

SUPPLEMENT TO “DYNAMIC MIXTURE-AVERSE PREFERENCES”  
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IN THIS SUPPLEMENT, we provide several supporting results that are used in the main paper. In Section S.1, we provide a general local expected-utility result for mixture-averse preferences that nests Proposition 1 in the main paper as a special case. In Section S.2, we establish the relationship between mixture-averse preferences and several prominent non-expected-utility theories, including rank-dependent utility, betweenness, disappointment aversion, and cautious expected utility. Section S.3 describes the implications of preference for diversification for insurance demand, and discusses how preference for diversification is equivalent to risk aversion for either rank-dependent utility or any preference that is quasiconcave in probabilities. Section S.4 establishes the existence of a value function for the optimal risk attitude representation. Proofs are contained in Section S.5.

S.1. LOCAL EXPECTED-UTILITY ANALYSIS

When applying the optimal risk attitude model, one important consideration is how properties of the set of transformations  $\Phi$  in the representation relate to properties of the corresponding risk preference. In this section, we show that the certainty equivalent for an ORA representation respects a stochastic order if and only if each of the transformations  $\phi \in \Phi$  also respects this order. This result is similar in spirit to the local expected-utility analysis introduced in the influential paper by Machina (1982). After presenting the main theorem of this section, we will make precise connections to Machina’s results and the many generalizations and extensions that appeared in the literature that followed. We will also describe how the expected-utility core recently developed by Cerreia-Vioglio, Maccheroni, and Marinacci (2017) can be related to the ORA representation.

The main result of this section applies to any convex utility representation on a set of lotteries  $\Delta(X)$ , where  $X$  is any compact metric space. Of particular interest is the special case where  $X$  is an interval, for example, a set of monetary outcomes or the set of continuation values for an ORA representation. We first state a general definition of stochastic orders generated by sets of functions.

DEFINITION S.1: Let  $\mathcal{C}$  be a set in the space of real-valued continuous functions  $C(X)$  for some compact metric space  $X$ . The order  $\succeq_c$  on  $\Delta(X)$  generated by  $\mathcal{C}$  is defined by

$$\mu \succeq_c \eta \iff \int \phi(x) d\mu(x) \geq \int \phi(x) d\eta(x), \quad \forall \phi \in \mathcal{C}.$$

A function  $W : \Delta(X) \rightarrow \mathbb{R}$  is *monotone with respect to the order  $\succeq_c$*  if  $\mu \succeq_c \eta$  implies  $W(\mu) \geq W(\eta)$ .

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This definition includes as special cases all of the stochastic orders typically used in economics. For example, if  $X$  is a subset of the real numbers, the first-order stochastic dominance order is generated by taking  $\mathcal{C}$  to be the set of all nondecreasing continuous functions; the second-order stochastic dominance order is generated by taking  $\mathcal{C}$  to be the set of all nondecreasing and concave continuous functions; and so on.

For any set  $\mathcal{C}$  of continuous functions in  $C(X)$ , let  $\langle \mathcal{C} \rangle$  denote smallest closed convex cone containing  $\mathcal{C}$  and all the constant functions, that is,  $\langle \mathcal{C} \rangle$  is the closed convex hull of the set of all affine transformations of functions in  $\mathcal{C}$ . It is easy to see that the stochastic order generated by  $\langle \mathcal{C} \rangle$  is the same as the stochastic order generated by  $\mathcal{C}$ . The following result shows that a convex function respects the order generated by a set  $\mathcal{C}$  if and only if it can be expressed as the supremum of some subset of  $\langle \mathcal{C} \rangle$ .

**THEOREM S.1:** *Suppose  $W : \Delta(X) \rightarrow \mathbb{R}$  for some compact metric space  $X$ , and suppose  $\mathcal{C} \subset C(X)$ . The following are equivalent:*

1.  *$W$  is lower semicontinuous in the topology of weak convergence, convex, and monotone with respect to the order  $\succeq_c$ .*
2. *There exists a set of functions  $\Phi \subset \langle \mathcal{C} \rangle$  such that*

$$W(\mu) = \sup_{\phi \in \Phi} \int \phi(x) d\mu(x). \quad (\text{S.1})$$

It is a standard result that a convex and lower semicontinuous function can be expressed as the supremum of a set of linear functions (Aliprantis and Border (2006, Theorem 7.6)). The new content of Theorem S.1 is that each of the linear functions in this set respects the same stochastic order as the original convex function.

Proposition 1 in the main text, which we now restate for convenience, is the special case of Theorem S.1 where  $X = [a, b]$  and the stochastic order is either first-order stochastic dominance (FOSD) or second-order stochastic dominance (SOSD).

**COROLLARY S.1:** *Suppose  $W : \Delta([a, b]) \rightarrow \mathbb{R}$  is lower semicontinuous in the topology of weak convergence and convex. Then:*

1.  *$W$  is monotone with respect to FOSD if and only if it satisfies Equation (S.1) for some collection  $\Phi$  of nondecreasing continuous functions  $\phi : [a, b] \rightarrow \mathbb{R}$ .*
2.  *$W$  is monotone with respect to SOSD if and only if it satisfies Equation (S.1) for some collection  $\Phi$  of nondecreasing and concave continuous functions  $\phi : [a, b] \rightarrow \mathbb{R}$ .*

Corollary S.1 is a variation of the main local expected-utility results from Machina (1982). Machina's approach was to assume Fréchet differentiability of the function  $W$  and relate the global properties of  $W$  to the local properties of its derivative. A number of papers have since explored relaxations of this differentiability assumption. Most recently, Cerreia-Vioglio, Maccheroni, and Marinacci (2017) showed that Machina's results can be extended to any Gateaux differentiable utility function and any integral stochastic order (as in Definition S.1). Since the ORA representation is in general not differentiable, these results will not suffice for the analysis in this paper. Theorem S.1 and Corollary S.1 complement the existing literature by showing that one can relax differentiability to the much weaker requirement of lower semicontinuity when dealing with convex functions.<sup>1</sup>

