Econometrica Supplementary Material

SUPPLEMENT TO "TESTING FOR SMOOTH STRUCTURAL CHANGES IN TIME SERIES MODELS VIA NONPARAMETRIC REGRESSION"

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MATHEMATICAL APPENDIX

THROUGHOUT THIS APPENDIX, $C \in (1, \infty)$ denotes a generic bounded constant.

PROOF OF THEOREM 1: First we decompose $Th^{1/2}\hat{Q}$:

$$\begin{aligned} Th^{1/2}\hat{Q} &= h^{1/2}\sum_{t=1}^{T}(\mathbf{X}_t'\hat{\alpha}_t - \mathbf{X}_t'\alpha)^2 + h^{1/2}\sum_{t=1}^{T}(\mathbf{X}_t'\hat{\alpha} - \mathbf{X}_t'\alpha)^2 \\ &\quad + 2h^{1/2}\sum_{t=1}^{T}(\mathbf{X}_t'\hat{\alpha}_t - \mathbf{X}_t'\alpha)(\mathbf{X}_t'\alpha - \mathbf{X}_t'\hat{\alpha}) \\ &= \hat{J}_1 + \hat{J}_2 + \hat{J}_3. \end{aligned}$$

The Theorem 1 proof consists of the proofs of Theorem A.1–A.3 below, which show that the nonparametric estimation determines the asymptotic mean and variance of $Th^{1/2}\hat{Q}$, and the estimation uncertainty of the parametric estimation is asymptotically negligible. *Q.E.D.*

THEOREM A.1: *Under the conditions of Theorem* 1, $\hat{H}_1 \equiv (\hat{J}_1 - \hat{A}_H)/\sqrt{\hat{B}_H} \xrightarrow{d} N(0,1)$.

THEOREM A.2: *Under the conditions of Theorem* 1, $\hat{J}_2 \xrightarrow{p} 0$.

THEOREM A.3: *Under the conditions of Theorem* 1, $\hat{J}_3 \xrightarrow{p} 0$.

PROOF OF THEOREM A.1: To show $\hat{H}_1 \xrightarrow{d} N(0,1)$, it suffices to show the following three propositions.

PROPOSITION A.1: *Under the conditions of Theorem* 1, $T^{-1}h^{-1} \times \sum_{s=1}^{T} k_{st}(\frac{s-t}{Th})^j \mathbf{X}_s\mathbf{X}_s' \xrightarrow{p} \mathbf{M}\int_{-1}^{1} u^j k(u)\,du$.

PROPOSITION A.2: *Under the conditions of Theorem* 1,

$$\hat{J}_1 = \hat{A}_H + 2\tilde{U} + o_P(1),$$

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where

$$\tilde{U} = T^{-1}h^{-1/2}\sum_{s=2}^{T}\sum_{t=1}^{s-1}\varepsilon_s\varepsilon_t\mathbf{X}_t'\mathbf{M}^{-1}\mathbf{X}_s w_{ts}$$

and $w_{ts} = \int_{-1}^{1} k(u)k(u+\frac{r-s}{Th})\,du$.

PROPOSITION A.3: *Under the conditions of Theorem* 1, $2\tilde{U}/\sqrt{\hat{B}_H} \xrightarrow{d} N(0,1)$.

PROOF OF PROPOSITION A.1: The proof is similar to the proof of (15.8) of Robinson (1989). Let X_{ps} be the pth element of $\mathbf{X}_s$ and let M_{pr} be the (p,r)th element of $\mathbf{M}$. For any p, r, we have

$$\begin{aligned}
&E\left|T^{-1}h^{-1}\sum_{s=1}^{T}k\left(\frac{s-t}{Th}\right)\left(\frac{s-t}{Th}\right)^{j}(X_{ps}X_{rs}-M_{pr})\right| \\
&\quad\le T^{-1}h^{-1}\left\{E\left[\sum_{|s-t|\le Th}k\left(\frac{s-t}{Th}\right)\left(\frac{s-t}{Th}\right)^{j}(X_{ps}X_{rs}-M_{pr})\right]^{2}\right\}^{1/2} \\
&\quad\le T^{-1}h^{-1}\left[\sum_{|s-t|\le Th}k^{2}\left(\frac{s-t}{Th}\right)\left(\frac{s-t}{Th}\right)^{2j}E(X_{ps}^{2}X_{rs}^{2})\right. \\
&\qquad+\sum_{|s-t|\le Th}\ \sum_{|s'-t|\le Th, s\ne s'}\left|k\left(\frac{s-t}{Th}\right)k\left(\frac{s'-t}{Th}\right)\left(\frac{s-t}{Th}\right)^{j}\left(\frac{s'-t}{Th}\right)^{j}\right| \\
&\qquad\left.\times\left(E|X_{ps}X_{rs}|^{2+\delta/2}\right)^{4/(4+\delta)}\beta(|s-s'|)^{\delta/(4+\delta)}\right]^{1/2} \\
&\quad\le C\left\{(Th)^{-1/2}+T^{-1}h^{-1}\left[Th\sum_{j\le Th}\beta(j)^{\delta/(4+\delta)}\right]^{1/2}\right\}=o(1).
\end{aligned}$$

Proposition A.1 follows by the Riemann sum approximation of an integral. *Q.E.D.*

PROOF OF PROPOSITION A.2: We first decompose

$$\begin{aligned}
\text{(A1)}\qquad \hat{J}_1 &= T^{-2}h^{-3/2}\sum_{t=1}^{T}\sum_{s=1}^{T}\varepsilon_t^2\mathbf{Q}_{tst}k_{st}^2 + 2T^{-2}h^{-3/2}\sum_{t=1}^{Th}\sum_{s=-T}^{T}\varepsilon_t^2\mathbf{Q}_{tst}k_{ts}k\left(\frac{t+s}{Th}\right) \\
&\quad + T^{-2}h^{-3/2}\sum_{t\ne s\ne r}\varepsilon_t\varepsilon_s\mathbf{Q}_{srt}k_{rs}k_{rt} + 2T^{-2}h^{-3/2}\sum_{t\ne s}\varepsilon_t\varepsilon_s\mathbf{Q}_{stt}k_{st}k(0)
\end{aligned}$$

$$+ 2T^{-2}h^{-3/2}\sum_{s=-T}^{-1}\sum_{t=-T,t\neq\pm s}^{2T}\sum_{r=1}^{T}\varepsilon_t\varepsilon_s\mathbf{Q}_{srt}k_{rs}k_{rt} + o_P(1)$$

$$= C_1 + C_2 + U_1 + R_1 + R_2 + o_P(1),$$

where $\mathbf{Q}_{srt} = \mathbf{X}_s'\mathbf{M}^{-1}\mathbf{X}_r\mathbf{X}_r'\mathbf{M}^{-1}\mathbf{X}_t$. We show that the first two terms determine the asymptotic mean, the third term determines the asymptotic variance, and the remainders are higher order terms.

We further decompose the first term as

$$\text{(A2)} \qquad C_1 = T^{-1}h^{-3/2}\sum_{j=1-T}^{T-1}(1-|j|/T)k^2\left(\frac{j}{Th}\right)C(j)$$
$$+ \left[T^{-2}h^{-3/2}\sum_{t=1}^{T}\sum_{s=1}^{T}\varepsilon_t^2\mathbf{Q}_{tst}k_{st}^2 - T^{-1}h^{-3/2}\sum_{j=1-T}^{T-1}(1-|j|/T)k^2\left(\frac{j}{Th}\right)C(j)\right]$$
$$= C_{11} + R_3,$$

where $C(j) = E(\varepsilon_{t-|j|}^2\mathbf{Q}_{t-|j|,t,t-|j|}) = E(\varepsilon_t^2\mathbf{X}_t'\mathbf{M}^{-1}\mathbf{X}_t) + E[\varepsilon_{t-|j|}^2\mathbf{X}_{t-|j|}'\mathbf{M}^{-1}(\mathbf{X}_t\mathbf{X}_t' - \mathbf{M})\mathbf{M}^{-1}\mathbf{X}_{t-|j|}]$. Similarly, we rewrite the second terms as

$$\text{(A3)} \qquad C_2 = T^{-1}h^{-1/2}\sum_{j=1-T}^{T-1}(1-|j|/T)C(j)k\left(\frac{j}{Th}\right)\int_{-1}^{1}k\left(\frac{j}{Th}+2u\right)du$$
$$+ \left\{2T^{-2}h^{-3/2}\sum_{t=1}^{Th}\sum_{s=-T}^{T}\varepsilon_t^2\mathbf{Q}_{tst}k_{ts}k\left(\frac{t+s}{Th}\right) - T^{-1}h^{-1/2}\sum_{j=1-T}^{T-1}(1-|j|/T)C(j)k\left(\frac{j}{Th}\right)\int_{-1}^{1}k\left(\frac{j}{Th}+2u\right)du\right\}$$
$$= C_{21} + R_4.$$

We note that $C_{11} + C_{21} = h^{-1/2}C_A E(\varepsilon_t^2\mathbf{X}_t'\mathbf{M}^{-1}\mathbf{X}_t)[1 + o_P(1)]$, where we have used the mixing inequality for the β-*mixing* process.

