SUPPLEMENT TO "COMMENTS ON 'CONVERGENCE PROPERTIES OF THE LIKELIHOOD OF COMPUTED DYNAMIC MODELS'"

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APPENDIX A: PROOFS OF (3), (4), AND (5)

DEFINITION 2: Let $c \neq 0$, and define

$$\chi(c) = \frac{1}{\sigma^2\sqrt{2\pi}} \max_z \left| \frac{\exp\left(-\frac{z^2}{2}\right) - \exp\left(-\frac{\left(z - \frac{c}{\sigma}\right)^2}{2}\right)}{\frac{c}{\sigma}} \right|.$$

REMARK 1: In the definition above, it was implicitly assumed that $\max_z |\exp(-z^2/2) - \exp(-(z - \frac{c}{\sigma})^2/2)/\frac{c}{\sigma}|$ is well defined. To confirm that it indeed is, define

$$\varphi(z) = \left| \exp\left(-\frac{z^2}{2}\right) - \exp\left(-\frac{\left(z - \frac{c}{\sigma}\right)^2}{2}\right) \Big/ \frac{c}{\sigma} \right|.$$

Note that (i) $\varphi(z) \to 0$ as $|z| \to \infty$ and (ii) $\varphi(0) > 0$. Therefore, we can find $B > 0$ sufficiently large that $\varphi(z) < \varphi(0)/2$ for all $|z| > B$. Now, over the compact set $\mathbb{B} = \{z : |z| \leq B\}$, the function is continuous and, therefore, there is some z^* at which the function $\varphi(\cdot)$ is maximized over $\mathbb{B}$. In other words,

$$\varphi(z^*) \geq \varphi(z) \quad \forall z \in \mathbb{B}. \tag{8}$$

Because $0 \in \mathbb{B}$, we should have $\varphi(z^*) \geq \varphi(0)$. But, for all $z \notin \mathbb{B}$, we have $\varphi(z) < \varphi(0)/2 < \varphi(0) \leq \varphi(z^*)$. In other words,

$$\varphi(z^*) \geq \varphi(z) \quad \forall z \notin \mathbb{B}. \tag{9}$$

Combining (8) and (9), we conclude that $\varphi(z^*) \geq \varphi(z)$ for all z. In other words, the maximum is attained.

Given the definition, we can write

$$\begin{aligned} &|p(y_t;\gamma) - p_j(y_t;\gamma)| \\ &\quad = \frac{1}{\sigma\sqrt{2\pi}} \left| \exp\left(-\frac{(y_t-\gamma)^2}{2\sigma^2}\right) - \exp\left(-\frac{(y_t-\delta-\gamma)^2}{2\sigma^2}\right) \right| \end{aligned}$$

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$$
\begin{aligned}
&= \frac{1}{\sigma\sqrt{2\pi}}\left|\exp\left(-\frac{\left(\frac{y_t}{\sigma}-\frac{\gamma}{\sigma}\right)^2}{2}\right)-\exp\left(-\frac{\left(\left(\frac{y_t}{\sigma}-\frac{\gamma}{\sigma}\right)-\frac{\delta}{\sigma}\right)^2}{2}\right)\right| \\
&= |\delta|\frac{1}{\sigma^2\sqrt{2\pi}}\left|\frac{\exp\left(-\frac{(\frac{y_t}{\sigma}-\frac{\gamma}{\sigma})^2}{2}\right)-\exp\left(-\frac{((\frac{y_t}{\sigma}-\frac{\gamma}{\sigma})-\frac{\delta}{\sigma})^2}{2}\right)}{\frac{\delta}{\sigma}}\right| \\
&\le |\delta|\chi(\delta),
\end{aligned}
$$

where $\frac{y_t}{\sigma}-\frac{\gamma}{\sigma}$ is interpreted as the z in the definition of $\chi(c)$. Note that this bound is sharp by the definition of $\chi(\cdot)$. In other words, there is a value of y_t (or analogously $y_t/\sigma-\gamma/\sigma$) such that the bound holds with equality.