<sup>1</sup>Local expected-utility results for convex functions have also been obtained elsewhere, but under the assumption of differentiability or else stronger forms of continuity. For example, Machina (1984) considered

Theorem S.1 also provides a way of relating the ORA representation to the expected-utility core analyzed by Cerreia-Vioglio (2009), Cerreia-Vioglio, Dillenberger, and Ortolova (2015), and Cerreia-Vioglio, Maccheroni, and Marinacci (2017).<sup>2</sup>

DEFINITION S.2: The *expected-utility core* of a function  $W : \Delta([a, b]) \rightarrow \mathbb{R}$  is the binary relation  $\succeq$  on  $\Delta([a, b])$  defined by

$$\mu \succeq \eta \iff W(\alpha\mu + (1-\alpha)\nu) \geq W(\alpha\eta + (1-\alpha)\nu), \quad \forall \alpha \in (0, 1], \forall \nu \in \Delta([a, b]).$$

It follows immediately from this definition that  $\succeq$  is consistent with  $W$  in the sense that

$$\mu \succeq \eta \implies W(\mu) \geq W(\eta).$$

If  $W$  satisfies independence,<sup>3</sup> then the converse is also true and hence  $\succeq$  is a complete and transitive binary relation on  $\Delta([a, b])$  that is represented by  $W$ . However, if  $W$  does not satisfy independence, then  $\succeq$  is necessarily incomplete. As discussed in Cerreia-Vioglio, Maccheroni, and Marinacci (2017), the expected-utility core  $\succeq$  is the largest relation that is consistent with  $W$  and satisfies independence.

Applying the results of Cerreia-Vioglio, Maccheroni, and Marinacci (2017) together with Theorem S.1 gives the following corollary.<sup>4</sup>

COROLLARY S.2: *Suppose  $W : \Delta([a, b]) \rightarrow \mathbb{R}$  is continuous in the topology of weak convergence and convex. Let  $\succeq$  denote the expected-utility core of  $W$ . Then:*

1. *There exists a collection of continuous functions  $\mathcal{C}$  such that  $\succeq = \succeq_{\mathcal{C}}$ , that is,*

$$\mu \succeq \eta \iff \int_a^b \phi(x) d\mu(x) \geq \int_a^b \phi(x) d\eta(x), \quad \forall \phi \in \mathcal{C}.$$

2. *If a collection of continuous functions  $\Phi$  satisfies Equation (S.1) for  $W$ , then  $\mathcal{C} \subset \langle \Phi \rangle$ .*
3. *There exists a collection of continuous functions  $\Phi^m$  that satisfies Equation (S.1) for  $W$  and satisfies  $\langle \mathcal{C} \rangle = \langle \Phi^m \rangle$ .*

Parts 1 and 2 of Corollary S.2 come from Cerreia-Vioglio, Maccheroni, and Marinacci (2017, Lemma 1). For part 3, note that  $\mu \succeq_{\mathcal{C}} \eta$  implies  $W(\mu) \geq W(\eta)$ . Therefore, Theorem S.1 implies there exists a set  $\Phi^m \subset \langle \mathcal{C} \rangle$  that satisfies Equation (S.1).<sup>5</sup> Note that parts 2 and 3 of this result together imply that  $\langle \Phi^m \rangle \subset \langle \Phi \rangle$  for any set of continuous functions  $\Phi$  that satisfies Equation (S.1). Thus the expected-utility core provides a way of identifying a set of transformations that is minimal in terms of the associated set of expected-utility preferences.

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convex and Fréchet differentiable functions and therefore was able to apply many results from his prior work (Machina (1982)). Chatterjee and Krishna (2011) relaxed the assumption of differentiability and obtained several local expected-utility results for concave and Lipschitz continuous functions.

<sup>2</sup>The expected-utility core is the risk counterpart of the revealed unambiguous preference relation studied by Ghirardato, Maccheroni, and Marinacci (2004), Cerreia-Vioglio et al. (2011), and Ghirardato and Siniscalchi (2012).

<sup>3</sup>Say that  $W$  satisfies independence if  $W(\mu) \geq W(\eta)$  implies  $W(\alpha\mu + (1-\alpha)\nu) \geq W(\alpha\eta + (1-\alpha)\nu)$ .

<sup>4</sup>I thank David Dillenberger for suggesting the connection to the expected-utility core and Simone Cerreia-Vioglio for detailed comments about how to formalize this result.

<sup>5</sup>Theorem 1 in Cerreia-Vioglio, Maccheroni, and Marinacci (2017) provides a similar result to part 3 of this corollary for Gateaux differentiable functions: If  $W$  is Gateaux differentiable, then  $\langle \nabla W \rangle = \langle \mathcal{C} \rangle$ . The ORA representation is in general not differentiable, and hence the connection between the dual representation of  $W$  and the set  $\mathcal{C}$  representing the expected-utility core instead relies on Theorem S.1.

## S.2. RELATED NON-EXPECTED-UTILITY PREFERENCES

In this section, we discuss the relationship between the optimal risk attitude representation and other non-expected-utility theories. The conclusions of this discussion are summarized in Figure 1 of the paper.

