SUPPLEMENT TO "CHEAP TALK WITH ENDOGENOUS CONFLICT OF INTEREST"

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APPENDIX B: PROOFS

LEMMA B.1: *Assumptions* 1–3 *are satisfied in Examples* 1–3. *Assumptions* 5 *and* 6 *both hold in Example* 1. *In Example* 2, *Assumption* 6 *holds but Assumption* 5 *need not. In Example* 3, *Assumptions* 4 *and* 5 *hold, but Assumption* 6 *need not.*

PROOF: *Example 1*

$$U(a, q, \omega) = -(a - \omega + q)^2.$$

The function $U(a, q, \omega)$ is strictly concave in q and in a. Complementarity holds because

$$U_{a\omega}(a, q, \omega) = 2 > 0.$$

The single-crossing property holds because

$$
\begin{aligned}
U(a, q, \omega) - U(a', q, \omega) &= (a' - \omega + q)^2 - (a - \omega + q)^2 \\
&= \big[(a' - \omega + q) + (a - \omega + q)\big]\big[(a' - \omega + q) - (a - \omega + q)\big] \\
&= (a' + a - 2\omega + 2q)(a' - a),
\end{aligned}
$$

which is increasing in q when $a' > a$.

Finally, Assumptions 5 and 6 both hold in Example 1 because when $U(a, q, \omega) = -(a - \omega + q)^2$ then $U_{aq} = -2$ and $U_a/U_q = 1$.

Example 2

$$U(a, q, \omega) = qa\omega - \frac{1}{2}(qa)^2.$$

The function $U(a, q, \omega)$ is strictly concave in q and in a. Complementarity holds because

$$U_{a\omega}(a, q, \omega) = q > 0.$$

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The single-crossing property holds because

$$U(a, q, \omega) - U(a', q, \omega) = q(a - a')\omega - \frac{1}{2}q^2\big[a^2 - (a')^2\big]$$

$$= (a - a')q\Big[\omega - \frac{1}{2}q(a + a')\Big].$$

The expression in brackets is decreasing in q, and so when $a' > a$ the entire expression is increasing in q.

Finally, Assumption 6 holds because $U_a/U_q = q/a > 0$ for any $q > 0$.

Example 3

$$U(a, q, \omega) = \omega\sqrt{aQ} + (1 - a)q.$$

The function $U(a, q, \omega)$ is concave (actually, linear) in q and strictly concave in a. Complementarity holds because

$$U_{a\omega}(a, q, \omega) = \frac{1}{2}\frac{\sqrt{Q}}{\sqrt{a}} > 0.$$

The single-crossing property holds because

$$U(a, q, \omega) - U(a', q, \omega) = \big[\omega\sqrt{aQ} + (1 - a)q\big] - \big[\omega\sqrt{a'Q} + (1 - a')q\big]$$

$$= \omega\sqrt{Q}(\sqrt{a} - \sqrt{a'}) + (a' - a)q.$$

When $a' > a$ this function is increasing in q and so the single-crossing property is satisfied.

Assumption 4 holds because $U_q = (1 - a) \geq 0$.

Finally, Assumption 5 holds because $U_{aq} = -1$.   *Q.E.D.*

LEMMA B.2: *If Assumptions* 1 *and* 3 *hold, the receiver's best response* a_k^* *defined in* (3) *is differentiable in* $(\omega_{k-1}, \omega_k, q_R)$, *and increasing in* ω_{k-1} *and in* ω_k.

PROOF: The receiver's best response to a partition element (b, c) is the $a^*(b, c, q_R)$ that solves

$$L(a; b, c, q_R) := \int_b^c U_a(a, q_R, \omega)\, dF(\omega) = 0.$$

Because of Assumption 1 $a^*(b, c, q_R)$ is unique. Since $U(\cdot, \cdot, \cdot)$ is twice continuously differentiable $L(a; b, c, q_R)$ is continuously differentiable in all its arguments, and then the implicit function theorem guarantees that $a^*(\cdot, \cdot, \cdot)$ is differentiable in its arguments. Using the implicit function theorem, and abbreviating $a^*(b, c) = a^*$, we get

$$0 = \frac{d}{db}L(a^*; b, c, q_R) = -U_a(a^*, q_R, b)f(b) + \frac{\partial a^*}{\partial b} \cdot \int_b^c U_{aa}(a^*, q_R, \omega)\, dF(\omega),$$

whence

$$\frac{\partial a^*}{\partial b} = \frac{U_a(a^*, q_R, b)f(b)}{\displaystyle\int_b^c U_{aa}(a^*, q_R, \omega)\, dF(\omega)}.$$

The denominator is negative because of Assumption 1. The numerator is negative because, whenever $b < c$:

$$0 = L\big(a^*; b, c, q_R\big)$$

$$> \Big[\min_\omega U_a\big(a^*, q_R, \omega\big)\Big] \cdot \int_b^c dF(\omega)$$

$$= U_a\big(a^*, q_R, b\big) \cdot \big[F(c) - F(b)\big],$$

where the first line comes from the definition of $a^*(b, c)$, and the strict inequality comes from Assumption 3 together with $b < c$. The proof in the case $a = b$ is trivial. Hence $\frac{\partial a^*}{\partial b} > 0$.