Next, we decompose U_1. Define

$$\phi(\xi_s, \xi_r, \xi_t) \equiv \varepsilon_t\varepsilon_s\mathbf{Q}_{srt}k_{rs}k_{rt} + \varepsilon_s\varepsilon_r\mathbf{Q}_{rts}k_{tr}k_{ts} + \varepsilon_r\varepsilon_t\mathbf{Q}_{tsr}k_{st}k_{sr},$$

$$\phi_t(\xi_s, \xi_r) \equiv \int\phi(\xi_s, \xi_r, \xi_t)\,dP(\xi_t) = \varepsilon_s\varepsilon_r\mathbf{X}_r'\mathbf{M}^{-1}\mathbf{X}_s k_{tr}k_{ts},$$

where $\xi_s = (\varepsilon_s, \mathbf{X}_s')$. We can rewrite U_1 as

$$U_1 = \frac{1}{3}T^{-2}h^{-3/2}\sum_{t\neq s\neq r}\phi_{srt} - 2T^{-2}h^{-3/2}\sum_{s\neq t}\varepsilon_s\varepsilon_t\mathbf{X}_t'\mathbf{M}\mathbf{X}_s k(0)k_{st} \\ + T^{-2}h^{-3/2}\sum_{s\neq t}\varepsilon_s\varepsilon_t\mathbf{X}_t'\mathbf{M}^{-1}\mathbf{X}_s\left(\frac{1}{Th}\sum_{r=1}^{T}k_{rt}k_{rs} - w_{ts}\right) \\ + T^{-2}h^{-3/2}\sum_{s\neq t}\varepsilon_s\varepsilon_t\mathbf{X}_t'\mathbf{M}^{-1}\mathbf{X}_s w_{ts}, \\ = R_5 - R_6 + R_7 + 2\tilde{U}, \tag{A4}$$

where $\phi_{srt} = [\phi(\xi_s, \xi_r, \xi_t) - \phi_t(\xi_s, \xi_r) - \phi_s(\xi_r, \xi_t) - \phi_r(\xi_s, \xi_t)]$. Proposition A.2 follows from the following lemma.

LEMMA A.1: *Let R_i be defined as in* (A1)–(A4), *where $i = 1 - 7$. Then $R_i = o_P(1)$.*

PROOF: The proofs of $R_i = o_P(1)$ are tedious. To save space, we only provide the proof for $i = 5$, which is the most involved. Other proofs are straightforward and available on request. We note that

$$ER_5^2 = CT^{-4}h^{-3}\sum_{1\le i_1<i_2<i_3\le T}\ \sum_{1\le i_4<i_5<i_6\le T} E\phi_{i_1i_2i_3}\phi_{i_4i_5i_6}.$$

First we consider the case where all indices are different from each other. Given the order of i_s, there are 20 different combinations. We consider the case where $1 \le i_1 < i_2 < i_3 < i_4 < i_5 < i_6 \le T$; other cases are similar. Let d_c be the cth largest difference among the adjacent indices. We have

$$\sum_{\substack{1\le i_1<i_2<i_3<i_4<i_5<i_6\le T\\ i_2-i_1=d_1}} \left|E\phi_{i_1i_2i_3}\phi_{i_4i_5i_6}\right| \\ \le \sum_{\substack{1\le i_1<i_2<i_3\le i_4<i_5<i_6\le T\\ i_2-i_1=d_1}} C\beta^{\delta/(1+\delta)}(d_1)W_{i_1\cdots i_6} \\ \le CT\sum_{j=1}^{T} j^4\beta^{\delta/(1+\delta)}(j),$$

where $W_{i_1\ldots i_6} = k_{i_1i_2}k_{i_1i_3}k_{i_4i_5}k_{i_4i_6} + k_{i_1i_2}k_{i_1i_3}k_{i_6i_5}k_{i_6i_4} + k_{i_1i_2}k_{i_1i_3}k_{i_5i_6}k_{i_5i_4} + k_{i_3i_1}k_{i_3i_2}k_{i_4i_5}k_{i_4i_6} + k_{i_3i_1}k_{i_3i_2}k_{i_6i_5}k_{i_6i_4} + k_{i_3i_1}k_{i_3i_2}k_{i_5i_6}k_{i_5i_4} + k_{i_2i_3}k_{i_2i_1}k_{i_4i_5}k_{i_4i_6} +$

$k_{i_2i_3}k_{i_2i_1}k_{i_6i_5}k_{i_6i_4} + k_{i_2i_3}k_{i_2i_1}k_{i_5i_6}k_{i_5i_4i_4i_6}$ and we have used the mixing inequality for the *β-mixing* process, Assumptions 1, 2, 4 and 5.

Similarly,

$$\sum_{\substack{1\le i_1<i_2<i_3<i_4<i_5<i_6\le T\\ i_6-i_5=d_1}} \left|E\phi_{i_1i_2i_3}\phi_{i_4i_5i_6}\right| \le CT\sum_{j=1}^{T} j^4\beta^{\delta/(1+\delta)}(j),$$

$$\sum_{\substack{1\le i_1<i_2<i_3<i_4<i_5<i_6\le T\\ i_2-i_1=d_2 \text{ or } i_6-i_5=d_2}} \left|E\phi_{i_1i_2i_3}\phi_{i_4i_5i_6}\right| \le CT^2h\sum_{j=1}^{T} j^3\beta^{\delta/(1+\delta)}(j),$$

$$\sum_{\substack{1\le i_1<i_2<i_3<i_4<i_5<i_6\le T\\ i_2-i_1=d_3 \text{ or } i_6-i_5=d_3}} \left|E\phi_{i_1i_2i_3}\phi_{i_4i_5i_6}\right| \le CT^3h^2\sum_{j=1}^{T} j^2\beta^{\delta/(1+\delta)}(j),$$

and

$$\begin{aligned}
&\sum_{\substack{1\le i_1<i_2<i_3<i_4<i_5<i_6\le T\\ \{i_2-i_1,i_6-i_5\}=\{d_4,d_5\}}} \left|E\phi_{i_1i_2i_3}\phi_{i_4i_5i_6}\right| \\
&\quad\le \sum_{\substack{1\le i_1<i_2<i_3<i_4<i_5<i_6\le T\\ \{i_2-i_1,i_6-i_5\}=\{d_4,d_5\}}} C\big[\beta^{\delta/(1+\delta)}(i_3-i_2)+\beta^{\delta/(1+\delta)}(i_4-i_3) \\
&\qquad\quad + \beta^{\delta/(1+\delta)}(i_5-i_4)\big] \\
&\quad\le CT^3h^2\sum_{j=1}^{T} j^2\beta^{\delta/(1+\delta)}(j).
\end{aligned}$$

For the cases where indices are not distinct from each other, we have

$$\sum_{\substack{1\le s,t,r,i,j\le T\\ s,t,r,i,j \text{ different}}} |E\phi_{str}\phi_{sij}| \le CT^3h^2\sum_{j=1}^{T} j\beta^{\delta/(1+\delta)}(j),$$

$$\sum_{\substack{1\le s,t,r,i\le T\\ s,t,r,i \text{ different}}} |E\phi_{str}\phi_{sti}| \le CT^3h^2\sum_{j=1}^{T} \beta^{\delta/(1+\delta)}(j),$$

and $\sum_{1\le s<t<r\le T}|E\phi_{str}^2| = O(T^3h^2)$. Then $R_5 = o_P(1)$ follows from Chebyshev inequality. *Q.E.D.*