### S.2.1. Probability Weighting and Rank-Dependent Utility

An important alternative to expected utility is the *rank-dependent utility* model (see, e.g., Quiggin (1982), Yaari (1987), Segal (1989)). For a probability measure  $\mu \in \Delta([a, b])$ , the certainty equivalent for rank-dependent utility takes the form

$$W(\mu) = h^{-1}\left(\int h(x) d(g \circ F_\mu)(x)\right),$$

where  $h : [a, b] \rightarrow \mathbb{R}$  is continuous and strictly increasing,  $F_\mu$  is the cumulative distribution function for the measure  $\mu$ , and  $g : [0, 1] \rightarrow [0, 1]$  is continuous, strictly increasing, and onto. The function  $g$  in the representation permits distortions of the cumulative probabilities. If  $g(p) = p$  for all  $p \in [0, 1]$ , then the expression above reduces to the certainty equivalent for expected utility. However, when  $g(F_\mu(x)) > F_\mu(x)$  for some  $x$ , the probability of obtaining an outcome below  $x$  is distorted upward, capturing the intuition that low-probability bad events may be overweighted. Reweighting of probabilities also played an important role in the prospect theory of Kahneman and Tversky (1979) and Tversky and Kahneman (1992).

Chew, Karni, and Safra (1987) showed that a rank-dependent utility preference is risk averse (dislikes mean-preserving spreads) if and only if both  $h$  and  $g$  are concave. In this case, the certainty equivalent  $W$  is a convex function (see Wakker (1994, Observation 2) or Chatterjee and Krishna (2011, Proposition 4.6)). The argument is as follows: For concave  $g$ ,

$$g(F_{\alpha\mu+(1-\alpha)\eta}(x)) = g(\alpha F_\mu(x) + (1-\alpha)F_\eta(x)) \geq \alpha g(F_\mu(x)) + (1-\alpha)g(F_\eta(x))$$

for all  $x \in [a, b]$ . Since  $h$  is increasing, this FOSD relationship between the transformed distributions implies

$$\int h(x) d(g \circ F_{\alpha\mu+(1-\alpha)\eta})(x) \leq \alpha \int h(x) d(g \circ F_\mu)(x) + (1-\alpha) \int h(x) d(g \circ F_\eta)(x).$$

Finally, concavity of  $h$  implies  $h^{-1}$  is convex, which yields the convexity of  $W$ . Since any recursive model with a convex certainty equivalent can be expressed as an ORA representation by Theorem 1 of the paper, this shows that Epstein–Zin preferences with a risk-averse RDU certainty equivalent are a special case of mixture-averse preferences.

The kinked transformation example from the main text is one of the special cases of the ORA representation that overlaps with recursive RDU. The following proposition formalizes the connection.

**PROPOSITION S.1:** Fix  $\theta \in [0, 1]$  and define  $\phi(x|\gamma, \theta)$  as in Equation (8) of the main text:

$$\phi(x|\gamma, \theta) = \begin{cases} \gamma + (1 + \theta)(x - \gamma) & \text{if } x \leq \gamma, \\ \gamma + (1 - \theta)(x - \gamma) & \text{if } x > \gamma. \end{cases}$$

Then, for any  $\mu \in \Delta([a, b])$ ,

$$\sup_{\gamma \in \mathbb{R}} \int \phi(x|\gamma, \theta) d\mu(x) = \int x d(g \circ F_\mu)(x),$$

where

$$g(\alpha) = \begin{cases} (1 + \theta)\alpha & \text{for } \alpha \leq 1/2, \\ (1 - \theta)\alpha + \theta & \text{for } \alpha > 1/2. \end{cases}$$

We close this section by pointing out another useful connection stemming from the relationship between mixture-averse preferences and RDU. In a recent paper, [Masatlioglu and Raymond \(2016\)](#) found a somewhat surprising relationship between rank-dependent utility and a model of endogenous reference points developed by [Kőszegi and Rabin \(2006, 2007\)](#). They showed that whenever the choice-acclimating personal equilibrium (CPE) concept for reference point formation from [Kőszegi and Rabin \(2006, 2007\)](#) leads to a risk preference that respects first-order stochastic dominance, that preference conforms to rank-dependent utility. The implication for our model is that any CPE representation that respects both FOSD and SOSD also satisfies mixture aversion.

### S.2.2. Disappointment Aversion, Betweenness, and Quasiconcave Risk Preferences

Another important class of non-expected-utility preferences are the *betweenness* preferences developed by [Chew \(1983\)](#) and [DeKel \(1986\)](#). One of the more widely used special cases of betweenness preferences is the *disappointment aversion* model of [Gul \(1991\)](#). [Grant, Kajii, and Polak \(2000, Lemma 2\)](#) showed that any betweenness preference that has a convex representation must be an expected-utility preference. Thus, in a dynamic setting, the only overlap of recursive betweenness preferences and mixture-averse preferences is EZKP expected utility.

Another intriguing related model is the *cautious expected utility* representation recently proposed by [Cerreia-Vioglio, Dillenberger, and Ortleva \(2015\)](#). The certainty equivalent for this model is the minimum of a set of expected-utility certainty equivalents. This representation has a nontrivial intersection with betweenness preferences that includes risk-averse disappointment aversion preferences. However, since cautious expected utility preferences are quasiconcave with respect to lotteries, they only overlap with mixture-averse preferences in the case of linear indifference curves, that is, betweenness preferences. Therefore, by the previous observations, the intersection of recursive cautious expected utility and mixture-averse preferences is again EZKP utility.

## S.3. PREFERENCE FOR DIVERSIFICATION

In this section, we consider random variables defined on some fixed probability space.<sup>6</sup> We will use  $\tilde{x}$  to denote a random variable and  $\mu_{\tilde{x}}$  to denote the distribution of that random variable. As stated in the paper, a risk preference with a certainty equivalent  $W$  satisfies preference for diversification if it is quasiconcave with respect to random variables.

