Econometrica Supplementary Material

SUPPLEMENT TO "LEVEL-k AUCTIONS: CAN A NONEQUILIBRIUM MODEL OF STRATEGIC THINKING EXPLAIN THE WINNER'S CURSE AND OVERBIDDING IN PRIVATE-VALUE AUCTIONS?": APPENDIX

(*Econometrica*, Vol. 75, No. 6, November 2007, 1721–1770)

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THIS APPENDIX PROVIDES background and more detail for the paper. It has five sections, the last of which, containing figures, is in a separate pdf document:

A. Interaction Between Value Adjustment and the Bidding Trade-off in First-Price Auctions
B. Calculation of First- and Second-Price Equilibrium, Cursed Equilibrium, and Level-k Bidding Strategies for Kagel and Levin's (1986), Avery and Kagel's (1997), and Goeree, Holt, and Palfrey's (2002) Experimental Designs
C. Estimates for Level-k Plus *Equilibrium* Models with versus Without Truthful Types
D. Estimates of Subject-Specific Precisions and Standard Errors
E. Logit Bid Densities for Random $L1$, Random $L2$, Truthful $L1$, Truthful $L2$, and *Equilibrium* and Representative Precisions

A. INTERACTION BETWEEN VALUE ADJUSTMENT AND THE BIDDING TRADE-OFF IN FIRST-PRICE AUCTIONS

This section studies the interaction between value adjustment and the bidding trade-off in first-price auctions for the level-k models introduced in Section 3, identifying the general principles that determine the bidding strategies in Section 4's examples.

First, isolate the bidding trade-off by considering the level-k bidding strategy with independent private values determined by (20) in the text, which balances the marginal benefit of increasing its bid (the value minus the bid times the increased probability of winning) against the marginal cost (the higher bid times the probability of winning), replacing $v(x,\cdot)$ by x and removing the conditioning from $f_Y(\cdot|x)F_Y(\cdot|x)$. (Private values suffice to isolate the bidding trade-off; we add independence for convenience.)

Now write $a_{k-1}(x,q)$ as a function of a parameter q, where increasing q shifts $a_{k-1}(x,q)$ up for all x and so shifts $a_{k-1}^{-1}(a,q)$ down for all a. With independent private values, (19) becomes

$$\max_a (x-a)F(a_{k-1}^{-1}(a,q)).$$

The first-price level-k bidding strategy is then

$$\begin{aligned}&\arg\max_a (x-a)F(a_{k-1}^{-1}(a,q))\\&\quad\equiv \arg\max_a \bigl\{\log(x-a)+\log\bigl[F(a_{k-1}^{-1}(a,q))\bigr]\bigr\},\end{aligned}$$

where log is the natural logarithm and $F(\cdot)>0$ near the optimum. Because q enters only the second term of the right-hand maximand, the optimal bid a is everywhere increasing in q if and only if (iff):

$$\frac{\partial^2 \log[F(a_{k-1}^{-1}(a,q))]}{\partial a\,\partial q} \equiv \frac{\partial\left[\dfrac{f(a_{k-1}^{-1}(a,q))}{F(a_{k-1}^{-1}(a,q))}\dfrac{\partial a_{k-1}^{-1}(a,q)}{\partial a}\right]}{\partial q} \geq 0 \text{ for all } a \text{ and } q,$$

or equivalently (given that all terms are positive) iff

$$(*)\qquad \frac{\partial\left[\left(\dfrac{F(a_{k-1}^{-1}(a,q))}{f(a_{k-1}^{-1}(a,q))}\right)\Big/\dfrac{\partial a_{k-1}^{-1}(a,q)}{\partial a}\right]}{\partial q} \leq 0 \quad \text{for all } a, x, \text{ and } q,$$

with an analogous condition for the optimal bid to be everywhere decreasing in q. The numerator in the square brackets on the left-hand side of $(*)$ is decreasing in q for most well-behaved distributions; but the denominator is also likely to be decreasing in q. Thus, neither the condition in $(*)$ nor its converse is satisfied for all plausible specifications of $F(\cdot)$ and $a_{k-1}(\cdot,q)$, and the condition is likely to be satisfied for some values of a, but not others: In general, the bidding trade-off may make bidders' bids strategic complements, strategic substitutes, or a mixture of both.

To see what determines the effect of the bidding trade-off more clearly, assume that $a_{k-1}(x)\equiv \gamma x+\delta$ with $\gamma>0$ as in KL's and AK's examples (Section 4), so that $a_{k-1}^{-1}(a)\equiv (a-\delta)/\gamma$ and $(\partial a_{k-1}^{-1}(a))/\partial a = 1/\gamma$. Given this and $(*)$, increasing γ increases the optimal bid a iff it decreases $\gamma F(\frac{a-\delta}{\gamma})/f(\frac{a-\delta}{\gamma})$, which will be the case, given $F(\underline{x})=0$, iff $F_Y(y)/f_Y(y)$ is convex in y. But an increase in δ will increase the optimal bid a iff it decreases $F(\frac{a-\delta}{\gamma})/f(\frac{a-\delta}{\gamma})$, which is the case for most well-behaved distributions. Thus, upward shifts in the slope of others' anticipated bidding strategy tend to make bidders' bids strategic complements (respectively, substitutes) iff $F_Y(y)/f_Y(y)$ is convex (concave) in y, while upward shifts in the level tend to make bidders' bids strategic complements in either case. In KL's example the bidding trade-off tends to make bids strategic complements because it increases the intercept δ but not the slope γ, which offsets the strategic substitutability of value adjustment. With an unconditionally uniform signal distribution, as in most independent-private-value examples, $F_Y(y)/f_Y(y)$ is linear and the bidding trade-off is neutral with respect to shifts in the slope.

The ambiguity just demonstrated for the independent-private-value case persists in the common-value case, where the bidding trade-off interacts with value adjustment: In general, comparing (20) with (2), the first-price level-k bidding strategy can be either higher than the first-price equilibrium bidding strategy, lower, or a mixture of both.

B. CALCULATION OF FIRST- AND SECOND-PRICE EQUILIBRIUM, CURSED-EQUILIBRIUM, AND LEVEL-k BIDDING STRATEGIES FOR KAGEL AND LEVIN'S (1986), AVERY AND KAGEL'S (1997), AND GOEREE, HOLT, AND PALFREY'S (2002) EXPERIMENTAL DESIGNS

This section calculates the first- and second-price equilibrium, cursed-equilibrium, and level-k bidding strategies for Kagel and Levin's (KL's), Avery and Kagel's (AK's) and Goeree, Holt, and Palfrey's (GHP's) examples, as background for the conclusions in Section 4.

1. *Kagel and Levin (1986): Common-Value First- and Second-Price Auctions*

a. *Setup*

- Number of bidders: n.
- Value function: $V_i = u(S, X) = S$, where $S \sim U[\underline{s}, \overline{s}]$.
- Signals: $X_i|S \sim U[s - \frac{a}{2}, s + \frac{a}{2}]$ independent and identically distributed (i.i.d.), $a > 0$. Density, distribution, and expected values of this variable conditional on S are: $f_{X|S} = \frac{1}{a}$, $F_{X|S} = \frac{x-s}{a} + \frac{1}{2}$, and $E[X|S] = s$.
- Also $S|X_i \sim U[x - \frac{a}{2}, x + \frac{a}{2}]$ with density and distribution: $f_{S|X} = \frac{1}{a}$, $F_{S|X} = \frac{s-x}{a} + \frac{1}{2}$, $r(x) = E[S|X = x] = x$.
- Define $Y = \max_{j\neq i}\{X_j\}$ (order statistic), the maximum of $(n-1)$ random draws from the signal distribution. Density and distribution functions of this variable conditional on S are $f_{Y|S} = \frac{(n-1)}{a}(\frac{y-s}{a} + \frac{1}{2})^{n-2}$ and $F_{Y|S} = (\frac{y-s}{a} + \frac{1}{2})^{n-1}$.
- Define $v(x, y) = E[S|X_i = x, Y = y]$, the expected value of the object given my private signal and the highest signal among the rest of individuals being y. We need two distribution functions and two expected values:
  - $f_{Y|X}(y|x)$
  - $f_{S|X,Y}(s|X_i = x, Y = y)$
  - $r(x) = E[s|X_i = x] = x$ (defined above)
  - $v(x, y) = E[s|X_i = x, Y = y]$.

To get the probability distribution function (pdf) of $Y|X$ as well as $S|X, Y$, we need to distinguish whether $X_i > Y$ or $X_i = Y$ or $X_i < Y$. For equilibrium analysis it turns out that the relation is given by $X_i = Y$, but for *level-k* thinking (nonequilibrium), to obtain the optimal strategy we need to consider the whole range $X_i \gtreqless Y$. We will get general expressions for these pdf's and expected values:

$$f_{Y|X}(y|x) = \left[-\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n-1}\right]_{\text{lowerbound}}^{\text{upperbound}},$$

$$f_{S|X,Y}(s|x,y) = \frac{\frac{(n-1)}{a}\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n-2}}{\left[-\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n-1}\right]_{\text{lowerbound}}^{\text{upperbound}}},$$

$$v(x,y) = \int s\frac{\frac{(n-1)}{a}\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n-2}}{\left[-\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n-1}\right]_{\text{lowerbound}}^{\text{upperbound}}}\,ds$$

$$= \frac{\int s\frac{(n-1)}{a}\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n-2}ds}{\left[-\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n-1}\right]_{\text{lowerbound}}^{\text{upperbound}}}.$$

For this last expression, define, $u(s) = s$ and $v'(s) = \frac{n-1}{a}(\frac{y-s}{a}+\frac{1}{2})^{n-2}$, where $v(s) = -(\frac{y-s}{a}+\frac{1}{2})^{n-1}$, and do integration by parts to obtain the expression

$$v(x,y) = \frac{\int s\frac{(n-1)}{a}\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n-2}ds}{\left[-\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n-1}\right]_{\text{lowerbound}}^{\text{upperbound}}}$$

$$= \frac{\left[-s\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n-1}-\frac{a}{n}\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n}\right]_{\text{lowerbound}}^{\text{upperbound}}}{\left[-\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n-1}\right]_{\text{lowerbound}}^{\text{upperbound}}}.$$

The bounds will depend on whether $X > Y$ or $Y > X$ or $Y = X$. We have three cases:

1. If $Y = X$, $[x - a/2, x + a/2]$. This is the case for equilibrium analysis:

$$f_{Y|X} = \frac{1}{a}\left[-\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n-1}\right]_{x-a/2}^{x+a/2} = \frac{1}{a}, \quad F_{Y|X} = \frac{1}{n},$$

$$f_{S|X,Y}(s|x,y) = \frac{\dfrac{(n-1)}{a}\left(\dfrac{y-s}{a}+\dfrac{1}{2}\right)^{n-2}}{\left[-\left(\dfrac{y-s}{a}+\dfrac{1}{2}\right)^{n-1}\right]_{x-a/2}^{x+a/2}}$$

$$= \frac{(n-1)}{a}\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n-2},$$

$$v(x,x) = \frac{\left[-s\left(\dfrac{x-s}{a}+\dfrac{1}{2}\right)^{n-1} - \dfrac{a}{n}\left(\dfrac{x-s}{a}+\dfrac{1}{2}\right)^{n}\right]_{x-a/2}^{x+a/2}}{\left[-\left(\dfrac{x-s}{a}+\dfrac{1}{2}\right)^{n-1}\right]_{x-a/2}^{x+a/2}}$$

$$= x - \frac{a}{2} + \frac{a}{n}.$$

2. If $Y < X$, $x - a < y < x$, $[x - a/2, y + a/2]$,

$$f_{Y|X} = \frac{1}{a}\left[-\left(\frac{y-s}{a}+\frac{1}{2}\right)^{n-1}\right]_{x-a/2}^{y+a/2} = \frac{1}{a}\left[\left(1-\frac{x-y}{a}\right)^{n-1}\right],$$

$$F_{Y|X} = \frac{1}{a}\left[\frac{a}{n}\left(1-\frac{x-y}{a}\right)^{n}\right],$$

$$f_{S|X,Y}(s|x,y) = \frac{\dfrac{(n-1)}{a}\left(\dfrac{y-s}{a}+\dfrac{1}{2}\right)^{n-2}}{\left[\left(1-\dfrac{x-y}{a}\right)^{n-1}\right]},$$

$$F_{S|X,Y}(s|x,y) = 1 - \frac{\left(\dfrac{y-s}{a}+\dfrac{1}{2}\right)^{n-1}}{\left[\left(1-\dfrac{x-y}{a}\right)^{n-1}\right]},$$

$$v(x,y) = \frac{\left[-s\left(\dfrac{y-s}{a}+\dfrac{1}{2}\right)^{n-1} - \dfrac{a}{n}\left(\dfrac{y-s}{a}+\dfrac{1}{2}\right)^{n}\right]_{x-a/2}^{y+a/2}}{\left[-\left(\dfrac{y-s}{a}+\dfrac{1}{2}\right)^{n-1}\right]_{x-a/2}^{y+a/2}}$$

$$= x - \frac{a}{2} + \frac{a}{n} - \frac{x-y}{n}.$$

3. If $Y > X$, $x < y < x + a$, $[y - a/2, x + a/2]$,

$$f_{Y|X} = \frac{1}{a}\left[-\left(\frac{y-s}{a} + \frac{1}{2}\right)^{n-1}\right]_{y-a/2}^{x+a/2} = \frac{1}{a}\left[1 - \left(\frac{y-x}{a}\right)^{n-1}\right],$$

$$F_{Y|X} = \frac{1}{a}\left[(y-x) - \frac{a}{n}\left(\frac{y-x}{a}\right)^{n}\right] + \frac{1}{n},$$

$$f_{S|X,Y}(s|x,y) = \frac{\dfrac{(n-1)}{a}\left(\dfrac{y-s}{a} + \dfrac{1}{2}\right)^{n-2}}{\left[1 - \left(\dfrac{y-x}{a}\right)^{n-1}\right]},$$

$$F_{S|X,Y}(s|x,y) = 1 - \frac{\left(\dfrac{y-s}{a} + \dfrac{1}{2}\right)^{n-1}}{\left[1 - \left(\dfrac{x-y}{a}\right)^{n-1}\right]},$$

$$v(x,y) = \frac{\left[-s\left(\dfrac{y-s}{a} + \dfrac{1}{2}\right)^{n-1} - \dfrac{a}{n}\left(\dfrac{y-s}{a} + \dfrac{1}{2}\right)^{n}\right]_{y-a/2}^{x+a/2}}{\left[-\left(\dfrac{y-s}{a} + \dfrac{1}{2}\right)^{n-1}\right]_{y-a/2}^{x+a/2}}$$

$$= y - \frac{a}{2} + \frac{a}{n} - \frac{\left(\dfrac{y-x}{a}\right)^{n-1}}{\left[1 - \left(\dfrac{y-x}{a}\right)^{n-1}\right]}\left(\frac{n-1}{n}\right)(x + a - y).$$

Summarizing:

$$f_{Y|X} = \begin{cases} \dfrac{1}{a}\left[\left(1 - \dfrac{x-y}{a}\right)^{n-1}\right], & x - a < y \le x, \\ \dfrac{1}{a}\left[1 - \left(\dfrac{y-x}{a}\right)^{n-1}\right], & x < y < x + a, \end{cases}$$

$$F_{Y|X} = \begin{cases} \dfrac{1}{a}\left[\dfrac{a}{n}\left(1 - \dfrac{x-y}{a}\right)^{n}\right], & x - a < y \le x, \\ \dfrac{1}{a}\left[(y-x) - \dfrac{a}{n}\left(\dfrac{y-x}{a}\right)^{n}\right] + \dfrac{1}{n}, & x < y < x + a, \end{cases}$$

$$v(x,y)=\begin{cases} x-\dfrac{a}{2}+\dfrac{a}{n}-\dfrac{x-y}{n}, \\ \quad x-a<y\le x, \\ y-\dfrac{a}{2}+\dfrac{a}{n}-\dfrac{\left(\dfrac{y-x}{a}\right)^{n-1}}{\left[1-\left(\dfrac{y-x}{a}\right)^{n-1}\right]}\left(\dfrac{n-1}{n}\right)(x+a-y), \\ \quad x<y<x+a. \end{cases}$$

So the range for the value of the object conditional on winning is $x-\frac{a}{2}\le v(x,y)\le x+\frac{a}{2}$.

b. *Equilibrium and cursed-equilibrium bidding strategies*

We will derive the equilibrium bidding strategy and remark the cursed-equilibrium bidding strategy, which rests on similar arguments. As in the paper, we use a for first-price and b for second-price bidding strategies.