Following the same steps we get

$$\frac{\partial a^*}{\partial c} = -\frac{U_a\big(a^*, q_R, c\big)f(c)}{\displaystyle\int_b^c U_{aa}\big(a^*, q_R, \omega\big)\,dF(\omega)}.$$

The denominator is negative because of Assumption 1. The numerator is positive because, whenever $b < c$:

$$0 = L\big(a^*; b, c, q_R\big)$$

$$< \Big[\max_\omega U_a\big(a^*, q_R, \omega\big)\Big] \cdot \int_b^c dF(\omega)$$

$$= U_a\big(a^*, q_R, c\big) \cdot \big[F(c) - F(b)\big].$$

Hence $\frac{\partial a^*}{\partial c} > 0$. *Q.E.D.*

LEMMA B.3—*Well-Definedness of Inverse Demand, and Properties of Expected Payoffs*: *Fix q_{-i}. Suppose the N-partition equilibrium correspondence $\Omega_N(\cdot, q_{-i})$ is defined on the interval (c, d). Then the function $V_i^*(y, \overline{q}_{-i}; \Omega_N)$ is differentiable at every $y \in (c, d)$ and the fundamental theorem of calculus applies. If, moreover, the correspondence $\Omega_N(\cdot, q_{-i})$ is extended continuously to $[c, d]$ then $V_i^*(\cdot, \overline{q}_{-i}; \Omega_N)$ is continuous on $[c, d]$.*

PROOF: Fix $\overline{q}_S$. For any $y \in (c, d)$ we have

$$V_R^*(y, \overline{q}_S; \Omega_N) = \sum_{k=1}^N \int_{\mathfrak{w}_{k-1}(y, \overline{q}_S)}^{\mathfrak{w}_k(y, \overline{q}_S)} U\big(\mathfrak{a}_k(y, \overline{q}_S), y, \omega\big)\,dF(\omega).$$

The functions $\mathfrak{w}_k(y, \overline{q}_S)$ are differentiable at y (refer to Section 2.7). By definition, $\mathfrak{a}_k(y, \overline{q}_S) = a(\mathfrak{w}_{k-1}(y, \overline{q}_S), \mathfrak{w}_k(y, \overline{q}_S), y)$ where the function $a(\cdot, \cdot, \cdot)$ was defined on page 7. This function was shown to be differentiable in all its arguments in Lemma B.2. Hence $\mathfrak{a}_k(y, q_S)$ is differentiable in y. The function U is differentiable by assumption, hence $U(\mathfrak{a}_k(y, \overline{q}_S), y, \omega)$ is differentiable in y. Therefore, the function $V_R^*(y, \overline{q}_S; \Omega_N)$ is differentiable at every $y \in (c, d)$. Therefore the fundamental theorem of calculus applies,

that is, for any $a, b \in (c, d)$ we have

$$V_R^*(b, \overline{q}_S; \Omega_N) - V_R^*(a, \overline{q}_S; \Omega_N) = \int_a^b \frac{\partial}{\partial y} V_R^*(y, \overline{q}_S; \Omega_N) \, dy$$

$$= \int_a^b D_R(y; \overline{q}_S, \Omega_N) \, dy.$$

Continuity of $V_R^*(\cdot, \overline{q}_S; \Omega_N)$ at the points c and d holds because if each $\mathfrak{w}_k(y, \overline{q}_S)$ is extended continuously to the closed interval $[c, d]$, then $a(\mathfrak{w}_{k-1}(y, \overline{q}_S), \mathfrak{w}_k(y, \overline{q}_S), y)$ also extends continuously, and since the function U is continuous by assumption, it follows that the function $V_R^*(y, \overline{q}_S; \Omega_N)$ also extends continuously.

The argument for the sender's expected payoff is identical.   Q.E.D.

