Appendix: Model Confidence Sets

Bootstrap Procedure, Inflation forecasting, Regression simulations and Taylor Rules

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1. Bootstrap Procedure

This section describes the bootstrap implementation of the MCS procedure used in the forecasting application.

1. **(Bootstrap indexes for resampling)**

   This is the first step because we need to use common random numbers for the bootstrap resamples in each iteration of the sequential test.

   (a) Choose the block-length bootstrap parameter, l. The optimal choice for l is tied to the persistence in $d_{i\cdot,t} = m^{-1}\sum_{j\in\mathcal{M}_0} d_{ij,t}$, $i = 1,\ldots,m$, which is difficult to estimate precisely when m is large. Instead one can use different choices for l, and verify that the result is not sensitive to the choice.

   (b) Generate B bootstrap resamples of $\{1,\ldots,n\}$. I.e., for $b = 1,\ldots,B$:

      i. Choose $\xi_{b_1} \sim U\{1,\ldots,n\}$ and set $(\tau_{b,1},\ldots,\tau_{b,l}) = (\xi_{b_1}, \xi_{b_1}+1,\ldots,\xi_{b_1}+l-1)$, with the convention $n+i = i$ for $i \geq 1$.

      ii. Choose $\xi_{b_2} \sim U\{1,\ldots,n\}$ and set $(\tau_{b,l+1},\ldots,\tau_{b,2l}) = (\xi_{b_2}, \xi_{b_2}+1,\ldots,\xi_{b_2}+l-1)$.

      iii. Continue until a sample size of n, is constructed.

      iv. This is repeated for all resamples $b = 1,\ldots,B$, using independent draws of the ξ's.

   (c) Save the full matrix of bootstrap indexes.

   Alternatively one can use a different bootstrap scheme, such as the stationary bootstrap of **?**.

2. **(Sample and Bootstrap Statistics)**

   (a) For each model and each point in time we evaluate the performance to obtain the variables $L_{i,t}$, for $i = 1,\ldots,m$, and $t = 1,\ldots,n$. These variables are used to calculate the sample averages for each model $\bar{L}_{i,\cdot} \equiv \frac{1}{n}\sum_{t=1}^{n} L_{i,t}$, $i = 1,\ldots,m$.

   (b) The corresponding bootstrap variables are now given by

   $$L^*_{b,i,t} = L_{i,\tau_{b,t}}, \quad \text{for} \quad b = 1,\ldots,B,\ i = 1,\ldots,m,\ \text{and } t = 1,\ldots,n,$$

   and calculate the bootstrap sample averages, $\bar{L}^*_{b,i} \equiv \frac{1}{n}\sum_{t=1}^{n} L^*_{b,i,t}$. The only variables that need to be stored are $\bar{L}_i$ and $\zeta^*_{b,i} \equiv \bar{L}^*_{b,i} - \bar{L}_i$, as all required statistics can be calculated from these two variables.

3. **(Sequential Testing)** Initialize by setting $\mathcal{M} = \mathcal{M}_0$.

   (a) Let m denote the number of elements in $\mathcal{M}$, and calculate

   $$\bar{L}_{\cdot} \equiv \frac{1}{m}\sum_{i=1}^{m}\bar{L}_i, \qquad \zeta^*_{b,\cdot} = \frac{1}{m}\sum_{i=1}^{m}\zeta^*_{b,i}, \qquad \text{and} \qquad \widehat{\text{var}}(\bar{d}_{i\cdot}) \equiv \frac{1}{B}\sum_{b=1}^{B}(\zeta^*_{b,i} - \zeta^*_{b,\cdot})^2.$$

Alternatively one can define $\widehat{\text{var}}(\cdot)$ to be its analytical value given the employed bootstrap scheme. Now define $t_{i\cdot} \equiv \bar{d}_{i\cdot}/\sqrt{\widehat{\text{var}}(\bar{d}_{i\cdot})}$ and calculate the test statistic $T_{\max} = \max_i t_{i\cdot}$.

(b) The bootstrap estimate of T_D's distribution is given by the empirical distribution of

$$T^*_{b,\max} = \max_i t^*_{b,i\cdot}, \quad \text{for} \quad b = 1, \ldots, B,$$

where $t^*_{b,i\cdot} \equiv (\zeta^*_{b,i} - \zeta^*_{b,\cdot})/\sqrt{\widehat{\text{var}}(\bar{d}_{i\cdot})}$.

(c) The p-value of $H_{0,\mathcal{M}}$ is given by

$$P_{H_{0,\mathcal{M}}} \equiv \frac{1}{B}\sum_{b=1}^{B} 1_{\left\{T_{\max} > T^*_{b,\max}\right\}},$$

where $1_{\{\cdot\}}$ is the indicator function.

(d) If $P_{H_{0,\mathcal{M}}} < \alpha$, where α is the level of the test, then $H_{0,\mathcal{M}}$ is rejected and $e_{\mathcal{M}} \equiv \arg\max_i t_{i\cdot}$ is eliminated from $\mathcal{M}$.

(e) The steps in 3.(a)-(d) are repeated until first 'acceptance'. The resulting set of models is denoted $\widehat{\mathcal{M}}^*_{1-\alpha}$ and referred to as the $(1-\alpha)$ MCS.

1.1. *Justification of bootstrap implementation*

Let $Z_t = (d_{1\cdot,t}, \ldots, d_{m\cdot,t})'$ then by Lemma 5 we have that $n^{1/2}(\bar{Z} - \psi) \overset{d}{\to} N_m(0, \Omega)$, where $\bar{Z} = \sum_{t=1}^{n} Z_t$. The bootstrap variables $\{Z^*_{b,t}\}$ are generated such that $n^{1/2}(\bar{Z}^*_b - \bar{Z}) \overset{d}{\to} N_m(0, \Omega)$, where the covariance matrix can be estimated by its analytical form under the bootstrap scheme, $\hat{\Omega}^*_n$ say, where $\hat{\Omega}^*_n$ is consistent for Ω as $n \to \infty$. Alternatively, Ω can be estimated directly from the resamples by $\hat{\Omega}_{n,B} \equiv n/B\sum_{b=1}^{B}(\bar{Z}^*_b - \bar{Z})(\bar{Z}^*_b - \bar{Z})'$, where $\hat{\Omega}_{n,B} \overset{p}{\to} \hat{\Omega}^*_n$ as $B \to \infty$ by the law of large numbers.

Our implementation is based on $\hat{\Omega}_{n.B}$, and the identity

$$\zeta^*_{b,i} - \zeta^*_{b,\cdot} = \bar{L}^*_{b,i} - \bar{L}_i - \frac{1}{m}\sum_{i=1}^{m}(\bar{L}^*_{b,i} - \bar{L}_i) = (\bar{L}^*_{b,i} - \bar{L}^*_{b,\cdot}) - (\bar{L}_i - \bar{L}_\cdot) = \bar{d}^*_{b,i\cdot} - \bar{d}_{i\cdot},$$

that shows that the diagonal elements of $\hat{\Omega}$ are given by

$$n/B\sum_{b=1}^{B}(\bar{Z}^*_{b,i} - \bar{Z}_i)^2 = n/B\sum_{b=1}^{B}(\bar{d}^*_{b,i\cdot} - \bar{d}_{i\cdot})^2 = \frac{n}{B}\sum_{b=1}^{B}(\zeta^*_{b,i} - \zeta^*_{b,\cdot})^2 = \widehat{\text{var}}(n^{1/2}\bar{d}_{i\cdot}).$$

Under the null hypothesis, the distribution of $T_{\max}$ is approximated by

$$\begin{aligned}
\max_i \left(\hat{D}^{-1/2} n^{1/2}(\bar{Z}^*_b - \bar{Z})\right)_i &= \max_i \left(\text{diag}(\widehat{\text{var}}(\bar{d}_{1\cdot}), \ldots, \widehat{\text{var}}(\bar{d}_{1\cdot}))^{-1/2}(\bar{Z}^*_b - \bar{Z})\right)_i \\
&= \max_i \frac{\bar{d}^*_{b,i\cdot} - \bar{d}_{i\cdot}}{\sqrt{\widehat{\text{var}}(\bar{d}_{i\cdot})}} = \max_i \frac{\zeta^*_{b,i} - \zeta^*_{b,\cdot}}{\sqrt{\widehat{\text{var}}(\bar{d}_{i\cdot})}} = \max t^*_{b,i\cdot} \\
&= T^*_{b,\max}.
\end{aligned}$$

2. Inflation Forecasting

Here we investigate the sensitivity of our MCS for Stock and Watson (JME,1999) to the choice of estimation scheme and equivalence test.

2.1. *Sensitivity Analysis of MCS's to estimation scheme and the choice of test for EPA*

The Tables corresponding to Table 2 in Stock and Watson (JME,1999) are as follows.