First price: The first-order conditions characterize the equilibrium bidding strategy:

$$a^{*\prime}(x)=(v(x,x)-a^*(x))\frac{f_{Y|X}(x|x)}{F_{Y|X}(x|x)},$$

$$a^*(x)=\frac{\displaystyle\int_{\underline{x}+a/2}^{x} v(x,x)\frac{f_{Y|X}(x|x)}{F_{Y|X}(x|x)}\left(\exp\int_{\underline{x}+a/2}^{x}\frac{f_{Y|X}(i|x)}{F_{Y|X}(i|x)}\,di\right)dx+k}{\left(\exp\displaystyle\int_{\underline{x}+a/2}^{x}\frac{f_{Y|X}(i|x)}{F_{Y|X}(i|x)}\,di\right)}.$$

Substitute $(f_{Y|X}(x|x))/(F_{Y|X}(x|x))=\frac{n}{a}$ and $v(x,x)=x-\frac{a}{2}+\frac{a}{n}$:

$$a^*(x)=\frac{\displaystyle\int_{\underline{x}+a/2}^{x}\left(x-\frac{a}{2}+\frac{a}{n}\right)\frac{n}{a}\left(\exp\int_{\underline{x}+a/2}^{x}\frac{n}{a}\,di\right)dx+k}{\left(\exp\displaystyle\int_{\underline{x}+a/2}^{x}\frac{n}{a}\,di\right)}.$$

First, $\exp\int_{\underline{x}+a/2}^{x}\frac{n}{a}\,di=\exp(\frac{n}{a}(x-\underline{x}-\frac{a}{2}))$. Now to solve for $\int_{\underline{x}+a/2}^{x}(x-\frac{a}{2}+\frac{a}{n})\frac{n}{a}\exp(\frac{n}{a}(x-\underline{x}-\frac{a}{2}))\,dx$, use integration by parts:

$$\int_{\underline{x}+a/2}^{x}\left(x-\frac{a}{2}+\frac{a}{n}\right)\frac{n}{a}\exp\left(\frac{n}{a}\left(x-\underline{x}-\frac{a}{2}\right)\right)dx$$
$$=\exp\left(\frac{n}{a}\left(x-\underline{x}-\frac{a}{2}\right)\right)\left[x-\frac{a}{2}\right]-\underline{x}.$$

So we obtain

$$a^*(x) = \frac{\exp\left(\frac{n}{a}\left(x - \underline{x} - \frac{a}{2}\right)\right)\left[x - \frac{a}{2}\right] - \underline{x} + k}{\exp\left(\frac{n}{a}\left(x - \underline{x} - \frac{a}{2}\right)\right)}$$
$$= x - \frac{a}{2} + (k - \underline{x})\exp\left(\frac{n}{a}\left(-x + \underline{x} + \frac{a}{2}\right)\right).$$

Now, to solve for k, use the initial condition $a^*(\underline{x} + \frac{a}{2}) = \underline{x} + \frac{a}{n+1}$ so

$$\left(\underline{x} + \frac{a}{2} - \frac{a}{2}\right) + (k - \underline{x})\exp\left(\frac{n}{a}\left(-\underline{x} - \frac{a}{2} + \underline{x} + \frac{a}{2}\right)\right) = \underline{x} + \frac{a}{n+1}.$$

Then $k - \underline{x} = \frac{a}{n+1}$.

For equilibrium and cursed equilibrium the bidding strategies are given by[1]

$$\boxed{a_*(x) = x - \frac{a}{2} + \frac{a}{n+1}\exp\left(-\frac{n}{a}\left(x - \underline{x} - \frac{a}{2}\right)\right),}$$

$$\boxed{\begin{aligned} a_\chi(x) = {} & \left[\chi x + (1-\chi)\left(x - \frac{a}{2} + \frac{a}{n}\right) - \frac{a}{n}\right] \\ & + \frac{a}{n+1}\exp\left(-\frac{n}{a}\left(x - \underline{x} - \frac{a}{2}\right)\right). \end{aligned}}$$

Second price: The optimal bidding strategy is given by $b^*(x) = v(x, x)$ and in this example

$$v(x, x) = x - \frac{a}{2} + \frac{a}{n}.$$

For equilibrium and cursed equilibrium the bidding strategy is given by

$$\boxed{b_*(x) = v(x, x) = x - \frac{a}{2} + \frac{a}{n},}$$

$$\boxed{b_\chi(x) = \chi x + (1-\chi)\left(x - \frac{a}{2} + \frac{a}{n}\right) = x - (1-\chi)a\frac{n-2}{2n}.}$$

[1]We are grateful to Yong Chao, John Kagel, and Dan Levin for correcting an error in our calculations that had previously convinced us that their version of this formula had a minor error.

If we compare equilibrium and cursed equilibrium, we see that the only difference between them is the level of cursedness χ and the difference between $v(x, x)$ and $r(x)$. In this specific example, if we compare these two expressions, we see that $v(x, x) = (x - \frac{a}{2} + \frac{a}{n}) \leq r(x) = x$ for $2 \leq n$, so fully-cursed-equilibrium bidders will overbid with respect to equilibrium if $n > 2$.

c. *Level-k bidding strategies*

We will look separately at specifications based on a random and a truthful $L0$.

c1. *Random $L1$ and random $L2$.* Random $L0$ is specified as uniform on the range of the values. In this specific example, the signal is informative about the possible range of the value and its own signal can be the lowest or highest possible; therefore, $b_0^r \sim U[x - a, x + a]$.

Random $L1$ bidders believe winning is not informative, so the value function considered is $r(x) = x$ and to calculate the bidding trade-off, the pdf and cumulative distribution function (cdf) considered are those of the highest uniform bid among $(n - 1)$ signals in the interval $[x - a, x + a]$:

$$f_{Z_1}(z) = (n-1)\left(\frac{z - x + a}{2a}\right)^{n-2} \frac{1}{2a} \quad \text{and}$$

$$F_{Z_1}(z) = \left(\frac{z - x + a}{2a}\right)^{n-1}.$$

First price: The bidding strategy is defined by the first-order conditions

$$(x - b) f_{Z_1}(b) - F_{Z_1}(b) = 0,$$

$$(x - b)(n-1)\left(\frac{b - x + a}{2a}\right)^{n-2} \frac{1}{2a} - \left(\frac{b - x + a}{2a}\right)^{n-1} = 0.$$

Solving for the optimal random $L1$ bidding strategy, we obtain

$$\boxed{a_1^r(x) = x - \frac{a}{n}.}$$

Random $L1$'s optimal bidding strategy approximately coincides with the fully-cursed equilibrium (ignoring the exponential part) and therefore random $L1$ bidders will overbid with respect to equilibrium when $n > 2$.

Second price: The bidding strategy is defined by the first-order conditions

$$(x - b) f_{Z_1}(b) = 0, \quad (x - b)(n-1)\left(\frac{b - x + a}{2a}\right)^{n-2} \frac{1}{2a} = 0.$$

Solving for the optimal random $L1$ bidding strategy, we obtain

$$\boxed{b_1^r(x) = x.}$$

This coincides with fully-cursed equilibrium and therefore random $L1$ will overbid with respect to equilibrium for $n > 2$.

Random $L2$ will best respond to random $L1$ bidders who are bidding $a_1^r(x) = x - a/n$ and $b_1^r(x) = x$ in first and second price, respectively:

First price: The first-order conditions that characterize the optimal bid are

$$\left(v(x, a_1^{r-1}(a)) - a\right) f_{Y|X}(a_1^{r-1}(a)|x) \frac{\partial a_1^{r-1}(a)}{\partial a} - F_{Y|X}(a_1^{r-1}(a)|x) = 0.$$

The optimal bid falls into the first region ($y > x$):

$$\left(x - \frac{a}{2}\left(\frac{n-2}{n}\right) - \frac{x - (b + a/n)}{n} - b\right)$$
$$= \frac{\frac{1}{a}\left[\frac{a}{n}\left(1 - \frac{x - (b + a/n)}{a}\right)^n\right]}{\frac{1}{a}\left[\left(1 - \frac{x - (b + a/n)}{a}\right)^{n-1}\right]},$$
$$\boxed{a_2^r(x) = x - \frac{a}{2}.}$$

Random $L2$ optimal bidding will approximately coincide with equilibrium bidding strategy (except the exponential part).

Second price: The first-order conditions that describe the optimal bidding strategy are

$$v(x, b_1^{r-1}(b)) - b = 0, \quad x - \frac{a}{2} + \frac{a}{n} - \frac{x - b_1^{r-1}(b)}{n} - b = 0,$$
$$\boxed{b_2^r(x) = x - \frac{a}{2}\left(\frac{n-2}{n-1}\right).}$$

Random $L2$ bidders will underbid relative to equilibrium.

c2. *Truthful $L1$ and truthful $L2$.* Truthful $L0$ will bid the value of the item given by its own signal or, more specifically, $b_0^t(x) = r(x) = x$. Both truthful $L1$ and truthful $L2$ bidders will condition on winning.

Truthful $L1$ will best respond to truthful $L0$ bidders who bid $b_0^t(x) = x$ both in first and second price.

First price: The first-order conditions given $b_0^{t-1}(b) = b$ simplify to

$$(v(x,b) - b) f_{Y|X}(b|x) - F_{Y|X}(b|x) = 0,$$

$$\left(x - \frac{a}{2} + \frac{a}{n} - \frac{x-b}{n} - b\right) = \frac{a}{n}\left(1 - \frac{x-b}{a}\right),$$

$$\boxed{a_1^t(x) = x - \frac{a}{2}.}$$

This approximately coincides with equilibrium (except for the exponential part).

Second price: Since truthful $L0$ coincides with random $L1$, then truthful $L1$ will coincide with random $L2$:

$$\boxed{b_1^t(x) = x - \frac{a}{2}\left(\frac{n-2}{n-1}\right).}$$

Truthful $L1$ bidders will underbid relative to equilibrium for $n > 2$.

Truthful $L2$ bidders will best respond to truthful $L1$ bidders who will bid $a_1^t(x) = x - \frac{a}{2}$ and $b_1^t(x) = x - \frac{a}{2}(\frac{n-2}{n-1})$ in first and second price, respectively.

First price: Truthful $L1$ bidders are already playing approximately equilibrium, so a best response to equilibrium will also be approximately equilibrium.

$$\boxed{b_2^t(x) = x - \frac{a}{2}.}$$

Truthful $L2$ will approximately coincide with equilibrium (except for the exponential part).

Second price: Truthful $L1$ bidders bid $b_1^t(x) = x - \frac{a}{2}(\frac{n-2}{n-1})$ so they are bidding less than in equilibrium. Define the inverse function

$$b_1^{t-1}(b) = b + \frac{a}{2}\left(\frac{n-2}{n-1}\right) = b + c.$$

First-order conditions for the second range ($x > y$) are

$$\big(v(x, b_1^{t-1}(b)) - b\big) f_{Y|X}(b_1^{t-1}(b)|x) \frac{\partial b_1^{t-1}(b)}{\partial b} = 0,$$

$$\frac{b + c - \frac{a}{2} + \frac{a}{n} - \left(\frac{b+c-x}{a}\right)^{n-1}\left(x + \frac{a}{2} + \frac{b+c-x}{n}\right)}{1 - \left(\frac{b+c-x}{a}\right)^{n-1}} - b = 0,$$

$$(b + c - x)^{n-1}\left(\frac{an}{2} + c - n(b - x) + b - x\right) = na^{n-1}\left(c - \frac{a}{2} + \frac{a}{n}\right).$$

Define $B = b - x$

$$\boxed{(B + c)^{n-1}\left(\frac{an}{2} + c - B(n - 1)\right) = na^{n-1}\left(c - \frac{a}{2} + \frac{a}{n}\right).}$$

We have a polynomial to solve and the degree of the polynomial will depend on the number of players. Truthful $L2$ bidders are bidding higher than their signal and, therefore, higher than even the fully-cursed-equilibrium bidders for $n > 2$.