PROOF OF COROLLARY 3: Suppose the interval (c, d) is in the support of $\Omega(\cdot, \overline{q}_{-i})$. Consider all the intervals $[c_\tau, d_\tau)$ defined in Definition 5 and number them such that $c \in [c_0, d_0)$ and $d \in [c_1, d_1)$. We have

$$V_i^*(d, \overline{q}_{-i}; \Omega) - V_i^*(c, \overline{q}_{-i}; \Omega) = \left[V_i^*(d_0, \overline{q}_{-i}; \Omega_0) - V_i^*(c, \overline{q}_{-i}; \Omega_0) \right]$$

$$+ \sum_{[c_\tau, d_\tau) \subset (c, d)} \left[V_i^*(d_\tau, \overline{q}_{-i}; \Omega_\tau) - V_i^*(c_\tau, \overline{q}_{-i}; \Omega_\tau) \right]$$

$$+ \left[V_i^*(d, \overline{q}_{-i}; \Omega_1) - V_i^*(c_1, \overline{q}_{-i}; \Omega_1) \right],$$

where Ω_τ denotes the particular N-partition correspondence associated to the interval $[d_\tau, d_\tau)$ by the equilibrium correspondence Ω. Note that Ω evaluated at d_τ is set equal to Ω_τ, using continuity of Ω. By Proposition 1, part 4, the sign of every term in brackets is determined by the sign of D_i. Thus, if the interval (c, d) lies inside the region where D_i is nonnegative (resp., nonpositive), then each term in brackets has the same sign and the infinite summation converges. Then $V_i^*(\cdot, \overline{q}_{-i}; \Omega)$ is nondecreasing (resp., nonincreasing).
Q.E.D.

APPENDIX C: TRADING WITH PRIVATE INFORMATION

There is an entrenched, controlling investor who owns θ_C share of the company. There is a continuum of noncontrolling investors, who are essentially noise traders that have perfectly elastic demand at some price p. An expert investor starts out with a share endowment denoted by t^*, he observes ω and then he trades. The expert investor can sell all but ε of his endowment, or buy more shares up to a maximum holding of $(1 - \theta_C)$, at the price set by the non-controlling investors, p. After observing the expert investor's trade, the two engage in cheap talk communication and the controlling investor chooses a.

C.1. *Model Discussion*

In this model, trading takes place under asymmetric information; therefore, trading serves as a signal of the expert's information. The assumption of constant price (i.e., noise traders with perfectly elastic demand) simplifies the analysis, but the analysis would not break down if we assumed a price function that is increasing in the expert's net trade. Finally, this model is most comparable to the entrenched shareholder setting discussed in Section 6.2, in that the controlling shareholder's holdings are fixed.

C.2. *Equilibrium Characterization*

The equilibrium is characterized by two regions. A low-ω region where the expert investor acquires less than $(1 - \theta_C)$; in this region the expert investor's trade fully reveals ω to the controlling investor. And, for some parameter values, a high-ω region where the expert investor acquires exactly $(1 - \theta_C)$ shares and then relies on cheap talk to convey information.

The analysis is inspired by the signaling model in Section 4 of Kartik (2007), where in our case, the expert investor signals through trading shares. However, in the present paper the cost/benefit of signaling is endogenous because the share value is a function of the receiver's action. So his results cannot be applied directly.

C.3. *Fully Separating Region*

In the fully-separating region, the controlling investor learns the exact state ω based on the expert investor's net trade after he learns the information. Denote the expert's ex post position after trading by $t(\omega)$. Suppose the expert investor retrades into $t(\widehat{\omega})$. If $t(\widehat{\omega})$ is fully separating, upon observing it the controlling investor correctly infers that the state is $\widehat{\omega}$ and she takes action $a^* = \widehat{\omega}/\theta_C$ (recall that $r_C = 1$). The expert investor's utility is then

$$-\frac{\sigma_X^2}{2r_E}\left(r_E t(\widehat{\omega})\frac{\widehat{\omega}}{\theta_C} - \omega\right)^2 - \left(t(\widehat{\omega}) - t^*\right)p, \tag{17}$$

where p denotes the price of shares. This is the utility after the expert (and now informed) investor has retraded his position using a separating trading strategy $t(\cdot)$. The term $(t(\widehat{\omega}) - t^*)p$ is the amount of money paid to achieve the new position $t(\widehat{\omega})$.

We want to show that the trading function described by the following differential equation:

$$t'(\omega) = -\frac{\left(\frac{r_E}{\theta_C}t(\omega) - 1\right)\omega}{\left(\frac{r_E}{\theta_C}t(\omega) - 1\right)\omega^2 + \frac{\theta_C}{\sigma_X^2}p}t(\omega) \quad \text{for } \omega > 0 \tag{18}$$

represents the separating region of an equilibrium.