Table A.1 use an expanding recursive estimation scheme.

Table A.2 use a rolling estimation scheme.

The Tables corresponding to Table 4 in Stock and Watson (JME,1999) are as follows.

Table A.3 use a expanding recursive estimation scheme.

Table A.4 use an rolling estimation scheme.

These tables also display the root MSE of each model.

3. Regression Simulation

Here we present simulation results for $\beta^2 = 0.1, 0.5, 0.9$ for the simulation experiment in section 5.2.

Tables A.5-A.7 report the fraction that each of the specifications is in the MCS for the KLIC, AIC* and BIC*.

Tables A.9-A.10 report the average MCS p-value for the KLIC, AIC* and BIC*.

4. Taylor Rules

The Tables are as follows.

Table A.11 gives MCS results when the models are estimated on a sample period covering 1979Q1 to

2006Q4.

Table A.12 gives MCS results when the sample period only span 1984Q1 to 2006Q4 .

References

Table A.1: MCS p-values for Stock and Watson JME (1999, table 2) (recursive scheme).

		PUNEW: 1970-1983				PUNEW: 1984-1996				GMDC: 1970-1983				GMDC: 1984-1996			
Variable	Trans	RMSE	p_6	p_9	p_{12}	RMSE	p_6	p_9	p_{12}	RMSE	p_6	p_9	p_{12}	RMSE	p_6	p_9	p_{12}
No change (month)		3.290	.001	.002	.003	2.140	.002	.003	.003	2.208	.013	.028	.035	1.751	.024	.025	.019
No change (year)	-	2.798	.007	.012	.013	1.207	1.00**	1.00**	1.00**	2.100	.024	.048	.056	0.888	1.00**	1.00**	1.00**
uniar	-	2.675	.004	.009	.011	1.360	.802**	.809**	.796**	1.941	.044	.069	.075	1.082	.205*	.213*	.208*
'Gaps' specifications																	
dtip	DT	2.519	.021	.029	.026	1.310	.845**	.871**	.868**	1.913	.053	.076	.077	1.043	.281**	.297**	.292**
dtgmpyq	DT	2.644	.004	.007	.008	1.446	.389**	.416**	.389**	2.067	.024	.047	.056	1.103	.144*	.145*	.134*
dtmsmtq	DT	2.341	.092	.095	.085	1.280	.845**	.871**	.868**	1.844	.061	.088	.084	1.007	.330**	.351**	.354**
dtlpnag	DT	2.482	.024	.030	.026	1.323	.835**	.871**	.868**	2.024	.040	.069	.075	1.012	.330**	.351**	.354**
ipxmca	LV	2.373	.055	.060	.057	1.264	.845**	.871**	.868**	1.887	.058	.087	.084	1.026	.330**	.351**	.354**
hsbp	LN	2.205	.763**	.766**	.765**	1.392	.765**	.782**	.779**	1.829	.061	.088	.084	0.993	.330**	.367**	.370**
lhmu25	LV	2.433	.026	.030	.026	1.401	.741**	.754**	.744**	1.937	.040	.070	.073	1.055	.295**	.308**	.314**
First difference specifications																	
ip	DLN	2.384	.047	.030	.026	1.429	.701**	.751**	.728**	1.819	.061	.088	.084	1.115	.144*	.145*	.131*
gmpyq	DLN	2.233	.496**	.453**	.385**	1.532	.256**	.292**	.270**	1.565	1.00**	1.00**	1.00**	1.149	.159*	.161*	.153*
msmtq	DLN	2.169	1.00**	1.00**	1.00**	1.353	.802**	.871**	.868**	1.778	.061	.088	.084	1.062	.289**	.303**	.314**
lpnag	DLN	2.308	.109*	.107*	.100	1.317	.845**	.871**	.868**	1.809	.061	.088	.084	1.009	.330**	.351**	.354**
dipxmca	DLV	2.355	.055	.060	.057	1.456	.536**	.549**	.517**	1.839	.059	.087	.083	1.128	.128*	.128*	.117*
dhsbp	DLN	2.701	.004	.007	.008	1.405	.741**	.754**	.744**	1.969	.035	.061	.064	1.077	.243*	.255**	.250**
dlhmu25	DLV	2.352	.055	.060	.057	1.474	.190*	.229*	.214*	1.878	.054	.078	.077	1.103	.137*	.141*	.130*
dlhur	DLV	2.321	.109*	.107*	.100	1.451	.283**	.308**	.288**	1.843	.059	.087	.084	1.088	.194*	.206*	.200*
Phillips curve																	
LHUR		2.387	.024	.030	.026	1.371	.741**	.754**	.744**	1.939	.047	.076	.077	1.050	.289**	.303**	.296**

RMSEs and MCS p-values for the different forecasts. For each p-value the bootstrap block length is identified by the subscript. The forecasts in $\widehat{\mathcal{M}}^*_{90\%}$ and $\widehat{\mathcal{M}}^*_{75\%}$ are identified by one and two asterisks, respectively.

Table A.2: MCS p-values for Stock and Watson JME (1999, table 2) (rolling scheme).

		PUNEW: 1970-1983				PUNEW: 1984-1996				GMDC: 1970-1983				GMDC: 1984-1996			
Variable	Trans	RMSE	p_6	p_9	p_{12}	RMSE	p_6	p_9	p_{12}	RMSE	p_6	p_9	p_{12}	RMSE	p_6	p_9	p_{12}
No change (month)		3.290	.000	.000	.001	2.140	.120*	.128*	.122*	2.208	.035	.041	.042	1.751	.106*	.116*	.113*
No change (year)	-	2.798	.003	.004	.006	1.207	1.00**	1.00**	1.00**	2.100	.077	.174*	.109*	0.888	1.00**	1.00**	1.00**
uniar	-	2.802	.001	.001	.004	1.330	.742**	.753**	.736**	2.026	.136*	.165*	.145*	1.070	.391**	.412**	.411**
'Gaps' specifications																	
dtip	DT	2.597	.022	.030	.059	1.475	.640**	.672**	.651**	2.103	.059	.092	.095	1.050	.391**	.412**	.411**
dtgmpyq	DT	2.751	.002	.004	.020	1.691	.249*	.302**	.299**	2.090	.149*	.106*	.157*	1.125	.307**	.323**	.317**
dtmsmtq	DT	2.202	.835**	.858**	.872**	1.704	.386**	.436**	.477**	1.806	.462**	.475**	.464**	1.046	.391**	.412**	.411**
dtlpnag	DT	2.591	.048	.060	.068	1.433	.688**	.706**	.694**	2.132	.059	.079	.075	1.026	.391**	.412**	.411**
ipxmca	LV	2.609	.044	.055	.034	1.318	.742**	.753**	.736**	2.040	.262**	.283**	.261**	1.034	.391**	.412**	.411**
hsbp	LN	2.114	1.00**	1.00**	1.00**	1.582	.549**	.590**	.579**	1.967	.352**	.378**	.364**	1.034	.391**	.412**	.411**
lhmu25	LV	2.968	.002	.004	.006	1.439	.640**	.672**	.651**	2.231	.027	.062	.061	1.040	.391**	.412**	.411**
First difference specifications																	
ip	DLN	2.344	.252**	.285**	.306**	1.393	.742**	.753**	.736**	1.946	.322**	.335**	.298**	1.058	.391**	.412**	.411**
gmpyq	DLN	2.306	.828**	.856**	.842**	1.524	.545**	.588**	.421**	1.709	1.00**	1.00**	1.00**	1.158	.304**	.322**	.317**
msmtq	DLN	2.158	.835**	.858**	.872**	1.391	.742**	.753**	.736**	1.857	.462**	.475**	.464**	1.066	.391**	.412**	.411**
lpnag	DLN	2.408	.385**	.413**	.430**	1.341	.742**	.753**	.736**	1.940	.341**	.342**	.298**	1.027	.391**	.412**	.411**
dipxmca	DLV	2.379	.099	.121*	.139*	1.353	.742**	.753**	.736**	1.903	.426**	.449**	.446**	1.041	.391**	.412**	.411**
dhsbp	DLN	2.850	.001	.002	.003	1.456	.664**	.683**	.665**	2.076	.066	.079	.075	1.070	.391**	.412**	.411**
dlhmu25	DLV	2.383	.130*	.154*	.169*	1.440	.640**	.672**	.579**	2.035	.122*	.100	.102*	1.065	.391**	.412**	.411**
dlhur	DLV	2.296	.594**	.621**	.631**	1.429	.687**	.706**	.691**	1.904	.415**	.363**	.330**	1.067	.391**	.412**	.411**
Phillips curve																	
LHUR		2.637	.024	.032	.034	1.388	.742**	.753**	.736**	2.076	.076	.097	.098	1.162	.317**	.333**	.325**

RMSEs and MCS p-values for the different forecasts. For each p-value the bootstrap block length is identified by the subscript. The forecasts in $\widehat{\mathcal{M}}^*_{90\%}$ and $\widehat{\mathcal{M}}^*_{75\%}$ are identified by one and two asterisks, respectively.