2. *Avery and Kagel (1997): Common-Value Second-Price Auction*

a. *Setup*

- Number of bidders $n = 2$.
- Value function: $V_i = u(X, S) = X_1 + X_2$.
- Signals: Two i.i.d. $X_i \sim U[\underline{x}, \overline{x}] = [1, 4]$ independent private signals.
- One important random variable is the highest signal among $(n - 1)$ individuals $Y = \max_{j \neq i}\{X_j\}$. Since we only have two signals, then

$$f_Y(y) = \frac{1}{\overline{x} - \underline{x}}, \quad F_Y = \left(\frac{y - \underline{x}}{\overline{x} - \underline{x}}\right).$$

- There are two important functions:
  ✓ $v(x, y) = E[u(X, S)|X_i = x, Y = y]$,
  ✓ $r(x) = E[u(X, S)|X_i = x]$;

$$\begin{aligned} v(x, y) &= \sum_{k=1}^{N} E[X_k|X_i = x, Y = y] = x + \sum_{k \neq i}^{N} E[X_k|X_i = x, Y = y] \\ &= x + (n - 1) \\ &\quad \times \left[\frac{1}{(n-1)} E[X_k|X_k = y] + \frac{(n-2)}{(n-1)} E[X_k|X_k \leq y]\right] \end{aligned}$$

$$= x + y\frac{n}{2} + \frac{(n-2)}{2}\underline{x}.$$

So in general, $x + \underline{x}(n-1) \leq v(x, y) \leq x + (\overline{x} + \underline{x})\frac{n}{2} - \underline{x}$. Since in this specific example, $n = 2$ and $X \sim U[1, 4]$, then this simplifies to $v(x, y) = x + y$ and $x + 1 \leq v(x, y) \leq x + 4$:

$$r(x) = E\left[\sum_{k=1,\dots,N} X_k | X_i = x\right] = x + (n-1)E[X_k]$$
$$= x + (n-1)\frac{\underline{x} + \overline{x}}{2}.$$

For our specific example, where $n = 2$ and $X \sim U[1, 4]$, the expression simplifies to $r(x) = x + \frac{5}{2}$.

Comparison between $v(x, x)$ and $r(x)$ for any n reveals

$$v(x, x) = x + x\frac{n}{2} + \frac{(n-2)}{2}\underline{x} \lesseqgtr x + (n-1)\frac{\overline{x} + \underline{x}}{2} = r(x),$$
$$v(x, x) = 2x \lesseqgtr x + \frac{5}{2} = r(x),$$
$$x \lesseqgtr \frac{(n-1)\overline{x} + \underline{x}}{n}, \quad x \lesseqgtr \frac{5}{2}.$$

So for high signal holders, $v(x, x) > r(x)$; for low signal holders, $v(x, x) < r(x)$. We will only consider second-price auctions.

b. *Equilibrium and cursed-equilibrium bidding strategies*

We will calculate equilibrium and just provide cursed equilibrium. Also, we will specialize to the special case of the experimental design, $n = 2$, and signals between $[1, 4]$.

Second price: $b_*(x) = v(x, x)$. We can directly substitute $y = x$ to obtain

$$\boxed{b_*(x) = v(x, x) = 2x,}$$
$$\boxed{b_\chi(x) = \chi\left(x + \frac{5}{2}\right) + (1-\chi)2x = x(2-\chi) + \chi\frac{5}{2},}$$
$$2x \leq b_\chi(x) \leq x + \frac{5}{2} \quad \text{for} \quad x < \frac{5}{2},$$
$$x + \frac{5}{2} \leq b_\chi(x) \leq 2x \quad \text{for} \quad x > \frac{5}{2}.$$

c. *Level-k bidding strategies*

We will calculate the optimal bidding strategies for both random and truthful specifications.

c1. *Random $L1$ and random $L2$.* Random $L0$ will be bidding according to the bidding function $b_0^r(x) \sim U[2, 8]$ because this is the range of the values (sum of two signals, where each is uniform in the interval $[1, 4]$).

Random $L1$ will best respond to random $L0$ bidders.

Second price: The optimal bidding strategy is given by

$$\boxed{b_1^r(x) = r(x) = x + \frac{5}{2}.}$$

Random $L1$ bidders will coincide with fully-cursed-equilibrium bidders in all second price. As we already mentioned, low signal holders will overbid relative to equilibrium and high signal holders will underbid relative to equilibrium.

Random $L2$ will best respond to random $L1$ bidders. Since random $L1$ bidders bid monotonically with respect to their signal, random $L2$ bidders will condition on winning.

Second price: The objective function becomes a linear function, so the optimal bidding for random $L2$ bidders will reduce to boundary solutions:

$$\boxed{b_2^r(x) = \begin{cases} b_1^r(\underline{x}) = 3.5, & \text{if } x < 5/2, \\ b_1^r(\overline{x}) = 6.5, & \text{if } x > 5/2. \end{cases}}$$

c2. *Truthful $L1$ and truthful $L2$.* Truthful $L0$ will be bidding according to the bidding function $b_0^t(x) = E[V_i|X_i = x] = r(x) = x + \frac{5}{2}$, so truthful $L1$ will coincide with random $L2$.

Truthful $L1$ will best respond to truthful $L0$ bidders.

Second price: Truthful $L0$ coincides with random $L1$, so truthful $L1$ will coincide with random $L2$:

$$\boxed{b_1^t(x) = \begin{cases} b_0^t(\underline{x}) = 3.5, & \text{if } x < 5/2, \\ b_0^t(\overline{x}) = 6.5, & \text{if } x > 5/2. \end{cases}}$$

Truthful $L2$ bidders best respond to truthful $L1$ bidders.

Second price: Since truthful $L1$ bidders bid monotonically with respect to their signal, then truthful $L2$ bidders will condition on winning. However, truthful $L1$ bidders also bid only two possible bids. This makes the bidding strategy of truthful $L2$ a little different. To calculate the optimal bidding, the following situations will have to be taken into account:

If $b < 3.5$, then the probability of winning is zero.

If $b = 3.5$, then truthful $L2$ bidders will win only if they encounter a low bidder ($\text{prob}(y < 2.5) = 1/2$), and, moreover, they will tie with the low bidder and the price the they will have to pay is 3.5. Their payoff will be given by $(1/4)(x + E[y|y \leq 2.5] - 3.5)$.

If $6.5 > b > 3.5$, then truthful $L2$ bidders will win with probability $1/2$ (if the opponent is a low bidder) and the price will be 3.5. The payoff will be given by $(1/2)(x + E[y|y \leq 2.5] - 3.5)$.

If $b = 6.5$, then truthful $L2$ bidders will beat low bidders (3.5) but will tie with high bidders. The payoff is given by $(1/2)(x + E[y|y \leq 2.5] - 3.5) + (1/4)(x + E[y|y > 2.5] - 6.5)$.

If $b > 6.5$, then truthful $L2$ bidders will win with probability 1, but depending on the partner's bid, the price they will pay is the low or high one. Their payoff is $(1/2)(x + E[y|y \leq 2.5] - 3.5) + (1/2)(x + E[y|y > 2.5] - 6.5)$.

Now, we can calculate both $E[y|y < 2.5] = 7/4$ and $E[y|y > 2.5] = 13/4$. So the objective function will reduce to

$$\pi_2^t(x) = \begin{cases} 0, & \text{for } b_2^t < 3.5, \\ \dfrac{1}{4}\left(x + \dfrac{7}{4} - 3.5\right), & \text{for } b_2^t = 3.5, \\ \dfrac{1}{2}\left(x + \dfrac{7}{4} - 3.5\right), & \text{for } 3.5 < b_2^t < 6.5, \\ \dfrac{1}{2}\left(x + \dfrac{7}{4} - 3.5\right) + \dfrac{1}{4}\left(x + \dfrac{13}{4} - 6.5\right), & \text{for } b_2^t = 6.5, \\ \dfrac{1}{2}\left(x + \dfrac{7}{4} - 3.5\right) + \dfrac{1}{2}\left(x + \dfrac{13}{4} - 6.5\right), & \text{for } b_2^t > 6.5. \end{cases}$$

So with this payoff function, low signal holders will bid lower than 3.5 and medium signal holders will bid slightly higher than 3.5 but lower than 6.5, and only the high signal holders will be willing to go above 6.5:

$$\boxed{b_2^t(x) = \begin{cases} < 3.5, & \text{if } x < 1.75, \\ 3.5 < b_2^t(x) < 6.5, & \text{if } 1.75 < x < 3.25, \\ > 6.5, & \text{if } x > 3.25. \end{cases}}$$

3. *Goeree, Holt, and Palfrey (2002): Independent-Private-Value First-Price Auction*

a. *Setup*

- Number of bidders $n = 2$.

- Private value auction: $V_i = u(S, X) = X_i$. In this experimental design (because it is private value), values and signals can be used equivalently.
- Values are i.i.d. uniform over the following discrete sets:
  - Low-value treatment: U on $\{0, 2, 4, 6, 8, 11\}$
  - High-value treatment: U on $\{0, 3, 5, 7, 9, 12\}$

  The probability of observing each value is 1/6.
- Bids are discrete.

b. *Equilibrium and cursed-equilibrium bidding strategies*

Since it is a private value auction, equilibrium and cursed equilibrium coincide. In GHP (2002), Table 1 and Table 2 summarize the optimal equilibrium for both treatments. It turns out that both treatments have the same equilibrium:

Low-Value Treatment	High-Value Treatment
$a_*(0) = 0$	$a_*(0) = 0$
$a_*(2) = 1$	$a_*(3) = 1$
$a_*(4) = 2$	$a_*(5) = 2$
$a_*(6) = 3$	$a_*(7) = 3$
$a_*(8) = 4$	$a_*(9) = 4$
$a_*(11) = 5$	$a_*(12) = 5$

c. *Level-k bidding strategies*

c1. *Random L*1 *and random L*2. Random $L0$ bidders take any bid with equal probability regardless of their own valuation, so the probability of observing any bid between 0 and 11 in the low-value treatment is 1/12 and the probability of observing any bid between 0 and 12 in the high-value treatment is 1/13. Random $L1$ bidders will best respond to random $L0$ bidders. We will calculate the expected payoffs for each specific signal and value except for bids that yield negative payoffs (represented by − for negative, when bidders go above their own valuation). In case of a tie we will assume the item is randomly assigned to the winners.

OPTIMAL BIDS FOR RANDOM $L1$ IN LOW-VALUE TREATMENT (**)

Values	0	1	2	3	4	5	6	7	8	9	10	11
$v=0$	0**	−0.12	−0.42	−0.87	−1.5	−2.29	−3.25	−4.37	−5.67	−7.12	−8.75	−10.54
$v=2$	0.08	0.12**	0	−0.29	−0.75	−1.37	−2.17	−3.12	−4.25	−5.54	−7	−8.62
$v=4$	0.17	0.37	0.42**	0.29	0	−0.46	−1.08	−1.87	−2.84	−3.96	−5.25	−6.71
$v=6$	0.25	0.62	0.84	0.87**	0.75	0.46	0	−0.62	−1.42	−2.37	−3.5	−4.79
$v=8$	0.34	0.87	1.25	1.46	1.5**	1.37	1.08	0.62	0	−0.79	−1.75	−2.87
$v=11$	0.46	1.25	1.87	2.34	2.62	2.75**	2.71	2.5	2.12	1.58	0.87	0

OPTIMAL BIDS FOR RANDOM $L1$ IN HIGH-VALUE TREATMENT (**)

Values	0	1	2	3	4	5	6	7	8	9	10	11	12
$V=0$	0**	−0.12	−0.38	−0.81	−1.38	−2.12	−3	−4.04	−5.23	−6.58	−8.08	−9.74	−11.54
$V=3$	0.12	0.23**	0.19	0	−0.35	−0.85	−1.5	−2.31	−3.27	−4.38	−5.65	−7.08	−8.65
$V=5$	0.19	0.46	0.57**	0.54	0.35	0	−0.5	−1.15	−1.96	−2.92	−4.04	−5.31	−6.73
$V=7$	0.27	0.69	0.96	1.08**	1.04	0.85	0.5	0	−0.65	−1.46	−2.42	−3.54	−4.81
$V=9$	0.35	0.92	1.35	1.62	1.73**	1.69	1.5	1.15	0.65	0	−0.81	−1.77	−2.88
$V=12$	0.46	1.27	1.92	2.42	2.77	2.96	3**	2.88	2.62	2.19	1.62	0.88	0

Random $L2$ bidders best respond to random $L1$ bidders.

OPTIMAL BIDS FOR RANDOM $L2$ IN LOW-VALUE TREATMENT (**)

Values	0	1	2	3	4	5	6	7	8	9	10	11
$v = 0$	0^{**}	−0.25	−0.84	−1.75	−3	−4.58	−6	−7	−8	−9	−10	−11
$v = 2$	0.17	0.25^{**}	0	−0.58	−1.5	−2.75	−4	−5	−6	−7	−8	−9
$v = 4$	0.34	0.75	0.84^{**}	0.58	0	−0.92	−2	−3	−4	−5	−6	−7
$v = 6$	0.5	1.25	1.67	1.75^{**}	1.5	0.92	0	−1	−2	−3	−4	−5
$v = 8$	0.67	1.75	2.5	2.92	3^{**}	2.75	2	1	0	−1	−2	−3
$v = 11$	0.92	2.5	3.75	4.67	5.25	5.5^{**}	5	4	3	2	1	0

OPTIMAL BIDS FOR RANDOM $L2$ IN HIGH-VALUE TREATMENT (**)

Values	0	1	2	3	4	5	6	7	8	9	10	11	12
$V = 0$	0^{**}	−0.25	−0.84	−1.75	−3	−4.17	−5.5	−7	−8	−9	−10	−11	−12
$V = 3$	0.25	0.5^{**}	0.42	0	−0.75	−1.67	−2.75	−4	−5	−6	−7	−8	−9
$V = 5$	0.42	1	1.25^{**}	1.17	0.75	0	−0.92	−2	−3	−4	−5	−6	−7
$V = 7$	0.58	1.5	2.08	2.34^{**}	2.25	1.67	0.92	0	−1	−2	−3	−4	−5
$V = 9$	0.75	2	2.92	3.5	3.75^{**}	3.34	2.75	2	1	0	−1	−2	−3
$V = 12$	1	2.75	4.17	5.25	6^{**}	5.84	5.5	5	4	3	2	1	0

Random bidders' optimal bids coincide with equilibrium except for the bidders who obtain a valuation equal to 12 in the high-value treatment, for which random $L1$ overbids and random $L2$ underbids with respect to equilibrium. However, the payoffs are different from equilibrium.

c2. *Truthful* $L1$ *and truthful* $L2$. Assume truthful $L0$ will choose truthfully, $a_0(v) = v$. Then we can calculate the optimal bidding strategy for truthful $L1$. We will calculate the expected payoffs for each specific signal and value except for bids that yield negative payoffs (represented by − for negative, when bidders go above their own valuation). In case of a tie, we will assume the item is randomly assigned to the winners.