LEMMA 1: *We have*

$$
\frac{1}{\sigma\sqrt{2\pi}} \le \liminf_{|c|\to\infty}|c|\chi(c) \le \limsup_{|c|\to\infty}|c|\chi(c) \le \frac{2}{\sigma\sqrt{2\pi}}.
$$

PROOF: By definition,

$$
\begin{aligned}
|c|\chi(c) &= \frac{|c|}{\sigma^2\sqrt{2\pi}}\max_z\left|\frac{\exp\left(-\frac{z^2}{2}\right)-\exp\left(-\frac{(z-\frac{c}{\sigma})^2}{2}\right)}{\frac{c}{\sigma}}\right| \\
&= \frac{1}{\sigma\sqrt{2\pi}}\max_z\left|\exp\left(-\frac{z^2}{2}\right)-\exp\left(-\frac{\left(z-\frac{c}{\sigma}\right)^2}{2}\right)\right| \\
&\le \frac{1}{\sigma\sqrt{2\pi}}\left[\max_z\left|\exp\left(-\frac{z^2}{2}\right)\right|+\max_z\left|\exp\left(-\frac{\left(z-\frac{c}{\sigma}\right)^2}{2}\right)\right|\right] \\
&\le \frac{2}{\sigma\sqrt{2\pi}},
\end{aligned}
$$

from which we obtain

$$
\limsup_{|c|\to\infty}|c|\chi(c) \le \frac{2}{\sigma\sqrt{2\pi}}.
$$

Next, note that

$$|c|\chi(c) = \frac{|c|}{\sigma^2\sqrt{2\pi}} \max_z \left| \frac{\exp\left(-\frac{z^2}{2}\right) - \exp\left(-\frac{\left(z-\frac{c}{\sigma}\right)^2}{2}\right)}{\frac{c}{\sigma}} \right|$$

$$\geq \frac{1}{\sigma\sqrt{2\pi}} \left| \exp\left(-\frac{0^2}{2}\right) - \exp\left(-\frac{\left(0-\frac{c}{\sigma}\right)^2}{2}\right) \right|$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \left| 1 - \exp\left(-\frac{\left(\frac{c}{\sigma}\right)^2}{2}\right) \right|,$$

from which we obtain

$$\frac{1}{\sigma\sqrt{2\pi}} \leq \liminf_{|c|\to\infty} |c|\chi(c). \qquad \textit{Q.E.D.}$$

LEMMA 2: *We have*

$$\chi(\delta) \leq \frac{\exp\left(-\frac{1}{2}\right)}{\sigma\sqrt{2\pi}}.$$

PROOF: By the mean value theorem, we have

$$\left| \exp\left(-\frac{z^2}{2}\right) - \exp\left(-\frac{(z-c)^2}{2}\right) \right| = |c| \left| \exp\left(-\frac{(z-c^*)^2}{2}\right)(z-c^*) \right|,$$

where c^* is on the line segment adjoining 0 and c. Note that the function $s \mapsto |\exp(-s^2/2)s|$ is bounded by $\exp(-\frac{1}{2})$ (it is maximized at $s = 1$). It follows that

$$\left| \frac{\exp\left(-\frac{z^2}{2}\right) - \exp\left(-\frac{(z-c)^2}{2}\right)}{c} \right| \leq \exp\left(-\frac{1}{2}\right),$$

from which we obtain

$$\max_z \left| \frac{\exp\left(-\frac{z^2}{2}\right) - \exp\left(-\frac{(z-c)^2}{2}\right)}{c} \right| \leq \exp\left(-\frac{1}{2}\right).$$

It follows that

$$\chi(c) = \frac{1}{\sigma^2\sqrt{2\pi}} \max_z \left| \frac{\exp\left(-\frac{z^2}{2}\right) - \exp\left(-\frac{\left(z-\frac{c}{\sigma}\right)^2}{2}\right)}{\frac{c}{\sigma}} \right|$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \max_z \left| \frac{\exp\left(-\frac{z^2}{2}\right) - \exp\left(-\frac{\left(z-\frac{c}{\sigma}\right)^2}{2}\right)}{c} \right|$$

$$\leq \frac{\exp\left(-\frac{1}{2}\right)}{\sigma\sqrt{2\pi}}. \qquad \textit{Q.E.D.}$$