<sup>6</sup>Note that we are presuming that the probability measure is known, and therefore we are still working within a framework of objective risk, as opposed to subjective beliefs.

DEFINITION S.3: A certainty equivalent  $W : \Delta([a, b]) \rightarrow [a, b]$  exhibits *preference for diversification* (PD) if, for any random variables  $\tilde{x}$  and  $\tilde{y}$  and any  $\alpha \in [0, 1]$ ,

$$W(\mu_{\tilde{x}}) \geq W(\mu_{\tilde{y}}) \implies W(\mu_{\alpha\tilde{x} + (1-\alpha)\tilde{y}}) \geq W(\mu_{\tilde{y}}).$$

Preference for diversification is a useful property of a model of risk preferences for several reasons: Together with homotheticity of preferences, PD permits representative agent analysis; it also implies the sufficiency of first-order conditions in maximization problems (e.g., portfolio choice). However, while each of these arguments supports PD as enhancing the analytic tractability of economic models, neither speaks to its descriptive realism, and in the paper we observed several compelling reasons for relaxing this condition. In Section 4.2 of the paper, we showed that relaxing PD permits heterogeneity in stock market participation even when agents have identical preferences. In this section, we show that obtaining the properties of demand for insurance discussed in Section 4.1 of the paper also requires violating PD. We then discuss the connection between PD and risk aversion.

### S.3.1. Preference for Diversification and Insurance Demand

Suppose, as in Example 3 from Section 4.1 of the main text, that an individual has wealth  $w$  and faces a loss of amount  $L$  with probability  $\pi$ . Let  $P(y)$  denote this individual's maximum willingness to pay (reservation price) for  $y \in [0, L]$  dollars of insurance coverage paid in the event of a loss. The following property relates the willingness to pay for additional insurance to the existing level of coverage.

DEFINITION S.4: An individual has a *nonincreasing marginal willingness to pay for insurance coverage* if  $P(y + \varepsilon) - P(y)$  is nonincreasing in  $y$  for every  $y, \varepsilon \geq 0$  such that  $y + \varepsilon \leq L$ .

In Section 4.1 of the paper, we argued that the above definition may be overly restrictive and that violations of this property may better match observed insurance choices—people often accept large increases in premiums in order to reduce their insurance deductibles (Sydnor (2010)). We also showed that the ORA representation can permit a marginal willingness to pay for additional insurance coverage that increases at some levels of coverage (while still maintaining risk aversion). In contrast, the following result shows that any risk preference that satisfies preference for diversification must exhibit nonincreasing marginal willingness to pay for insurance.

PROPOSITION S.2: *If the certainty equivalent  $W$  for an individual's risk preferences exhibits preference for diversification, then this individual has a nonincreasing marginal willingness to pay for insurance coverage.*

The implications of Proposition S.2 are immediate in the case of expected utility. If an individual does not have a nonincreasing marginal willingness to pay for insurance coverage, then she must violate preference for diversification. For an expected-utility maximizer, preference for diversification is satisfied if and only if her Bernoulli utility function is concave. Thus, in order to have a marginal willingness to pay for insurance coverage that increases at some levels of coverage (e.g., near full coverage), the individual must violate risk aversion. The next section shows that this conclusion also extends to many non-expected-utility models.

### S.3.2. Preference for Diversification and SOSD

The connection between preference for diversification and risk aversion (monotonicity with respect to SOSD) has been well documented for a number of models. [Dekel \(1989\)](#) showed that preference for diversification implies risk aversion for any risk preference. He also showed that the converse is true for preferences that are quasiconcave in probabilities. The following proposition summarizes his result as well as related observations for rank-dependent utility from [Chew, Karni, and Safra \(1987\)](#).

PROPOSITION S.3:

1. ([Dekel \(1989\)](#)) *If  $W$  is quasiconcave in probabilities and respects SOSD, then it satisfies preference for diversification.*
2. ([Chew, Karni, and Safra \(1987\)](#)) *If  $W$  is a rank-dependent utility certainty equivalent and it respects SOSD, then it satisfies preference for diversification.*<sup>7</sup>

Note that Proposition S.3 applies to all of the risk preferences discussed in Section S.2. Thus, other than our model of mixture-averse preferences, most of the non-expected-utility preferences considered in the literature are encompassed by this result. The only prominent theory that we are aware of that is not covered by this result is the *quadratic utility* model of [Chew, Epstein, and Segal \(1991\)](#). To our knowledge, it remains an open question whether quadratic utility can violate PD while still respecting SOSD.

### S.4. EXISTENCE OF A VALUE FUNCTION

The value function  $V$  is included explicitly in the definition of the ORA representation. However, it may be desirable to obtain such a value function from the other parameters  $(u, \Phi, \beta)$  of the representation. Using similar techniques to [Epstein and Zin \(1989\)](#), the following result shows that this is possible.<sup>8</sup>

PROPOSITION S.4: *Suppose  $\beta \in (0, 1)$  and  $u : C \rightarrow \mathbb{R}$  is a continuous and nonconstant function. Let  $[\hat{a}, \hat{b}] = u(C)$  where  $-\infty < \hat{a} < \hat{b} < +\infty$ ,<sup>9</sup> and let  $a = \frac{\hat{a}}{1-\beta}$ ,  $b = \frac{\hat{b}}{1-\beta}$ . Let  $\Phi$  be any collection of continuous and nondecreasing functions  $\phi : [a, b] \rightarrow \mathbb{R}$  that satisfies  $\sup_{\phi \in \Phi} \phi(x) = x$  for all  $x \in [a, b]$ . Then, there exists a bounded and lower semicontinuous function  $V : D \rightarrow [a, b]$  that satisfies Equation (3) in the paper, that is,*

$$V(c, m) = u(c) + \beta \sup_{\phi \in \Phi} \int_D \phi(V(\hat{c}, \hat{m})) dm(\hat{c}, \hat{m}),$$

for all  $(c, m) \in D$ .