OPTIMAL BIDS FOR TRUTHFUL $L1$ IN LOW-VALUE TREATMENT (**)

Values	0	1	2	3	4	5	6	7	8	9	10	11
$v = 0$	0^{**}	−0.17	−0.5	−1	−1.67	−2.5	−3.5	−4.67	−6	−7.5	−8.34	−10.08
$v = 2$	0.17^{**}	0.17^{**}	0	−0.34	−0.84	−1.5	−2.34	−3.34	−4.5	−5.83	−6.67	−8.5
$v = 4$	0.34	0.5^{**}	0.5^{**}	0.34	0	−0.5	−1.17	−2	−3	−4.17	−5	−6.42
$v = 6$	0.5	0.84	1^{**}	1^{**}	0.84	0.5	0	−0.67	−1.5	−2.5	−3.34	−4.58
$v = 8$	0.67	1.17	1.5	1.67^{**}	1.67^{**}	1.5	1.17	0.67	0	−0.84	−1.67	−2.75
$v = 11$	0.92	1.67	2.25	2.67	2.92	3^{**}	2.92	2.67	2.25	1.67	0.84	0

For the low-value treatment, truthful $L1$ will predict equilibrium bidding as well as underbidding with respect to equilibrium bidding.

OPTIMAL BIDS FOR TRUTHFUL $L1$ IN HIGH-VALUE TREATMENT (**)

Values	0	1	2	3	4	5	6	7	8	9	10	11	12
$V = 0$	0**	−0.17	−0.34	−0.75	−1.34	−2.08	−3	−4.08	−5.34	−6.75	−8.34	−9.17	−11
$V = 3$	0.25	0.34**	0.17	0	−0.34	−0.84	−1.5	−2.34	−3.34	−4.5	−5.84	−6.67	−8.25
$V = 5$	0.42	0.67**	0.5	0.5	0.34	0	−0.5	−1.17	−2	−3	−4.17	−5	−6.42
$V = 7$	0.58	1**	0.84	1**	1**	0.84	0.5	0	−0.67	−1.5	−2.5	−3.34	−4.58
$V = 9$	0.75	1.34	1.17	1.5	1.67**	1.67**	1.5	1.17	0.67	0	−0.84	−1.67	−2.75
$V = 12$	1	1.84	1.67	2.25	2.67	2.92	3**	2.92	2.67	2.25	1.67	0.84	0

For the high-value treatment, truthful $L1$ will predict mostly overbidding with respect to equilibrium.

Truthful $L2$ will best respond to truthful $L1$. If there are multiple optimal bids, we assume that each of them will be made with equal probability. We will calculate the expected payoffs for each specific signal and value except for bids that yield negative payoffs (represented by − for negative). In case of a tie, we will assume the item is randomly assigned to the winners.

OPTIMAL BIDS FOR TRUTHFUL $L2$ IN LOW-VALUE TREATMENT (**)

Values	0	1	2	3	4	5	6	7	8	9	10	11
$V = 0$	0**	−0.34	−1	−2	−3.17	−4.58	−6	−7	−8	−9	−10	−11
$v = 2$	0.25	0.34**	0	−0.67	−1.58	−2.75	−4	−5	−6	−7	−8	−9
$v = 4$	0.5	1**	1**	0.67	0	−0.92	−2	−3	−4	−5	−6	−7
$v = 6$	0.75	1.67	2**	2**	1.58	0.92	0	−1	−2	−3	−4	−5
$v = 8$	1	2.34	3	3.34**	3.17	2.75	2	1	0	−1	−2	−3
$V = 11$	1.37	3.34	4.5	5.34	5.54**	5.5	5	4	3	2	1	0

Truthful $L2$ will tend to underbid with respect to equilibrium in low-value treatments.

OPTIMAL BIDS FOR TRUTHFUL $L2$ IN HIGH-VALUE TREATMENT (**)

Values	0	1	2	3	4	5	6	7	8	9	10	11	12
$V = 0$	0**	−0.36	−1.12	−1.75	−2.72	−3.96	−5.5	−7	−8	−9	−10	−11	−12
$V = 3$	0.25	0.73**	0.56	0	−0.68	−1.58	−2.75	−4	−5	−6	−7	−8	−9
$V = 5$	0.42	1.45	1.67**	1.17	0.68	0	−0.92	−2	−3	−4	−5	−6	−7
$V = 7$	0.58	2.17	2.78**	2.34	2.04	1.58	0.92	0	−1	−2	−3	−4	−5
$V = 9$	0.75	2.89	3.89**	3.5	3.40	3.17	2.75	2	1	0	−1	−2	−3
$V = 12$	1	3.97	5.56**	5.25	5.45	5.54	5.5	5	4	3	2	1	0

Truthful $L2$ bidders will underbid with respect to equilibrium in high-value treatments.

4. *Objective Functions for Level-k Decision Rules*

a. *Kagel and Levin (1986)*

a1. *Random L1.*

First price:

$$\pi_1^r(b_1^r, x) = \begin{cases} 0, & b_1^r \le x - a, \\ [x - b_1^r]\left(\dfrac{b_1^r - x + a}{2a}\right)^{n-1}, & x - a < b_1^r < x + a, \\ x - b_1^r, & b_1^r \ge x + a. \end{cases}$$

Second price:

$$\pi_1^r(b_1^r, x) = \begin{cases} 0, & b_1^r \le x - a, \\ [x - b_1^r]\left(\dfrac{b_1^r - x + a}{2a}\right)^{n-1} + \dfrac{2a}{n}\left(\dfrac{b_1^r - x + a}{2a}\right)^{n}, & x - a < b_1^r < x + a, \\ \dfrac{a(2-n)}{n}, & b_1^r \ge x + a. \end{cases}$$

a2. *Random L2, truthful L1, and truthful L2.*

First price:

$$\pi_k(b_k, x) = \begin{cases} 0, & b_{k-1}^{-1}(b_k) \le x - a, \\ \frac{1}{n}\left(1 - \frac{x - b_{k-1}^{-1}(b_k)}{a}\right)^n \\ \quad \times \left[x - \frac{a}{2} + \frac{a}{(n+1)} - \frac{x - b_{k-1}^{-1}(b_k)}{(n+1)} - b_k\right], \\ \qquad x - a < b_{k-1}^{-1}(b_k) \le x, \\ \frac{1}{a}\left[\frac{b_{k-1}^{-1}(b_k)^2}{2} - \left(\frac{a}{2} - \frac{a}{n}\right)(b_{k-1}^{-1}(b_k) - x) \right. \\ \quad - \left(x + \frac{a}{2}\right)\frac{a}{n}\left(\frac{b_{k-1}^{-1}(b_k) - x}{a}\right)^n - \frac{(b_{k-1}^{-1}(b_k) - x)^{n+1}}{n(n+1)a^{n-1}} \\ \quad \left. - \frac{x^2}{2} - b_k\left(b_{k-1}^{-1}(b_k) - x - \frac{a}{n}\left(\frac{b_{k-1}^{-1}(b_k) - x}{a}\right)^n\right)\right] \\ \quad + \left(x - \frac{a}{2} + \frac{a}{n+1} - b_k\right)\frac{1}{n}, \\ \qquad x < b_{k-1}^{-1}(b_k) < x + a, \\ x - b_k, & b_{k-1}^{-1}(b_k) \ge x + a. \end{cases}$$

Second price:

$$
\pi_k(b_k,x)=\begin{cases}
0, & b_{k-1}^{-1}(b_k)\le x-a,\\[4pt]
\dfrac{1}{n}\left(1-\dfrac{x-b_{k-1}^{-1}(b_k)}{a}\right)^{n} & \\
\quad\times\left(x-\dfrac{a}{2}+\dfrac{2a}{n+1}-[x-b_{k-1}^{-1}(b_k)]\left(\dfrac{2}{n+1}\right)-b_k\right), & \\
 & x-a<b_{k-1}^{-1}(b_k)\le x,\\[4pt]
\dfrac{1}{a}\Bigg[b_{k-1}^{-1}(b_k)^2-\left(\dfrac{a}{2}-\dfrac{a}{n}\right)(b_{k-1}^{-1}(b_k)-x) & \\
\quad-\left(x+\dfrac{a}{2}\right)\dfrac{a}{n}\left(\dfrac{b_{k-1}^{-1}(b_k)-x}{a}\right)^{n}-\dfrac{(b_{k-1}^{-1}(b_k)-x)^{n+1}}{n(n+1)a^{n-1}} & \\
\quad-b_k\left[b_{k-1}^{-1}(b_k)-\dfrac{a}{n}\left(\dfrac{b_{k-1}^{-1}(b_k)-x}{a}\right)^{n}\right] & \\
\quad-\dfrac{a^2}{n(n+1)}\left(\dfrac{b_{k-1}^{-1}(b_k)-x}{a}\right)^{n+1}-x^2+b_{k-1}(x)x\Bigg] & \\
\quad+\dfrac{1}{n}\left(x-\dfrac{a}{2}+\dfrac{2a}{n+1}-b_{k-1}(x)\right), & \\
 & x<b_{k-1}^{-1}(b_k)<x+a,\\[4pt]
\dfrac{1}{a}\left[2ax+\dfrac{a^2}{2}-b_{k-1}(x+a)\left[x+a\dfrac{n-1}{n}\right]+b_{k-1}(x)x\right] & \\
\quad-\dfrac{b_{k-1}(x)}{n}, & b_{k-1}^{-1}(b_k)\ge x+a.
\end{cases}
$$

b. *Avery and Kagel (1997)*

b1. *Random L*1.

$$
\pi_1^r(b_1^r,x)=\begin{cases}
\left(x+\dfrac{5}{2}-b_1^r\right)\left(\dfrac{b_1^r-2}{6}\right)+\dfrac{(b_1^r-2)^2}{12}, & 2<b_1^r(x)<8,\\[4pt]
\left(x-\dfrac{5}{2}\right), & b_1^r(x)>8.
\end{cases}
$$

b2. *Random L2 and truthful L1.*

$$\pi_k(b_k, x) = \begin{cases} \left(\dfrac{b_{k-1}^{-1}(b_k) - 1}{3}\right)\Bigg[x + b_{k-1}^{-1}(b_k) - b_k \\ \quad - \left(1 - \dfrac{\partial b_{k-1}(y)}{\partial y}\right)\left(\dfrac{b_{k-1}^{-1}(b_k) - 1}{2}\right)\Bigg], \\ \quad 1 < b_{k-1}^{-1}(b_k) < 4, \\ [x + 4 - b_{k-1}(4)] - \left[1 - \dfrac{\partial b_{k-1}(y)}{\partial y}\right]\dfrac{3}{2}, \\ \quad b_{k-1}^{-1}(b_k) > 4. \end{cases}$$

b3. *Truthful L2.*

$$\pi_2^t(b_2^t, x) = \begin{cases} 0, & b_2^t < 3.5, \\ \dfrac{1}{4}\left(x + \dfrac{7}{4} - 3.5\right), & b_2^t = 3.5, \\ \dfrac{1}{2}\left(x + \dfrac{7}{4} - 3.5\right), & 3.5 < b_2^t < 6.5, \\ \dfrac{1}{2}\left(x + \dfrac{7}{4} - 3.5\right) + \dfrac{1}{4}\left(x + \dfrac{13}{4} - 6.5\right), & b_2^t = 6.5, \\ \dfrac{1}{2}\left(x + \dfrac{7}{4} - 3.5\right) + \dfrac{1}{2}\left(x + \dfrac{13}{4} - 6.5\right), & b_2^t > 6.5. \end{cases}$$

C. ESTIMATES FOR LEVEL-k PLUS *EQUILIBRIUM* MODELS WITH VERSUS WITHOUT TRUTHFUL TYPES

This section compares the Level-k plus *Equilibrium* estimates reported in Tables IIIa–IIId with the estimates when Truthful types are excluded from the specification.

TABLE Ci

LEVEL-k PLUS *EQUILIBRIUM* ESTIMATES WITH VERSUS WITHOUT TRUTHFUL TYPES FOR KAGEL AND LEVIN FIRST-PRICE

	Level-k Plus *Equilibrium* with Truthful Types						Level-k Plus *Equilibrium* Without Truthful Types				
Specification	Subject-Specific Precision (λ_i)	Type-Specific Precision (λ_k)		Constant Precision (λ)		Specification	Subject-Specific Precision (λ_i)	Type-Specific Precision (λ_k)		Constant Precision (λ)	
	$\hat{\pi}_k$	$\hat{\pi}_k$	$\hat{\lambda}_k$	$\hat{\pi}_k$	$\hat{\lambda}$		$\hat{\pi}_k$	$\hat{\pi}_k$	$\hat{\lambda}_k$	$\hat{\pi}_k$	$\hat{\lambda}$
Random $L0$	0.04	0	—	0	—	Random $L0$	0.04	0	—	0	—
Random $L1$	0.59	0.35	1	0.49	1.62	Random $L1$	0.63	0.46	1.29	0.53	1.52
Random $L2$	0.04	0.03	280.90	0	1.62	Random $L2$	0.18	0.06	139.89	0.35	1.52
Truthful $L1$	0.18	0.54	1.21	0.29	1.62						
Truthful $L2$	~Eq.	~Eq.	~Eq.	~Eq.	~Eq.						
Equilibrium	0.16	0.08	11.09	0.22	1.62	*Equilibrium*	0.16	0.48	1.03	0.13	1.52
Log-likelihood	−1,658.30	−1,739.6		−1,753.54		Log-likelihood	−1,659.06	−1,746.65		−1,753.54	

TABLE Cii

LEVEL-k PLUS *EQUILIBRIUM* ESTIMATES WITH VERSUS WITHOUT TRUTHFUL TYPES FOR KAGEL AND LEVIN SECOND-PRICE

	Level-k Plus *Equilibrium* with Truthful Types						Level-k Plus *Equilibrium* Without Truthful Types				
Specification	Subject-Specific Precision (λ_i)	Type-Specific Precision (λ_k)		Constant Precision (λ)		Specification	Subject-Specific Precision (λ_i)	Type-Specific Precision (λ_k)		Constant Precision (λ)	
	$\hat{\pi}_k$	$\hat{\pi}_k$	$\hat{\lambda}_k$	$\hat{\pi}_k$	$\hat{\lambda}$		$\hat{\pi}_k$	$\hat{\pi}_k$	$\hat{\lambda}_k$	$\hat{\pi}_k$	$\hat{\lambda}$
Random $L0$	0	0	—	0	—	Random $L0$	0.18	0.49	—	0	—
Random $L1$	0.21	0.10	95.84	0.62	8.91	Random $L1$	0.32	0.44	25.66	0	1.31
Random $L2$	0.21	0.27	2.50	0.11	8.91	Random $L2$	0.21	0.10	24.51	0.47	1.31
Truthful $L1$	$\sim R.L2$	$\sim R.L2$	$\sim R.L2$	$\sim R.L2$	$\sim R.L2$						
Truthful $L2$	0.32	0.33	6.10	0.27	8.91						
Equilibrium	0.25	0.30	49.76	0	8.91	*Equilibrium*	0.29	0	—	0.53	1.31
Log-likelihood	−918.26	−967.80		−973.81		Log-likelihood	−940.47	−983.48		−995.14	