APPENDIX B: PROOF OF (6)

For the joint likelihood, we have

$$\prod_{t=1}^{T} p(y_t; \gamma) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^T \exp\left(-\frac{\sum_{t=1}^{T}(y_t-\gamma)^2}{2\sigma^2}\right)$$

$$= \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^T \exp\left(-\frac{\sum_{t=1}^{T}(y_t-\overline{y})^2 + T(\overline{y}-\gamma)^2}{2\sigma^2}\right)$$

$$= \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^T \exp\left(-\frac{\sum_{t=1}^{T}(y_t-\overline{y})^2}{2\sigma^2}\right)\exp\left(-\frac{T(\overline{y}-\gamma)^2}{2\sigma^2}\right)$$

and, likewise,

$$\prod_{t=1}^{T} p_j(y_t; \gamma) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^T \times \exp\left(-\frac{\sum_{t=1}^{T}(y_t-\overline{y})^2}{2\sigma^2}\right)\exp\left(-\frac{T(\overline{y}-\delta-\gamma)^2}{2\sigma^2}\right).$$

Therefore,

$$
\begin{aligned}
&\left|\prod_{t=1}^{T} p_j(y_t;\gamma) - \prod_{t=1}^{T} p(y_t;\gamma)\right| \\
&= \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^{T} \exp\left(-\frac{\sum_{t=1}^{T}(y_t-\overline{y})^2}{2\sigma^2}\right) \\
&\quad \times \left|\exp\left(-\frac{T(\overline{y}-\gamma)^2}{2\sigma^2}\right) - \exp\left(-\frac{T(\overline{y}-\delta-\gamma)^2}{2\sigma^2}\right)\right| \\
&= \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^{T} \exp\left(-\frac{\sum_{t=1}^{T}(y_t-\overline{y})^2}{2\sigma^2}\right)\left|\exp\left(-\frac{\left(\frac{\sqrt{T}\overline{y}}{\sigma}-\frac{\sqrt{T}\gamma}{\sigma}\right)^2}{2}\right)\right. \\
&\quad \left. - \exp\left(-\frac{\left(\left(\frac{\sqrt{T}\overline{y}}{\sigma}-\frac{\sqrt{T}\gamma}{\sigma}\right)-\frac{\sqrt{T}\delta}{\sigma}\right)^2}{2}\right)\right| \\
&= \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^{T-1} \exp\left(-\frac{\sum_{t=1}^{T}(y_t-\overline{y})^2}{2\sigma^2}\right)|\sqrt{T}\delta|\frac{1}{\sigma^2\sqrt{2\pi}} \\
&\quad \times \left|\frac{\exp\left(-\frac{\left(\frac{\sqrt{T}\overline{y}}{\sigma}-\frac{\sqrt{T}\gamma}{\sigma}\right)^2}{2}\right) - \exp\left(-\frac{\left(\left(\frac{\sqrt{T}\overline{y}}{\sigma}-\frac{\sqrt{T}\gamma}{\sigma}\right)-\frac{\sqrt{T}\delta}{\sigma}\right)^2}{2}\right)}{\frac{\sqrt{T}\delta}{\sigma}}\right| \\
&\le \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^{T-1} \exp\left(-\frac{\sum_{t=1}^{T}(y_t-\overline{y})^2}{2\sigma^2}\right)|\sqrt{T}\delta|\chi(\sqrt{T}\delta) \\
&\le \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^{T-1} |\sqrt{T}\delta|\chi(\sqrt{T}\delta),
\end{aligned}
$$

where now $(\frac{\sqrt{T}\overline{y}}{\sigma}-\frac{\sqrt{T}\gamma}{\sigma})$ is interpreted as the z in the definition of $\chi(c)$. By the definition of $\chi(c)$, the first inequality will hold with equality at some value of $\overline{y}$. The second inequality holds with equality by setting $y_t=\overline{y}$ for all t. Hence this bound is sharp. *Q.E.D.*