For the ORA representation to be well-defined, the functions  $\phi \in \Phi$  must be defined everywhere on the set  $V(D)$ . However, if  $V$  is not known and needs to be determined from the other parameters of the representation  $(u, \Phi, \beta)$ , then the relevant domain of the functions  $\phi \in \Phi$  is not known a priori. Nonetheless, Proposition S.4 shows that the

<sup>7</sup>[Chew, Karni, and Safra \(1987\)](#) assumed Gateaux differentiability in their result. However, [Grant, Kajii, and Polak \(2000, Lemma 3\)](#) showed that the same result holds true without any differentiability assumptions.

<sup>8</sup>As with most other recursive non-expected-utility models, it is not possible to apply the standard techniques from [Blackwell \(1965\)](#) to prove existence of a value function.

<sup>9</sup>Since  $C$  is compact and connected and  $u$  is continuous,  $u(C)$  is a closed and bounded interval in  $\mathbb{R}$ .

range of  $V$  can be determined from the range of  $u$ , and hence it suffices to consider functions  $\phi$  defined on this interval  $[a, b]$ . In particular, if  $u \geq 0$  ( $u \leq 0$ , respectively), then it suffices to define each  $\phi$  on  $\mathbb{R}_+$  ( $\mathbb{R}_-$ , respectively).

There are two noticeable gaps in Proposition S.4. First, it does not ensure the uniqueness of the function  $V$ . Second, it does not ensure that the function  $V$  is continuous, only lower semicontinuous. There are similar limitations to the existence results in Epstein and Zin (1989). Since resolving these issues is not central to the analysis in this paper, obtaining a stronger version of this result is left as an open question for future research. However, it is worth noting that in the case of homothetic preferences, it is possible to ensure both uniqueness and continuity of the value function using recent results from Marinacci and Montrucchio (2010).

## S.5. PROOFS

### S.5.1. Proof of Theorem S.1

Theorem S.1 will be proved by means of a separation argument. Since  $\geq_c = \geq_{(C)}$ , it is without loss of generality to assume that  $C$  is a closed convex cone in the space of continuous functions  $C(X)$  that contains the constant functions. Let  $ca(X)$  denote the set of all signed (countably-additive) Borel measures of bounded variation on the compact metric space  $X$ . Consider the following subset of  $ca(X)$ :

$$K \equiv \left\{ \mu \in ca(X) : \int \phi(x) d\mu(x) \geq 0 \text{ for every } \phi \in C \right\}. \quad (\text{S.2})$$

Note that  $K$  is a cone in  $ca(X)$ . In addition, since the constant functions identically equal to 1 and  $-1$  are both in  $C$ ,  $\mu(X) = 0$  for all  $\mu \in K$ . The following lemma makes some other simple observations about  $K$  that will be used in the proof of the proposition.

LEMMA S.1: *The set  $K$  defined in Equation (S.2) is a weak\* closed convex cone in  $ca(X)$ , and for any  $\mu, \eta \in \Delta(X)$ ,*

$$\mu \geq_c \eta \iff \mu - \eta \in K.$$

PROOF: For any  $\phi \in C$ , the set

$$K_\phi \equiv \left\{ \mu \in ca(X) : \int \phi(x) d\mu(x) \geq 0 \right\}$$

is weak\* closed and convex. Since  $K = \bigcap_{\phi \in C} K_\phi$  is the intersection of closed and convex sets, it is also closed and convex. The equivalence in the displayed equation follows directly from the definition of  $\geq_c$ . *Q.E.D.*

Continuing the proof of Theorem S.1, it is immediate that 2 implies 1. To prove that 1 implies 2, it suffices to show that for any  $\mu \in \Delta(X)$  and any  $\alpha < W(\mu)$ , there exists a function  $\phi_{\mu, \alpha} \in C$  such that  $\alpha \leq \int \phi_{\mu, \alpha}(x) d\mu(x)$  and  $\int \phi_{\mu, \alpha}(x) d\eta(x) \leq W(\eta)$  for all  $\eta \in \Delta(X)$ . Then, letting

$$\Phi = \left\{ \phi_{\mu, \alpha} : \mu \in \Delta(X), \alpha < W(\mu) \right\},$$



it follows directly that

$$W(\mu) = \sup_{\phi \in \Phi} \int \phi(x) d\mu(x).$$

Fix any  $\mu \in \Delta(X)$  and any  $\alpha < W(\mu)$ . The proof is completed by showing the existence of a function  $\phi_{\mu,\alpha}$  as described above. This is accomplished using a separation argument similar to standard duality results for convex functions (see, e.g., [Ekeland and Turnbull \(1983\)](#) or [Phelps \(1993\)](#)). The epigraph of  $W$  is defined as follows:

$$\text{epi}(W) = \{(\eta, t) \in \Delta(X) \times \mathbb{R} : t \geq W(\eta)\}.$$

Since  $W$  is convex with a convex domain  $\Delta(X)$ ,  $\text{epi}(W)$  is a convex subset of  $ca(X) \times \mathbb{R}$ . Moreover, as a weak\* lower semicontinuous function with a weak\* closed domain, it is a standard result that  $\text{epi}(W)$  is a closed subset of  $ca(X) \times \mathbb{R}$ .<sup>10</sup> Now, define a set  $K_{\mu,\alpha}$  as follows:

$$K_{\mu,\alpha} \equiv (\{\mu\} + K) \times \{\alpha\} = \{\mu + \nu : \nu \in K\} \times \{\alpha\}.$$