TABLE Ciii

LEVEL-k PLUS *EQUILIBRIUM* ESTIMATES WITH VERSUS WITHOUT TRUTHFUL TYPES FOR AVERY AND KAGEL SECOND-PRICE

	Level-k Plus *Equilibrium* with Truthful Types				
Specification	Subject-Specific Precision (λ_i)	Type-Specific Precision (λ_k)		Constant Precision (λ)	
	$\hat{\pi}_k$	$\hat{\pi}_k$	$\hat{\lambda}_k$	$\hat{\pi}_k$	$\hat{\lambda}$
Random $L0$	0	0	—	0	—
Random $L1$	0.65	0.56	12.75	0.94	4.30
Random $L2$	0.09	0	—	0.06	4.30
Truthful $L1$	$\sim R.L2$	$\sim R.L2$	$\sim R.L2$	$\sim R.L2$	$\sim R.L2$
Truthful $L2$	0.22	0.05	633.01	0	4.30
Equilibrium	0.04	0.39	0.63	0	4.30
Log-likelihood	−668.23	−702.44		−710.53	

	Level-k Plus *Equilibrium* Without Truthful Types				
Specification	Subject-Specific Precision (λ_i)	Type-Specific Precision (λ_k)		Constant Precision (λ)	
	$\hat{\pi}_k$	$\hat{\pi}_k$	$\hat{\lambda}_k$	$\hat{\pi}_k$	$\hat{\lambda}$
Random $L0$	0	0	—	0	—
Random $L1$	0.74	0.62	11.47	0.94	4.30
Random $L2$	0.09	0	—	0.06	4.30
Equilibrium	0.17	0.38	0.65	0	4.30
Log-likelihood	−674.39	−703.57		−710.53	

TABLE Civ

LEVEL-k PLUS *EQUILIBRIUM* ESTIMATES WITH VERSUS WITHOUT TRUTHFUL TYPES FOR GOEREE, HOLT, AND PALFREY FIRST-PRICE

	Level-k Plus *Equilibrium* with Truthful Types						Level-k Plus *Equilibrium* Without Truthful Types				
Specification	Subject-Specific Precision (λ_i)	Type-Specific Precision (λ_k)		Constant Precision (λ)		Specification	Subject-Specific Precision (λ_i)	Type-Specific Precision (λ_k)		Constant Precision (λ)	
	$\hat{\pi}_k$	$\hat{\pi}_k$	$\hat{\lambda}_k$	$\hat{\pi}_k$	$\hat{\lambda}$		$\hat{\pi}_k$	$\hat{\pi}_k$	$\hat{\lambda}_k$	$\hat{\pi}_k$	$\hat{\lambda}$
Random $L0$	0	0	—	0	—	Random $L0$	0	0	—	0	—
Random $L1$	0.65	0.98	8.54	0.99	8.71	Random $L1$	0.78	0.98	8.54	0.99	8.71
Random $L2$	0.04	0	—	0	8.71	Random $L2$	0.04	0	—	0	8.71
Truthful $L1$	0.14	0	—	0	8.71						
Truthful $L2$	0.01	0	—	0	8.71						
Equilibrium	0.16	0.02	29.84	0.01	8.71	*Equilibrium*	0.19	0.02	29.84	0.01	8.71
Log-likelihood	−569.53	−642.91		−644.12		Log-likelihood	−575.41	−642.91		−644.12	

D. ESTIMATES OF SUBJECT-SPECIFIC PRECISIONS AND STANDARD ERRORS

This section reports subject-specific precisions and standard errors for the estimates reported in Table IIIa–IIId. In defining the jackknife for subject-specific precisions, we fixed each subject's type at its original estimated value and did the jackknife by excluding one period at a time. Because it is difficult to imagine that precision does not vary with type, this yields estimates with a clearer interpretation than the alternative of allowing a subject's estimated type to vary within the jackknife. However, it also precludes using the jackknife to estimate the standard errors of the type frequencies $\hat{\pi}_k$.

TABLE Di

MODELS AND ESTIMATES FOR KAGEL AND LEVIN FIRST-PRICE

	Level-k Plus Equilibrium			Cursed Equilibrium		
Subject	Type	Subject-Specific Precision (λ_i)	Standard Error	χ	Subject-Specific Precision (λ_i)	Standard Error
1	$R.L1$	99.75	34.96	1	16.89	6.17
2	$R.L1$	30.00	51.46	1	5.71	12.17
3	$R.L1$	7.90	15.41	1	1.73	3.61
4	$R.L2$	0.47	0.25	0.8	0.55	3.14
5	$R.L1$	0.21	0.14	0	0.05	0.06
6	$R.L2$	0.97	0.65	0.3	0.91	0.86
7	$T.L1$	49.04	22.40	0.3	61.23	58.87
8	$R.L1$	1.26	0.50	1	0.34	0.14
9	EQ	1.42	3.28	0	1.42	3.28
10	$R.L1$	1.65	4.96	1	0.55	0.98
11	$R.L1$	3.28	2.00	1	0.88	0.41
12	$R.L1$	49.20	19.93	0.8	14.00	3.41
13	$T.L1$	27.44	7.56	0.4	19.89	8.16
14	$R.L1$	11.21	11.09	1	2.22	1.54
15	$R.L1$	2.29	0.41	1	0.58	0.10
16	$R.L1$	1.86	1.51	1	0.55	0.34
17	EQ	1.35	1.48	0	1.35	1.48
18	$R.L1$	2.51	0.85	1	0.65	0.17
19	$R.L1$	1.64	3.80	1	0.54	0.77
20	$R.L1$	533.86	107.99	0.8	83.74	15.29
21	$R.L1$	462.91	396.98	1	19.44	11.63
22	$R.L1$	9.24	9.88	1	0.87	0.49
23	EQ	1.65	2.20	0	1.65	3.09
24	EQ	244.04	1656.06	0	244.03	806.35
25	$T.L1$	2.74	1.73	0.8	4.02	2.30
26	$R.L1$	13.75	3.26	1	0.92	0.19
27	EQ	0.23	0.22	0	0.23	0.22
28	$T.L1$	6.12	11.95	0	1.67	1.48
29	$T.L1$	3.14	2.10	0.2	1.41	0.47

Continues

TABLE Di—*Continued*

	Level-k Plus Equilibrium			Cursed Equilibrium		
Subject	Type	Subject-Specific Precision (λ_i)	Standard Error	χ	Subject-Specific Precision (λ_i)	Standard Error
30	*R.L*0	0	0	*R.L*0	0	0
31	*R.L*1	35.44	80.81	0.7	1.76	24.14
32	*R.L*1	1.10	0.63	*R.L*0	0	0
33	*R.L*1	8.65	102.39	1	1.02	3.63
34	*R.L*1	33.64	3.81	1	2.01	0.28
35	*T.L*1	208.44	107.81	0.2	29.05	10.66
36	*T.L*1	8.11	0.00	0.7	3585.00	1448.97
37	*R.L*1	178.37	198.97	1	8.59	9.16
38	*R.L*1	295.83	94.83	0.9	14.61	2.16
39	*T.L*1	7.90	3.16	0.7	29.13	16.51
40	*EQ*	10.67	613.43	0	10.67	613.44
41	*R.L*1	1.59	0.62	1	0.08	0.07
42	*R.L*1	6.29	4.43	1	0.64	0.32
43	*R.L*1	4.50	111.20	1	0.67	3.57
44	*T.L*1	1.97	3.44	0.4	1.44	1.68
45	*R.L*1	51.15	42.21	1	3.04	1.84
46	*R.L*1	1.13	0.46	1	0.31	0.14
47	*R.L*1	1.16	2.04	1	0.37	0.40
48	*R.L*1	3.46	0.94	1	0.82	0.22
49	*R.L*0	0	0	1	0	0
50	*EQ*	0.62	1.12	0	0.62	1.12
51	*EQ*	82.06	67.60	0	82.07	33.13

TABLE Dii

MODELS AND ESTIMATES FOR KAGEL AND LEVIN SECOND-PRICE

	Level-k Plus Equilibrium			Cursed Equilibrium		
Subject	Type	Subject-Specific Precision (λ_i)	Standard Error	χ	Subject-Specific Precision (λ_i)	Standard Error
1	*EQ*	143.71	123.00	0.6	12.26	6.51
2	*T.L*2	4.24	3.00	0	3.98	4.21
3	*R.L*1	10.48	7.05	1	3.91	1.85
4	*R.L*1	116.77	114.23	1	22.18	21.56
5	*T.L*2	9.10	3.34	*R.L*0	0	0
6	*R.L*2	3.05	0.81	0.5	11.55	21.57
7	*R.L*2	1.13	1.04	0.2	3.38	0.66
8	*R.L*1	105.26	27.22	0.9	22.11	3.42
9	*T.L*2	10.06	22.46	*R.L*0	0	0
10	*R.L*2	15.38	284.14	0.2	5.65	1.17
11	*T.L*2	5.41	0.97	0	13.57	11.12
12	*EQ*	24.79	470.14	0	24.79	470.14
13	*T.L*2	2.88	0.98	1	0.53	5.59
14	*R.L*2	22.68	435.99	0.2	3.75	0.56
15	*R.L*1	102.58	82.95	0.9	24.27	17.76
16	*EQ*	36.26	56.14	1	4.73	3.57
17	*T.L*2	5.29	2.98	*R.L*0	0	0
18	*R.L*1	47.17	30.58	0.9	10.01	5.25
19	*T.L*2	180.63	36.79	*R.L*0	0	0
20	*EQ*	78.98	171.37	0	78.98	171.37
21	*T.L*2	6.81	3.94	*R.L*0	0	0
22	*R.L*1	25.44	29.97	0.8	6.83	12.22
23	*EQ*	57.13	51.42	0	57.12	51.43
24	*R.L*2	2.50	2.58	0.4	5.20	1.68
25	*T.L*2	5.57	1.76	1	0.87	1.03
26	*EQ*	82.64	118.18	0.1	19.96	7.06
27	*R.L*2	76.01	21.95	0.1	17.69	12.25
28	*EQ*	212.31	112.34	0.6	13.98	5.51

TABLE Diii

MODELS AND ESTIMATES FOR AVERY AND KAGEL SECOND-PRICE

	Level-k Plus Equilibrium			Cursed Equilibrium		
Subject	Type	Subject-Specific Precision (λ_i)	Standard Error	χ	Subject-Specific Precision (λ_i)	Standard Error
1	$R.L1$	133.31	104.89	1	53.83	82.54
2	EQ	51.07	113.80	0.7	88.87	45.01
3	$R.L1$	2.81	1.14	1	1.22	0.41
4	$T.L2$	3.90	1.76	0.5	1.72	1.09
5	$R.L1$	8.21	4.37	1	5.76	1.43
6	$T.L2$	900.00	158.44	0	9.80	11.29
7	$T.L2$	3.49	1.65	0.4	2.25	0.84
8	$R.L1$	23.25	2.52	1	12.65	2.60
9	$R.L1$	84.44	24.78	0.7	321.00	193.68
10	$R.L1$	0.90	2.36	$R.L0$	0	0
11	$R.L2$	0.29	0.77	1	1.11	3.95
12	$R.L1$	0.79	1.56	$R.L0$	0	0
13	$R.L1$	25.33	6.59	0.6	26.48	4.62
14	$R.L2$	0.05	1.05	$R.L0$	0	0
15	$R.L1$	11.65	9.30	1	4.91	6.72
16	$R.L1$	21.46	10.39	1	9.87	5.56
17	$T.L2$	3.20	1.13	0.1	2.08	2.47
18	$R.L1$	7.76	6.73	1	7.09	2.58
19	$R.L1$	7.48	2.49	0.7	4.12	2.88
20	$R.L1$	10.15	6.32	1	6.65	1.52
21	$R.L1$	17.72	12.54	1	24.13	9.85
22	$T.L2$	410.95	81.53	0.2	18.74	45.18
23	$R.L1$	4.54	2.95	0.5	2.29	1.06

TABLE Div

MODELS AND ESTIMATES FOR GOEREE, HOLT, AND PALFREY FIRST-PRICE

	Level-k Plus Equilibrium			QRE	
Subject	Type	Subject-Specific Precision (λ_i)	Standard Error	Subject-Specific Precision (λ_i)	Standard Error
1	$R.L1$	6.73	2.57	2.63	0.44
2	$R.L1$	4.26	0.99	1.36	0.22
3	$R.L1$	5, 78	1.48	2.51	0.51
4	$R.L1$	15.82	8.58	3.26	3.95
5	EQ	19.20	71.10	14.07	72.72
6	EQ	21.50	219.59	12.57	3.83
7	$R.L1$	4.10	30.32	2.32	15.63
8	$R.L1$	47.29	8.68	3.36	0.44
9	$R.L1$	8.01	1.09	2.03	0.38
10	EQ	6.64	0.83	5.14	0.28
11	EQ	2.96	3.06	3.20	4.08
12	$R.L1$	33.56	18.60	6.02	3.50
13	$R.L1$	21.40	6.34	4.85	1.71
14	$R.L1$	14.86	2.01	2.78	1.06
15	$R.L1$	43.99	448.88	15.38	105.48
16	$R.L1$	14.85	2.37	3.88	0.60
17	$R.L1$	44.82	430.39	15.38	94.72
18	$R.L1$	7.20	6.19	2.31	2.63
19	$R.L1$	15.76	2.28	4.10	0.69
20	$R.L1$	5.24	2.04	1.68	0.45
21	$R.L1$	10.64	9.14	3.43	0.66
22	EQ	374.00	89.95	177.32	4.85
23	$T.L2$	5.44	16.75	4.19	32.90
24	$R.L1$	3.58	1.34	1.88	0.56
25	EQ	11.57	3.59	11.73	10.93
26	$R.L1$	7.36	2.67	3.14	1.17
27	$R.L1$	6.48	2.05	2.43	0.39
28	$R.L1$	10.87	10.55	4.09	4.29
29	$R.L1$	2.65	0.33	1.07	0.16
30	$R.L1$	95.80	253.19	7.52	79.30
31	$R.L1$	4.93	0.92	1.87	0.25
32	$R.L1$	21.40	6.24	4.81	1.55
33	$R.L1$	66.32	20.34	7.58	17.49
34	$R.L1$	22.40	6.60	4.95	1.67
35	$R.L1$	7.58	2.67	2.47	0.80
36	$R.L1$	3.06	1.02	1.23	0.22
37	$R.L1$	6.04	5.74	2.31	2.61
38	EQ	4.54	2.44	4.29	1.76
39	$R.L1$	7.10	2.57	3.09	1.16
40	EQ	12.20	4.33	8.93	2.36
41	$R.L1$	20.54	19.12	9.52	99.67
42	$R.L1$	7.72	1.12	4.05	0.59
43	$R.L1$	8.76	3.42	2.58	2.06