Table A.3: MCS p-values for Stock and Watson JME (1999, table 4) (recursive scheme).

	PUNEW: 1970-1983				PUNEW: 1984-1996				GMDC: 1970-1983				GMDC: 1984-1996			
Variable	RMSE	p_6	p_9	p_{12}	RMSE	p_6	p_9	p_{12}	RMSE	p_6	p_9	p_{12}	RMSE	p_6	p_9	p_{12}
No change (month)	3.290	.006	.010	.007	2.140	.000	.000	.000	2.208	.000	.002	.004	1.751	.000	.000	.000
No change (year)	2.798	.010	.017	.021	1.207	1.00**	1.00**	1.00**	2.100	.002	.005	.011	0.888	1.00**	1.00**	1.00**
Univariate	2.675	.010	.017	.021	1.360	.741**	.757**	.749**	1.941	.026	.054	.075	1.082	.140*	.136*	.125*
Panel A. All indicators																
Mul. factors	2.158	.256**	.286**	.290**	1.291	.923**	.941**	.944**	1.894	.085	.102*	.128*	0.964	.576**	.601**	.596**
1 factor	2.069	.699**	.722**	.714**	1.274	.923**	.941**	.944**	1.692	1.00**	1.00**	1.00**	1.002	.568**	.596**	.596**
Comb. mean	2.439	.011	.017	.021	1.289	.923**	.941**	.944**	1.853	.110*	.110*	.128*	1.036	.437**	.470**	.469**
Comb. median	2.550	.011	.017	.021	1.316	.904**	.917**	.920**	1.895	.077	.092	.106*	1.063	.236*	.241*	.232*
Comb. ridge reg.	2.209	.062	.066	.054	1.280	.923**	.941**	.944**	1.842	.116*	.117*	.130*	1.019	.430**	.456**	.452**
Panel B. Real activity indicators																
Mul. factors	2.019	1.00**	1.00**	1.00**	1.357	.797**	.820**	.820**	1.792	.156*	.174*	.194*	0.946	.576**	.601**	.596**
1 factor	2.079	.699**	.722**	.714**	1.281	.923**	.941**	.944**	1.753	.235*	.271**	.292**	1.017	.542**	.575**	.573**
Comb. mean	2.346	.016	.025	.029	1.284	.923**	.941**	.944**	1.807	.132*	.130*	.148*	1.020	.430**	.456**	.452**
Comb. median	2.381	.012	.019	.029	1.299	.923**	.917**	.920**	1.831	.122*	.117*	.130*	1.036	.265**	.241*	.232*
Comb. ridge reg.	2.192	.114*	.124*	.116*	1.298	.923**	.941**	.944**	1.773	.174*	.174*	.194*	1.022	.430**	.456**	.452**
Panel C. Interest rates																
Mul. factors	2.585	.011	.017	.021	1.495	.054	.030	.014	1.976	.058	.075	.097	1.173	.111*	.114*	.103*
1 factor	2.524	.016	.025	.029	1.495	.009	.005	.001	2.038	.001	.004	.010	1.077	.209*	.213*	.204*
Comb. mean	2.424	.016	.025	.029	1.341	.844**	.862**	.862**	1.900	.077	.092	.106*	1.079	.132*	.128*	.118*
Comb. median	2.513	.011	.017	.021	1.336	.873**	.888**	.891**	1.912	.061	.078	.099	1.078	.187*	.149*	.138*
Comb. ridge reg.	2.432	.016	.025	.029	1.368	.513**	.460**	.384**	1.943	.007	.010	.017	1.123	.102*	.099	.088
Panel D. Money																
Mul. factors	2.679	.010	.017	.019	1.360	.619**	.592**	.532**	1.933	.058	.071	.086	1.080	.152*	.164*	.153*
1 factor	2.679	.010	.017	.021	1.360	.715**	.727**	.715**	1.933	.058	.072	.090	1.080	.168*	.187*	.176*
Comb. mean	2.664	.010	.017	.021	1.350	.769**	.789**	.786**	1.964	.003	.012	.020	1.066	.326**	.355**	.347**
Comb. median	2.670	.010	.017	.021	1.348	.816**	.833**	.828**	1.954	.010	.025	.037	1.070	.267**	.274**	.265**
Comb. ridge reg.	2.638	.010	.017	.021	1.385	.260**	.205*	.150*	1.934	.058	.075	.097	1.121	.124*	.107*	.096
Phillips curve																
LHUR	2.387	.012	.019	.024	1.371	.479**	.428**	.358**	1.939	.062	.081	.106*	1.050	.398**	.340**	.328**

RMSEs and MCS p-values for the different forecasts. For each p-value the bootstrap block length is identified by the subscript. The forecasts in $\widehat{\mathcal{M}}^*_{90\%}$ and $\widehat{\mathcal{M}}^*_{75\%}$ are identified by one and two asterisks, respectively.

Table A.4: MCS p-values for Stock and Watson JME (1999, table 4) (rolling scheme).

	PUNEW: 1970-1983				PUNEW: 1984-1996				GMDC: 1970-1983				GMDC: 1984-1996			
Variable	RMSE	p_6	p_9	p_{12}	RMSE	p_6	p_9	p_{12}	RMSE	p_6	p_9	p_{12}	RMSE	p_6	p_9	p_{12}
No change (month)	3.290	.005	.007	.006	2.140	.000	.000	.000	2.208	.001	.002	.006	1.751	.000	.000	.000
No change (year)	2.798	.010	.019	.020	1.207	1.00**	1.00**	1.00**	2.100	.070	.100*	.120*	0.888	1.00**	1.00**	1.00**
Univariate	2.802	.008	.011	.012	1.330	.702**	.725**	.718**	2.026	.017	.030	.046	1.070	.360**	.369**	.378**
Panel A. All indicators																
Mul. factors	2.367	.246*	.266**	.266**	1.407	.059	.089	.069	2.105	.041	.065	.088	1.013	.528**	.566**	.570**
1 factor	2.106	1.00**	1.00**	1.00**	1.351	.125*	.171*	.186*	1.746	1.00**	1.00**	1.00**	1.038	.528**	.566**	.570**
Comb. mean	2.423	.119*	.126*	.093	1.269	.844**	.866**	.869**	1.880	.521**	.557**	.585**	1.030	.528**	.566**	.570**
Comb. median	2.585	.028	.030	.030	1.294	.844**	.866**	.869**	1.939	.270**	.310**	.323**	1.055	.499**	.532**	.530**
Comb. ridge reg.	2.121	.971**	.974**	.975**	1.318	.844**	.866**	.869**	1.918	.294**	.316**	.518**	1.013	.528**	.566**	.570**
Panel B. Real activity indicators																
Mul. factors	2.245	.778**	.783**	.768**	1.416	.013	.022	.022	1.959	.294**	.316**	.323**	0.990	.528**	.566**	.570**
1 factor	2.115	.971**	.974**	.975**	1.347	.302**	.353**	.358**	1.774	.684**	.713**	.720**	1.041	.528**	.566**	.570**
Comb. mean	2.284	.597**	.615**	.615**	1.263	.844**	.866**	.869**	1.827	.646**	.685**	.698**	1.012	.528**	.566**	.570**
Comb. median	2.329	.442**	.476**	.495**	1.284	.844**	.866**	.869**	1.854	.584**	.628**	.647**	1.038	.514**	.550**	.553**
Comb. ridge reg.	2.160	.952**	.954**	.953**	1.326	.826**	.851**	.855**	1.888	.543**	.578**	.518**	1.013	.528**	.566**	.570**
Panel C. Interest rates																
Mul. factors	2.828	.019	.016	.019	1.512	.003	.004	.005	2.215	.001	.003	.008	1.294	.003	.007	.008
1 factor	2.776	.033	.033	.030	1.463	.001	.003	.003	2.111	.001	.003	.007	1.102	.029	.085	.161*
Comb. mean	2.474	.161*	.165*	.092	1.349	.087	.114*	.123*	1.935	.263**	.308**	.323**	1.060	.484**	.514**	.522**
Comb. median	2.567	.033	.033	.077	1.377	.046	.033	.034	1.974	.211*	.257**	.290**	1.066	.486**	.518**	.418**
Comb. ridge reg.	2.436	.189*	.199*	.164*	1.372	.013	.069	.069	1.962	.144*	.185*	.216*	1.052	.499**	.532**	.530**
Panel D. Money																
Mul. factors	2.801	.010	.012	.015	1.340	.358**	.592**	.597**	2.028	.005	.011	.020	1.075	.097	.049	.057
1 factor	2.805	.010	.016	.013	1.352	.148*	.177*	.186*	2.027	.010	.020	.031	1.104	.006	.013	.026
Comb. mean	2.742	.010	.016	.019	1.390	.013	.022	.022	2.033	.003	.006	.012	1.088	.014	.026	.015
Comb. median	2.752	.010	.016	.019	1.340	.605**	.389**	.386**	2.032	.002	.004	.008	1.077	.181*	.223*	.095
Comb. ridge reg.	2.721	.010	.016	.019	1.446	.003	.006	.007	2.013	.041	.065	.088	1.088	.003	.009	.010
Phillips curve																
LHUR	2.637	.033	.033	.030	1.388	.013	.022	.022	2.076	.010	.020	.031	1.162	.334**	.429**	.423**

RMSEs and MCS p-values for the different forecasts. For each p-value the bootstrap block length is identified by the subscript. The forecasts in $\widehat{\mathcal{M}}^*_{90\%}$ and $\widehat{\mathcal{M}}^*_{75\%}$ are identified by one and two asterisks, respectively.