APPENDIX C: PROOF OF THEOREM 2

Because $Q_0(\gamma)$ is continuous,[1] Γ is compact, and γ_0 is the unique maximizer of $Q_0(\gamma)$, we can find $\varepsilon > 0$ such that $\sup_{|\gamma-\gamma_0|>\varepsilon} Q_0(\gamma) < Q_0(\gamma_0)$ and $Q_0''(\gamma) < 0$ for $|\gamma - \gamma_0| \leq \varepsilon$. We can then find $\eta > 0$ sufficiently small such that $\sup_{|\gamma-\gamma_0|>\varepsilon} Q_0(\gamma) < Q_0(\gamma_0) - 3\eta$ and $Q_0''(\gamma) < -3\eta$ for $|\gamma - \gamma_0| \leq \varepsilon$.

We now show that $|\gamma_j - \gamma_0| \leq \varepsilon$ for j sufficiently large, say for all $j \geq J$. By NM (Lemma 2.4), for example, we have $Q_0(\gamma)$ continuous and

$$\sup_{\gamma\in\Gamma}\left|\frac{1}{T}\sum_{t=1}^{T}\log p(y_t;\gamma) - Q_0(\gamma)\right| = o_p(1).$$

Likewise, we also have $Q_j(\gamma)$ continuous and

$$\sup_{\gamma\in\Gamma}\left|\frac{1}{T}\sum_{t=1}^{T}\log p_j(y_t;\gamma) - Q_j(\gamma)\right| = o_p(1).$$

Because of the definition of the bound Δ_j and Condition 1, we then have $|Q_j(\gamma) - Q_0(\gamma)| \leq \eta$, $|Q_j'(\gamma) - Q_0'(\gamma)| \leq \eta$, and $|Q_j''(\gamma) - Q_0''(\gamma)| \leq \eta$ for j sufficiently large. Because $-\eta \leq Q_j(\gamma) - Q_0(\gamma) \leq \eta$, we have $Q_j(\gamma) \leq Q_0(\gamma) + \eta$, in particular for $|\gamma - \gamma_0| > \varepsilon$. We therefore obtain

$$\sup_{|\gamma-\gamma_0|>\varepsilon} Q_j(\gamma) \leq \sup_{|\gamma-\gamma_0|>\varepsilon} Q_0(\gamma) + \eta. \tag{10}$$

We also have

$$\sup_{|\gamma-\gamma_0|>\varepsilon} Q_0(\gamma) < Q_0(\gamma_0) - 3\eta. \tag{11}$$

Combining (10) and (11), we obtain $\sup_{|\gamma-\gamma_0|>\varepsilon} Q_j(\gamma) \leq Q_0(\gamma_0) - 2\eta$ or

$$Q_0(\gamma_0) \geq \sup_{|\gamma-\gamma_0|>\varepsilon} Q_j(\gamma) + 2\eta. \tag{12}$$

Because $Q_j(\gamma) \geq Q_0(\gamma) - \eta$ for $|\gamma - \gamma_0| \leq \varepsilon$, we have $\sup_{|\gamma-\gamma_0|\leq\varepsilon} Q_j(\gamma) \geq \sup_{|\gamma-\gamma_0|\leq\varepsilon} Q_0(\gamma) - \eta$. But because $\sup_{|\gamma-\gamma_0|\leq\varepsilon} Q_0(\gamma) = Q_0(\gamma_0)$, we have

$$\sup_{|\gamma-\gamma_0|\leq\varepsilon} Q_j(\gamma) \geq Q_0(\gamma_0) - \eta. \tag{13}$$

Combining (12) and (13), we obtain

$$\sup_{|\gamma-\gamma_0|\leq\varepsilon} Q_j(\gamma) \geq \sup_{|\gamma-\gamma_0|>\varepsilon} Q_j(\gamma) + \eta,$$

[1] See, for example, NM (Lemma 2.4).

and the maximizer γ_j of $Q_j(\gamma)$ satisfies $|\gamma_j - \gamma_0| \le \varepsilon$.