This completes the proof of Proposition A.2. *Q.E.D.*

PROOF OF PROPOSITION A.3: Let

$$R_s = T^{-1}h^{-1/2}\sum_{r=1}^{s-1}\varepsilon_s\varepsilon_r\mathbf{X}_r'M^{-1}\mathbf{X}_s w_{rs} = T^{-1}h^{-1/2}\sum_{r=1}^{s-1}\varphi_{rs}w_{rs},$$

where $\varphi_{rs} = \varepsilon_s\varepsilon_r\mathbf{X}_r'\mathbf{M}^{-1}\mathbf{X}_s$. We apply Brown's (1971) martingale limit theorem, which states $\mathrm{var}(2\tilde{U})^{-1/2}2\tilde{U} \xrightarrow{d} N(0,1)$ if

$$\text{(A5)} \qquad \mathrm{var}(2\tilde{U})^{-1}\sum_{s=1}^{T}(2R_s)^2\mathbf{1}\big[|2R_s| > \eta\cdot\mathrm{var}(2\tilde{U})^{1/2}\big] \to 0 \quad \forall\eta > 0,$$

$$\text{(A6)} \qquad \mathrm{var}(2\tilde{U})^{-1}\sum_{s=1}^{T}E[(2R_s)^2|\mathcal{F}_{s-1}] \xrightarrow{p} 1.$$

First, we compute the variance

$$\begin{aligned}\text{(A7)} \qquad \mathrm{var}(2\tilde{U}) &= 4T^{-2}h^{-1}\sum_{s=1}^{T}\sum_{r=1}^{s-1}E(\varepsilon_r^2\varepsilon_s^2\mathbf{X}_r'\mathbf{M}^{-1}\mathbf{X}_s\mathbf{X}_s'\mathbf{M}^{-1}\mathbf{X}_r w_{rs}^2)\\ &\quad + 4T^{-2}h^{-1}\\ &\quad \times\sum_{s=1}^{T}\sum_{r_1=1}^{s-1}\sum_{r_2=1,r_1\neq r_2}^{s-1}E(\varepsilon_s^2\varepsilon_{r_1}\varepsilon_{r_2}\mathbf{X}_{r_1}'\mathbf{M}^{-1}\mathbf{X}_s\mathbf{X}_s'\mathbf{M}^{-1}\mathbf{X}_{r_2}w_{r_1 s}w_{r_2 s})\\ &= V_1 + V_2.\end{aligned}$$

For the first term, we have

$$\begin{aligned}\text{(A8)} \qquad V_1 &= 4T^{-2}h^{-1}\sum_{s=1}^{T}\sum_{r=1}^{s-1}w_{rs}^2\,\mathrm{trace}(\Omega M^{-1}\Omega M^{-1})\\ &\quad + 4T^{-2}h^{-1}\\ &\quad \times\sum_{s=1}^{T}\sum_{r=1}^{s-1}w_{rs}^2\,\mathrm{trace}\big\{E[\mathbf{M}^{-1}(\varepsilon_s^2\mathbf{X}_s\mathbf{X}_s' - \Omega)\mathbf{M}^{-1}(\varepsilon_r^2\mathbf{X}_r\mathbf{X}_r' - \Omega)]\big\}\\ &= 4C_B\,\mathrm{trace}(\Omega M^{-1}\Omega M^{-1}) + o(1),\end{aligned}$$

where we have used the change of variables and the mixing inequality.

For the second term,

$$\begin{aligned}\text{(A9)} \qquad V_2 &= 4T^{-1}h^{-1}\sum_{j=1}^{T-1}\sum_{l=1,j\neq l}^{T-1}(1-j/T)(1-l/T)\,\mathrm{trace}[C(j,l)]\\ &\quad \times\int_{-1}^{1}k(u)k\left(u+\frac{j}{Th}\right)du\int_{-1}^{1}k(u)k\left(u+\frac{l}{Th}\right)du = o(1),\end{aligned}$$

where $C(j,l) = E[\mathbf{M}^{-1}(\varepsilon_s^2\mathbf{X}_s\mathbf{X}_s' - \Omega)\mathbf{M}^{-1}\varepsilon_{s-j}\mathbf{X}_{s-j}\varepsilon_{s-l}\mathbf{X}_{s-l}']$, and we have used the change of variables and the mixing inequality. Given (A8) and (A9), we have $\text{var}(2\tilde{U}) = O(1)$.

We now verify condition (A5). Since we have

$$\sum_{s=2}^{T} E(R_s^4) = T^{-4}h^{-2}\sum_{s=2}^{T} E\Bigg(\sum_{i=1}^{s-1}\varphi_{is}^4 w_{is}^4 + 6\sum_{1\le i<j<s}\varphi_{is}^2\varphi_{js}^2 w_{is}^2 w_{js}^2$$
$$+ 4\sum_{t=1}^{s-1}\sum_{1\le i<j<s}\varphi_{ts}^2\varphi_{is}\varphi_{js}w_{ts}^2 w_{is}w_{js}$$
$$+ 4\sum_{1\le i<j<s,1\le t<r<s}\varphi_{is}\varphi_{js}\varphi_{ts}\varphi_{rs}w_{is}w_{js}w_{ts}w_{rs}\Bigg)$$
$$= O(T^{-2}h^{-1}) + O(T^{-1}) + O(h) + O(h),$$

$[\text{var}(2\tilde{U})]^{-2}\sum_{s=1}^{T} E(R_s^4) \to 0$ and (A5) holds.

Next we verify condition (A6). Let $Q_s = \sum_{r=1}^{s-1}\varepsilon_r\mathbf{X}_r' w_{ts}$. Then we have

$$\begin{aligned}E(R_s^2|\mathcal{F}_{s-1}) &= T^{-2}h^{-1}Q_s\mathbf{M}^{-1}E(\varepsilon_s^2\mathbf{X}_s\mathbf{X}_s'|\mathcal{F}_{s-1})\mathbf{M}^{-1}Q_s' \\ &= T^{-2}h^{-1}Q_s\mathbf{M}^{-1}[E(\varepsilon_s^2\mathbf{X}_s\mathbf{X}_s'|\mathcal{F}_{s-1}) - \Omega]\mathbf{M}^{-1}Q_s' \\ &\quad + T^{-2}h^{-1}Q_s\mathbf{M}^{-1}\Omega M^{-1}Q_s' \\ &= V_{1s} + R_{1s}, \quad \text{say.}\end{aligned} \tag{A10}$$

We further decompose

$$\begin{aligned}R_{1s} &= T^{-2}h^{-1}[Q_s\mathbf{M}^{-1}\Omega M^{-1}Q_s' - E(Q_s\mathbf{M}^{-1}\Omega M^{-1}Q_s')] \\ &\quad + T^{-2}h^{-1}E\left(\sum_{r=1}^{s-1}\varepsilon_r^2\mathbf{X}_r'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_r w_{rs}^2\right) \\ &= R_{2s} + T^{-2}h^{-1}\,\text{trace}(\Omega M^{-1}\Omega M^{-1})\sum_{r=1}^{s-1} w_{rs}.\end{aligned} \tag{A11}$$

Then we write

$$\begin{aligned}R_{2s} &= T^{-2}h^{-1}\sum_{r=1}^{s-1}[\varepsilon_r^2\mathbf{X}_r'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_r - E(\varepsilon_r^2\mathbf{X}_r'\mathbf{M}^{-1}\mathbf{X}_r)]w_{rs}^2 \\ &\quad + 2T^{-2}h^{-1}\sum_{r_1=1}^{s-1}\sum_{r_2=1}^{r_1}\varepsilon_{r_1}\mathbf{X}_{r_1}'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_{r_2}\varepsilon_{r_2}w_{r_1 s}w_{r_2 s} \\ &= V_{2s} + V_{3s}, \quad \text{say.}\end{aligned} \tag{A12}$$

It follows from (A10)–(A12) that $\sum_{s=1}^{T}\{E[(2R_s)^2|\mathcal{F}_{s-1}] - E[(2R_s)^2]\} = \sum_{i=1}^{3}\sum_{s=1}^{T} 4V_{is} + V_1$. It suffices to show Lemmas A.2–A.4 below, which imply $E|\sum_{s=1}^{T}E[(2R_s)^2|\mathcal{F}_{s-1}] - E[(2R_s)^2]|^2 = o(1)$. Thus, condition (A6) holds, and so $2\tilde{U}/\sqrt{\hat{B}_H} \to^d N(0,1)$ by Brown's (1971) theorem. *Q.E.D.*

This completes the proof of Theorem A.1. *Q.E.D.*

LEMMA A.2: *Let V_{1s} be defined as in* (A10). *Then* $E(\sum_{s=1}^{T} V_{1s})^2 = o(1)$.

LEMMA A.3: *Let V_{2s} be defined as in* (A11). *Then* $E(\sum_{s=1}^{T} V_{2s})^2 = o(1)$.

LEMMA A.4: *Let V_{3s} be defined as in* (A12). *Then* $E(\sum_{s=1}^{T} V_{3s})^2 = o(1)$.

