdot)$ is the distribution of income shocks. To find aggregate consumption, we use

$$c_t = \int_0^{\hat{c}_t - x_t} (x_t + v) dF(v) + \hat{c}_t(1 - F(\hat{c}_t - x_t)).$$

Finally, total savings are

$$a_{t+1} = x_t + \int v_{it} di - c_t = \int_{\hat{c}_t - x_t}^{\infty} (x_t + v - \hat{c}_t) dF(v).$$

We prefer our approach based on taste shocks to this alternative approach based on income shocks for two reasons. First, with Pareto-distribution taste shocks, the wedge Δ_t can be computed in closed form. Second, with income taste shocks, one can easily show that average savings are necessarily below average income, so the model would require additional sources of heterogeneity to match the average liquid asset holdings observed in the data.

I.5. Expense Shocks

Suppose instead that individual agents are subject to idiosyncratic expense shocks, so they maximize

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \int \log(c_{it} - v_{it}) di,$$

where $c_{it} - v_{it}$ is the amount consumed net of the required expense shocks. The constraints are s.t.

$$\begin{aligned} x_t &= (1 + r_{t-1})(a_t - b_t) + y + b_{t+1}, \\ b_{t+1} &\leq \bar{b}, \\ c_{it} &\leq x_t, \end{aligned}$$

$$a_{t+1} = x_t - \int c_{it} \, di.$$

The first-order conditions are

$$\frac{1}{c_{it} - v_{it}} = \beta(1 + r_t)\mathbb{E}_t \lambda_{t+1} + \xi_{it},$$

and

$$\begin{aligned} \lambda_t &= \beta(1 + r_t)\mathbb{E}_t \lambda_{t+1} + \int \xi_{it} \, di, \\ \lambda_t &= \beta(1 + r_t)\mathbb{E}_t \lambda_{t+1} + \mu_t. \end{aligned}$$

The constraint is that

$$c_{it} \leq x_t.$$

Let

$$\tilde{c}_{it} = c_{it} - v_{it},$$

which implies that the Euler equation is

$$\frac{1}{\tilde{c}_{it}} = \beta(1 + r_t)\mathbb{E}_t \lambda_{t+1} + \xi_{it},$$

and the liquidity constraint is

$$\tilde{c}_{it} \leq x_t - v_{it}.$$

We need to bound the support of v here due to the Inada conditions. So assume $F(v) = (\frac{v}{\bar{v}})^\alpha$, where \bar{v} is the upper bound and α determines the shape of the distribution.

The wedge Δ_t satisfies

$$\Delta_t = \frac{\lambda_t}{\beta(1 + r_t)\lambda_{t+1}} - 1 = \frac{\int \xi_{it} \, di}{\beta(1 + r_t)\lambda_{t+1}}.$$

Since

$$\frac{\xi_{it}}{\beta(1 + r_t)\lambda_{t+1}} = \frac{\hat{c}_t}{c_{it} - v_{it}} - 1,$$

and

$$c_{it} = \min(\hat{c}_t + v_{it}, x_t),$$

where

$$\hat{c}_t = \frac{1}{\beta(1+r_t)\lambda_{t+1}},$$

we have

$$\Delta_t = \int_{x_t - \hat{c}_t}^{\bar{v}} \left(\frac{\hat{c}_t}{x_t - v} - 1 \right) dF(v).$$

Consumption is

$$c_t = \int_0^{x_t - \hat{c}_t} (\hat{c}_t + v) dF(v) + x_t \left(1 - \left(\frac{x_t - \hat{c}_t}{\bar{v}} \right)^\alpha \right),$$

or

$$c_t = x_t - \frac{1}{1+\alpha} (x_t - \hat{c}_t) \left(\frac{x_t - \hat{c}_t}{\bar{v}} \right)^\alpha.$$

So savings are

$$a_{t+1} = x_t - c_t = \frac{x_t - \hat{c}_t}{1+\alpha} \left(\frac{x_t - \hat{c}_t}{\bar{v}} \right)^\alpha.$$

Though more tractable, we found this approach less numerically stable and therefore less well-suited for estimation.

APPENDIX J: MODEL WITH DEFAULT

Here we describe in greater detail the model with default. For expositional convenience, we abstract from regional heterogeneity.