LEMMA C.1—Features of the Solution to the Differential Equation: *Let $t(\cdot)$ solve the differential equation* (18) *with initial condition* $t(0) = \varepsilon$, *where* $0 < \varepsilon < \theta_C/r_E$. *Then* $t'(0) = 0$; $t'(\omega) > 0$ *for all* $\omega > 0$; *and the function* $t(\omega)$ *achieves* θ_C/r_E *asymptotically, but not for finite* ω.

PROOF: Denote

$$f(\omega) = \frac{r_E}{\theta_C}t(\omega) \quad \text{and} \quad k_0 = \frac{\theta_C}{\sigma_X^2}p,$$

so that equation (18) rewrites as

$$f'(\omega) = \frac{(1 - f(\omega))\omega}{(f(\omega) - 1)\omega^2 + k_0}f(\omega). \tag{19}$$

The following equation is an implicit solution of equation (19):

$$2f(\omega) + 2\log(1 - f(\omega)) + \frac{\omega^2}{k_0}f(\omega)^2 + c_0 = 0, \tag{20}$$

where k_0 is fixed by the parameters of the problem, and $c_0 = -2[\log(1 - \frac{r_E}{\theta_C}\varepsilon) + \frac{r_E}{\theta_C}\varepsilon] > 0$ is chosen to satisfy our initial condition $t(0) = \varepsilon$. This can be verified by differentiating (20) with respect to ω.

We now show that $f(\cdot)$ is strictly increasing. Fom (19), this is the case if

$$-\frac{k_0}{\omega^2} < f(\omega) - 1 < 0. \tag{21}$$

Let us first focus on the right-hand inequality of (21). Since $t(0) = \varepsilon < \theta_C/r_E$, at $\omega = 0$ we have $f(\omega) - 1 < 0$. Furthermore, for any finite ω it must be $f(\omega) - 1 < 0$; indeed, at any $\widehat{\omega} < \infty$ such that $f(\widehat{\omega}) = 1$, equation (20) implies

$$2 + 2\log(0) + \frac{(\widehat{\omega})^2}{k_0} + c_0 = 0,$$

which is a contradiction since the LHS is infinite. Let us now focus on the left-hand inequality of (21). This inequality holds at $\omega = 0$ because $f(\omega) - 1 > -\frac{k_0}{\omega^2} = -\infty$. Furthermore, this inequality holds for any finite ω; indeed, by contradiction, let $\widetilde{\omega}$ be the smallest value of ω at which the left-hand inequality of (21) fails:

$$(f(\widetilde{\omega}) - 1)\widetilde{\omega}^2 + k_0 = 0.$$

Then equation (20) reads

$$2\left(1 - \frac{k_0}{\widetilde{\omega}^2}\right) + 2\log\left(\frac{k_0}{\widetilde{\omega}^2}\right) + \frac{\widetilde{\omega}^2}{k_0}\left(1 - \frac{k_0}{\widetilde{\omega}^2}\right)^2 + c_0 = 0.$$

Denoting $x = \frac{k_0}{\widetilde{\omega}^2} > 0$, this equation rewrites as

$$2(1 - x) + 2\log(x) + \frac{1}{x}(1 - x)^2 + c_0 = 0.$$

The above function is decreasing in x (its derivative equals $-(1 - x)^2/x^2$), and it equals $c_0 > 0$ at $x = 1$. Therefore, the function is positive for all $x \in (0, 1]$. But we know that $x = 1 - f(\widetilde{\omega}) < 1$ because by definition of $\widetilde{\omega}$, equation (21) holds for all $\omega \in (0, \widetilde{\omega})$, whence $f(\widetilde{\omega}) > 0$. Therefore, there is no $\widetilde{\omega}$ that solves equation (20). This establishes that $f(\cdot)$ is strictly increasing.