PROOF OF LEMMA A.2: Let

$$\begin{aligned}
&\Omega(\zeta_{r_1}, \zeta_s, \zeta_{r_2}) \\
&\quad = \varepsilon_{r_1}\mathbf{X}'_{r_1}\mathbf{M}^{-1}[E(\varepsilon_s^2\mathbf{X}_s\mathbf{X}'_s|\mathcal{F}_{s-1}) - \Omega]\mathbf{M}^{-1}\mathbf{X}_{r_2}\varepsilon_{r_2}w_{r_1 s}w_{r_2 s} \\
&\qquad + \varepsilon_{r_2}\mathbf{X}'_{r_2}\mathbf{M}^{-1}\big[E\big(\varepsilon_{r_1}^2\mathbf{X}_{r_1}\mathbf{X}'_{r_1}|\mathcal{F}_{r_1-1}\big) - \Omega\big]\mathbf{M}^{-1}\mathbf{X}_s\varepsilon_s w_{r_1 r_2}w_{r_1 s} \\
&\qquad + \varepsilon_s\mathbf{X}'_s\mathbf{M}^{-1}\big[E\big(\varepsilon_{r_2}^2\mathbf{X}_{r_2}\mathbf{X}'_{r_2}|\mathcal{F}_{r_2-1}\big) - \Omega\big]\mathbf{M}^{-1}\mathbf{X}_{r_1}\varepsilon_{r_1}w_{r_2 s}w_{r_1 r_2},
\end{aligned}$$

where $\zeta_s = (\mathbf{X}_s, \varepsilon_s)$.

Then we have

$$E\left(\sum_{s=1}^{T} V_{1s}\right)^2 = CT^{-4}h^{-2}E\bigg[\sum_{s\neq r_1\neq r_2}\Omega\big(\zeta_{r_1}, \zeta_s, \zeta_{r_2}\big)\bigg]^2 = O(T^{-1}) = o(1),$$

where we have used a similar argument as that for $R_5 = o_P(1)$. *Q.E.D.*

PROOF OF LEMMA A.3: We have

$$\begin{aligned}
&E\left(\sum_{s=1}^{T} V_{2s}\right)^2 \\
&\quad = T^{-4}h^{-2}E \\
&\qquad \times \left\{\sum_{r=1}^{T}\sum_{r=1}^{s-1}[\varepsilon_r^2\mathbf{X}'_r\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_r - E(\varepsilon_r^2\mathbf{X}'_r\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_r)]w_{rs}^2\right\}^2 \\
&\quad = T^{-2}E\left\{\sum_{r=1}^{T}[\varepsilon_r^2\mathbf{X}'_r\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_r - E(\varepsilon_r^2\mathbf{X}'_r\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_r)]\right.
\end{aligned}$$

$$\times T^{-1}h^{-1}\sum_{j=1}^{T-1}\left(1-\frac{j}{T}\right)w^2\left(\frac{j}{Th}\right)\Bigg\}^2$$

$$= T^{-2}E\sum_{r=1}^{T}[\varepsilon_r^2\mathbf{X}_r'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_r - E(\varepsilon_r^2\mathbf{X}_r'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_r)]^2$$

$$\times\int w^2(u)\,du$$

$$+ T^{-2}\sum_{r\neq s}E[\varepsilon_r^2\mathbf{X}_r'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_r - E(\varepsilon_r^2\mathbf{X}_r'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_r)]$$

$$\times[\varepsilon_s^2\mathbf{X}_s'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_s - E(\varepsilon_s^2\mathbf{X}_s'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_s)] + o(1)$$

$$= O(T^{-1}) = o(1). \qquad \textit{Q.E.D.}$$

PROOF OF LEMMA A.4: We have

$$E\left(\sum_{s=1}^{T}V_{3s}\right)^2$$

$$= T^{-4}h^{-2}E\left(\sum_{s=1}^{T}\sum_{r_1=1}^{s-1}\sum_{r_2=1}^{r_1-1}\varepsilon_{r_1}\mathbf{X}_{r_1}'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_{r_2}\varepsilon_{r_2}w_{r_1s}w_{r_2s}\right)^2$$

$$= T^{-2}E\left(\sum_{r_1=1}^{T-1}\sum_{r_2=1}^{r_1-1}\varepsilon_{r_1}\mathbf{X}_{r_1}'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_{r_2}\varepsilon_{r_2}a_{r_1r_2}\right) + o(1)$$

$$= T^{-2}E\sum_{r_1=1}^{T-1}\sum_{r_2=1}^{r_1-1}\varepsilon_{r_1}^2\mathbf{X}_{r_1}'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_{r_2}\mathbf{X}_{r_2}'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_{r_1}'\varepsilon_{r_2}^2a_{r_1r_2}^2$$

$$+ T^{-2}E\sum_{r_1=1}^{T-1}\sum_{r_2=1}^{r_1-1}\sum_{r_3=1,r_3\neq r_2}^{r_1-1}\varepsilon_{r_1}^2\mathbf{X}_{r_2}'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_{r_1}\mathbf{X}_{r_1}'\mathbf{M}^{-1}\Omega M^{-1}\mathbf{X}_{r_3}'$$

$$\times\varepsilon_{r_2}\varepsilon_{r_3}a_{r_1r_2}a_{r_1r_3}$$

$$= O(h) + O(T^{-1}) = o(1),$$

where $a_{r_1r_2} = \int\int k(u)k(u+v)\,du\int k(u)k(u+v+\frac{r_1-r_2}{Th})\,du\,dv.$ *Q.E.D.*

PROOF OF THEOREM A.2: We have

$$\hat{J}_2 = h^{1/2}\sqrt{T}(\hat{\alpha}-\alpha)'\mathbf{M}_T\sqrt{T}(\hat{\alpha}-\alpha)$$

$$= o_P(1),$$

where $\mathbf{M}_T = \frac{1}{T}\sum_{t=1}^T \mathbf{X}_t\mathbf{X}_t'$ and we have used the fact $\sqrt{T}(\hat{\alpha}-\alpha) = O_P(1)$ and $\frac{1}{T}\sum_{t=1}^T \mathbf{X}_t\mathbf{X}_t' = O_p(1)$. *Q.E.D.*

PROOF OF THEOREM A.3: We have

$$\begin{aligned}\hat{J}_3 &= -2h^{1/2}\sqrt{T}(\hat{\alpha}-\alpha)' \\ &\quad \times \left(\frac{1}{Th}\sum_{t=1}^T k_{st}\mathbf{X}_t\mathbf{X}_t'\right)\left(\frac{1}{Th}\sum_{s=1}^T k_{st}\mathbf{X}_s\mathbf{X}_s'\right)^{-1}\left(\frac{1}{\sqrt{T}}\sum_{s=1}^T \varepsilon_s\mathbf{X}_s'\right)' \\ &= o_P(1),\end{aligned}$$

where we have used the fact $\sqrt{T}(\hat{\alpha}-\alpha) = O_P(1)$, $\mathbf{M}_T = O_p(1)$, and $\frac{1}{Th} \times \sum_{t=1}^T k_{st}\mathbf{X}_t\mathbf{X}_t' = O_P(1)$. *Q.E.D.*

PROOF OF THEOREM 2: Under the alternative hypothesis,

$$\begin{aligned}\hat{Q} &= \frac{1}{T}\sum_{t=1}^T(\hat{\alpha}-\alpha_t)'\mathbf{X}_t\mathbf{X}_t'(\hat{\alpha}-\alpha_t) + \frac{1}{T}\sum_{t=1}^T(\hat{\alpha}_t-\alpha_t)'\mathbf{X}_t\mathbf{X}_t'(\hat{\alpha}_t-\alpha_t) \\ &\quad - \frac{2}{T}\sum_{t=1}^T(\hat{\alpha}_t-\alpha_t)'\mathbf{X}_t\mathbf{X}_t'(\hat{\alpha}-\alpha_t) \\ &= \hat{J}_4+\hat{J}_5+\hat{J}_6.\end{aligned} \tag{A13}$$

