*Econometrica Supplementary Material*

# SUPPLEMENT TO “THE COLLECTIVE MODEL OF HOUSEHOLD CONSUMPTION: A NONPARAMETRIC CHARACTERIZATION”

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THIS SUPPLEMENTARY DOCUMENT generalizes our main results for two-member households toward $M$-member households: Section S1 contains the nonparametric characterization of collective rationality, Section S2 discusses the minimum number of goods and observations that enable rejection of $M$-member collective rationality, and Section S3 presents the testable collective rationality conditions. In addition, Section S4 clarifies the bargaining power interpretation of the “situation-dependent dictatorship” solution that underlies the sufficiency result (for two-member households) in Proposition 4 of the main text.

## S1. A CHARACTERIZATION OF COLLECTIVE RATIONALITY FOR M-MEMBER HOUSEHOLDS

The discussion in the main text restricts to two-member households. In this supplemental material, we consider the case with $M$ household members. Note that this general case includes the two-member model ($M = 2$) and the unitary model ($M = 1$) as special cases.

The household's observed aggregate quantities $\mathbf{q}$ are now decomposed into $M$ quantities $\mathbf{q}^m$ ($m = 1, \dots, M$) that capture private consumption and quantities $\mathbf{q}^h$ that represent public consumption. The different quantities are interrelated as follows:

$$\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2 + \cdots + \mathbf{q}^M + \mathbf{q}^h.$$

Each member $m$ ($m = 1, \dots, M$) is further characterized by own preferences that are represented by a nonsatiated utility function $U^m(\mathbf{q}^1, \mathbf{q}^2, \dots, \mathbf{q}^M, \mathbf{q}^h)$ that is nondecreasing in its arguments.

As was the case for two-member households, we assume a set of $T$ observations of prices and quantities, where $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \dots, T\}$ denotes the set of observations. To generalize Definition 1 of the main text, for observed quantities $\mathbf{q}_j$, we define *feasible personalized quantities* $\widehat{\mathbf{q}}_j$ as

$$\text{(S1.1)} \quad \widehat{\mathbf{q}}_j = (\mathfrak{q}_j^1, \dots, \mathfrak{q}_j^M, \mathfrak{q}_j^h) \quad \text{with}$$
$$\mathfrak{q}_j^1, \dots, \mathfrak{q}_j^M, \mathfrak{q}_j^h \in \Re_+^n \quad \text{and} \quad \mathfrak{q}_j^1 + \cdots + \mathfrak{q}_j^M + \mathfrak{q}_j^h = \mathbf{q}_j.$$

The interpretation is directly analogous to that for the two-member model. Using (S1.1), we have the following definition.

DEFINITION S1: Let $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \dots, T\}$ be a set of observations. A combination of $M$ utility functions $U^1, \dots, U^M$ provides an *M-member collective rationalization* (CR-M) of $S$ if for each observation $j$ there exist feasible personalized quantities $\widehat{\mathbf{q}}_j = (\mathfrak{q}_j^1, \dots, \mathfrak{q}_j^M, \mathfrak{q}_j^h)$ and $\mu_j^2, \dots, \mu_j^M \in \Re_{++}$ such that

$$U^1(\widehat{\mathbf{q}}_j) + \sum_{m=2}^{M} \mu_j^m U^m(\widehat{\mathbf{q}}_j) \geq U^1(\widehat{\mathbf{z}}) + \sum_{m=2}^{M} \mu_j^m U^m(\widehat{\mathbf{z}})$$

for all $\widehat{\mathbf{z}} = (\mathfrak{z}^1, \dots, \mathfrak{z}^M, \mathfrak{z}^h)$ with $\mathfrak{z}^1, \dots, \mathfrak{z}^M, \mathfrak{z}^h \in \Re_+^n$ and $\mathbf{p}_j'(\mathfrak{z}^1 + \cdots + \mathfrak{z}^M + \mathfrak{z}^h) \leq \mathbf{p}_j'\mathbf{q}_j$.

Analogous to the two-member case, optimal household quantities result from the maximization of a weighted sum of household member utilities, with weights representing the bargaining power of the household members. Once more, optimality is to be understood in a Pareto efficiency sense.