Continues

TABLE Div—*Continued*

CONTINUED

Subject	Type	Level-k Plus Equilibrium: Subject-Specific Precision (λ_i)	Level-k Plus Equilibrium: Standard Error	QRE: Subject-Specific Precision (λ_i)	QRE: Standard Error
44	*R.L*1	8.27	1.58	4.33	0.75
45	*R.L*1	4.38	0.97	2.23	0.54
46	*R.L*1	2.55	0.46	1.17	0.29
47	*R.L*1	5.97	2.50	2.08	0.65
48	*T.L*1	48.96	197.12	6.13	105.12
49	*T.L*1	11.80	205.19	4.10	9.13
50	*R.L*1	6.84	4.53	3.73	2.43
51	*R.L*1	6.00	2.32	2.77	1.20
52	*EQ*	8.50	4.71	9.95	6.09
53	*EQ*	17.30	1.44	20.06	2.37
54	*EQ*	8.99	6.84	10.56	5.15
55	*R.L*1	36.35	20.15	21.46	5.31
56	*R.L*1	10.37	2.17	3.66	1.01
57	*R.L*2	11.44	14.75	9.34	4.56
58	*T.L*1	14.67	3.10	5.52	2.05
59	*R.L*1	18.34	11.95	8.61	3.03
60	*T.L*1	20.33	220.67	3.24	83.36
61	*T.L*1	5.24	1.12	1.52	0.21
62	*EQ*	7.70	3.57	9.36	4.39
63	*T.L*1	9.52	8.10	3.41	3.75
64	*R.L*1	5.74	12.21	2.09	5.68
65	*R.L*1	9.87	2.52	5.49	1.26
66	*T.L*1	343.00	0	31.03	11.87
67	*T.L*1	10.46	1.93	3.10	0.51
68	*T.L*1	9.68	7.55	3.53	3.58
69	*T.L*1	19.77	6.55	9.43	6.46
70	*R.L*1	26.69	143.67	10.38	8.27
71	*EQ*	29.94	8.21	48.41	63.26
72	*R.L*1	4.86	1.79	2.22	0.65
73	*R.L*1	5.09	1.50	1.93	0.51
74	*R.L*1	2.73	0.56	1.30	0.32
75	*R.L*1	2.19	1.61	1.36	0.63
76	*R.L*1	5.44	5.07	2.05	3.09
77	*R.L*1	24.11	4.71	5.18	5.67
78	*R.L*2	8.23	2.76	6.17	1.37
79	*T.L*1	6.77	1.13	1.84	0.44
80	*R.L*2	18.43	1.97	17.88	1.38

E. LOGIT BID DENSITIES FOR RANDOM $L1$, RANDOM $L2$, TRUTHFUL $L1$, TRUTHFUL $L2$, AND *EQUILIBRIUM* AND REPRESENTATIVE PRECISIONS

This section graphs the logit bid densities for Random $L1$, Random $L2$, Truthful $L1$, Truthful $L2$, and *Equilibrium* and representative precisions, to il-

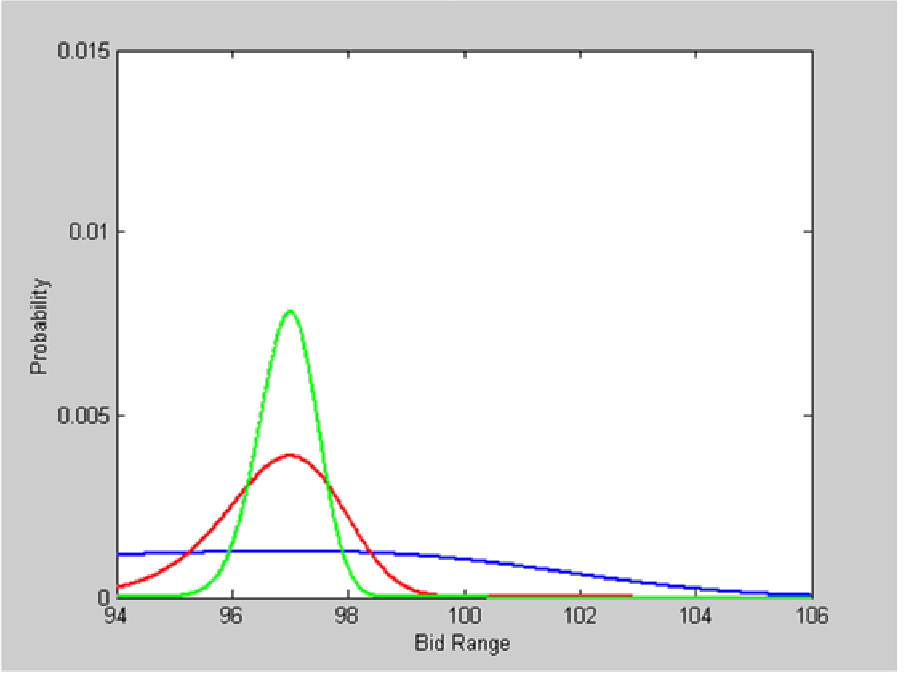

FIGURE E.1.—Kagel and Levin first-price: *Random* $L1$ with private signal $x = 100$ (logit bid densities for precisions 1.5, 50, and 200 in blue, red, and green, respectively).

lustrate the implications of the precision estimates reported in Tables IIIa–IIId in the paper and Tables Di–Div in Section D above.

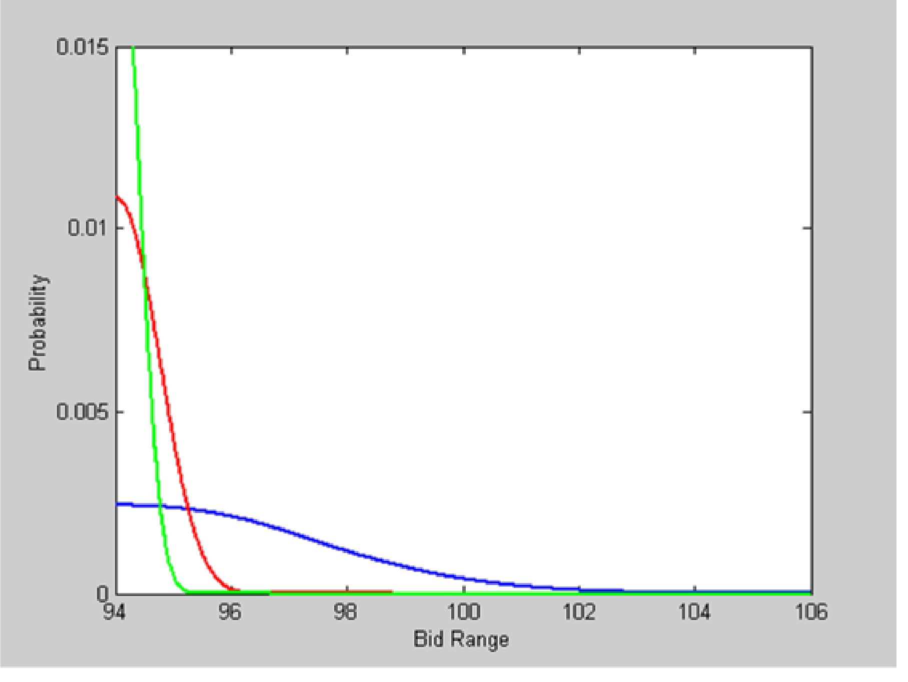

FIGURE E.2.—Kagel and Levin first-price: *Random L2* with private signal $x = 100$ (logit bid densities for precisions 1.5, 50, and 200 in blue, red, and green, respectively).

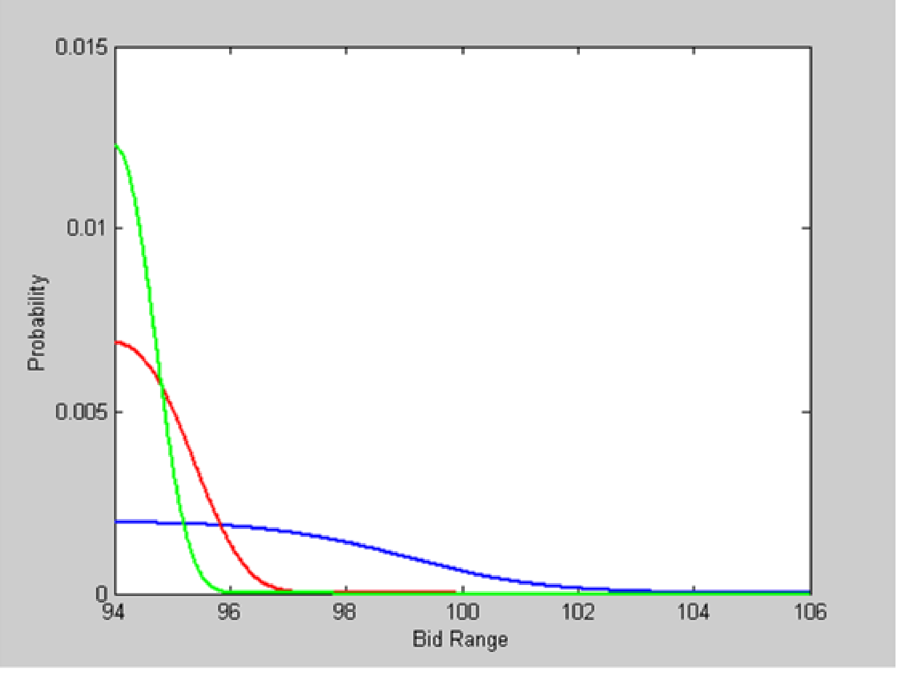

FIGURE E.3.—Kagel and Levin first-price: *Truthful* $L1$ with private signal $x = 100$ (logit bid densities for precisions 1.5, 50, and 200 in blue, red, and green, respectively).

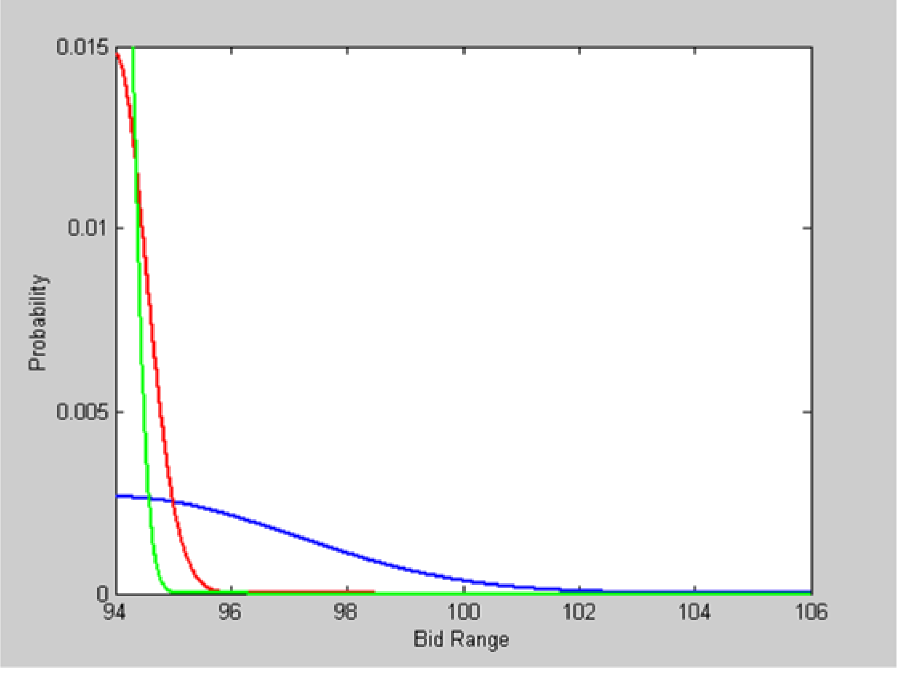

FIGURE E.4.—Kagel and Levin first-price: *Equilibrium*/*Truthful* $L2$ with private signal $x = 100$ (logit bid densities for precisions 1.5, 50, and 200 in blue, red, and green, respectively).

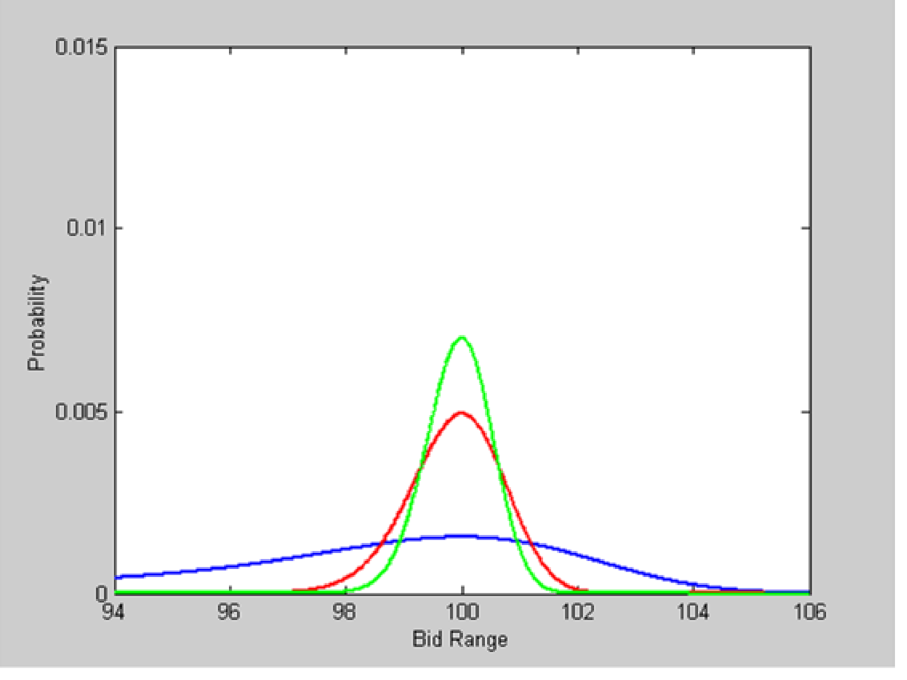

FIGURE E.5.—Kagel and Levin second-price: *Random L*1 with private signal $x = 100$ (logit bid densities for precisions 5, 50, and 100 in blue, red, and green, respectively).