The first term in (A13) is related to the parametric estimator

$$\begin{aligned}\hat{J}_4 &= \frac{1}{T}\sum_{t=1}^T(\alpha^*-\alpha_t)'\mathbf{X}_t\mathbf{X}_t'(\alpha^*-\alpha_t) + \frac{2}{T}\sum_{t=1}^T(\alpha^*-\alpha_t)'\mathbf{X}_t\mathbf{X}_t'(\hat{\alpha}-\alpha^*) \\ &\quad + (\hat{\alpha}-\alpha^*)'\mathbf{M}_T(\hat{\alpha}-\alpha^*) \\ &= \alpha^{*\prime}\mathbf{M}_T\alpha^* - \frac{2}{T}\sum_{t=1}^T \alpha_t'\mathbf{X}_t\mathbf{X}_t'\alpha^* + \frac{1}{T}\sum_{t=1}^T \alpha_t'\mathbf{X}_t\mathbf{X}_t'\alpha_t \\ &\quad + \frac{2}{\sqrt{T}}\left[\frac{1}{T}\sum_{t=1}^T(\alpha^*-\alpha_t)'\mathbf{X}_t\mathbf{X}_t'\right]\sqrt{T}(\hat{\alpha}-\alpha^*) \\ &\quad + \frac{1}{T}\sqrt{T}(\hat{\alpha}-\alpha^*)'\mathbf{M}_T\sqrt{T}(\hat{\alpha}-\alpha^*) \\ &= \alpha^{*\prime}\mathbf{M}\alpha^* - 2\int_0^1 \alpha'(u)\,du\,\mathbf{M}\alpha^* + \int_0^1 \alpha'(u)\mathbf{M}\alpha(u)\,du + o_P(1),\end{aligned}$$

where $\alpha^* = p\lim_{T\to\infty}\hat{\alpha}$ as defined in Assumption 4. The second term in (A13) is related to the nonparametric estimator

$$\begin{aligned}
\hat{J}_5 &= \frac{1}{T}\sum_{t=1}^{T}\left[W_t^{-1}\sum_{s=t-\lfloor Th\rfloor}^{t+\lfloor Th\rfloor} k_{st}\mathbf{X}_s\mathbf{X}_s'(\alpha_s-\alpha_t)\right]' \\
&\quad\times \mathbf{X}_t\mathbf{X}_t'\left[W_t^{-1}\sum_{s=t-\lfloor Th\rfloor}^{t+\lfloor Th\rfloor} k_{st}\mathbf{X}_s\mathbf{X}_s'(\alpha_s-\alpha_t)\right] \\
&\quad+\frac{2}{T}\sum_{t=1}^{T}\left[W_t^{-1}\sum_{s=t-\lfloor Th\rfloor}^{t+\lfloor Th\rfloor} k_{st}\mathbf{X}_s\mathbf{X}_s'(\alpha_s-\alpha_t)\right]' \\
&\quad\times \mathbf{X}_t\mathbf{X}_t'\left[W_t^{-1}\sum_{s=t-\lfloor Th\rfloor}^{t+\lfloor Th\rfloor} k_{st}\mathbf{X}_s\mathbf{X}_s'^{-1}\sum_{s=t-\lfloor Th\rfloor}^{t+\lfloor Th\rfloor} k_{st}\mathbf{X}_s\varepsilon_s\right] \\
&\quad+\frac{1}{T}\sum_{t=1}^{T}\left(W_t^{-1}\sum_{s=t-\lfloor Th\rfloor}^{t+\lfloor Th\rfloor} k_{st}\mathbf{X}_s\varepsilon_s\right)' \\
&\quad\times \mathbf{X}_t\mathbf{X}_t'\left(W_t^{-1}\sum_{s=t-\lfloor Th\rfloor}^{t+\lfloor Th\rfloor} k_{st}\mathbf{X}_s\varepsilon_s\right)+o_P(1) \\
&= \hat{J}_{51}+\hat{J}_{52}+\hat{J}_{53},
\end{aligned}$$

where $W_t = \sum_{s=t-\lfloor Th\rfloor}^{t+\lfloor Th\rfloor} k_{st}\mathbf{X}_s\mathbf{X}_s'$. For $\hat{J}_{51}$, we have

$$\begin{aligned}
&E\left\|\frac{1}{Th}\sum_{s=t-\lfloor Th\rfloor}^{t+\lfloor Th\rfloor} k_{st}\mathbf{X}_s\mathbf{X}_s'(\alpha_s-\alpha_t)\right\| \\
&\quad\le \sup_{|s-t|\le Th}\|\alpha_s-\alpha_t\|\frac{1}{Th}\sum_{s=t-\lfloor Th\rfloor}^{t+\lfloor Th\rfloor} k_{st}\operatorname{trace}(\mathbf{M}),
\end{aligned}$$

which converges to 0 if t/T belongs to continuity points and to C if t/T belongs to discontinuity points. By noting that the number of discontinuity points is finite, we have $\hat{J}_{51} = o_P(1)$. Similarly, we have $\hat{J}_{52} = o_P(1)$. Following the proof of Theorem A.1, we have $\hat{J}_{53} = o_P(1)$. Using a similar argument, we can show that the cross-product term in (A13) vanishes to 0 asymptotically. Moreover, it is easy to see $T^{-1}h^{-1/2}\hat{A}_H = o_P(1)$ and $\hat{B}_H = O_P(1)$. Hence, it follows that for any sequence $\{M_T = o(T\sqrt{h})\}$, we have $P(\hat{H} > M_T) \to 1$. *Q.E.D.*

PROOF OF THEOREM 3: The proof is similar to that of Theorem 2, but we make use of the local specifications.

(i) As $\alpha(u) = \alpha + j_T g(u)$ for all $u \in [0, 1]$, it holds for t/T. For notational simplicity, we use α_t and g_t to denote $a(t/T)$ and $g(t/T)$, respectively. The first term in (A13) is related to the OLS estimator

$$\text{(A14)} \quad T\hat{J}_4 = \sum_{t=1}^{T}\Bigg[j_T g_t - \mathbf{M}_T^{-1}\Bigg(T^{-1}\sum_{s=1}^{T}\mathbf{X}_s\mathbf{X}_s' g_s j_T\Bigg) - \mathbf{M}_T^{-1}\Bigg(T^{-1}\sum_{s=1}^{T}\mathbf{X}_s\varepsilon_s\Bigg)\Bigg]'\mathbf{X}_t\mathbf{X}_t' \times\Bigg[j_T g_t - \mathbf{M}_T^{-1}\Bigg(T^{-1}\sum_{s=1}^{T}\mathbf{X}_s\mathbf{X}_s' g_s j_T\Bigg) - \mathbf{M}_T^{-1}\Bigg(T^{-1}\sum_{s=1}^{T}\mathbf{X}_s\varepsilon_s\Bigg)\Bigg]$$

$$= \sum_{t=1}^{T} j_T\Bigg[g_t - \mathbf{M}_T^{-1}\Bigg(T^{-1}\sum_{s=1}^{T}\mathbf{X}_s\mathbf{X}_s' g_s\Bigg)\Bigg]' \times \mathbf{X}_t\mathbf{X}_t'\Bigg[g_t - \mathbf{M}_T^{-1}\Bigg(T^{-1}\sum_{s=1}^{T}\mathbf{X}_s\mathbf{X}_s' g_s\Bigg)\Bigg]$$

$$- 2j_T\sum_{t=1}^{T}\Bigg[g_t - \mathbf{M}_T^{-1}\Bigg(T^{-1}\sum_{s=1}^{T}\mathbf{X}_s\mathbf{X}_s' g_s\Bigg)\Bigg]' \times \mathbf{X}_t\mathbf{X}_t'\mathbf{M}_T^{-1}\Bigg(T^{-1}\sum_{s=1}^{T}\mathbf{X}_s\varepsilon_s\Bigg)$$

$$+ T^{-1}\Bigg(\sum_{s=1}^{T}\mathbf{X}_s\varepsilon_s\Bigg)'\mathbf{M}_T^{-1}\Bigg(\sum_{s=1}^{T}\mathbf{X}_s\varepsilon_s\Bigg)$$

$$= L_{11} + L_{12} + L_{13}.$$

For the first term in (A14), we have

$$\text{(A15)} \quad L_{11} = j_T^2\sum_{t=1}^{T} g_t'\mathbf{X}_t\mathbf{X}_t' g_t - 2j_T^2 T^{-1}\Bigg(\sum_{s=1}^{T}\mathbf{X}_s\mathbf{X}_s' g_s\Bigg)'\mathbf{M}_T^{-1}\Bigg(\sum_{s=1}^{T}\mathbf{X}_s\mathbf{X}_s' g_s\Bigg) + j_T^2 T^{-2}\Bigg(\sum_{s=1}^{T}\mathbf{X}_s\mathbf{X}_s' g_s\Bigg)'\mathbf{M}_T^{-1}\Bigg(\sum_{t=1}^{T}\mathbf{X}_t\mathbf{X}_t'\Bigg)\mathbf{M}_T^{-1}\Bigg(\sum_{s=1}^{T}\mathbf{X}_s\mathbf{X}_s' g_s\Bigg)$$

$$
\begin{aligned}
&= Tj_T^2\left(T^{-1}\sum_{t=1}^{T} g_t'\mathbf{X}_t\mathbf{X}_t'g_t\right) - 2Tj_T^2\left(T^{-1}\sum_{s=1}^{T}\mathbf{X}_s\mathbf{X}_s'g_s\right)' \\
&\quad \times \mathbf{M}_T^{-1}\left(T^{-1}\sum_{s=1}^{T}\mathbf{X}_s\mathbf{X}_s'g_s\right) \\
&\quad + Tj_T^2\left(T^{-1}\sum_{s=1}^{T}\mathbf{X}_s\mathbf{X}_s'g_s\right)\mathbf{M}_T^{-1}\left(T^{-1}\sum_{s=1}^{T}\mathbf{X}_s\mathbf{X}_s'g_s\right) \\
&= h^{-1/2}\delta_1 + o_P\big(h^{-1/2}\big),
\end{aligned}
$$

where the first term yields the noncentrality parameter under $\mathbb{H}_{1A}(j_T)$.