To introduce the collective rationalization conditions for the $M$-member case, we define *feasible personalized prices* $(\widehat{\mathbf{p}}_j^1, \dots, \widehat{\mathbf{p}}_j^M)$ as

$$\widehat{\mathbf{p}}_j^m = (\mathfrak{p}_j^{1,m}, \dots, \mathfrak{p}_j^{M,m}, \mathfrak{p}_j^{h,m}) \quad \text{for} \quad m = 1, \dots, M-1,$$

$$\widehat{\mathbf{p}}_j^M = \left(\mathbf{p}_j - \sum_{m=1}^{M-1} \mathfrak{p}_j^{1,m}, \dots, \mathbf{p}_j - \sum_{m=1}^{M-1} \mathfrak{p}_j^{M,m}, \mathbf{p}_j - \sum_{m=1}^{M-1} \mathfrak{p}_j^{h,m}\right),$$

with $\mathfrak{p}_j^{1,m}, \mathfrak{p}_j^{2,m}, \mathfrak{p}_j^{h,m} \in \Re_+^n (m = 1, \dots, M-1)$ so that $\sum_{m=1}^{M-1} \mathfrak{p}_j^{c,m} \leq \mathbf{p}_j$ $(c = 1, \dots, M, h)$; and define a *set of feasible personalized prices and quantities*

$$\widehat{S} = \{(\widehat{\mathbf{p}}_j^1, \dots, \widehat{\mathbf{p}}_j^M; \widehat{\mathbf{q}}_j); j = 1, \dots, T\}. \tag{S1.2}$$

Once more, the interpretation is analogous to that for the two-member case. We then have the following result, which generalizes Proposition 1 of the main text.

PROPOSITION S1: *Let $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \dots, T\}$ be a set of observations. The following conditions are equivalent:*

(i) *There exists a combination of $M$ concave and continuous utility functions $U^1, \dots, U^M$ that provides a CR-M of $S$.*

(ii) *There exists a set of feasible personalized prices and quantities $\widehat{S}$ such that the sets $\{(\widehat{\mathbf{p}}_j^m; \widehat{\mathbf{q}}_j); j = 1, \dots, T\}$ $(m = 1, \dots, M)$ all satisfy the generalized axiom of revealed preference (GARP).*

(iii) *There exists a set of feasible personalized prices and quantities $\widehat{S}$ and numbers $U_j^m, \lambda_j^m > 0$ $(m = 1, \dots, M)$, such that for all $i, j \in \{1, \dots, T\}$, $U_i^m - U_j^m \leq \lambda_j^m (\widehat{\mathbf{p}}_j^m)'(\widehat{\mathbf{q}}_i - \widehat{\mathbf{q}}_j)$.*

The construction of the proof is directly analogous to that of Proposition 1 in the main text.

## S2. MINIMUM NUMBER OF GOODS AND OBSERVATIONS TO ENABLE REJECTION OF M-MEMBER COLLECTIVE RATIONALITY

Let us then regard the minimal empirical conditions for possible rejection of the CR-M conditions in Proposition S1. These are given in the following result, which generalizes Proposition 3 of the main text.

PROPOSITION S2: *There does not always exist a combination of utility functions* $U^1, \ldots, U^M$ *that provide a CR-M of the set of observations* $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ *if and only if* (i) *the number of goods* $n \geq M + 1$ *and* (ii) *the number of observations* $T \geq M + 1$.

PROOF—Necessity: We sketch only the basic intuition for the result that there is always data consistency with the CR-M conditions if $n = M$ or $T = M$. First, consistency with the CR-M conditions for $M$ goods can always be achieved for an intrahousehold allocation with each $m$th ($m \in \{1, \ldots, M\}$) member consuming exclusively the $m$th commodity. Next, consistency with the CR-M conditions for $M$ observations can always be achieved for an intrahousehold allocation with each $m$th ($m \in \{1, \ldots, M\}$) member as the "dictator" for the $m$th household observation. Compare with the argument following Proposition 3 in the main text.

Sufficiency: We show that a CR-M of the set $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, M + 1\}$ is impossible if the following conditions are met:

$$\text{(S2.1)} \qquad \forall j \in \{1, \ldots, M + 1\}, \quad \mathbf{p}_j' \mathbf{q}_j > \mathbf{p}_j' \left( \sum_{\substack{i=1 \\ i \neq j}}^{M+1} \mathbf{q}_i \right).$$

As a preliminary step, we note that for all sets $\widehat{S}$ we have $\forall i, j \in \{1, \ldots, M + 1\}$,

$$\text{(S2.2)} \qquad \sum_{m=1}^{M} (\widehat{\mathbf{p}}_j^m)' \widehat{\mathbf{q}}_i = \mathbf{p}_j' \mathbf{q}_i;$$

this follows from the definitions of $\widehat{\mathbf{p}}_j^m$ and $\widehat{\mathbf{q}}_i$, and will prove useful in our subsequent discussion.