We now show that $\lim_{\omega\to\infty} f(\omega) = 1$, which proves that $t(\omega)$ achieves θ_C/r_E asymptotically. Sending ω to infinity in equation (20) causes the term involving ω^2 to approach $+\infty$ which, recalling that $f(\omega) > 0$, requires that the log term converges to $-\infty$. Thus $\lim_{\omega\to\infty} 1 - f(\omega) = 0$.   *Q.E.D.*

COROLLARY 5: *Let $t(\cdot)$ solve the differential equation* (18) *with initial condition $t(0) = \varepsilon$. Then $t(1) \leq (\theta_C/r_E)$.*

As $t(\omega) \uparrow \frac{\theta_C}{r_E}$ the function $t'(\omega)$ goes to zero hence $t(\omega)$ becomes very flat. This means that slight variations in shares convey a lot of information about the expert's signal. If $\frac{\theta_C}{r_E} < (1 - \theta_C)$ this "efficient signaling"takes place at ownership levels below $1 - \theta_C$. In this case, the signaling equilibrium can be perfectly separating (and revealing) for all ω. For the equilibrium to involve some pooling it must be that

$$\frac{\theta_C}{r_E} > (1 - \theta_C).$$

COROLLARY 6: *Since the $t(\cdot)$ that solves the differential equation* (18) *with initial condition* $t(0) = \varepsilon$ *is positive and nondecreasing in* ω, *the product* $t(\omega)\omega$ *is strictly increasing for all* ω.

LEMMA C.2—Best Response Property: *Let* $t(\cdot)$ *solve the differential equation* (18) *with initial condition* $t(0) = \varepsilon$. *Suppose the controlling investor expects type* ω *to play* $t(\omega)$. *Then any expert type* ω *prefers* $t(\omega)$ *to* $t(\widehat{\omega})$ *for any* $\widehat{\omega} > 0$.

PROOF: Suppose the expert investor who knows the state ω acquires $t(\widehat{\omega})$; then his utility is

$$u(\widehat{\omega}; \omega) = -\frac{\sigma_X^2}{2r_E}\left(r_E t(\widehat{\omega})\frac{\widehat{\omega}}{\theta_C} - \omega\right)^2 - \left(t(\widehat{\omega}) - t^*\right)p. \tag{22}$$

Differentiating this utility function with respect to $\widehat{\omega}$ yields the following first-order condition:

$$u_1(\widehat{\omega}; \omega) = -\left[\frac{\sigma_X^2}{\theta_C}\left(r_E t(\widehat{\omega})\frac{\widehat{\omega}}{\theta_C} - \omega\right)\left(\frac{\partial}{\partial\widehat{\omega}}t(\widehat{\omega})\widehat{\omega}\right)\right] - t'(\widehat{\omega})p = 0. \tag{23}$$

For $\widehat{\omega} = \omega$ to be a maximum, this first-order condition must hold at $\widehat{\omega} = \omega$. We now show that the condition holds if $t(\cdot)$ solves the differential equation (18). To see this, set $\widehat{\omega} = \omega$ and rewrite the first-order condition as follows:

$$
\begin{aligned}
u_1(\omega; \omega) &= -\left[\frac{\sigma_X^2}{\theta_C}\left(r_E t(\omega)\frac{\omega}{\theta_C} - \omega\right)\left(\frac{\partial}{\partial\omega}t(\omega)\omega\right)\right] - t'(\omega)p \\
&= -\left(\frac{r_E}{\theta_C}t(\omega)\omega - \omega\right)\left[t'(\omega)\omega + t(\omega)\right] - \frac{\theta_C}{\sigma_X^2}t'(\omega)p \\
&= -\left[\left(\frac{r_E}{\theta_C}t(\omega)\omega - \omega\right)\omega + \frac{\theta_C}{\sigma_X^2}p\right]t'(\omega) - \left(\frac{r_E}{\theta_C}t(\omega)\omega - \omega\right)t(\omega) = 0, \quad (24)
\end{aligned}
$$

which holds if $t(\cdot)$ solves the differential equation (18). In other words, $t(\omega)$ solves the differential equation (18) for $\omega \in [0, \infty)$ if and only if

$$u_1(\omega; \omega) = 0 \quad \text{for all } \omega \in [0, \infty). \tag{25}$$

Now suppose $t(\omega)$ solves the differential equation (18), and let us check the second-order conditions for a maximum. These conditions require that the expert's utility function (22) be single-peaked. Denoting this function by $u(\widehat{\omega}, \omega)$, the first-order conditions (23) can be written as

$$u_1(\widehat{\omega}, \omega)|_{\widehat{\omega}=\omega} = 0.$$

Using (23), we can write

$$u_1(\widehat{\omega}, \omega) = u_1(\widehat{\omega}, \widehat{\omega}) + \frac{\sigma_X^2}{\theta_C}(\omega - \widehat{\omega})\left(\frac{\partial}{\partial\widehat{\omega}}t(\widehat{\omega})\widehat{\omega}\right)$$

$$= \frac{\sigma_X^2}{\theta_C}(\omega - \widehat{\omega})\left(\frac{\partial}{\partial\widehat{\omega}}t(\widehat{\omega})\widehat{\omega}\right),$$

where the second equality holds because of (25). Since by Corollary 6 $\frac{\partial}{\partial\widehat{\omega}}t(\widehat{\omega})\widehat{\omega} > 0$, this expression shows that the first derivative is is positive for $\widehat{\omega} < \omega$ and negative otherwise. Hence $u(\widehat{\omega}, \omega)$ is indeed single-peaked as a function of $\widehat{\omega}$ and it attains a maximum at $\widehat{\omega} = \omega$.   Q.E.D.