The cross-product term in (A14) is

$$
\begin{aligned}
L_{12} &= -2j_T\left(\sum_{t=1}^{T} g_t'\mathbf{X}_t\mathbf{X}_t'\right)\mathbf{M}_T^{-1}\left(T^{-1}\sum_{s=1}^{T}\mathbf{X}_s\varepsilon_s\right) \\
&\quad - 2j_T T^{-1}\left(\sum_{s=1}^{T} g_s'\mathbf{X}_s\mathbf{X}_s'\right)\mathbf{M}_T^{-1}\left(\sum_{s=1}^{T}\mathbf{X}_s\varepsilon_s\right) \\
&= O_P(\sqrt{T}j_T) = o_P(h^{-1/4}),
\end{aligned}
$$

and $L_{13} = O_P(1)$.

The second term in (A13) is related to the nonparametric estimation

$$
\begin{aligned}
T\hat{J}_5 &= \left\{\sum_{t=1}^{T}\left\{j_T W_t^{-1}\sum_{s=t-\lfloor Th\rfloor}^{t+\lfloor Th\rfloor} k_{st}\mathbf{X}_s\mathbf{X}_s'\right.\right. \\
&\quad \left.\left.\times\left[dg_t(u)/du\left(\frac{s-t}{T}\right) + d^2g_{\bar{s}}(u)/du^2\left(\frac{s-t}{T}\right)^2\right]\right\}' \right. \\
&\quad \times \mathbf{X}_t\mathbf{X}_t'\left\{j_T W_t^{-1}\sum_{s=t-\lfloor Th\rfloor}^{t+\lfloor Th\rfloor} k_{st}\mathbf{X}_s\mathbf{X}_s'\right. \\
&\quad \left.\times\left[dg_t(u)/du\left(\frac{s-t}{T}\right) + d^2g_{\bar{s}}(u)/du^2\left(\frac{s-t}{T}\right)^2\right]\right\} \\
&\quad + 2\sum_{t=1}^{T}\left\{j_T W_t^{-1}\sum_{s=t-\lfloor Th\rfloor}^{t+\lfloor Th\rfloor} k_{st}\mathbf{X}_s\mathbf{X}_s'\right.
\end{aligned}
$$

$$\times \left[dg_t(u)/du\left(\frac{s-t}{T}\right) + d^2 g_{\bar{s}}(u)/du^2 \left(\frac{s-t}{T}\right)^2 \right] \Bigg\}'$$

$$\times \mathbf{X}_t \mathbf{X}_t' W_t^{-1} \sum_{s=t-\lfloor Th \rfloor}^{t+\lfloor Th \rfloor} k_{st} \mathbf{X}_s \varepsilon_s$$

$$+ \sum_{t=1}^{T} \left(\sum_{s=t-\lfloor Th \rfloor}^{t+\lfloor Th \rfloor} k_{st} \mathbf{X}_s \varepsilon_s \right)' W_t^{-1} \mathbf{X}_t \mathbf{X}_t' W_t^{-1} \left(\sum_{s=t-\lfloor Th \rfloor}^{t+\lfloor Th \rfloor} k_{st} \mathbf{X}_s \varepsilon_s \right) \Bigg\}$$

$$\times [1 + o_P(1)]$$

$$= O_P(Th^4 j_T^2) + O_P\big(T^{1/2} h^{3/2} j_T\big) + \sum_{t=1}^{T} \left(\sum_{s=t-\lfloor Th \rfloor}^{t+\lfloor Th \rfloor} k_{st} \mathbf{X}_s \varepsilon_s \right)'$$

$$\times W_t^{-1} \mathbf{X}_t \mathbf{X}_t' W_t^{-1} \left(\sum_{s=t-\lfloor Th \rfloor}^{t+\lfloor Th \rfloor} k_{st} \mathbf{X}_s \varepsilon_s \right),$$

where the last term determines the asymptotic mean and variance under $\mathbb{H}_0$. The last term in (A13) is a cross-product term,

$$T\hat{J}_6 = 2 \sum_{t=1}^{T} \left[j_T g_t - \mathbf{M}_T^{-1} \left(T^{-1} \sum_{s=1}^{T} \mathbf{X}_s \mathbf{X}_s' g_s j_T \right) \right.$$

$$\left. - \mathbf{M}_T^{-1} \left(T^{-1} \sum_{s=1}^{T} \mathbf{X}_s \varepsilon_s \right) \right]' \mathbf{X}_t \mathbf{X}_t'$$

$$\times \Bigg\{ j_T W_t^{-1} \sum_{s} k_{st} \mathbf{X}_s \mathbf{X}_s'$$

$$\times \left[dg_t(u)/du\left(\frac{s-t}{T}\right) + d^2 g_{\bar{s}}(u)/du^2 \left(\frac{s-t}{T}\right)^2 \right]$$

$$+ W_t^{-1} \left(\sum_{s} k_{st} \mathbf{X}_s \varepsilon_s \right) \Bigg\}$$

$$= O_P(Th^2 j_T^2) + O_P(Th^2 j_T^2) + O_P\big(T^{1/2} h^2 j_T\big) + O_P(1)$$

$$= o_P\big(h^{-1/2}\big),$$

and hence Theorem 3(i) follows.

(ii) Similar to (i), we have

$$
\begin{aligned}
\text{(A16)} \quad T\hat{J}_4 = {} & \sum_t b_T^2 \left[f_{t,t_0} - \mathbf{M}_T^{-1} \left(T^{-1} \sum_s \mathbf{X}_s \mathbf{X}_s' f_{s,t_0} \right) \right]' \\
& \times \mathbf{X}_t \mathbf{X}_t' \left[f_{t,t_0} - \mathbf{M}_T^{-1} \left(T^{-1} \sum_s \mathbf{X}_s \mathbf{X}_s' f_{s,t_0} \right) \right] \\
& - 2b_T \sum_t \left[f_{t,t_0} - \mathbf{M}_T^{-1} \left(T^{-1} \sum_s \mathbf{X}_s \mathbf{X}_s' f_{s,t_0} \right) \right]' \\
& \times \mathbf{X}_t \mathbf{X}_t' \mathbf{M}_T^{-1} \left(T^{-1} \sum_s \mathbf{X}_s \varepsilon_s \right) \\
& + T^{-1} \left(\sum_s \mathbf{X}_s \varepsilon_s \right)' \mathbf{M}_T^{-1} \left(\sum_s \mathbf{X}_s \varepsilon_s \right) \\
= {} & h^{-1/2} \delta_2 + o_P\left(h^{-1/2}\right) + o_P\left(h^{-1/4}\right) + O_P(1).
\end{aligned}
$$