Let us then rewrite the CR-M conditions (iii) of Proposition S1 as (for each $i, j \in \{1, \ldots, M + 1\}$ and $m \in \{1, \ldots, M\}$)

$$\text{(S2.3)} \qquad \frac{1}{\lambda_j^m} (U_i^m - U_j^m) \leq (\widehat{\mathbf{p}}_j^m)' (\widehat{\mathbf{q}}_i - \widehat{\mathbf{q}}_j).$$

Next observe that if there are $M + 1$ observations and given that there are only $M$ household members, then for any possible ordering of each member $m$'s ($m = 1, \dots, M$) "utilities" $U_k^m$ ($k = 1, \dots, M + 1$) there is at least one observation $j \in \{1, \dots, M + 1\}$ of which each $m$th ($m = 1, \dots, M$) household member is dominated in utility terms by some other observation $i(m) \in \{1, \dots, M + 1\}$; that is, $\exists j \in \{1, \dots, M + 1\}$, $\forall m \in \{1, \dots, M\}$, $\exists i(m) \in \{1, \dots, M + 1\}$, $i(m) \neq j$: $U_j^m \leq U_{i(m)}^m$.

Let us then concentrate on such an observation $j$ when constructing necessary conditions for a CR-M of the set $S$. For all $m = 1, \dots, M$, it holds that (see (S2.3))

$$0 \leq \frac{1}{\lambda_j^m}(U_{i(m)}^m - U_j^m) \leq (\widehat{\mathbf{p}}_j^m)'(\widehat{\mathbf{q}}_{i(m)} - \widehat{\mathbf{q}}_j)$$

or

$$(\widehat{\mathbf{p}}_j^m)'\widehat{\mathbf{q}}_j \leq (\widehat{\mathbf{p}}_j^m)'\widehat{\mathbf{q}}_{i(m)}.$$

Using (S2.2), we thus have

$$\text{(S2.4)} \qquad \mathbf{p}_j'\mathbf{q}_j = \sum_{m=1}^{M}(\widehat{\mathbf{p}}_j^m)'\widehat{\mathbf{q}}_j \leq \sum_{m=1}^{M}(\widehat{\mathbf{p}}_j^m)'\widehat{\mathbf{q}}_{i(m)},$$

which provides a lower bound for $\sum_{m=1}^{M}(\widehat{\mathbf{p}}_j^m)'\widehat{\mathbf{q}}_{i(m)}$.

On the other hand, an upper bound can be constructed on the basis of (S2.2), which implies for any subset $\mathbf{M} \subseteq \{1, \dots, M\}$ that

$$\sum_{l \in \mathbf{M}}(\widehat{\mathbf{p}}_j^l)'\widehat{\mathbf{q}}_{i(m)} \leq \sum_{l=1}^{M}(\widehat{\mathbf{p}}_j^l)'\widehat{\mathbf{q}}_{i(m)} \leq \mathbf{p}_j'\mathbf{q}_{i(m)}, \quad \forall i(m)\ (m \in \{1, \dots, M\}).$$

Define $\mathbf{M}_i = \{m \in \{1, \dots, M\} | i(m) = i\}$ for all $i \in \{1, \dots, M + 1\}$. Note that $\mathbf{M}_j = \emptyset$ by construction. Then

$$\text{(S2.5)} \qquad \sum_{m=1}^{M}(\widehat{\mathbf{p}}_j^m)'\widehat{\mathbf{q}}_{i(m)} = \sum_{l \in \mathbf{M}_1}(\widehat{\mathbf{p}}_j^l)'\widehat{\mathbf{q}}_1 + \cdots + \sum_{l \in \mathbf{M}_{M+1}}(\widehat{\mathbf{p}}_j^l)'\widehat{\mathbf{q}}_{M+1} \leq \mathbf{p}_j'\left(\sum_{\substack{i=1 \\ i \neq j}}^{M+1}\mathbf{q}_i\right).$$

From (S2.4) and (S2.5), we derive a necessary condition for a CR-M of the set $S$,

$$\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\left(\sum_{\substack{i=1 \\ i \neq j}}^{M+1}\mathbf{q}_i\right),$$

which conflicts with the property (S2.1) of the observed prices and quantities under consideration.