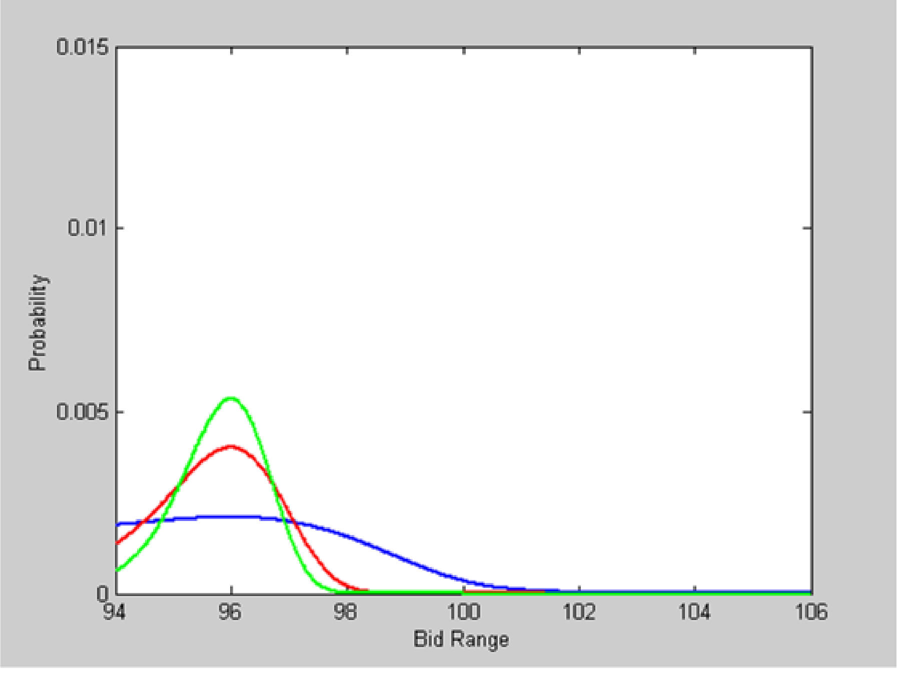

FIGURE E.6.—Kagel and Levin second-price: *Random* $L2$/*Truthful* $L1$ with private signal $x = 100$ (logit bid densities for precisions 5, 50, and 100 in blue, red, and green, respectively).

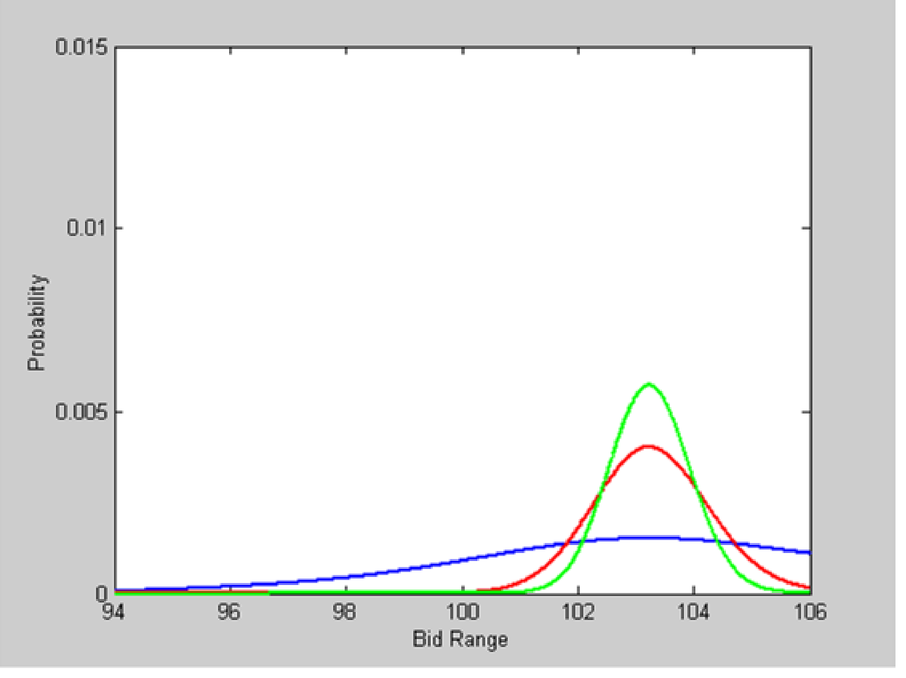

FIGURE E.7.—Kagel and Levin second-price: *Truthful L*2 with private signal $x = 100$ (logit bid densities for precisions 5, 50, and 100 in blue, red, and green, respectively).

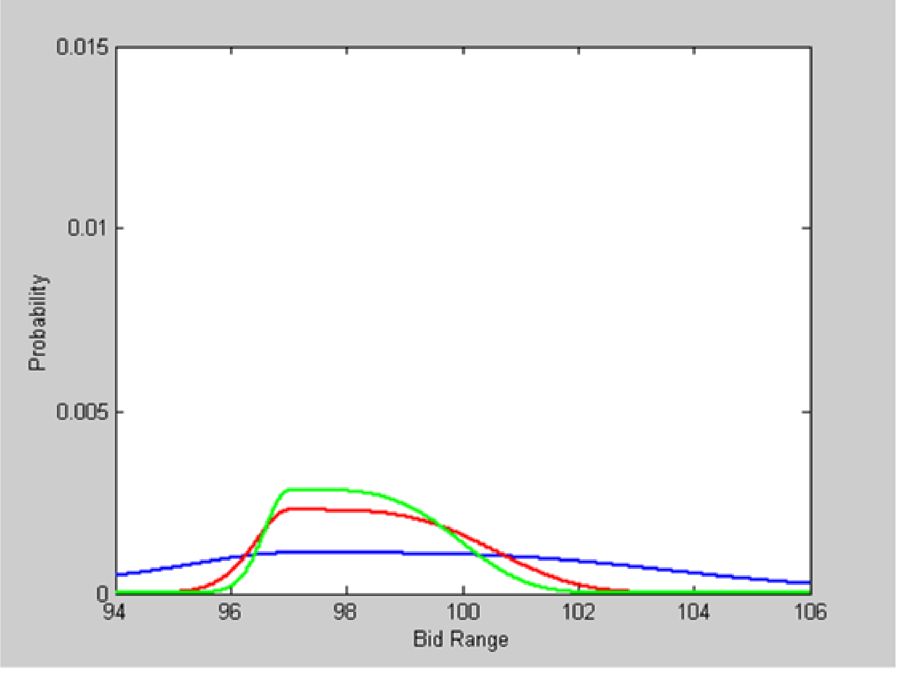

FIGURE E.8.—Kagel and Levin second-price: *Equilibrium* with private signal $x = 100$ (logit bid densities for precisions 5, 50, and 100 in blue, red, and green, respectively).

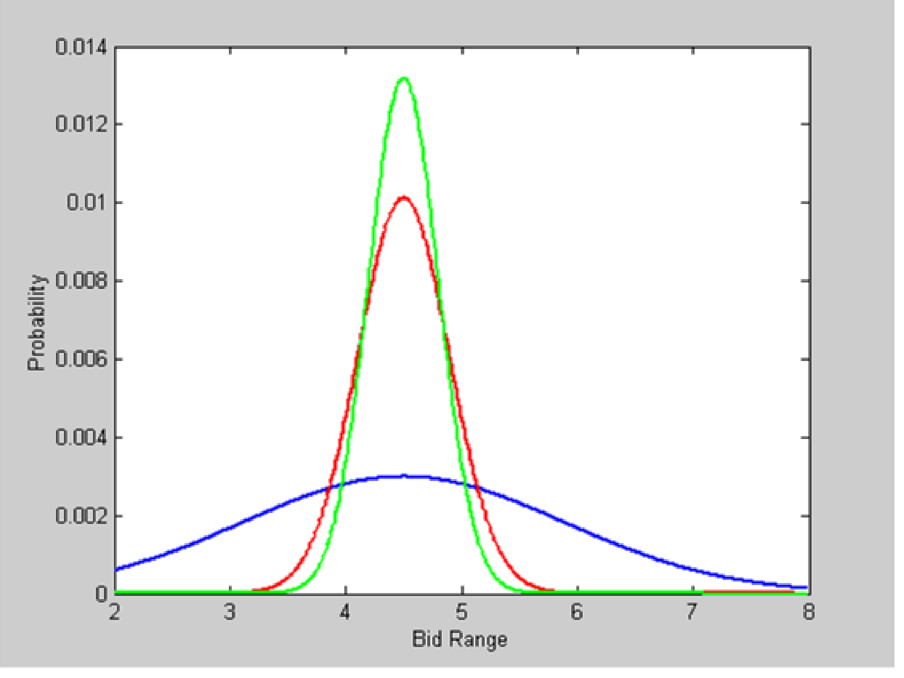

FIGURE E.9.—Avery and Kagel second-price: *Random* $L1$ with private signal $x = 2$ (logit bid densities for precisions 4, 50, and 85 in blue, red, and green, respectively).

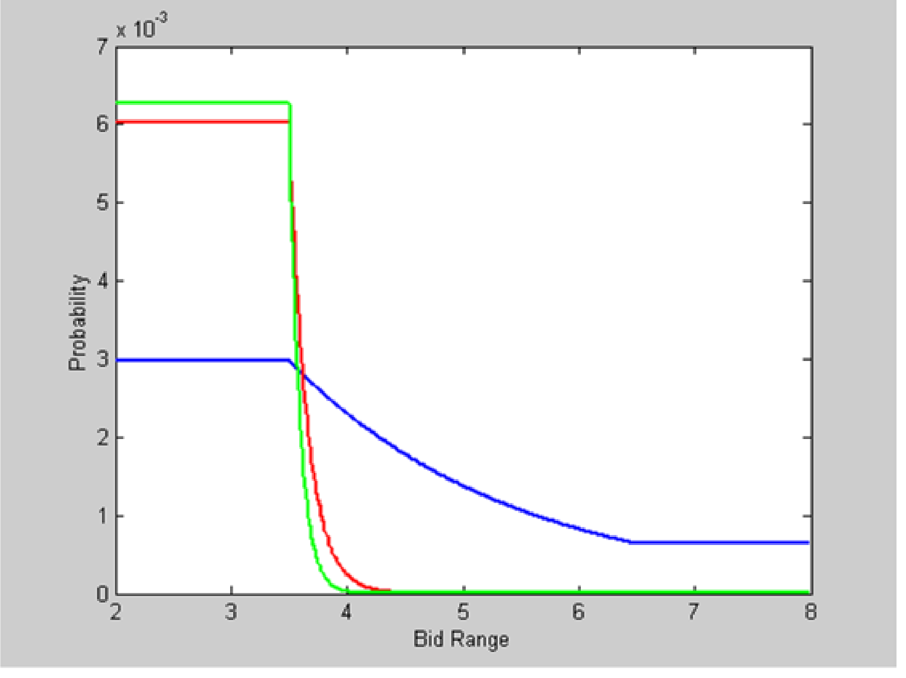

FIGURE E.10.—Avery and Kagel second-price: *Random L*2/*Truthful L*1 with private signal $x = 2$, (logit bid densities for precisions 4, 50, and 85 in blue, red, and green respectively).

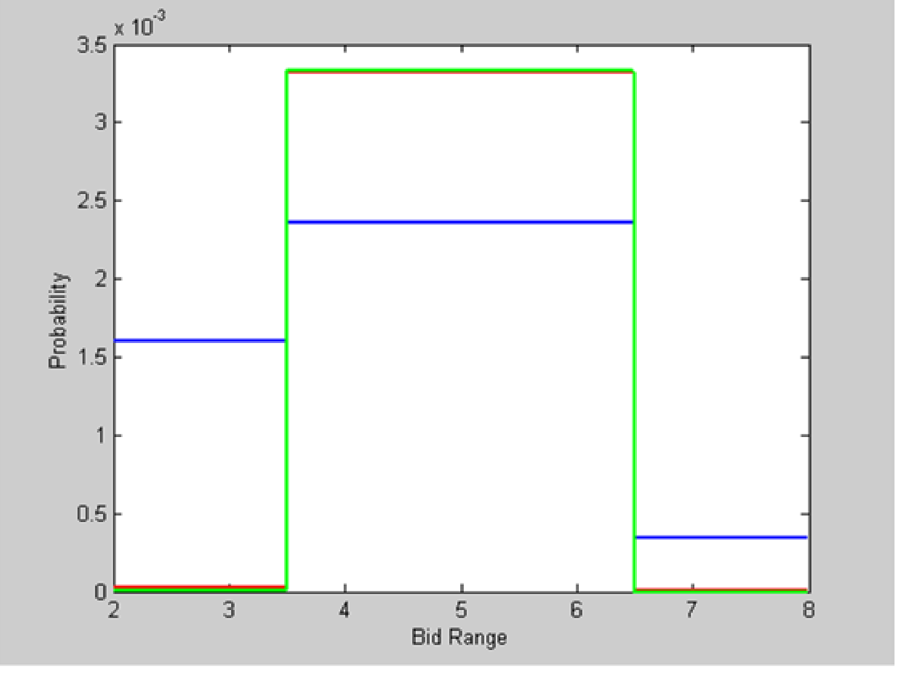

FIGURE E.11.—Avery and Kagel second-price: *Truthful* $L2$ with private signal $x = 2$ (logit bid densities for precisions 4, 50, and 85 in blue, red, and green, respectively).