Table A.5: Simulation Experiment II: $\beta^2 = 0.1$, fraction in MCS

	$Q(\mathcal{Z}_j, \hat{\theta}_j)$		$\hat{k}^*$		KLIC		AIC* (TIC)		BIC*	
Panel A: $n = 50$										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	47.8	48.1	2.00	1.99	0.974	0.956	0.985	0.980	0.988	0.989
X_0, X_1	41.4	41.8	3.03	3.03	1.000	0.999	1.000	0.999	0.979	0.960
$X_0, \ldots, X_2$	40.3	40.8	4.09	4.09	1.000	0.999	0.998	0.999	0.928	0.922
$X_0, \ldots, X_3$	39.3	39.7	5.19	5.19	1.000	1.000	0.997	0.998	0.840	0.832
$X_0, \ldots, X_4$	38.2	38.5	6.33	6.33	1.000	1.000	0.994	0.996	0.676	0.653
$X_0, \ldots, X_5$	37.0	37.4	7.51	7.51	1.000	1.000	0.987	0.989	0.482	0.435
$X_0, \ldots, X_6$	35.8	36.2	8.75	8.74	1.000	1.000	0.970	0.965	0.312	0.250
X_0, X_2	46.3	42.8	3.03	3.03	0.979	0.997	0.984	0.999	0.944	0.959
X_0, X_2, X_3	45.0	41.5	4.09	4.09	0.982	0.998	0.978	0.999	0.812	0.914
$X_0, X_2, \ldots, X_4$	43.7	40.3	5.19	5.18	0.983	0.999	0.966	0.998	0.612	0.808
$X_0, X_2, \ldots, X_5$	42.5	39.2	6.33	6.32	0.983	0.999	0.946	0.995	0.422	0.609
$X_0, X_2, \ldots, X_6$	41.3	38.0	7.51	7.51	0.983	0.999	0.906	0.987	0.271	0.396
Panel B: $n = 100$										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	97.9	98.2	2.00	1.99	0.859	0.806	0.903	0.886	0.930	0.943
X_0, X_1	86.2	86.7	3.01	3.01	1.000	0.999	1.000	1.000	0.996	0.991
$X_0, \ldots, X_2$	85.2	85.6	4.03	4.03	1.000	1.000	0.994	1.000	0.926	0.973
$X_0, \ldots, X_3$	84.2	84.6	5.07	5.07	1.000	1.000	0.990	1.000	0.864	0.900
$X_0, \ldots, X_4$	83.1	83.5	6.12	6.12	1.000	1.000	0.987	1.000	0.677	0.684
$X_0, \ldots, X_5$	82.0	82.4	7.19	7.19	1.000	1.000	0.980	0.999	0.447	0.412
$X_0, \ldots, X_6$	80.9	81.4	8.28	8.27	1.000	1.000	0.972	0.993	0.268	0.220
X_0, X_2	95.9	88.7	3.01	3.00	0.881	0.994	0.900	0.998	0.847	0.988
X_0, X_2, X_3	94.4	87.2	4.03	4.03	0.894	0.997	0.884	0.999	0.628	0.957
$X_0, X_2, \ldots, X_4$	93.1	86.0	5.07	5.07	0.901	0.998	0.865	0.998	0.406	0.837
$X_0, X_2, \ldots, X_5$	91.8	84.9	6.12	6.12	0.904	0.998	0.826	0.997	0.242	0.591
$X_0, X_2, \ldots, X_6$	90.5	83.7	7.19	7.18	0.906	0.998	0.770	0.991	0.140	0.343
Panel C: $n = 500$										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	498	498	2.00	2.00	0.002	0.001	0.005	0.003	0.015	0.018
X_0, X_1	444	444	3.00	3.00	1.000	1.000	1.000	1.000	1.000	1.000
$X_0, \ldots, X_2$	443	443	4.01	4.01	1.000	1.000	0.957	0.999	0.215	0.779
$X_0, \ldots, X_3$	442	442	5.01	5.01	1.000	1.000	0.937	0.998	0.109	0.554
$X_0, \ldots, X_4$	441	441	6.02	6.02	1.000	1.000	0.903	0.996	0.054	0.208
$X_0, \ldots, X_5$	440	440	7.03	7.03	1.000	1.000	0.859	0.992	0.031	0.085
$X_0, \ldots, X_6$	439	439	8.04	8.04	1.000	1.000	0.796	0.986	0.014	0.035
X_0, X_2	493	454	3.00	3.00	0.007	0.849	0.011	0.886	0.013	0.717
X_0, X_2, X_3	489	451	4.00	4.00	0.011	0.915	0.013	0.916	0.008	0.478
$X_0, X_2, \ldots, X_4$	486	449	5.01	5.01	0.016	0.932	0.015	0.909	0.003	0.241
$X_0, X_2, \ldots, X_5$	484	448	6.02	6.02	0.019	0.939	0.016	0.886	0.002	0.100
$X_0, X_2, \ldots, X_6$	483	447	7.03	7.02	0.022	0.943	0.016	0.843	0.001	0.042

The average value of the maximized log-likelihood function multiplied by minus two is reported in the first two columns. The next pair of columns has the average estimate of the degrees of freedom. The last three pairs of columns report the frequency that a particular regression model is in the $\widehat{\mathcal{M}}^*_{90\%}$ for each of the three criteria, KLIC, AIC* and BIC*.