We conclude that it is impossible to construct $U^1, \ldots, U^M$ that provide a CR-M of a set $S$ that satisfies (S2.1). This shows sufficiency for (at least) $M + 1$ observations. Sufficiency for (at least) $M + 1$ goods follows from Example S1 (CR-M rejection), which follows. *Q.E.D.*

In words, as soon as there are more goods and observations than household members, the collective model can be rejected. If one of the conditions (i) and (ii) in Proposition S2 is not fulfilled, then a CR-M of the set of observations $S$ is always possible.

To further illustrate, we next provide a general price-quantity data structure that cannot be collectively rationalized.

EXAMPLE S1—CR-M Rejection: In the proof of Proposition S2, we established that a CR-M of the set $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, M + 1\}$ is impossible if the following conditions are met:

$$\text{(S2.6)} \qquad \forall j \in \{1, \ldots, M + 1\}, \quad \mathbf{p}_j'\mathbf{q}_j > \mathbf{p}_j'\left(\sum_{\substack{i=1 \\ i \neq j}}^{M+1} \mathbf{q}_i\right).$$

We investigate these conditions for $\mathbf{p}_j \in \Re_{++}^{M+1}$ and $\mathbf{q}_j \in \Re_{+}^{M+1}$ $(j = 1, \ldots, M + 1)$ that have the structure

$$\mathbf{p}_j = \begin{pmatrix} 1 \\ \vdots \\ 1 \\ p \\ 1 \\ \vdots \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{q}_j = \begin{pmatrix} 1 \\ \vdots \\ 1 \\ q \\ 1 \\ \vdots \\ 1 \end{pmatrix},$$

where $p$ and $q$ always appear as the $j$th row elements of, respectively, $\mathbf{p}_j$ and $\mathbf{q}_j$. This specific set of observations $S$ obtains

$$\mathbf{p}_j'\mathbf{q}_j = pq + M \quad \forall j \in \{1, \ldots, M + 1\},$$
$$\mathbf{p}_j'\mathbf{q}_i = p + q + M - 1 \quad \forall i, j \in \{1, \ldots, M + 1\}, \quad j \neq i.$$

Hence, the set $S$ meets (S2.6) if and only if

$$\text{(S2.7)} \qquad pq + M > M(p + q + M - 1).$$

Rewriting (S2.7) as

$$p(q - M) > M(q + M - 2),$$

it is easy to see that for all $q > M$ there exists $p$ such that (S2.7) is met.

To give a numerical example, we reject collective rationality for $M = 5$ if $q = 10$ and $p = 14$. Similar constructions are conceivable for alternative $M$ values.

## S3. TESTABLE COLLECTIVE RATIONALITY RESTRICTIONS

We next generalize the testable collective rationality restrictions of Section 3 of the main text. First, as for the necessity restrictions, we can establish results similar to Lemmas 1 and 2. For compactness, we abstract from a formal statement, but the analogy with the two-member case is easy. Using this, we have the following result.

PROPOSITION S3: *Suppose that there exists a combination of utility functions $U^1, \ldots, U^M$ that provide a CR-M of the set of observations $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$. Then there exist hypothetical relations $H_0^m$ and $H^m$ for each member $m \in \{1, \ldots, M\}$ such that*:

(i) *If $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$, then $\mathbf{q}_i H_0^m \mathbf{q}_j$ for some $m$.*

(ii) *if $\mathbf{q}_i H_0^m \mathbf{q}_k, \mathbf{q}_k H_0^m \mathbf{q}_l, \ldots, \mathbf{q}_z H_0^m \mathbf{q}_j$ for some* (*possibly empty*) *sequence $(k, l, \ldots, z)$, then $\mathbf{q}_i H^m \mathbf{q}_j$;*

(iii) *if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ and $\mathbf{q}_j H^m \mathbf{q}_i$ for all $m \in \mathbf{M} \subseteq \{1, \ldots, M\}$, then $\mathbf{q}_i H_0^l \mathbf{q}_j$ for $l \notin \mathbf{M}$.*

(iv) *If $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'(\sum_{k=1}^{M} \mathbf{q}_{j_k})$ and $\mathbf{q}_{j_{k_1}} H^m \mathbf{q}_i$ for some $k_1 \in \{1, \ldots, M\}$, then $\mathbf{q}_i H_0^l \mathbf{q}_{j_{k_2}}$ (with $m \neq l$) for some $k_2 \in \{1, \ldots, M\}$.*

(v) *$\mathbf{p}_j'\mathbf{q}_j \leq \sum_{\mathbf{q}_i \in R_j} \mathbf{p}_j'\mathbf{q}_i$ for each set $R_j = \{\bigcup_{m=1,\ldots,M} \{\mathbf{q}_{i_m}\} | \forall m \in \{1, \ldots, M\}: \exists i_m \in \{1, \ldots, T\}$ such that $\mathbf{q}_{i_m} H^m \mathbf{q}_j\}$.*

The construction of the proof is directly analogous to that of Proposition 2 in the main text.