By Lemma S.1,  $K_{\mu,\alpha}$  is a closed and convex subset of  $ca(X) \times \mathbb{R}$ . Establishing the following claim allows the separating hyperplane theorem to be applied.<sup>11</sup>

**CLAIM S.1:** *For  $\alpha < W(\mu)$ , the set  $\text{epi}(W) - K_{\mu,\alpha}$  is convex, and  $(0, 0) \notin \text{cl}(\text{epi}(W) - K_{\mu,\alpha})$ .*

**PROOF OF CLAIM S.1:** First, note that  $K_{\mu,\alpha} \cap \text{epi}(W) = \emptyset$ . To see this, take any  $(\eta, t) \in K_{\mu,\alpha}$ . Then, by definition,  $t = \alpha$  and  $\eta - \mu \in K$ . If  $\eta \notin \Delta(X)$ , then it is trivial that  $(\eta, t) \notin \text{epi}(W)$ . Alternatively, if  $\eta \in \Delta(X)$ , then Lemma S.1 implies  $\eta \geq_c \mu$ . In this case,  $W(\eta) \geq W(\mu) > \alpha = t$ , so again  $(\eta, t) \notin \text{epi}(W)$ . Thus,  $K_{\mu,\alpha}$  and  $\text{epi}(W)$  are disjoint, closed, and convex sets.

Since  $K_{\mu,\alpha}$  and  $\text{epi}(W)$  are convex and disjoint,  $\text{epi}(W) - K_{\mu,\alpha}$  is convex and  $(0, 0) \notin \text{epi}(W) - K_{\mu,\alpha}$ . Since  $W$  is weak\* lower semicontinuous and has a weak\* compact domain  $\Delta(X)$ , it attains a minimum value  $\underline{W}$ . Therefore,  $\text{epi}(W)$  can be written as the union of the following two sets:

$$B_1 \equiv \text{epi}(W) \cap (\Delta(X) \times [\underline{W}, W(\mu)]) = \{(\eta, t) \in \Delta(X) \times \mathbb{R} : W(\mu) \geq t \geq W(\eta)\},$$

$$B_2 \equiv \text{epi}(W) \cap (\Delta(X) \times [W(\mu), +\infty)) = \{(\eta, t) \in \Delta(X) \times \mathbb{R} : t \geq \max\{W(\eta), W(\mu)\}\}.$$

As the intersection of a closed set and a compact set,  $B_1$  is compact, and as the intersection of two closed sets,  $B_2$  is closed. Since the difference of a compact set and a closed set is closed,  $B_1 - K_{\mu,\alpha}$  is closed. Since  $B_1 - K_{\mu,\alpha} \subset \text{epi}(W) - K_{\mu,\alpha}$ , this set does not contain  $(0, 0)$ . Also note that for every  $(\nu, t) \in B_2 - K_{\mu,\alpha}$ , it must be that  $t \geq W(\mu) - \alpha > 0$ . Therefore,  $B_2 - K_{\mu,\alpha} \subset ca(X) \times [W(\mu) - \alpha, +\infty)$ , a closed set not containing  $(0, 0)$ . Thus,  $\text{epi}(W) - K_{\mu,\alpha}$  is contained in the union of the closed sets  $B_1 - K_{\mu,\alpha}$  and  $ca(X) \times [W(\mu) - \alpha, +\infty)$ , each of which does not contain  $(0, 0)$ . *Q.E.D.*

<sup>10</sup>The set  $ca(X) \times \mathbb{R}$  is endowed with the product topology generated by the weak\* topology on  $ca(X)$  and the Euclidean topology on  $\mathbb{R}$ .

<sup>11</sup>Although  $\text{epi}(W)$  and  $K_{\mu,\alpha}$  are disjoint, closed, and convex sets, standard separation theorems require that at least one of the sets either be compact or have a nonempty interior. Therefore, a slightly more involved argument is required here.

Continuing the proof of Theorem S.1, note that  $ca(X) \times \mathbb{R}$  is a locally convex Hausdorff space (Theorem 5.73 in Aliprantis and Border (2006)). Therefore, the separating hyperplane theorem (Theorem 5.79 in Aliprantis and Border (2006)) implies there exists a weak\* continuous linear functional  $F : ca(X) \rightarrow \mathbb{R}$  and  $\lambda \in \mathbb{R}$  such that

$$F(v) + \lambda t < F(0) + \lambda 0 = 0, \quad \forall (v, t) \in \text{epi}(W) - K_{\mu, \alpha}.$$

For any  $(\eta, t) \in \text{epi}(W)$  and  $\nu \in K$ , we have  $(\mu + \nu, \alpha) \in K_{\mu, \alpha}$  and therefore  $(\eta - \mu - \nu, t - \alpha) \in \text{epi}(W) - K_{\mu, \alpha}$ . Thus

$$F(\eta) + \lambda t < F(\mu) + F(\nu) + \lambda \alpha, \quad \forall (\eta, t) \in \text{epi}(W), \forall \nu \in K. \quad (\text{S.3})$$

Taking  $(\eta, t) = (\mu, W(\mu))$  and  $\nu = 0$ , it follows that  $\lambda W(\mu) < \lambda \alpha$ . Since  $\alpha < W(\mu)$ , this implies  $\lambda < 0$ . Therefore, setting  $\nu = 0$  in Equation (S.3), conclude that for all  $\eta \in \Delta(X)$ ,

$$F(\eta) + \lambda W(\eta) < F(\mu) + \lambda \alpha \implies W(\eta) > -\frac{F(\eta)}{\lambda} + \frac{F(\mu)}{\lambda} + \alpha.$$