LEMMA C.3—Initial Condition of Equilibrium Trading Strategy: *There are potentially many signaling equilibria, each associated with a different value of $t(0)$. The controling investor is indifferent among them all. The one that is most preferred by the expert investor is the one with the smallest $t(0) = \varepsilon$.*

PROOF: The family of strategies identified by differential equation (18) indexed by its initial conditions $t(0)$, give rise to a family of signaling equilibria. Higher initial conditions result in a pointwise-higher strategies $t(\cdot)$. Irrespective of the initial condition $t(0)$, all strategies identified by differential equation (18) induce the same fully informed controlling agent's action. Therefore, the expert investor's preferred equilibrium within this family of signaling equilibria is the one where his holdings $t(\omega)$ are closest to the expert investor's *preferred holdings conditional on the controlling agent being fully informed*. We now show that the expert investor's preferred holdings conditional on the controlling agent being fully informed are lower than any equilibrium signaling strategy. To see this, notice that if the controlling investor knows ω the expert's problem is

$$\max_t -\frac{\sigma_X^2}{2r_E}\left(r_E t\frac{\omega}{\theta_C} - \omega\right)^2 - (t - t^*)p.$$

The first-order conditions read

$$-\frac{\sigma_X^2}{\theta_C}\left(\frac{r_E}{\theta_C}t^{\mathrm{FI}} - 1\right)\omega^2 - p = 0,$$

where t^{FI} denotes the full-information optimal holdings for the expert investor. A slight rearrangement yields

$$\left(\frac{r_E}{\theta_C}t^{\mathrm{FI}} - 1\right)\omega^2 + \frac{\theta_C}{\sigma_X^2}p = 0.$$

From (21), which must hold in equilibrium, we have

$$\left(\frac{r_E}{\theta_C}t(\omega) - 1\right)\omega^2 + \frac{\theta_C}{\sigma_X^2}p > 0,$$

which in comparison to the previous equation verifies that for any ω, t^{FI} is smaller than the equilibrium trading level $t(\omega)$. Therefore, the equilibrium that is most preferred by the expert investor is the one with the smallest $t(0) = \varepsilon$.   Q.E.D.

PROPOSITION C.1—*Characterization of Fully Separating Equilibrium*: *Let $t(\cdot)$ solve the differential equation* (18) *with initial condition $t(0) \to 0$.*

1. *Suppose $t(1) \le 1 - \theta_C$. Then there is a fully separating equilibrium where all expert types in $[0, 1]$ trade according to $t(\cdot)$. The amount of shares acquired after retrading cannot exceed θ_C / r_E.*

C.4. *Comparative Statics for the Fully Separating Equilibrium Strategy*

Denote by $t(\omega; r_E, \theta_C, \sigma_X^2, p)$ solve the differential equation (18) with initial condition $t(0) = \varepsilon$. We want to see whether, as the parameters r_E, θ_C, σ_X^2, p change, whether the effect on $t(\cdot)$ is monotonic. To do this, we will compare two solutions $t(\omega; p)$ and $t(\omega; p')$ with $p' > p$. If whenever $t(\omega; p) = t(\omega; p')$, we have $t'(\omega; p) > t'(\omega; p')$ then the two solutions never cross and we have our monotonicity result.

LEMMA C.4: *$p' > p$ implies $t(\omega; p) > t(\omega; p')$.*

PROOF: Suppose $t(\omega; p) = t(\omega; p') = t$. Then from differential equation (18), we have

$$t'(\omega; p) = -\frac{\left(\dfrac{r_E}{\theta_C}t - 1\right)\omega}{\left(\dfrac{r_E}{\theta_C}t - 1\right)\omega^2 + \dfrac{\theta_C}{\sigma_X^2}p}t > -\frac{\left(\dfrac{r_E}{\theta_C}t - 1\right)\omega}{\left(\dfrac{r_E}{\theta_C}t - 1\right)\omega^2 + \dfrac{\theta_C}{\sigma_X^2}p'}t = t'(\omega; p').$$

It follows that $t(\omega; p) > t(\omega; p')$.   Q.E.D.