We now get back to the proof of Theorem 2. By the first order condition, we have $0 = Q_j'(\gamma_j)$. By the mean value theorem, we obtain $0 = Q_j'(\gamma_0) + Q_j''(\gamma_j^*)(\gamma_j - \gamma_0)$, where γ_j^* is on the line segment adjoining γ_j and γ_0. We therefore have $\gamma_j - \gamma_0 = -Q_j'(\gamma_0)/Q_j''(\gamma_j^*)$. Because $|\gamma_j^* - \gamma_0| \le |\gamma_j - \gamma_0| \le \varepsilon$, we can see that $Q_0''(\gamma_j^*) < -3\eta$. This means that $Q_j''(\gamma_j^*) < -2\eta$ and that the division is well defined. Hence,

$$|\gamma_j - \gamma_0| \le |Q_j'(\gamma_0)|/2\eta \le \frac{\Delta_j}{2\eta} \tag{14}$$

for all $j \ge J$. (Roughly speaking, this inequality indicates that when the approximation is sufficiently precise, the difference between γ_j and γ_0 depends on the degree of approximation and the concavity of the objective function at γ_0.)

For $j < J$, let

$$\varrho = \max_{1\le j<J}\left\{\frac{|\gamma_j - \gamma_0|}{\Delta_j} 1(\Delta_j > 0)\right\}, \tag{15}$$

where $1(\cdot)$ denotes the indicator function. Let $\zeta = \max(\frac{1}{2\eta}, \varrho)$ (note that ζ does not depend on T).

Combining (14) and (15), we conclude that

$$|\gamma_j - \gamma_0| \le \zeta \cdot \Delta_j$$

for all j. *Q.E.D.*

APPENDIX D: PROOF OF THEOREM 3

Note that

$$\sup_{\gamma\in\Gamma}\left|\frac{1}{T}\sum_{t=1}^{T}\log p_j(y_t;\gamma) - \frac{1}{T}\sum_{t=1}^{T}\log p(y_t;\gamma)\right| \le \Delta_j$$

by definition and

$$\sup_{\gamma\in\Gamma}\left|\frac{1}{T}\sum_{t=1}^{T}\log p(y_t;\gamma) - Q(\gamma)\right| = o_p(1).$$

This implies that

$$\sup_{\gamma\in\Gamma}\left|\frac{1}{T}\sum_{t=1}^{T}\log p_j(y_t;\gamma) - Q(\gamma)\right| \le \Delta_j + o_p(1) = o_p(1) \tag{16}$$

by the assumption that $\Delta_j = o(1)$. Combining (16) with Conditions 2 and 4, and using NM (Theorem 2.5), we obtain the desired conclusion. *Q.E.D.*

APPENDIX E: PROOF OF THEOREM 4

Recalling that Theorem 1 implies that $\text{plim}_{T\to\infty} \widehat{\gamma}_j = \gamma_j$, consider

$$\widehat{\gamma}_j - \gamma_0 = (\widehat{\gamma}_j - \gamma_j) + (\gamma_j - \gamma_0). \tag{17}$$

We first characterize the asymptotic distribution of $\sqrt{T}(\widehat{\gamma}_j - \gamma_j)$. Note that γ_j and $\widehat{\gamma}_j$ solve

$$0 = E[\nabla_\gamma \log p_j(y_t; \gamma_j)],$$
$$0 = \frac{1}{T}\sum_{t=1}^{T} \nabla_\gamma \log p_j(y_t; \widehat{\gamma}_j).$$

Expanding the second equality around γ_j and using the mean value theorem, we obtain

$$0 = \frac{1}{T}\sum_{t=1}^{T} \nabla_\gamma \log p_j(y_t; \gamma_j) + \left(\frac{1}{T}\sum_{t=1}^{T} \nabla_{\gamma\gamma} \log p_j(y_t; \widetilde{\gamma}_j)\right)(\widehat{\gamma}_j - \gamma_j),$$

where $\widetilde{\gamma}_j$ is on the line segment adjoining $\widehat{\gamma}_j$ and γ_j. It follows that

$$\sqrt{T}(\widehat{\gamma}_j - \gamma_j) = -\left(\frac{1}{T}\sum_{t=1}^{T} \nabla_{\gamma\gamma} \log p_j(y_t; \widetilde{\gamma}_j)\right)^{-1} \times \frac{1}{\sqrt{T}}\sum_{t=1}^{T} \nabla_\gamma \log p_j(y_t; \gamma_j). \tag{18}$$