Consider the weak\* continuous linear functional  $\eta \mapsto -\frac{F(\eta)}{\lambda}$  defined on  $ca(X)$ . Since the weak\* topology on  $ca(X)$  is generated by  $C(X)$ , every weak\* continuous linear functional on  $ca(X)$  corresponds to some  $\psi \in C(X)$  (Theorem 5.93 in Aliprantis and Border (2006)). In particular, there exists  $\psi \in C(X)$  such that  $-\frac{F(\eta)}{\lambda} = \int \psi(x) d\eta(x)$  for all  $\eta \in ca(X)$ . Define  $\phi_{\mu, \alpha} \in C(X)$  by  $\phi_{\mu, \alpha}(x) = \psi(x) + \frac{F(\mu)}{\lambda} + \alpha$  for  $x \in X$ . Then, for every  $\eta \in \Delta(X)$ ,

$$W(\eta) > \int \psi(x) d\eta(x) + \frac{F(\mu)}{\lambda} + \alpha = \int \left[ \psi(x) + \frac{F(\mu)}{\lambda} + \alpha \right] d\eta(x) = \int \phi_{\mu, \alpha}(x) d\eta(x).$$

In addition,

$$\int \phi_{\mu, \alpha}(x) d\mu(x) = -\frac{F(\mu)}{\lambda} + \frac{F(\mu)}{\lambda} + \alpha = \alpha.$$

The final step in the proof is to show that  $\phi_{\mu, \alpha} \in \mathcal{C}$ . Fix any  $\nu \in K$ , and note that  $r\nu \in K$  for all  $r > 0$ . Therefore, Equation (S.3) implies

$$\begin{aligned} F(\mu) + \lambda W(\mu) &< F(\mu) + F(r\nu) + \lambda \alpha \\ &= F(\mu) + rF(\nu) + \lambda \alpha. \end{aligned}$$

For this to be true for every  $r > 0$ , it must be that  $F(\nu) \geq 0$ . That is,  $\int \psi(x) d\nu(x) \geq 0$  for all  $\nu \in K$ . Thus there is no  $\nu \in ca(X)$  such that

$$\int \psi(x) d\nu(x) < 0 \quad \text{and} \quad \int \phi(x) d\nu(x) \geq 0, \quad \forall \phi \in \mathcal{C}.$$

An infinite-dimensional version of Farkas's lemma (Corollary 5.84 in Aliprantis and Border (2006)) therefore implies  $\psi \in \mathcal{C}$ . Since  $\mathcal{C}$  is a convex cone that contains all constant functions, conclude also that  $\phi_{\mu, \alpha} \in \mathcal{C}$ . This completes the proof.

S.5.2. *Proof of Proposition S.1*

Ben-Tal and Teboule (2007) showed that the supremum for this parameterized kinked transformation function is attained by taking  $\gamma^*$  equal to the median of  $\mu$ . More precisely, the maximizer is any  $\gamma^* \in [a, b]$  such that  $F_\mu(\gamma^*-) \leq 1/2 \leq F_\mu(\gamma^*)$ , where  $F_\mu(\gamma^*-) = \lim_{x \nearrow \gamma^*} F_\mu(x)$  denotes the left-hand limit for  $F_\mu$  at  $\gamma^*$ . Thus, fixing any such  $\gamma^*$ , the left side of the equality in the statement of the proposition is equal to

$$\begin{aligned} & \sup_{\gamma \in \mathbb{R}} \int \phi(x|\gamma, \theta) d\mu(x) \\ &= \int_{[a, b]} \phi(x|\gamma^*, \theta) dF_\mu(x) \\ &= \gamma^* + \int_{[a, \gamma^*]} (1 + \theta)(x - \gamma^*) dF_\mu(x) + \int_{(\gamma^*, b]} (1 - \theta)(x - \gamma^*) dF_\mu(x) \\ &= \theta\gamma^*(1 - 2F_\mu(\gamma^*)) + \int_{[a, \gamma^*]} (1 + \theta)x dF_\mu(x) + \int_{(\gamma^*, b]} (1 - \theta)x dF_\mu(x). \end{aligned}$$

The right side of the equality in the statement of the proposition is equal to

$$\begin{aligned} & \int x d(g \circ F_\mu)(x) \\ &= \gamma^*[g(F_\mu(\gamma^*)) - g(F_\mu(\gamma^*-))] + \int_{[a, \gamma^*]} (1 + \theta)x dF_\mu(x) + \int_{(\gamma^*, b]} (1 - \theta)x dF_\mu(x) \\ &= (1 - \theta)\gamma^*(F_\mu(\gamma^*) - 1/2) + (1 + \theta)\gamma^*(1/2 - F_\mu(\gamma^*-)) \\ & \quad + \int_{[a, \gamma^*]} (1 + \theta)x dF_\mu(x) + \int_{(\gamma^*, b]} (1 - \theta)x dF_\mu(x) \\ &= \theta\gamma^*(1 - 2F_\mu(\gamma^*)) + (1 + \theta)\gamma^*(F_\mu(\gamma^*) - F_\mu(\gamma^*-)) \\ & \quad + \int_{[a, \gamma^*]} (1 + \theta)x dF_\mu(x) + \int_{(\gamma^*, b]} (1 - \theta)x dF_\mu(x) \\ &= \theta\gamma^*(1 - 2F_\mu(\gamma^*)) + \int_{[a, \gamma^*]} (1 + \theta)x dF_\mu(x) + \int_{(\gamma^*, b]} (1 - \theta)x dF_\mu(x), \end{aligned}$$

which is identical to the expression in the equation above.