LEMMA C.5: *$r_E' > r_E$ implies $t(\omega; r_E) > t(\omega; r_E')$.*

PROOF: Since $r_E' > r_E$, the following inequalities hold for any $t > 0$:

$$\underbrace{\left(\frac{r_E}{\theta_C}t - 1\right)}_{-}\underbrace{\left[\left(\frac{r_E'}{\theta_C}t - 1\right)\omega^2 + \frac{\theta_C}{\sigma_X^2}p\right]}_{+} < \underbrace{\left(\frac{r_E'}{\theta_C}t - 1\right)}_{-}\underbrace{\left[\left(\frac{r_E}{\theta_C}t - 1\right)\omega^2 + \frac{\theta_C}{\sigma_X^2}p\right]}_{+},$$

$$\frac{\left(\dfrac{r_E}{\theta_C}t - 1\right)\omega}{\left(\dfrac{r_E}{\theta_C}t - 1\right)\omega^2 + \dfrac{\theta_C}{\sigma_X^2}p} < \frac{\left(\dfrac{r_E'}{\theta_C}t - 1\right)\omega}{\left(\dfrac{r_E'}{\theta_C}t - 1\right)\omega^2 + \dfrac{\theta_C}{\sigma_X^2}p},$$

$$-\frac{\left(\dfrac{r_E}{\theta_C}t - 1\right)\omega}{\left(\dfrac{r_E}{\theta_C}t - 1\right)\omega^2 + \dfrac{\theta_C}{\sigma_X^2}p}t > -\frac{\left(\dfrac{r_E'}{\theta_C}t - 1\right)\omega}{\left(\dfrac{r_E'}{\theta_C}t - 1\right)\omega^2 + \dfrac{\theta_C}{\sigma_X^2}p}t.$$

If $t(\omega; r_E) = t(\omega; r_E') = t$ then the above inequality reads

$$t'(\omega; r_E) > t'\left(\omega; r_E'\right).$$

So $r_E' > r_E$ implies that if $t(\omega; r_E) = t(\omega; r_E') = t$ then $t'(\omega; r_E) > t'(\omega; r_E')$. It follows that $t(\omega; r_E) > t(\omega; r_E')$.   Q.E.D.

LEMMA C.6: $\theta'_C > \theta_C$ implies $t(\omega; \theta_C) > t(\omega; \theta'_C)$ if and only if $t(\omega) < \frac{\theta_C}{2r_E}$.

PROOF: Suppose $t(\omega; p) = t(\omega; p') = t$. Then in light of differential equation (18) the following inequalities are equivalent:

$$t'(\omega; \theta_C) > t'(\omega; \theta'_C),$$

$$-\frac{\left(\frac{r_E}{\theta_C}t - 1\right)\omega}{\left(\frac{r_E}{\theta_C}t - 1\right)\omega^2 + \frac{\theta_C}{\sigma_X^2}p}t > -\frac{\left(\frac{r_E}{\theta'_C}t - 1\right)\omega}{\left(\frac{r_E}{\theta'_C}t - 1\right)\omega^2 + \frac{\theta'_C}{\sigma_X^2}p}t,$$

$$\frac{\left(\frac{r_E}{\theta_C}t - 1\right)}{\left(\frac{r_E}{\theta_C}t - 1\right)\omega^2 + \frac{\theta_C}{\sigma_X^2}p} < \frac{\left(\frac{r_E}{\theta'_C}t - 1\right)}{\left(\frac{r_E}{\theta'_C}t - 1\right)\omega^2 + \frac{\theta'_C}{\sigma_X^2}p},$$

$$\left(\frac{r_E}{\theta_C}t - 1\right)\left[\left(\frac{r_E}{\theta'_C}t - 1\right)\omega^2 + \frac{\theta'_C}{\sigma_X^2}p\right] < \left(\frac{r_E}{\theta'_C}t - 1\right)\left[\left(\frac{r_E}{\theta_C}t - 1\right)\omega^2 + \frac{\theta_C}{\sigma_X^2}p\right],$$

$$\underbrace{\left(\frac{r_E}{\theta_C}t - 1\right)}_{-}\theta'_C < \underbrace{\left(\frac{r_E}{\theta'_C}t - 1\right)}_{-}\theta_C,$$

$$\frac{1}{\theta_C}\underbrace{\left(\frac{r_E}{\theta_C}t - 1\right)}_{-} < \frac{1}{\theta'_C}\underbrace{\left(\frac{r_E}{\theta'_C}t - 1\right)}_{-}.$$