Let q_t be the price of a mortgage loan, which pays one unit next period if the borrower does not default, and h_t denote the housing stock. We assume that in addition to the idiosyncratic liquidity shocks, members of the representative household experience idiosyncratic shocks to the quality of housing they own, ω_{it} , which are i.i.d. draws from $G(w)$ with $\int_0^{\bar{\omega}} \omega dG(\omega) = 1$. Each member therefore has housing wealth $\omega_{it} e_t h_t$ and is responsible for an equal share of the family's debt b_t . An individual member has the option to default on its debt and does so if the value of its home is below the value of its mortgage debt:

$$\omega_{it} e_t h_t < b_t.$$

This determines a threshold

$$\hat{\omega}_t = \frac{b_t}{e_t h_t},$$

below which the agent defaults. The budget constraint, integrated over all members, is

$$x_t + e_t h_{t+1} = w_t n_t + \int_{\hat{\omega}_t}^{\bar{\omega}} (\omega e_t h_t - b_t) dG(\omega) + q_t b_{t+1} + (1 + r_{t-1}) a_t.$$

Financial intermediaries are perfectly competitive and owned by the representative household. In period t , the intermediary receives liquid assets from households and lends these funds in the mortgage market at price q_t .

We assume a dead-weight loss from default. When the lender seizes the collateral on a property that defaults, with value $\omega e_t h_t$, it only recovers a fraction $\theta \leq 1$ of it.

J.1. Bond Price

The expected value of what the lender will receive next period, in exchange for lending $q_t b_{t+1}$, is

$$\beta \mathbb{E}_t \lambda_{t+1} \left[(1 - G(\hat{\omega}_{t+1})) b_{t+1} + \theta e_{t+1} h_{t+1} \int_0^{\hat{\omega}_{t+1}} \omega dG(\omega) \right],$$

or

$$\beta \mathbb{E}_t \lambda_{t+1} \left[1 - G(\hat{\omega}_{t+1}) + \frac{\theta}{\hat{\omega}_{t+1}} \int_0^{\hat{\omega}_{t+1}} \omega dG(\omega) \right] b_{t+1}$$

next periods in exchange for lending $q_t b_{t+1}$ to the household. The intermediary borrows these resources from households who save in the liquid asset at an interest rate r_t , so its cost of lending is

$$\beta \mathbb{E}_t \lambda_{t+1} (1 + r_t) q_t b_{t+1},$$

where λ_t is the shadow value of wealth of the representative household. We also assume that there is a transaction cost of issuing new loans, proportional to the loan amount, so the total cost of the intermediary is

$$\beta \mathbb{E}_t \lambda_{t+1} (1 + r_t) q_t b_{t+1} + \tau_t \lambda_t q_t b_{t+1}.$$

We think of τ_t as capturing, in a parsimonious way, various frictions that lead to fluctuations in the spread at which lenders are willing to lend in the mortgage market, in short, *credit supply* shocks.

The expected profits of the intermediary are therefore

$$\beta \mathbb{E}_t \lambda_{t+1} \left[1 - G(\hat{\omega}_{t+1}) + \frac{\theta}{\hat{\omega}_{t+1}} \int_0^{\hat{\omega}_{t+1}} \omega dG(\omega) \right] b_{t+1} - \beta \mathbb{E}_t \lambda_{t+1} (1 + r_t) q_t b_{t+1} - \tau_t \lambda_t q_t b_{t+1}.$$

Competition drives these expected profits to zero, so we have

$$\beta \mathbb{E}_t \lambda_{t+1} \left[1 - G(\hat{\omega}_{t+1}) + \frac{\theta}{\hat{\omega}_{t+1}} \int_0^{\hat{\omega}_{t+1}} \omega dG(\omega) \right] = [\beta \mathbb{E}_t \lambda_{t+1} (1 + r_t) + \tau_t \lambda_t] q_t.$$

Recall that the households' FOC for savings in the liquid account is

$$\lambda_t = \beta (1 + r_t) (1 + \Delta_t) \mathbb{E}_t \lambda_{t+1},$$

where Δ_t depends on the multipliers on the liquidity constraint. Using this expression, we can express the bond price as

$$q_t = \frac{1 + \Delta_t}{1 + (1 + \Delta_t) \tau_t} \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left(1 - G(\hat{\omega}_{t+1}) + \frac{\theta}{\hat{\omega}_{t+1}} \int_0^{\hat{\omega}_{t+1}} \omega dG(\omega) \right).$$