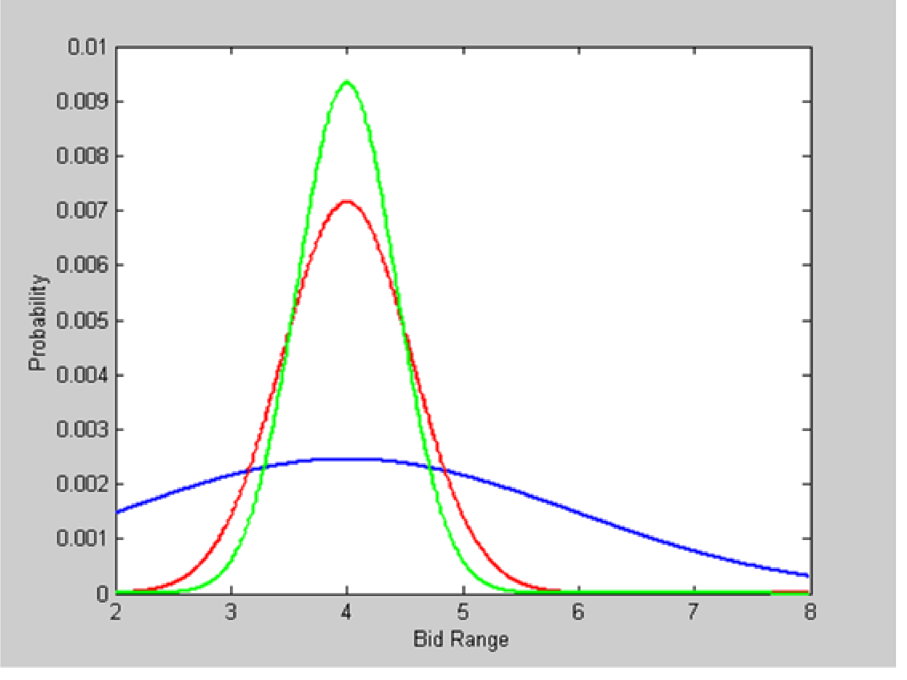

FIGURE E.12.—Avery and Kagel second-price: *Equilibrium* with private signal $x = 2$ (logit bid densities for precisions 4, 50, and 85 in blue, red, and green, respectively).

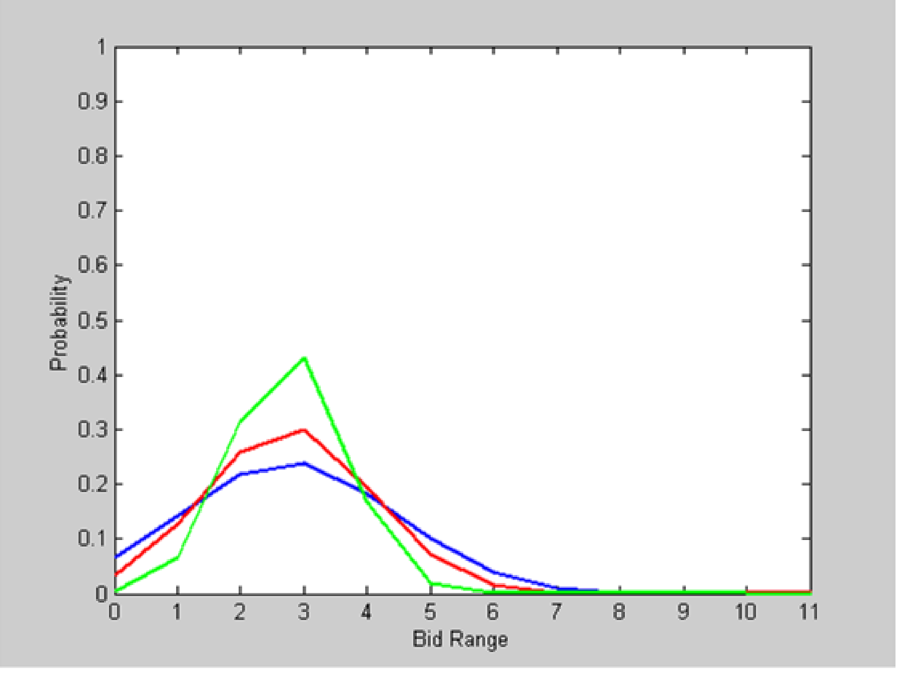

FIGURE E.13.—Goeree, Holt, and Palfrey first-price: *Random L*1 with private signal $x = 2$ (logit bid densities for precisions 6, 10, and 22 in blue, red, and green, respectively).

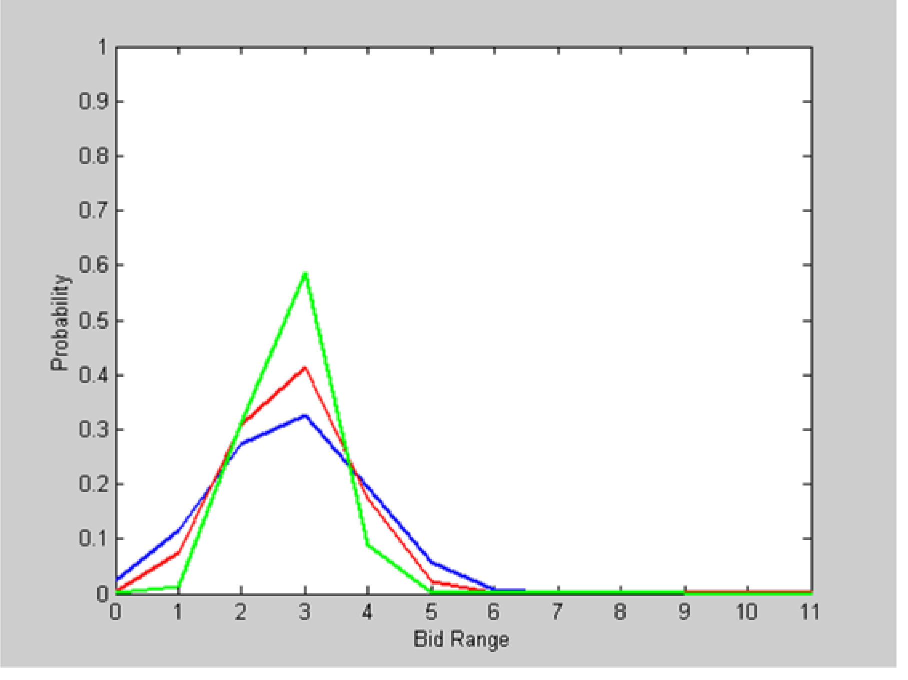

FIGURE E.14.—Goeree, Holt, and Palfrey first-price: *Random* $L2$ with private signal $x = 2$ (logit bid densities for precisions 6, 10, and 22 in blue, red, and green, respectively).

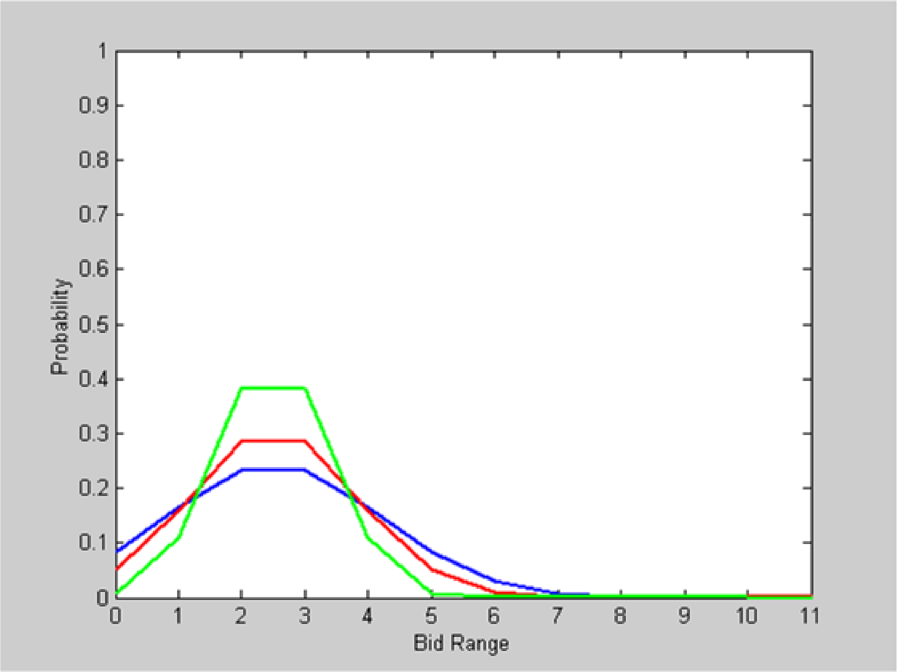

FIGURE E.15.—Goeree, Holt, and Palfrey first-price: *Truthful* $L1$ with private signal $x = 2$ (logit bid densities for precisions 6, 10, and 22 in blue, red, and green, respectively).

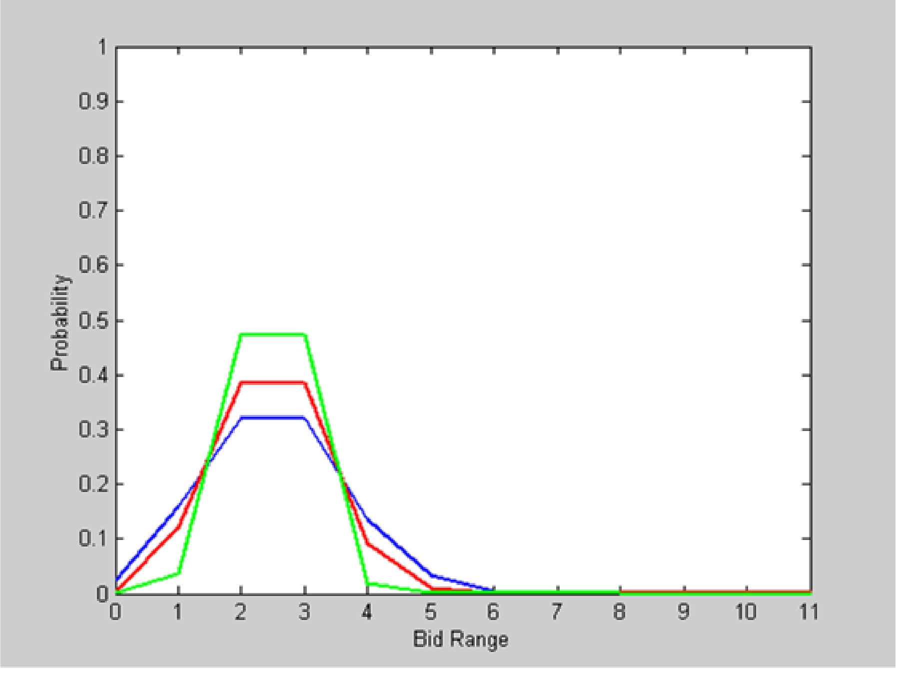

FIGURE E.16.—Goeree, Holt, and Palfrey first-price: *Truthful* $L2$ with private signal $x = 2$ (logit bid densities for precisions 6, 10, and 22 in blue, red, and green, respectively).

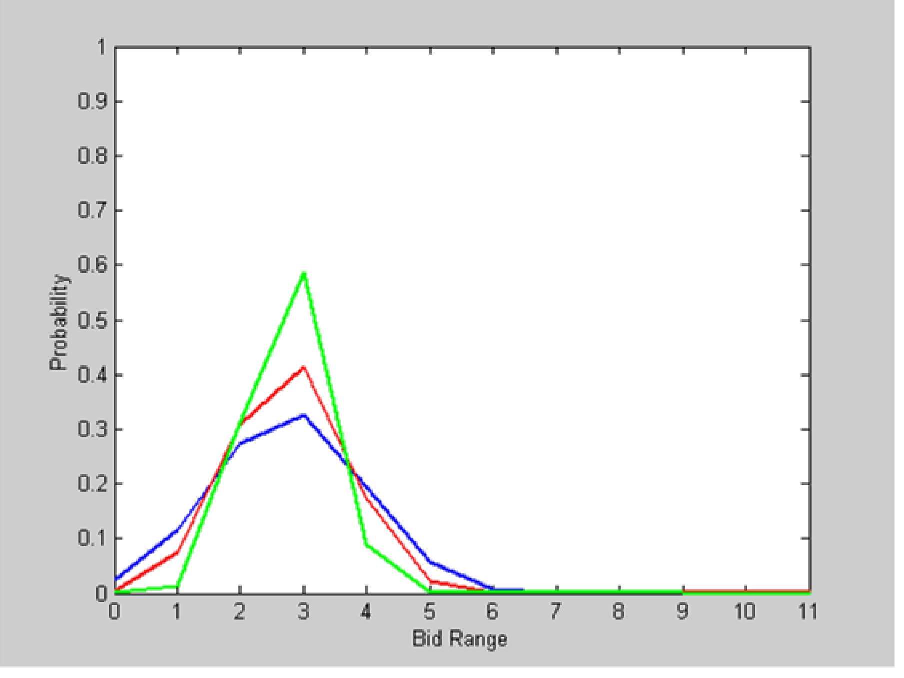

FIGURE E.17.—Goeree, Holt, and Palfrey first-price: *Equilibrium* with private signal $x = 2$ (logit bid densities for precisions 6, 10, and 22 in blue, red, and green, respectively).

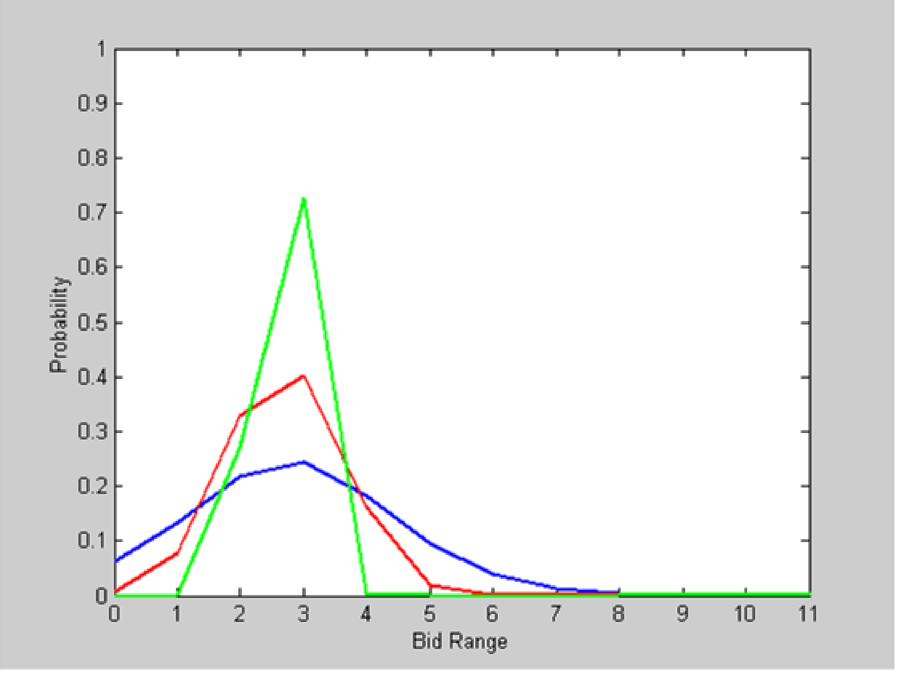

FIGURE E.18.—Goeree, Holt, and Palfrey first-price: *QRE* with private signal $x = 2$ (logit bid densities for precisions 3, 9, and 50 in blue, red, and green, respectively).

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Manuscript received October, 2005; final revision received July, 2007.