This necessary condition has an interpretation directly similar to its two-member analogue. Rules (i)–(iv) contain restrictions on the specification of the hypothetical relations $H_0^m$ and $H^m$ for the given set of observations $S$. Rule (v), which complies with rules (v) and (vi) in Proposition 2, subsequently states that if each household member $m$ prefers $\mathbf{q}_{i_m}$ over $\mathbf{q}_j$, then $\mathbf{q}_j$ cannot be more expensive than the combination of these preferred quantities under the prices $\mathbf{p}_j$. It is easy to verify that this condition reduces to the unitary GARP condition for $M = 1$ (i.e., there is only one household member).

We next define the complementary sufficiency condition for a CR-M of the set of observations $S$.

PROPOSITION S4: *Suppose that for the set of observations $S = \{(\mathbf{p}_j; \mathbf{q}_j); j = 1, \ldots, T\}$ there exist hypothetical relations $H_0^m$ and $H^m$ for each member $m \in \{1, \ldots, M\}$ that satisfy rules* (i)–(v) *in Proposition* S3 *and in addition allow for* *constructing sets $S^1, \dots, S^M$ with $\bigcup_m S^m = S$ and $S^m \cap S^l = \emptyset$ for $m \neq l$ such that*:

(vi) $S^m = \{(\mathbf{p}_j; \mathbf{q}_j) \in S | \mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ *whenever* $\mathbf{q}_i H^m \mathbf{q}_j\}$;

(vii) *for each* $(\mathbf{p}_i; \mathbf{q}_i), (\mathbf{p}_j; \mathbf{q}_j) \in S^m$, $\mathbf{q}_i H_0^m \mathbf{q}_j$ *whenever* $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$.

*Then there exists a combination of utility functions $U^1, \dots, U^M$ that provides a CR-M of the set $S$.*

PROOF: It can be shown that if the set of observations $S$ meets the sufficiency condition, then the conditions for a CR-M of the data are met for the following specification of the set of feasible personalized prices and quantities $\widehat{S}$: (i) we specify the feasible personalized quantities such that for $(\mathbf{p}_i; \mathbf{q}_i) \in S^m$, $\mathfrak{q}_i^m = \mathbf{q}_i$ and $\mathfrak{q}_i^l = \mathbf{0}$ for $l \neq m$; (ii) for each observation $i$ we specify feasible personalized prices such that $\mathfrak{p}_i^{m,m} = \mathbf{p}_i$ for $m \in \{1, \dots, M-1\}$ and $\mathfrak{p}_i^{l,m} = \mathbf{0}$ for $l \neq m$. The construction of the proof is directly analogous to that of Proposition 4 in the main text. *Q.E.D.*

Like in the two-member case, this sufficient condition can be interpreted in terms of a *situation-dependent dictatorship* model. Just like the necessity condition, the sufficiency condition reduces to the GARP condition for $M = 1$. In that case, the only feasible personalized prices and quantities are the observed aggregate prices and quantities, and the necessary and sufficient conditions for rational household behavior always coincide. In the more general case ($M > 1$), we may expect the necessity condition to converge toward the sufficiency condition when the sample size increases; compare with our discussion in Section 4 of the main text.

Finally, because the necessary and sufficient conditions in Propositions S3 and S4 only require aggregate prices $\mathbf{p}_j$ and quantities $\mathbf{q}_j$, they enable operational collective rationality tests that apply to the general case of $T$ observations. Finite algorithms for verifying the conditions are directly similar to those for the two-member case (see the Appendix in the main text).

## S4. BARGAINING POWER AND SITUATION-DEPENDENT DICTATORSHIP

This section shows that the requirement of a strictly positive bargaining power for each household member (i.e., $\infty > \mu_j > 0$ in Definition 1 of the main text) is compatible with the situation-dependent dictatorship solution that underlies the sufficiency result in Proposition 4 of the main text. The following argument concentrates on two-member households for simplicity, but it can directly be generalized to hold for $M$-member households.