S.5.3. *Proof of Proposition S.2*

Fix any  $w, L, \pi$  and any  $y, y' \in [0, L]$ . By definition, the individual is indifferent between insurance coverage  $y$  at premium  $P(y)$  and insurance coverage  $y'$  at premium  $P(y')$ :

$$\begin{aligned} W \begin{pmatrix} w - P(y) - L + y & \pi \\ w - P(y) & 1 - \pi \end{pmatrix} &= W \begin{pmatrix} w - P(y') - L + y' & \pi \\ w - P(y') & 1 - \pi \end{pmatrix} \\ &= W \begin{pmatrix} w - L & \pi \\ w & 1 - \pi \end{pmatrix}. \end{aligned}$$

Preference for diversification therefore implies that for any  $\alpha \in [0, 1]$ ,

$$W \begin{pmatrix} w - \alpha P(y) - (1 - \alpha)P(y') - L + \alpha y + (1 - \alpha)y' & \pi \\ w - \alpha P(y) - (1 - \alpha)P(y') & 1 - \pi \end{pmatrix} \geq W \begin{pmatrix} w - L & \pi \\ w & 1 - \pi \end{pmatrix}.$$

Since  $W$  respects FOSD, this implies  $P(\alpha y + (1 - \alpha)y') \geq \alpha P(y) + (1 - \alpha)P(y')$ , so  $P$  is concave. Thus the individual has a nonincreasing marginal willingness to pay for insurance coverage.

#### S.5.4. Proof of Proposition S.4

As in the statement of the proposition, let  $[\hat{a}, \hat{b}] = u(C)$  and  $a = \frac{\hat{a}}{1-\beta}$ ,  $b = \frac{\hat{b}}{1-\beta}$ . Let  $L$  denote the space of all lower semicontinuous functions from  $D$  to  $[a, b]$ :

$$L \equiv \{f : D \rightarrow [a, b] : f \text{ is lower semicontinuous}\}.$$

Define an operator  $T$  on  $L$  by

$$Tf(c, m) = u(c) + \beta \sup_{\phi \in \Phi} \int_D \phi(f(\hat{c}, \hat{m})) dm(\hat{c}, \hat{m}),$$

for  $(c, m) \in D$ .

The first step is to show that  $Tf \in L$  for all  $f \in L$ , and hence  $T : L \rightarrow L$ . Fix any  $f \in L$ . Since  $f$  is bounded by  $a$  and  $b$  and each  $\phi$  is nondecreasing, it follows that

$$\phi(a) \leq \int_D \phi(f(\hat{c}, \hat{m})) dm(\hat{c}, \hat{m}) \leq \phi(b), \quad \forall m \in \Delta(D), \phi \in \Phi.$$

Taking the supremum of each expression and using the property  $\sup_{\phi \in \Phi} \phi(x) = x$  gives

$$a \leq \sup_{\phi \in \Phi} \int_D \phi(f(\hat{c}, \hat{m})) dm(\hat{c}, \hat{m}) \leq b.$$

Since  $(1 - \beta)a \leq u(c) \leq (1 - \beta)b$  for all  $c \in C$ , this implies  $a \leq Tf \leq b$ . Next, the lower semicontinuity of  $f$  implies that  $\phi \circ f$  is lower semicontinuous for all  $\phi \in \Phi$ , since each  $\phi$  is continuous and nondecreasing. This in turn implies that the mapping  $m \mapsto \int_D \phi(f(\hat{c}, \hat{m})) dm(\hat{c}, \hat{m})$  is lower semicontinuous (see Theorem 15.5 in [Aliprantis and Border \(2006\)](#)). It is a standard result that the supremum of any collection of lower semicontinuous functions is lower semicontinuous. Together with the continuity of  $u$ , conclude that  $Tf$  is lower semicontinuous. Hence  $Tf \in L$  for all  $f \in L$ .

The proof is completed by showing that  $T$  has a fixed point  $V \in L$ . To show the existence of a fixed point, first construct a sequence as follows: Let  $V_1(c, m) = a$  for all  $(c, m) \in D$ , and let  $V_{n+1} = TV_n$  for all  $n \in \mathbb{N}$ . Since each  $\phi \in \Phi$  is nondecreasing, it follows immediately that  $T$  is monotone:  $f \leq g$  implies  $Tf \leq Tg$ . Note that  $V_1 \leq V_2$ , since  $V_1 \leq g$  for any  $g \in L$  by definition. Thus  $V_2 = TV_1 \leq TV_2 = V_3$ . Proceeding by induction,  $V_n \leq V_{n+1}$  for all  $n \in \mathbb{N}$ . Since  $\{V_n\}$  is an increasing sequence of bounded functions, it converges pointwise to some function  $V : D \rightarrow [a, b]$ . Moreover, since  $V$  is equal to the supremum of the collection of lower semicontinuous functions  $\{V_n : n \in \mathbb{N}\}$ , it is lower semicontinuous. Hence  $V \in L$ .

The last step is to show  $TV = V$ . Since  $V_n \leq V$  for all  $n$ , monotonicity of the operator  $T$  implies  $V_{n+1} = TV_n \leq TV$ . Taking limits gives  $V \leq TV$ . To establish the opposite inequality, note first that for any  $m \in \Delta(D)$  and  $\phi \in \Phi$ , the mapping  $f \mapsto \int_D \phi(f(\hat{c}, \hat{m})) dm(\hat{c}, \hat{m})$

is continuous in the product topology (i.e., the topology of pointwise convergence) by the dominated convergence theorem. This implies the mapping  $f \mapsto Tf(c, m)$  is lower semi-continuous for all  $(c, m) \in D$ . Thus  $V_n \rightarrow V$  implies

$$TV(c, m) \leq \liminf_n TV_n(c, m) = \liminf_n V_{n+1}(c, m) = V(c, m),$$

for all  $(c, m) \in D$ . Hence  $TV = V$ , which completes the proof.

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