Table A.6: Simulation Experiment II: $\beta^2 = 0.5$, fraction in MCS

	$Q(\mathcal{Z}_j, \hat{\theta}_j)$		$\hat{k}^*$		KLIC		AIC* (TIC)		BIC*	
Panel A: n = 50										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	48.1	48.1	1.99	2.00	0.058	0.038	0.085	0.070	0.118	0.124
X_0, X_1	12.4	12.4	3.02	3.02	0.998	0.999	1.000	1.000	1.000	1.000
$X_0, \ldots, X_2$	11.3	11.3	4.08	4.08	0.998	0.999	0.962	0.999	0.566	0.940
$X_0, \ldots, X_3$	10.2	10.2	5.18	5.18	0.999	0.999	0.940	0.998	0.469	0.912
$X_0, \ldots, X_4$	9.09	9.04	6.32	6.32	1.000	1.000	0.905	0.997	0.367	0.803
$X_0, \ldots, X_5$	7.95	7.88	7.50	7.50	1.000	1.000	0.867	0.994	0.279	0.598
$X_0, \ldots, X_6$	6.77	6.69	8.73	8.74	1.000	1.000	0.806	0.990	0.203	0.400
X_0, X_2	44.7	21.0	3.02	3.02	0.086	0.905	0.100	0.935	0.099	0.877
X_0, X_2, X_3	42.3	18.1	4.08	4.08	0.106	0.948	0.107	0.949	0.077	0.806
$X_0, X_2, \ldots, X_4$	40.4	16.3	5.18	5.18	0.120	0.958	0.105	0.938	0.054	0.665
$X_0, X_2, \ldots, X_5$	38.8	14.8	6.32	6.32	0.132	0.962	0.100	0.913	0.036	0.501
$X_0, X_2, \ldots, X_6$	37.2	13.4	7.50	7.51	0.145	0.964	0.094	0.869	0.022	0.348
Panel B: n = 100										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	98.0	98.1	1.99	1.99	0.000	0.000	0.000	0.000	0.000	0.000
X_0, X_1	27.6	27.8	3.00	3.00	0.998	1.000	1.000	1.000	1.000	1.000
$X_0, \ldots, X_2$	26.6	26.7	4.03	4.03	0.999	1.000	0.959	0.982	0.402	0.675
$X_0, \ldots, X_3$	25.5	25.7	5.07	5.06	0.999	1.000	0.939	0.975	0.276	0.619
$X_0, \ldots, X_4$	24.4	24.6	6.12	6.12	1.000	1.000	0.908	0.960	0.174	0.545
$X_0, \ldots, X_5$	23.4	23.6	7.19	7.18	1.000	1.000	0.864	0.942	0.101	0.390
$X_0, \ldots, X_6$	22.3	22.5	8.28	8.27	1.000	1.000	0.800	0.920	0.059	0.238
X_0, X_2	92.4	45.1	3.00	3.01	0.000	0.548	0.000	0.585	0.000	0.490
X_0, X_2, X_3	88.8	40.4	4.03	4.03	0.000	0.691	0.000	0.666	0.000	0.443
$X_0, X_2, \ldots, X_4$	86.1	38.1	5.07	5.07	0.000	0.736	0.000	0.675	0.000	0.338
$X_0, X_2, \ldots, X_5$	83.9	36.3	6.12	6.12	0.000	0.759	0.000	0.655	0.000	0.236
$X_0, X_2, \ldots, X_6$	82.0	34.8	7.19	7.19	0.001	0.772	0.000	0.631	0.000	0.143
Panel C: n = 500										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	498	498	2.00	2.00	0.000	0.000	0.000	0.000	0.000	0.000
X_0, X_1	151	151	3.00	3.00	0.999	0.999	1.000	1.000	1.000	1.000
$X_0, \ldots, X_2$	150	150	4.00	4.00	0.999	0.999	0.958	0.960	0.207	0.206
$X_0, \ldots, X_3$	149	149	5.01	5.01	0.999	1.000	0.938	0.938	0.100	0.099
$X_0, \ldots, X_4$	148	148	6.02	6.01	1.000	1.000	0.907	0.901	0.044	0.042
$X_0, \ldots, X_5$	147	147	7.03	7.02	1.000	1.000	0.858	0.852	0.020	0.017
$X_0, \ldots, X_6$	145	146	8.04	8.03	1.000	1.000	0.790	0.792	0.006	0.008
X_0, X_2	474	238	3.00	3.00	0.000	0.000	0.000	0.000	0.000	0.000
X_0, X_2, X_3	460	219	4.00	4.00	0.000	0.002	0.000	0.002	0.000	0.002
$X_0, X_2, \ldots, X_4$	451	211	5.01	5.01	0.000	0.004	0.000	0.004	0.000	0.001
$X_0, X_2, \ldots, X_5$	444	206	6.02	6.01	0.000	0.006	0.000	0.006	0.000	0.001
$X_0, X_2, \ldots, X_6$	439	203	7.03	7.02	0.000	0.008	0.000	0.007	0.000	0.000

The average value of the maximized log-likelihood function multiplied by minus two is reported in the first two columns. The next pair of columns has the average estimate of the degrees of freedom. The last three pairs of columns report the frequency that a particular regression model is in the $\widehat{\mathcal{M}}^*_{90\%}$ for each of the three criteria, KLIC, AIC* and BIC*.

Table A.7: Simulation Experiment II: $\beta^2 = 0.9$, fraction in MCS

	$Q(\mathcal{Z}_j, \hat{\theta}_j)$		$\hat{k}^*$		KLIC		AIC* (TIC)		BIC*	
Panel A: $n = 50$										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	47.9	48.1	2.00	1.99	0.000	0.000	0.000	0.000	0.000	0.000
X_0, X_1	-68.4	-68.3	3.03	3.03	0.999	0.999	1.000	1.000	1.000	1.000
$X_0, \ldots, X_2$	-69.5	-69.3	4.09	4.09	0.999	0.999	0.959	0.962	0.521	0.521
$X_0, \ldots, X_3$	-70.6	-70.4	5.19	5.19	0.999	0.999	0.939	0.941	0.405	0.406
$X_0, \ldots, X_4$	-71.7	-71.5	6.33	6.32	1.000	0.999	0.909	0.908	0.283	0.289
$X_0, \ldots, X_5$	-72.8	-72.7	7.51	7.51	1.000	1.000	0.858	0.864	0.190	0.202
$X_0, \ldots, X_6$	-74.0	-73.9	8.75	8.75	1.000	1.000	0.786	0.797	0.119	0.135
X_0, X_2	42.6	-18.5	3.03	3.02	0.000	0.005	0.000	0.007	0.000	0.009
X_0, X_2, X_3	39.2	-27.2	4.09	4.08	0.000	0.021	0.000	0.023	0.000	0.016
$X_0, X_2, \ldots, X_4$	36.5	-31.4	5.19	5.18	0.000	0.036	0.000	0.032	0.000	0.018
$X_0, X_2, \ldots, X_5$	34.3	-34.2	6.33	6.32	0.000	0.048	0.000	0.036	0.000	0.014
$X_0, X_2, \ldots, X_6$	32.3	-36.4	7.51	7.50	0.000	0.056	0.000	0.038	0.000	0.010
Panel B: $n = 100$										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	98.0	98.0	1.99	1.99	0.000	0.000	0.000	0.000	0.000	0.000
X_0, X_1	-134	-133	3.01	3.00	0.999	0.999	1.000	1.000	1.000	1.000
$X_0, \ldots, X_2$	-135	-134	4.03	4.02	1.000	0.999	0.958	0.958	0.400	0.400
$X_0, \ldots, X_3$	-136	-135	5.07	5.06	1.000	1.000	0.937	0.937	0.277	0.275
$X_0, \ldots, X_4$	-137	-137	6.12	6.11	1.000	1.000	0.903	0.904	0.176	0.166
$X_0, \ldots, X_5$	-138	-138	7.19	7.18	1.000	1.000	0.855	0.859	0.103	0.100
$X_0, \ldots, X_6$	-139	-139	8.28	8.27	1.000	1.000	0.796	0.796	0.057	0.053
X_0, X_2	88.5	-33.9	3.00	3.00	0.000	0.000	0.000	0.000	0.000	0.000
X_0, X_2, X_3	82.7	-50.2	4.02	4.03	0.000	0.000	0.000	0.000	0.000	0.000
$X_0, X_2, \ldots, X_4$	78.5	-57.4	5.06	5.06	0.000	0.000	0.000	0.000	0.000	0.000
$X_0, X_2, \ldots, X_5$	75.2	-61.8	6.11	6.12	0.000	0.000	0.000	0.000	0.000	0.000
$X_0, X_2, \ldots, X_6$	72.4	-64.8	7.18	7.19	0.000	0.000	0.000	0.000	0.000	0.000
Panel C: $n = 500$										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	499	499	2.00	2.00	0.000	0.000	0.000	0.000	0.000	0.000
X_0, X_1	-654	-654	3.00	3.00	0.999	0.999	1.000	1.000	1.000	1.000
$X_0, \ldots, X_2$	-656	-655	4.01	4.00	1.000	1.000	0.957	0.956	0.206	0.202
$X_0, \ldots, X_3$	-657	-656	5.01	5.01	1.000	1.000	0.937	0.936	0.097	0.093
$X_0, \ldots, X_4$	-658	-657	6.02	6.02	1.000	1.000	0.902	0.901	0.040	0.037
$X_0, \ldots, X_5$	-659	-658	7.03	7.03	1.000	1.000	0.858	0.852	0.019	0.015
$X_0, \ldots, X_6$	-660	-659	8.04	8.04	1.000	1.000	0.796	0.789	0.006	0.006
X_0, X_2	455	-156	3.00	3.00	0.000	0.000	0.000	0.000	0.000	0.000
X_0, X_2, X_3	430	-233	4.00	4.00	0.000	0.000	0.000	0.000	0.000	0.000
$X_0, X_2, \ldots, X_4$	413	-264	5.01	5.01	0.000	0.000	0.000	0.000	0.000	0.000
$X_0, X_2, \ldots, X_5$	401	-282	6.01	6.02	0.000	0.000	0.000	0.000	0.000	0.000
$X_0, X_2, \ldots, X_6$	392	-293	7.02	7.02	0.000	0.000	0.000	0.000	0.000	0.000

The average value of the maximized log-likelihood function multiplied by minus two is reported in the first two columns. The next pair of columns has the average estimate of the degrees of freedom. The last three pairs of columns report the frequency that a particular regression model is in the $\widehat{\mathcal{M}}^*_{90\%}$ for each of the three criteria, KLIC, AIC* and BIC*.