To formally explain the compatibility, we first recall the sufficiency part ((iii) $\Rightarrow$ (i)) in the proof of Proposition 1 in the main text. That proof shows that if the data are consistent with the inequalities in part (iii) of Proposition 1, then consistency with a collective rationalization of the observed set $S$ is possible for $\mu_j = (\lambda_j^1/\lambda_j^2)$.

The proof of Proposition 4 then shows consistency with the GARP conditions in part (ii) of Proposition 1 for the following specification of the feasible personalized quantities and prices:

$$\text{if} \quad (\mathbf{p}_j; \mathbf{q}_j) \in S^1, \quad \text{then} \quad \mathfrak{q}_j^1 = \mathbf{q}_j;$$

$$\text{if} \quad (\mathbf{p}_j; \mathbf{q}_j) \in S^2, \quad \text{then} \quad \mathfrak{q}_j^2 = \mathbf{q}_j;$$

$$\mathfrak{p}_j^1 = \mathbf{p}_j, \quad \mathfrak{p}_j^2 = \mathfrak{p}_j^h = \mathbf{0} \quad \text{for all } (\mathbf{p}_j; \mathbf{q}_j) \in S.$$

We note that consistency with these GARP conditions implies consistency with the inequalities in part (iii) of Proposition 1. In fact, it can be verified that for the given specification of the feasible personalized quantities and prices, consistency with these inequalities does not in any way impose $\lambda_j^1 = 0$ or $\lambda_j^2 = 0$. For example, let us give a specific solution for the inequalities for member 1:

$$\text{(S4.1)} \quad \text{for all } (\mathbf{p}_i; \mathbf{q}_i) \in S^1, \quad (\mathbf{p}_j; \mathbf{q}_j) \in S^1: \quad U_i^1 - U_j^1 \leq \lambda_j^1 (\mathbf{p}_j)'(\mathbf{q}_i - \mathbf{q}_j),$$

$$\text{(S4.2)} \quad \text{for all } (\mathbf{p}_i; \mathbf{q}_i) \in S^1, \quad (\mathbf{p}_j; \mathbf{q}_j) \in S^2: \quad U_i^1 - U_j^1 \leq \lambda_j^1 (\mathbf{p}_j)'(\mathbf{q}_i),$$

$$\text{(S4.3)} \quad \text{for all } (\mathbf{p}_i; \mathbf{q}_i) \in S^2, \quad (\mathbf{p}_j; \mathbf{q}_j) \in S^1: \quad U_i^1 - U_j^1 \leq -\lambda_j^1 (\mathbf{p}_j)'(\mathbf{q}_j),$$

$$\text{(S4.4)} \quad \text{for all } (\mathbf{p}_i; \mathbf{q}_i) \in S^2, \quad (\mathbf{p}_j; \mathbf{q}_j) \in S^2: \quad U_i^1 - U_j^1 \leq 0.$$

To construct the solution, we first take, for all $(\mathbf{p}_i; \mathbf{q}_i) \in S^1$, $U_i^1 > 0$ and $\lambda_i^1 > 0$ that obtain consistency with (S4.1); this is possible because the subset $S^1$ is consistent with GARP. Subsequently, we specify for all $(\mathbf{p}_j; \mathbf{q}_j) \in S^2$ that $U_j^1 = \overline{U}$ with $\min_{(\mathbf{p}_i; \mathbf{q}_i) \in S^1} U_i^1 > \overline{U}$ ($>0$) (note that this guarantees consistency with (S4.4)). In that case, we can always choose $\lambda_j^1 > 0$ for all $(\mathbf{p}_j; \mathbf{q}_j) \in S$; see in particular (S4.2) and (S4.3). We conclude that the set of inequalities (S4.1) and (S4.4) does not impose $\lambda_j^1 = 0$ for any $j$. A directly analogous argument obtains that we can always set $\lambda_j^2 > 0$ for any $j$ when the sufficiency condition in Proposition 4 is met.

As a result, we can always specify (for all observations $j$) $\mu_j$ such that $\infty > \mu_j > 0$ to obtain consistency with Definition 1. We thus conclude that our specification of the personalized prices and quantities (i.e., the situation-dependent dictatorship solution) for establishing the sufficiency result in Proposition 4 is consistent with the requirement that $\infty > \mu_j > 0$ in Definition 1.

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