The inequality holds if

$$0 < \frac{\partial}{\partial\theta}\left(\frac{r_E}{\theta^2}t - \frac{1}{\theta}\right)$$

$$= -\frac{2}{\theta^3}r_E t + \frac{1}{\theta^2},$$

which is equivalent to $t < \frac{\theta}{2r_E}$.   *Q.E.D.*

C.5. *Pooling Region*

The pooling region has the form $[\overline{\omega}, 1]$. On that region all types purchase the maximum amount of available shares (in our case, $1 - \theta_C$) and so no signal is conveyed by share position. Instead, communication takes place via cheap talk. Type $\overline{\omega}$ has to be indifferent between buying $t(\overline{\omega})$ and being perfectly revealed, or buying $1 - \theta_C$ and pooling with the lowest interval in the cheap talk equilibrium partition. Given an equilibrium characterized by a threshold $\overline{\omega}$, a full-revelation trading function $t(\cdot)$, and a partition $\{\overline{\omega}, \omega_1, \omega_2, \ldots\}$ of the pooling region, type ω's payoff from buying $t(x)$ for any $x < \overline{\omega}$ is, by (22):

$$\underline{U}(x; \omega) = -\frac{\sigma_X^2}{2r_E}\left(r_E t(x)\frac{x}{\theta_C} - \omega\right)^2 - (t(x) - t^*)p + \frac{\sigma_X^2}{2r_E}\omega^2 \quad \text{for any } x < \overline{\omega},$$

whereas type ω's payoff from pooling at $1 - \theta_C$ and inducing any action $y\theta_C > \overline{\omega}\theta_C$ is

$$\overline{U}(y; \omega) = -\frac{\sigma_X^2}{2r_E}\left(r_E(1 - \theta_C)\frac{y}{\theta_C} - \omega\right)^2 - (1 - \theta_C - t^*)p + \frac{\sigma_X^2}{2r_E}\omega^2 \quad \text{for any } y > \overline{\omega}.$$

LEMMA C.7—*Higher Types Are More Inclined to Pool: Fix $x < \overline{\omega} < y$. Then $\overline{U}(y; \omega) - \underline{U}(x; \omega)$ is increasing in ω.*

PROOF:

$$\overline{U}(y; \omega) - \underline{U}(x; \omega)$$

$$= -\frac{\sigma_X^2}{2r_E}\left(r_E(1 - \theta_C)\frac{y}{\theta_C} - \omega\right)^2 - (1 - \theta_C - t^*)p$$

$$+ \frac{\sigma_X^2}{2r_E}\left(r_E t(x)\frac{x}{\theta_C} - \omega\right)^2 + (t(x) - t^*)p$$

$$= -\frac{\sigma_X^2}{2r_E}\left(r_E(1 - \theta_C)\frac{y}{\theta_C} - \omega\right)^2 + \frac{\sigma_X^2}{2r_E}\left(r_E t(x)\frac{x}{\theta_C} - \omega\right)^2 + (t(x) - 1 + \theta_C)p.$$

The derivative with respect to ω is

$$\frac{\sigma_X^2}{r_E}\left(r_E(1 - \theta_C)\frac{y}{\theta_C} - \omega\right) - \frac{\sigma_X^2}{r_E}\left(r_E t(x)\frac{x}{\theta_C} - \omega\right)$$

$$= \frac{\sigma_X^2}{r_E}\left(r_E(1 - \theta_C)\frac{y}{\theta_C} - r_E t(x)\frac{x}{\theta_C}\right),$$

which is positive because $(1 - \theta_C) > t(x)$ and $y > x$.   *Q.E.D.*

From this, we have that the equilibrium is characterized by a cutoff type, which is indifferent between separating and pooling, that is, a type $\overline{\omega}$ which solves

$$0 = -\frac{\sigma_X^2}{2r_E}\left(r_E t(\overline{\omega})\frac{\overline{\omega}}{\theta_C} - \overline{\omega}\right)^2 - (t(x) - t^*)p$$

$$+ \frac{\sigma_X^2}{2r_E}\left(r_E(1 - \theta_C)\frac{y_1(\overline{\omega})}{\theta_C} - \overline{\omega}\right)^2 + (1 - \theta_C - t^*)p$$

$$= -\frac{\sigma_X^2}{2r_E}\left[\left(r_E t(\overline{\omega})\frac{\overline{\omega}}{\theta_C} - \overline{\omega}\right)^2 - \left(r_E(1 - \theta_C)\frac{y_1(\overline{\omega})}{\theta_C} - \overline{\omega}\right)^2\right] + (1 - \theta_C - t(\overline{\omega}))p. \quad (26)$$