This simplifies to

$$q_t = \frac{1 + \Delta_t}{1 + (1 + \Delta_t) \tau_t} \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left(1 - \left(1 - \frac{\theta}{\bar{\omega}} \right) \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}} \right)^x \right),$$

where we made use of

$$G(\omega) = \left(\frac{\omega}{\bar{\omega}} \right)^\chi,$$

and

$$\int_0^{\hat{\omega}} \omega \, dG(\omega) = \left(\frac{\hat{\omega}}{\bar{\omega}} \right)^{1+\chi}.$$

J.2. Optimal Choice of Debt

Consider next the household's optimal debt choice. We follow Hatchondo and Martinez and study a Markov perfect equilibrium. Since there are no refinancing frictions, the price at which the household borrows tomorrow, q_{t+1} , does not depend on the amount it borrows today, b_{t+1} . The borrowing FOC is therefore

$$\lambda_t \left[q_t + \frac{\partial q_t}{\partial b_{t+1}} b_{t+1} \right] = \beta \mathbb{E}_t \lambda_{t+1} \int_{\hat{\omega}_{t+1}}^{\bar{\omega}} dG(\omega).$$

Using

$$\int_{\hat{\omega}}^{\bar{\omega}} dG(\omega) = 1 - G(\hat{\omega}) = 1 - \left(\frac{\hat{\omega}}{\bar{\omega}} \right)^\chi$$

allows us to write

$$\lambda_t \left[q_t + \frac{\partial q_t}{\partial b_{t+1}} b_{t+1} \right] = \beta \mathbb{E}_t \lambda_{t+1} \left[1 - \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}} \right)^\chi \right].$$

Since

$$\frac{\partial q_t}{\partial b_{t+1}} = -\chi \left(1 - \frac{\theta}{\bar{\omega}} \right) \frac{1 + \Delta_t}{1 + (1 + \Delta_t)\tau_t} \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}} \right)^\chi \frac{1}{b_{t+1}},$$

we can write the debt FOC as

$$q_t - \chi \left(1 - \frac{\theta}{\bar{\omega}} \right) \frac{1 + \Delta_t}{1 + (1 + \Delta_t)\tau_t} \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}} \right)^\chi = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[1 - \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}} \right)^\chi \right],$$

which implies that

$$q_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[1 - \left(1 - \chi \left(1 - \frac{\theta}{\bar{\omega}} \right) \frac{1 + \Delta_t}{1 + (1 + \Delta_t)\tau_t} \right) \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}} \right)^\chi \right].$$

Though we no longer assume a limit on how much the household can borrow, the household recognizes that by borrowing more, it increases the interest rate, which leads to an interior solution for b_{t+1} . We now attribute the fluctuations in household credit in the data to credit shocks, τ_t .

J.3. Optimal Choice of Housing

The housing FOC is

$$\lambda_t e_t = \beta \mathbb{E}_t \frac{\eta_h}{h_{t+1}} + \beta \mathbb{E}_t \lambda_{t+1} e_{t+1} \left(1 - \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}} \right)^{1+\chi} \right) + \lambda_t \frac{\partial q_t}{\partial h_{t+1}} b_{t+1}.$$

Since

$$\frac{\partial q_t}{\partial h_{t+1}} = \chi \left(1 - \frac{\theta}{\bar{\omega}} \right) \frac{1 + \Delta_t}{1 + (1 + \Delta_t) \tau_t} \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}} \right)^\chi \frac{1}{h_{t+1}},$$

we can write

$$\begin{aligned} \lambda_t e_t &= \beta \mathbb{E}_t \frac{\eta_h}{h_{t+1}} + \beta \mathbb{E}_t \lambda_{t+1} e_{t+1} \left(1 - \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}} \right)^{1+\chi} \right) \\ &\quad + \chi \left(1 - \frac{\theta}{\bar{\omega}} \right) \frac{1 + \Delta_t}{1 + (1 + \Delta_t) \tau_t} \beta \mathbb{E}_t \lambda_{t+1} \left(\frac{\hat{\omega}_{t+1}}{\bar{\omega}} \right)^\chi \frac{b_{t+1}}{h_{t+1}}. \end{aligned}$$

The rest of the model is identical to that described in text. In the aggregate, the total amount of liquid assets, a_{t+1} , is equal to the overall amount of mortgage debt, $q_t b_{t+1}$.

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