The second term in (A13) under $\mathbb{H}_{2A}(b_T, r_T)$ is

$$
\begin{aligned}
\text{(A17)} \quad T\hat{J}_5 = {} & \sum_t \Bigg\{ b_T W_t^{-1} \sum_s k_{st} \mathbf{X}_s \mathbf{X}_s' \\
& \times \left[df_{t,t_0}(z)/dz \left(\frac{s-t}{Tr_T} \right) + d^2 f_{\bar{s},t_0}(z)/dz^2 \left(\frac{s-t}{Tr_T} \right)^2 \right] \Bigg\}' \\
& \times \mathbf{X}_t \mathbf{X}_t' \Bigg\{ b_T W_t^{-1} \sum_s k_{st} \mathbf{X}_s \mathbf{X}_s' \\
& \times \left[df_{t,t_0}(z)/dz \left(\frac{s-t}{Tr_T} \right) + d^2 f_{\bar{s},t_0}(z)/dz^2 \left(\frac{s-t}{Tr_T} \right)^2 \right] \Bigg\} \\
& + 2 \sum_t \Bigg\{ b_T W_t^{-1} \sum_s k_{st} \mathbf{X}_s \mathbf{X}_s' \\
& \times \left[df_{t,t_0}(z)/dz \left(\frac{s-t}{Tr_T} \right) + d^2 f_{\bar{s},t_0}(z)/dz^2 \left(\frac{s-t}{Tr_T} \right)^2 \right] \Bigg\}' \\
& \times \mathbf{X}_t \mathbf{X}_t' W_t^{-1} \sum_s k_{st} \mathbf{X}_s \varepsilon_s + \sum_t \left(\sum_s k_{st} \mathbf{X}_s \varepsilon_s \right)' W_t^{-1} \\
& \times \mathbf{X}_t \mathbf{X}_t' W_t^{-1} \left(\sum_s k_{st} \mathbf{X}_s \varepsilon_s \right)
\end{aligned}
$$

$$= O_P(Th^4 b_T^2 r_T^{-3}) + O_P\big(T^{1/2} h^2 b_T r_T^{-3/2}\big) + \sum_t \bigg(\sum_s k_{st} \mathbf{X}_s \varepsilon_s\bigg)' W_t^{-1}$$

$$\times \mathbf{X}_t \mathbf{X}_t' W_t^{-1} \bigg(\sum_s k_{st} \mathbf{X}_s \varepsilon_s\bigg),$$

where the last term determines the asymptotic mean and variance under $\mathbb{H}_0$. The last term in (A13) is

$$\text{(A18)} \qquad T\hat{J}_6 = 2 \sum_t \bigg[b_T f_{t,t_0} - \mathbf{M}_T^{-1} \bigg(T^{-1} \sum_s \mathbf{X}_s \mathbf{X}_s' f_{s,t_0} b_T \bigg)$$

$$- \mathbf{M}_T^{-1} \bigg(T^{-1} \sum_s \mathbf{X}_s \varepsilon_s \bigg) \bigg]' \mathbf{X}_t \mathbf{X}_t'$$

$$\times \bigg\{ b_T W_t^{-1} \sum_s k_{st} \mathbf{X}_s \mathbf{X}_s'$$

$$\times \bigg[df_{t,t_0}(z)/dz \bigg(\frac{s-t}{Tr_T} \bigg) + d^2 f_{\bar{s},t_0}(z)/dz^2 \bigg(\frac{s-t}{Tr_T} \bigg)^2 \bigg]$$

$$+ W_t^{-1} \bigg(\sum_s k_{st} \mathbf{X}_s \varepsilon_s \bigg) \bigg\}$$

$$= o_P(1) + O_P(Th^2 b_T^2 r_T^{-1}) + O_P(Th^2 b_T^2 r_T^{-1}) + O_P\big(T^{1/2} h^2 b_T r_T^{-1}\big)$$

$$+ O_P\big(b_T T^{1/2} r_T^{1/2}\big) + O_P\big(T^{1/2} b_T r_T\big) + O_P(1).$$

Hence, Theorem 3(ii) follows from (A16)–(A18). *Q.E.D.*

PROOF OF THEOREM 4: (i) Following a similar proof as that of Theorem 3(i), we can show that under $\mathbb{H}_{1A}(j_T)$, the generalized Chow test $\hat{C} \to^d N(\delta_1/\sqrt{B_C}, 1)$ as $T \to \infty$, where $B_C = 4\,\mathrm{trace}(\mathbf{M}^{-1}\Omega\mathbf{M}^{-1}\Omega) \int_0^1 [2k(v) - \int_{-1}^1 k(u)k(u+v)\,du]^2\,dv$.

The Pitman asymptotic relative efficiency of the $\hat{H}$ test over the $\hat{C}$ test is the limit of sample sizes for the two tests to have the same asymptotic power at the same significance level and under the same local alternative (see Pitman (1979, Chapter 7)). Specifically, suppose T_1 and T_2 are the sample sizes for the $\hat{H}$ test and the $\hat{C}$ test, respectively. Then Pitman asymptotic relative efficiency of $\hat{H}$ to $\hat{C}$ is defined as

$$\mathrm{RE}(\hat{H} : \hat{C}) = \lim_{T_1, T_2 \to \infty} \frac{T_2}{T_1}$$

under the condition that $\hat{H}$ and $\hat{C}$ have the same asymptotic power when the local alternatives

$$\lim_{T_1,T_2\to\infty} \frac{j_{T_1} g_1(u)}{j_{T_2} g_2(u)} = 1.$$

Assuming $h_j = CT_j^{-\lambda}$ for $j = 1, 2$, we have

$$\lim_{T_1,T_2\to\infty} \left(\frac{T_1}{T_2}\right)^{-(2-\lambda)/4} = \frac{g_2(u)}{g_1(u)}, \tag{A19}$$

and to have the same asymptotic power, we have

$$\frac{\int_0^1 g_1(u)'\mathbf{M}g_1(u)\,du - \int_0^1 g_1(u)'\,du\,\mathbf{M}\int_0^1 g_1(u)\,du}{\sqrt{4\,\mathrm{trace}(\mathbf{M}^{-1}\Omega\mathbf{M}^{-1}\Omega)\int_0^1\left[\int_{-1}^1 k(u)k(u+v)\,du\right]^2 dv}} \tag{A20}$$
$$= \frac{\int_0^1 g_2(u)'\mathbf{M}g_2(u)\,du - \int_0^1 g_2(u)'\,du\,\mathbf{M}\int_0^1 g_2(u)\,du}{\sqrt{4\,\mathrm{trace}(\mathbf{M}^{-1}\Omega\mathbf{M}^{-1}\Omega)\int_0^1\left[2k(v) - \int_{-1}^1 k(u)k(u+v)\,du\right]^2 dv}}.$$

Combining (A19) and (A20), we have

$$\lim_{T_1,T_2\to\infty} \left(\frac{T_1}{T_2}\right)^{-(2-\lambda)/2} = \left(\frac{\int_0^1\left[2k(v) - \int_{-1}^1 k(u)k(u+v)\,du\right]^2 dv}{\int_0^1\left[\int_{-1}^1 k(u)k(u+v)\,du\right]^2 dv}\right)^{1/2}.$$

Rearranging terms, we get

$$\mathrm{RE}(\hat{H}:\hat{C}) = \lim_{T_1,T_2\to\infty} \frac{T_2}{T_1}$$
$$= \left(\frac{\int_0^1\left[2k(v) - \int_{-1}^1 k(u)k(u+v)\,du\right]^2 dv}{\int_0^1\left[\int_{-1}^1 k(u)k(u+v)\,du\right]^2 dv}\right)^{1/(2-\lambda)}.$$

Following a similar proof as that of Theorem 3(ii), we can show that under $\mathbb{H}_{2A}(b_T, r_T)$, the generalized Chow test $\hat{C} \to^d N(\delta_2/\sqrt{B_C}, 1)$ as $T \to \infty$.

From the same local alternative, we have

$$\lim_{T_1, T_2 \to \infty} \frac{b_{T_1} f_1\left(\dfrac{u - u_0}{r_{T_1}}\right)}{b_{T_2} f_2\left(\dfrac{u - u_0}{r_{T_2}}\right)} = 1$$

for all $u \in [0, 1]$. Therefore, we can integrate out u and have

$$\begin{aligned}
&\lim_{T_1, T_2 \to \infty} \frac{b_{T_1}^2 \displaystyle\int_0^1 f_1\left(\frac{u - u_0}{r_{T_1}}\right)' \mathbf{M} f_1\left(\frac{u - u_0}{r_{T_1}}\right) du}{b_{T_2} \displaystyle\int_0^1 f_2\left(\frac{u - u_0}{r_{T_2}}\right)' \mathbf{M} f\left(\frac{u - u_0}{r_{T_2}}\right) du} \\
&= \lim_{T_1, T_2 \to \infty} \frac{b_{T_1}^2 r_{T_1} \displaystyle\int_{-u_0/r_{T_1}}^{(1-u_0)/r_{T_1}} f_1(z)' \mathbf{M} f_1(z)\, dz}{b_{T_2}^2 r_{T_2} \displaystyle\int_{-u_0/r_{T_2}}^{(1-u_0)/r_{T_2}} \int_0^1 f_2(z)' \mathbf{M} f_2(z)\, dz} \\
&= \lim_{T_1, T_2 \to \infty} \frac{b_{T_1}^2 r_{T_1} \displaystyle\int_{-\infty}^{\infty} f_1(z)' \mathbf{M} f_1(z)\, dz}{b_{T_2}^2 r_{T_2} \displaystyle\int_{-\infty}^{\infty} f_2(z)' \mathbf{M} f_2(z)\, dz} \\
&= \lim_{T_1, T_2 \to \infty} \frac{T_1^{-(2-\lambda)/4} \displaystyle\int_{-\infty}^{\infty} f_1(z)' \mathbf{M} f_1(z)\, dz}{T_2^{-(2-\lambda)/4} \displaystyle\int_{-\infty}^{\infty} f_2(z)' \mathbf{M} f_2(z)\, dz} = 1,
\end{aligned}$$