Table A.8: Simulation Experiment II: $\beta^2 = 0.1$, average MCS p-value

	$Q(\mathcal{Z}_j, \hat{\theta}_j)$		$\hat{k}^*$		KLIC		AIC* (TIC)		BIC*	
Panel A: $n = 50$										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	47.8	48.1	2.00	1.99	0.682	0.659	0.729	0.739	0.744	0.768
X_0, X_1	41.4	41.8	3.03	3.03	0.927	0.918	0.938	0.908	0.841	0.794
$X_0, \ldots, X_2$	40.3	40.8	4.09	4.09	0.905	0.921	0.784	0.820	0.504	0.515
$X_0, \ldots, X_3$	39.3	39.7	5.19	5.19	0.905	0.925	0.738	0.765	0.366	0.359
$X_0, \ldots, X_4$	38.2	38.5	6.33	6.33	0.907	0.928	0.690	0.705	0.258	0.242
$X_0, \ldots, X_5$	37.0	37.4	7.51	7.51	0.905	0.927	0.635	0.635	0.176	0.158
$X_0, \ldots, X_6$	35.8	36.2	8.75	8.74	0.903	0.919	0.575	0.560	0.117	0.099
X_0, X_2	46.3	42.8	3.03	3.03	0.702	0.880	0.676	0.848	0.474	0.696
X_0, X_2, X_3	45.0	41.5	4.09	4.09	0.711	0.893	0.631	0.791	0.334	0.486
$X_0, X_2, \ldots, X_4$	43.7	40.3	5.19	5.18	0.716	0.900	0.579	0.738	0.234	0.339
$X_0, X_2, \ldots, X_5$	42.5	39.2	6.33	6.32	0.717	0.901	0.522	0.675	0.161	0.228
$X_0, X_2, \ldots, X_6$	41.3	38.0	7.51	7.51	0.714	0.897	0.465	0.606	0.108	0.150
Panel B: $n = 100$										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	97.9	98.2	2.00	1.99	0.421	0.390	0.485	0.490	0.521	0.552
X_0, X_1	86.2	86.7	3.01	3.01	0.919	0.920	0.964	0.936	0.950	0.893
$X_0, \ldots, X_2$	85.2	85.6	4.03	4.03	0.883	0.916	0.732	0.814	0.450	0.490
$X_0, \ldots, X_3$	84.2	84.6	5.07	5.07	0.883	0.921	0.689	0.768	0.324	0.333
$X_0, \ldots, X_4$	83.1	83.5	6.12	6.12	0.882	0.925	0.650	0.720	0.224	0.215
$X_0, \ldots, X_5$	82.0	82.4	7.19	7.19	0.880	0.925	0.609	0.663	0.148	0.134
$X_0, \ldots, X_6$	80.9	81.4	8.28	8.27	0.879	0.921	0.566	0.603	0.094	0.080
X_0, X_2	95.9	88.7	3.01	3.00	0.451	0.824	0.471	0.802	0.345	0.674
X_0, X_2, X_3	94.4	87.2	4.03	4.03	0.467	0.849	0.444	0.756	0.227	0.447
$X_0, X_2, \ldots, X_4$	93.1	86.0	5.07	5.07	0.476	0.858	0.412	0.712	0.149	0.298
$X_0, X_2, \ldots, X_5$	91.8	84.9	6.12	6.12	0.483	0.861	0.378	0.660	0.095	0.190
$X_0, X_2, \ldots, X_6$	90.5	83.7	7.19	7.18	0.488	0.859	0.343	0.605	0.058	0.117
Panel C: $n = 500$										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	498	498	2.00	2.00	0.006	0.005	0.008	0.006	0.013	0.014
X_0, X_1	444	444	3.00	3.00	0.909	0.925	0.962	0.968	0.997	0.984
$X_0, \ldots, X_2$	443	443	4.01	4.01	0.868	0.883	0.646	0.706	0.114	0.264
$X_0, \ldots, X_3$	442	442	5.01	5.01	0.866	0.883	0.588	0.670	0.053	0.149
$X_0, \ldots, X_4$	441	441	6.02	6.02	0.866	0.882	0.535	0.635	0.031	0.080
$X_0, \ldots, X_5$	440	440	7.03	7.03	0.862	0.880	0.480	0.598	0.021	0.045
$X_0, \ldots, X_6$	439	439	8.04	8.04	0.858	0.880	0.427	0.559	0.015	0.026
X_0, X_2	493	454	3.00	3.00	0.009	0.410	0.011	0.427	0.011	0.284
X_0, X_2, X_3	489	451	4.00	4.00	0.011	0.490	0.012	0.455	0.008	0.168
$X_0, X_2, \ldots, X_4$	486	449	5.01	5.01	0.013	0.521	0.012	0.441	0.005	0.088
$X_0, X_2, \ldots, X_5$	484	448	6.02	6.02	0.014	0.537	0.013	0.417	0.003	0.046
$X_0, X_2, \ldots, X_6$	483	447	7.03	7.02	0.016	0.546	0.013	0.388	0.002	0.026

The average value of the maximized log-likelihood function multiplied by minus two is reported in the first two columns. The next pair of columns has the average estimate of the degrees of freedom. The last three pairs of columns report the average MCS p-value for each of the three criteria, KLIC, AIC* and BIC*.

Table A.9: Simulation Experiment II: $\beta^2 = 0.5$, average MCS p-value

	$Q(\mathcal{Z}_j, \hat{\theta}_j)$		$\hat{k}^*$		KLIC		AIC* (TIC)		BIC*	
Panel A: $n = 50$										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	48.1	48.1	1.99	2.00	0.027	0.021	0.035	0.031	0.043	0.046
X_0, X_1	12.4	12.4	3.02	3.02	0.897	0.918	0.955	0.958	0.983	0.971
$X_0, \ldots, X_2$	11.3	11.3	4.08	4.08	0.858	0.885	0.644	0.725	0.293	0.438
$X_0, \ldots, X_3$	10.2	10.2	5.18	5.18	0.857	0.888	0.584	0.690	0.204	0.345
$X_0, \ldots, X_4$	9.09	9.04	6.32	6.32	0.856	0.889	0.529	0.650	0.148	0.258
$X_0, \ldots, X_5$	7.95	7.88	7.50	7.50	0.852	0.888	0.470	0.605	0.108	0.189
$X_0, \ldots, X_6$	6.77	6.69	8.73	8.74	0.848	0.887	0.417	0.554	0.079	0.133
X_0, X_2	44.7	21.0	3.02	3.02	0.035	0.491	0.040	0.494	0.038	0.383
X_0, X_2, X_3	42.3	18.1	4.08	4.08	0.041	0.566	0.041	0.511	0.032	0.314
$X_0, X_2, \ldots, X_4$	40.4	16.3	5.18	5.18	0.045	0.594	0.040	0.491	0.024	0.241
$X_0, X_2, \ldots, X_5$	38.8	14.8	6.32	6.32	0.048	0.608	0.038	0.457	0.018	0.177
$X_0, X_2, \ldots, X_6$	37.2	13.4	7.50	7.51	0.051	0.615	0.037	0.418	0.013	0.125
Panel B: $n = 100$										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	98.0	98.1	1.99	1.99	0.001	0.000	0.001	0.001	0.001	0.001
X_0, X_1	27.6	27.8	3.00	3.00	0.898	0.914	0.957	0.964	0.990	0.990
$X_0, \ldots, X_2$	26.6	26.7	4.03	4.03	0.862	0.873	0.652	0.667	0.213	0.290
$X_0, \ldots, X_3$	25.5	25.7	5.07	5.06	0.862	0.873	0.592	0.618	0.130	0.229
$X_0, \ldots, X_4$	24.4	24.6	6.12	6.12	0.863	0.870	0.539	0.574	0.081	0.174
$X_0, \ldots, X_5$	23.4	23.6	7.19	7.18	0.860	0.868	0.482	0.535	0.051	0.126
$X_0, \ldots, X_6$	22.3	22.5	8.28	8.27	0.858	0.865	0.429	0.496	0.031	0.086
X_0, X_2	92.4	45.1	3.00	3.01	0.001	0.207	0.001	0.230	0.001	0.183
X_0, X_2, X_3	88.8	40.4	4.03	4.03	0.001	0.286	0.001	0.276	0.001	0.164
$X_0, X_2, \ldots, X_4$	86.1	38.1	5.07	5.07	0.002	0.319	0.001	0.280	0.001	0.124
$X_0, X_2, \ldots, X_5$	83.9	36.3	6.12	6.12	0.002	0.338	0.002	0.271	0.001	0.087
$X_0, X_2, \ldots, X_6$	82.0	34.8	7.19	7.19	0.002	0.351	0.002	0.256	0.000	0.056
Panel C: $n = 500$										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	498	498	2.00	2.00	0.000	0.000	0.000	0.000	0.000	0.000
X_0, X_1	151	151	3.00	3.00	0.903	0.905	0.960	0.962	0.997	0.997
$X_0, \ldots, X_2$	150	150	4.00	4.00	0.864	0.864	0.644	0.648	0.112	0.110
$X_0, \ldots, X_3$	149	149	5.01	5.01	0.864	0.865	0.587	0.592	0.050	0.050
$X_0, \ldots, X_4$	148	148	6.02	6.01	0.864	0.864	0.533	0.536	0.023	0.022
$X_0, \ldots, X_5$	147	147	7.03	7.02	0.860	0.859	0.480	0.482	0.011	0.011
$X_0, \ldots, X_6$	145	146	8.04	8.03	0.858	0.855	0.430	0.432	0.005	0.006
X_0, X_2	474	238	3.00	3.00	0.000	0.000	0.000	0.000	0.000	0.001
X_0, X_2, X_3	460	219	4.00	4.00	0.000	0.002	0.000	0.002	0.000	0.002
$X_0, X_2, \ldots, X_4$	451	211	5.01	5.01	0.000	0.004	0.000	0.004	0.000	0.002
$X_0, X_2, \ldots, X_5$	444	206	6.02	6.01	0.000	0.005	0.000	0.005	0.000	0.001
$X_0, X_2, \ldots, X_6$	439	203	7.03	7.02	0.000	0.006	0.000	0.005	0.000	0.001