Note that

$$\left|\frac{1}{T}\sum_{t=1}^{T} \nabla_{\gamma\gamma} \log p_j(y_t; \widetilde{\gamma}_j) - \frac{1}{T}\sum_{t=1}^{T} \nabla_{\gamma\gamma} \log p(y_t; \widetilde{\gamma}_j)\right| \le \Delta_j \tag{19}$$

by definition. We also have

$$\left|\frac{1}{T}\sum_{t=1}^{T} \nabla_{\gamma\gamma} \log p(y_t; \widetilde{\gamma}_j) - Q_0''(\widetilde{\gamma}_j)\right| = o_p(1) \tag{20}$$

by NM (Lemma 2.4), for example. Finally, because $\widetilde{\gamma}_j = \gamma_0 + o_p(1)$ (since $\widetilde{\gamma}_j$ is on the line segment between γ_0 and γ_j), and because $Q_0''(\gamma)$ is continuous by dominated convergence, we have

$$Q_0''(\widetilde{\gamma}_j) = Q_0''(\gamma_0) + o_p(1). \tag{21}$$

Combining (19), (20), and (21), and the assumption that $\Delta_j \to 0$, we obtain that

$$\frac{1}{T}\sum_{t=1}^{T} \nabla_{\gamma\gamma} \log p_j(y_t; \widetilde{\gamma}_j) = Q_0''(\gamma_0) + o_p(1). \tag{22}$$

We also note that

$$E\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T} \nabla_{\gamma} \log p_j(y_t; \gamma_j) - \frac{1}{\sqrt{T}}\sum_{t=1}^{T} \nabla_{\gamma} \log p(y_t; \gamma_0)\right] = 0, \tag{23}$$

since by definition γ_j and γ_0 maximize $Q_j(\gamma)$ and $Q_0(\gamma)$, respectively. In addition, since $\Delta_j \to 0$,

$$\begin{aligned} &\operatorname{Var}\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T} \nabla_{\gamma} \log p_j(y_t; \gamma_j) - \frac{1}{\sqrt{T}}\sum_{t=1}^{T} \nabla_{\gamma} \log p(y_t; \gamma_0)\right) \\ &\quad = E\big[(\nabla_{\gamma} \log p_j(y_t; \gamma_j) - \nabla_{\gamma} \log p(y_t; \gamma_0))^2\big] = o(1). \end{aligned} \tag{24}$$

Combining (23) and (24) and applying Chebyshev's inequality, we conclude that

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T} \nabla_{\gamma} \log p_j(y_t; \gamma_j) = \frac{1}{\sqrt{T}}\sum_{t=1}^{T} \nabla_{\gamma} \log p(y_t; \gamma_0) + o_p(1). \tag{25}$$

Now, (18), (22), and (25) imply that

$$\begin{aligned} \sqrt{T}(\widehat{\gamma}_j - \gamma_j) &= -Q_0''(\gamma_0)^{-1}\frac{1}{\sqrt{T}}\sum_{t=1}^{T} \nabla_{\gamma} \log p(y_t; \gamma_0) + o_p(1) \\ &\Rightarrow N(0, -Q_0''(\gamma_0)^{-1}) \end{aligned} \tag{26}$$

as $T \to \infty$ and $\Delta_j \to 0$. The second line in (26) uses the central limit theorem and information equality.

Last, note that Theorem 2 implies that $\gamma_j - \gamma_0 = O(\Delta_j)$ or

$$\sqrt{T}(\gamma_j - \gamma_0) = O(\sqrt{T}\Delta_j).$$

Combining this with (17) and (26), we conclude that

$$\sqrt{T}(\widehat{\gamma}_j - \gamma_0) \Rightarrow N(0, -Q_0''(\gamma_0)^{-1})$$

as $T \to \infty$ and $\sqrt{T}\Delta_j \to 0$. *Q.E.D.*

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