The average value of the maximized log-likelihood function multiplied by minus two is reported in the first two columns. The next pair of columns has the average estimate of the degrees of freedom. The last three pairs of columns report the average MCS p-value for each of the three criteria, KLIC, AIC* and BIC*.

Table A.10: Simulation Experiment II: $\beta^2 = 0.9$, average MCS p-value

	$Q(\mathcal{Z}_j, \hat{\theta}_j)$		$\hat{k}^*$		KLIC		AIC* (TIC)		BIC*	
Panel A: n = 50										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	47.9	48.1	2.00	1.99	0.000	0.000	0.000	0.000	0.000	0.000
X_0, X_1	-68.4	-68.3	3.03	3.03	0.902	0.899	0.957	0.955	0.984	0.983
$X_0, \ldots, X_2$	-69.5	-69.3	4.09	4.09	0.862	0.861	0.644	0.647	0.279	0.285
$X_0, \ldots, X_3$	-70.6	-70.4	5.19	5.19	0.860	0.860	0.582	0.584	0.188	0.191
$X_0, \ldots, X_4$	-71.7	-71.5	6.33	6.32	0.860	0.859	0.524	0.526	0.127	0.131
$X_0, \ldots, X_5$	-72.8	-72.7	7.51	7.51	0.855	0.856	0.465	0.469	0.085	0.090
$X_0, \ldots, X_6$	-74.0	-73.9	8.75	8.75	0.851	0.850	0.411	0.413	0.057	0.061
X_0, X_2	42.6	-18.5	3.03	3.02	0.000	0.005	0.000	0.006	0.000	0.007
X_0, X_2, X_3	39.2	-27.2	4.09	4.08	0.000	0.013	0.000	0.013	0.000	0.010
$X_0, X_2, \ldots, X_4$	36.5	-31.4	5.19	5.18	0.000	0.018	0.000	0.016	0.000	0.010
$X_0, X_2, \ldots, X_5$	34.3	-34.2	6.33	6.32	0.000	0.021	0.000	0.017	0.000	0.009
$X_0, X_2, \ldots, X_6$	32.3	-36.4	7.51	7.50	0.000	0.024	0.000	0.018	0.000	0.007
Panel B: n = 100										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	98.0	98.0	1.99	1.99	0.000	0.000	0.000	0.000	0.000	0.000
X_0, X_1	-134	-133	3.01	3.00	0.902	0.900	0.960	0.958	0.991	0.990
$X_0, \ldots, X_2$	-135	-134	4.03	4.02	0.861	0.863	0.644	0.654	0.211	0.214
$X_0, \ldots, X_3$	-136	-135	5.07	5.06	0.861	0.862	0.586	0.592	0.127	0.128
$X_0, \ldots, X_4$	-137	-137	6.12	6.11	0.861	0.862	0.531	0.536	0.079	0.079
$X_0, \ldots, X_5$	-138	-138	7.19	7.18	0.857	0.858	0.477	0.480	0.048	0.047
$X_0, \ldots, X_6$	-139	-139	8.28	8.27	0.855	0.854	0.426	0.428	0.028	0.028
X_0, X_2	88.5	-33.9	3.00	3.00	0.000	0.000	0.000	0.000	0.000	0.000
X_0, X_2, X_3	82.7	-50.2	4.02	4.03	0.000	0.000	0.000	0.000	0.000	0.000
$X_0, X_2, \ldots, X_4$	78.5	-57.4	5.06	5.06	0.000	0.000	0.000	0.000	0.000	0.000
$X_0, X_2, \ldots, X_5$	75.2	-61.8	6.11	6.12	0.000	0.000	0.000	0.000	0.000	0.000
$X_0, X_2, \ldots, X_6$	72.4	-64.8	7.18	7.19	0.000	0.001	0.000	0.000	0.000	0.000
Panel C: n = 500										
$\rho =$	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9	0.3	0.9
X_0	499	499	2.00	2.00	0.000	0.000	0.000	0.000	0.000	0.000
X_0, X_1	-654	-654	3.00	3.00	0.908	0.910	0.962	0.963	0.997	0.997
$X_0, \ldots, X_2$	-656	-655	4.01	4.00	0.868	0.868	0.646	0.646	0.111	0.110
$X_0, \ldots, X_3$	-657	-656	5.01	5.01	0.866	0.866	0.588	0.587	0.047	0.047
$X_0, \ldots, X_4$	-658	-657	6.02	6.02	0.866	0.864	0.535	0.532	0.022	0.021
$X_0, \ldots, X_5$	-659	-658	7.03	7.03	0.862	0.860	0.480	0.478	0.011	0.010
$X_0, \ldots, X_6$	-660	-659	8.04	8.04	0.858	0.854	0.427	0.426	0.005	0.005
X_0, X_2	455	-156	3.00	3.00	0.000	0.000	0.000	0.000	0.000	0.000
X_0, X_2, X_3	430	-233	4.00	4.00	0.000	0.000	0.000	0.000	0.000	0.000
$X_0, X_2, \ldots, X_4$	413	-264	5.01	5.01	0.000	0.000	0.000	0.000	0.000	0.000
$X_0, X_2, \ldots, X_5$	401	-282	6.01	6.02	0.000	0.000	0.000	0.000	0.000	0.000
$X_0, X_2, \ldots, X_6$	392	-293	7.02	7.02	0.000	0.000	0.000	0.000	0.000	0.000

The average value of the maximized log-likelihood function multiplied by minus two is reported in the first two columns. The next pair of columns has the average estimate of the degrees of freedom. The last three pairs of columns report the average MCS p-value for each of the three criteria, KLIC, AIC* and BIC*.

Table A.11: MCS for Taylor Rules: 1979:Q1 to 2006:Q4

Model Specification			$Q(\mathcal{Z}_j, \hat{\theta}_j)$	$\hat{k}^{\star}$	KLIC	AIC*	BIC*
R_{t-1}			93.15	13.74	106.89 (0.30)**	120.63 (0.47)**	157.99 (0.63)**
	π_{t-1}	y_{t-1}	284.82	11.44	296.25 (0.00)	307.69 (0.00)	338.79 (0.00)
	$\pi_{t-j},\ _{j=1,2}$	$y_{t-j},\ _{j=1,2}$	258.95	14.66	273.61 (0.00)	288.28 (0.01)	328.14 (0.01)
	π_{t-1}	ur_{t-1}	289.65	10.20	299.84 (0.00)	310.04 (0.00)	337.75 (0.00)
	$\pi_{t-j},\ _{j=1,2}$	$ur_{t-j},\ _{j=1,2}$	268.90	12.82	281.72 (0.00)	294.53 (0.00)	329.37 (0.01)
	π_{t-1}	$rulc_{t-1}$	289.99	9.89	299.88 (0.00)	309.77 (0.00)	336.67 (0.01)
	$\pi_{t-j},\ _{j=1,2}$	$rulc_{t-j},\ _{j=1,2}$	266.07	12.12	278.19 (0.00)	290.31 (0.01)	323.26 (0.01)
	y_{t-1}	ur_{t-1}	387.45	17.04	404.49 (0.00)	421.54 (0.00)	467.86 (0.00)
	$y_{t-j},\ _{j=1,2}$	$ur_{t-j},\ _{j=1,2}$	385.86	23.42	409.28 (0.00)	432.69 (0.00)	496.35 (0.00)
	y_{t-1}	$rulc_{t-1}$	386.47	14.92	401.39 (0.00)	416.32 (0.00)	456.89 (0.00)
	$y_{t-j},\ _{j=1,2}$	$rulc_{t-j},\ _{j=1,2}$	385.43	19.44	404.87 (0.00)	424.31 (0.00)	477.16 (0.00)
	ur_{t-1}	$rulc_{t-1}$	386.21	15.41	401.62 (0.00)	417.02 (0.00)	458.90 (0.00)
	$ur_{t-j},\ _{j=1,2}$	$rulc_{t-j},\ _{j=1,2}$	384.82	19.86	404.68 (0.00)	424.54 (0.00)	478.52 (0.00)
R_{t-1}	π_{t-1}	y_{t-1}	68.57	17.71	86.28 (0.86)**	103.98 (1.00)**	152.12 (0.64)**
R_{t-1}	$\pi_{t-j},\ _{j=1,2}$	$y_{t-j},\ _{j=1,2}$	62.11	22.11	84.22 (1.00)**	106.32 (0.93)**	166.43 (0.41)**
R_{t-1}	π_{t-1}	ur_{t-1}	77.57	16.32	93.89 (0.72)**	110.22 (0.89)**	154.60 (0.64)**
R_{t-1}	$\pi_{t-j},\ _{j=1,2}$	$ur_{t-j},\ _{j=1,2}$	73.27	18.79	92.07 (0.80)**	110.86 (0.89)**	161.95 (0.57)**
R_{t-1}	π_{t-1}	$rulc_{t-1}$	72.80	16.06	88.86 (0.86)**	104.92 (0.93)**	148.58 (1.00)**
R_{t-1}	$\pi_{t-j},\ _{j=1,2}$	$rulc_{t-j},\ _{j=1,2}$	69.21	19.26	88.47 (0.86)**	107.73 (0.92)**	160.09 (0.58)**
R_{t-1}	y_{t-1}	ur_{t-1}	86.16	19.16	105.33 (0.33)**	124.49 (0.38)**	176.59 (0.16)*
R_{t-1}	$y_{t-j},\ _{j=1,2}$	$ur_{t-j},\ _{j=1,2}$	85.51	24.32	109.83 (0.28)**	134.16 (0.18)*	200.28 (0.02)
R_{t-1}	y_{t-1}	$rulc_{t-1}$	89.42	18.92	108.35 (0.29)**	127.27 (0.31)**	178.72 (0.15)*
R_{t-1}	$y_{t-j},\ _{j=1,2}$	$rulc_{t-j},\ _{j=1,2}$	88.11	22.42	110.53 (0.28)**	132.94 (0.20)*	193.88 (0.03)
R_{t-1}	ur_{t-1}	$rulc_{t-1}$	87.42	18.07	105.49 (0.33)**	123.55 (0.38)**	172.66 (0.21)*
R_{t-1}	$ur_{t-j},\ _{j=1,2}$	$rulc_{t-j},\ _{j=1,2}$	85.93	21.32	107.25 (0.30)**	128.56 (0.28)**	186.51 (0.06)

We report the maximized log-likelihood function (multiplied by minus two), the effective degress of freedom, and the three criteria, KLIC, AIC* and BIC*, along with the corresponding MCS p-values. The regression models in $\widehat{\mathcal{M}}^{*}_{90\%}$ and $\widehat{\mathcal{M}}^{*}_{75\%}$ are identified by one and two asterisks, respectively. See the text and Table 6 for variable mnemonics and definitions.

Table A.12: MCS for Taylor Rules: 1984:Q1 to 2006:Q4

Model Specification			$Q(\mathcal{Z}_j, \hat{\theta}_j)$	$\hat{k}^\star$	KLIC	AIC*	BIC*
R_{t-1}			-38.90	6.97	-31.93 (0.56)**	-24.96 (0.93)**	-7.39 (1.00)**
	π_{t-1}	y_{t-1}	208.37	11.57	219.93 (0.00)	231.50 (0.00)	260.68 (0.00)
	$\pi_{t-j},\ _{j=1,2}$	$y_{t-j},\ _{j=1,2}$	190.30	13.76	204.06 (0.00)	217.83 (0.00)	252.54 (0.00)
	π_{t-1}	ur_{t-1}	227.79	11.91	239.70 (0.00)	251.61 (0.00)	281.63 (0.00)
	$\pi_{t-j},\ _{j=1,2}$	$ur_{t-j},\ _{j=1,2}$	220.78	14.10	234.88 (0.00)	248.97 (0.00)	284.52 (0.00)
	π_{t-1}	$rulc_{t-1}$	228.07	10.34	238.41 (0.00)	248.75 (0.00)	274.83 (0.00)
	$\pi_{t-j},\ _{j=1,2}$	$rulc_{t-j},\ _{j=1,2}$	213.53	12.06	225.59 (0.00)	237.65 (0.00)	268.07 (0.00)
	y_{t-1}	ur_{t-1}	226.30	12.82	239.12 (0.00)	251.93 (0.00)	284.25 (0.00)
	$y_{t-j},\ _{j=1,2}$	$ur_{t-j},\ _{j=1,2}$	216.60	16.04	232.65 (0.00)	248.69 (0.00)	289.15 (0.00)
	y_{t-1}	$rulc_{t-1}$	225.63	12.38	238.01 (0.00)	250.39 (0.00)	281.62 (0.00)
	$y_{t-j},\ _{j=1,2}$	$rulc_{t-j},\ _{j=1,2}$	216.68	14.39	231.07 (0.00)	245.46 (0.00)	281.76 (0.00)
	ur_{t-1}	$rulc_{t-1}$	238.39	12.46	250.85 (0.00)	263.31 (0.00)	294.74 (0.00)
	$ur_{t-j},\ _{j=1,2}$	$rulc_{t-j},\ _{j=1,2}$	233.41	14.90	248.31 (0.00)	263.21 (0.00)	300.78 (0.00)
R_{t-1}	π_{t-1}	y_{t-1}	-66.21	14.74	-51.47 (0.78)**	-36.73 (0.96)**	0.44 (0.92)**
R_{t-1}	$\pi_{t-j},\ _{j=1,2}$	$y_{t-j},\ _{j=1,2}$	-70.85	16.93	-53.92 (1.00)**	-37.00 (1.00)**	5.69 (0.86)**
R_{t-1}	π_{t-1}	ur_{t-1}	-45.39	9.63	-35.76 (0.64)**	-26.13 (0.93)**	-1.84 (0.92)**
R_{t-1}	$\pi_{t-j},\ _{j=1,2}$	$ur_{t-j},\ _{j=1,2}$	-45.55	13.23	-32.31 (0.56)**	-19.08 (0.74)**	14.30 (0.57)**
R_{t-1}	π_{t-1}	$rulc_{t-1}$	-51.84	11.31	-40.53 (0.72)**	-29.22 (0.93)**	-0.71 (0.92)**
R_{t-1}	$\pi_{t-j},\ _{j=1,2}$	$rulc_{t-j},\ _{j=1,2}$	-53.68	12.74	-40.94 (0.72)**	-28.21 (0.93)**	3.91 (0.88)**
R_{t-1}	y_{t-1}	ur_{t-1}	-58.05	13.50	-44.55 (0.72)**	-31.05 (0.93)**	2.99 (0.89)**
R_{t-1}	$y_{t-j},\ _{j=1,2}$	$ur_{t-j},\ _{j=1,2}$	-62.18	16.65	-45.53 (0.72)**	-28.88 (0.93)**	13.12 (0.60)**
R_{t-1}	y_{t-1}	$rulc_{t-1}$	-58.72	14.20	-44.52 (0.72)**	-30.32 (0.93)**	5.50 (0.86)**
R_{t-1}	$y_{t-j},\ _{j=1,2}$	$rulc_{t-j},\ _{j=1,2}$	-64.74	15.81	-48.94 (0.78)**	-33.13 (0.93)**	6.74 (0.86)**
R_{t-1}	ur_{t-1}	$rulc_{t-1}$	-50.00	12.00	-37.99 (0.64)**	-25.99 (0.93)**	4.28 (0.87)**
R_{t-1}	$ur_{t-j},\ _{j=1,2}$	$rulc_{t-j},\ _{j=1,2}$	-50.96	15.73	-35.22 (0.62)**	-19.49 (0.74)**	20.19 (0.35)**

We report the maximized log-likelihood function (multiplied by minus two), the effective degress of freedom, and the three criteria, KLIC, AIC* and BIC*, along with the corresponding MCS p-values. The regression models in $\widehat{\mathcal{M}}^*_{90\%}$ and $\widehat{\mathcal{M}}^*_{75\%}$ are identified by one and two asterisks, respectively. See the text and Table 6 for variable mnemonics and definitions.