where we have used the fact that $\frac{1-u_0}{r_{T_j}} \to \infty$ and $-\frac{u_0}{r_{T_j}} \to -\infty$ as $T_j \to \infty$ for $j = 1, 2$. It follows that

$$\text{(A21)} \qquad \lim_{T_1, T_2 \to \infty} \left(\frac{T_1}{T_2}\right)^{-(2-\lambda)/4} = \frac{\int_{-\infty}^{\infty} f_2(z)' \mathbf{M} f_2(z)\, dz}{\int_{-\infty}^{\infty} f_1(z)' \mathbf{M} f_1(z)\, dz},$$

and to have the same asymptotic power, we have

$$\text{(A22)}\qquad \frac{\int f_1(z)'\mathbf{M}f_1(z)\,dz}{\sqrt{4\,\text{trace}(\mathbf{M}^{-1}\Omega\mathbf{M}^{-1}\Omega)\int_0^1\left[\int_{-1}^1 k(u)k(u+v)\,du\right]^2 dv}}$$
$$= \frac{\int f_2(z)'\mathbf{M}f_2(z)\,dz}{\sqrt{4\,\text{trace}(\mathbf{M}^{-1}\Omega\mathbf{M}^{-1}\Omega)\int_0^1\left[2k(v)-\int_{-1}^1 k(u)k(u+v)\,du\right]^2 dv}}.$$

Combining (A21) and (A22), we have

$$\begin{aligned}\text{RE}(\hat{H}:\hat{C}) &= \lim_{T_1,T_2\to\infty}\frac{T_2}{T_1}\\ &= \left(\frac{\int_0^1\left[2k(v)-\int_{-1}^1 k(u)k(u+v)\,du\right]^2 dv}{\int_0^1\left[\int_{-1}^1 k(u)k(u+v)\,du\right]^2 dv}\right)^{1/(2-\lambda)}.\end{aligned}$$

Thus $\hat{H}$ is more efficient than $\hat{C}$ if

$$\text{(A23)}\qquad \int_{-1}^1\left[2k(v)-\int_{-1}^1 k(u)k(u+v)\,du\right]^2 dv > \int_{-1}^1\left[\int_{-1}^1 k(u)k(u+v)\,du\right]^2 dv.$$

Equation (A23) was shown in Hong and Zhang (2006). Now we verify this condition:

$$\begin{aligned}&\int_{-1}^1\left[2k(v)-\int_{-1}^1 k(u)k(u+v)\,du\right]^2 dv\\ &\quad -\int_{-1}^1\left[\int_{-1}^1 k(u)k(u+v)\,du\right]^2 dv\\ &= 4\left[\int_{-1}^1 k^2(v)\,dv-\int_{-1}^1\int_{-1}^1 k(v)k(u)k(u+v)\,du\,dv\right]\\ &\ge 4\left\{\int_{-1}^1 k^2(v)\,dv\right.\end{aligned}$$

$$- \int_{-1}^{1} k(u) \left[\int_{-1}^{1} k^2(v)\, dv \right]^{1/2} \left[\int_{-1}^{1} k^2(v+u)\, dv \right]^{1/2} du \Bigg\}$$

$$> 4 \left[\int_{-1}^{1} k^2(v)\, dv - \int_{-1}^{1} k(u)\, du \int_{-1}^{1} k^2(v)\, dv \right]$$

$$= 0.$$

(ii) By Theorem 1 in Chen and Hong (2008), $\hat{C}$ and $\hat{H}$ are asymptotic $N(0,1)$ under $\mathbb{H}_0$. Hence their asymptotic p-values are $1 - \Phi(\hat{C})$ and $1 - \Phi(\hat{H})$, respectively, where $\Phi(\cdot)$ is the cumulative distribution function of $N(0,1)$. Using Theorem 2 in Chen and Hong (2008), we have

$$\text{(A24)} \quad -2T^{-2}h^{-1} \ln[1 - \Phi(\hat{C})] = \left\{ 4\, \mathrm{trace}(\mathbf{M}^{-1} \Omega \mathbf{M}^{-1} \Omega) \int_0^1 \left[2k(v) - \int_{-1}^{1} k(u)k(u+v)\, du \right]^2 dv \right\}^{-1} \times \left(\alpha^{*\prime} \mathbf{M} \alpha^* - 2 \int_0^1 \alpha'(u)\, du\, \mathbf{M} \alpha^* + \int_0^1 \alpha'(u) \mathbf{M} \alpha(u)\, du \right) + o_P(1)$$

and

$$\text{(A25)} \quad -2T^{-2}h^{-1} \ln[1 - \Phi(\hat{H})] = \left\{ 4\, \mathrm{trace}(\mathbf{M}^{-1} \Omega \mathbf{M}^{-1} \Omega) \int_0^1 \left[\int_{-1}^{1} k(u)k(u+v)\, du \right]^2 dv \right\}^{-1} \times \left(\alpha^{*\prime} \mathbf{M} \alpha^* - 2 \int_0^1 \alpha'(u)\, du\, \mathbf{M} \alpha^* + \int_0^1 \alpha'(u) \mathbf{M} \alpha(u)\, du \right) + o_P(1).$$

Following Bahadur (1960), $\{4\, \mathrm{trace}(\mathbf{M}^{-1}\Omega\mathbf{M}^{-1}\Omega) \int_0^1 [2k(v) - \int_{-1}^1 k(u)k(u+v)\, du]^2\, dv\}^{-1} (\alpha^{*\prime}\mathbf{M}\alpha^* - 2\int_0^1 \alpha'(u)\, du\, \mathbf{M}\alpha^* + \int_0^1 \alpha'(u)\mathbf{M}\alpha(u)\, du)$ and $\{4 \times \mathrm{trace}(\mathbf{M}^{-1}\Omega\mathbf{M}^{-1}\Omega) \int_0^1 [\int_{-1}^1 k(u)k(u+v)\, du]^2\, dv\}^{-1} (\alpha^{*\prime}\mathbf{M}\alpha^* - 2\int_0^1 \alpha'(u)\, du \times \mathbf{M}\alpha^* + \int_0^1 \alpha'(u)\mathbf{M}\alpha(u)\, du)$ can be regarded as the "asymptotic slope" of the tests $\hat{C}$ and $\hat{H}$. Given (A24) and (A25), and $h = cT^{-\lambda}$ for $\lambda \in (0,1)$, Bahadur's asymptotic relative efficiency $\mathrm{RE}(\hat{H} : \hat{C})$ of $\hat{H}$ to $\hat{C}$ is

$$\mathrm{RE}(\hat{H} : \hat{C}) = \left\{ \frac{\int_{-1}^{1} \left[2k(v) - \int_{-1}^{1} k(u)k(u+v)\, du \right]^2 dv}{\int_{-1}^{1} \left[\int_{-1}^{1} k(u)k(u+v)\, du \right]^2 dv} \right\}^{1/(2-\lambda)},$$

and $\hat{H}$ is more efficient than $\hat{C}$ in terms of the Bahadur asymptotic efficiency criterion given (A23). *Q.E.D.*

FIGURES AND ADDITIONAL TABLES

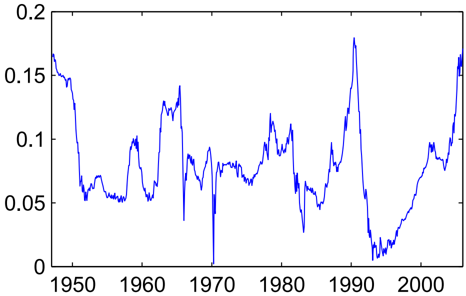

FIGURE 1.—Log dividend price ratio.

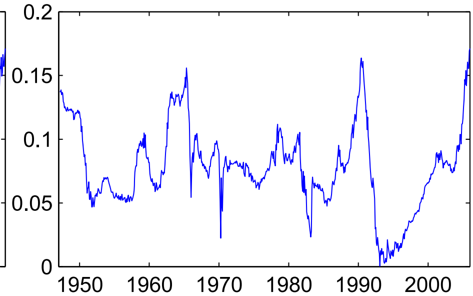

FIGURE 2.—Log dividend yield.

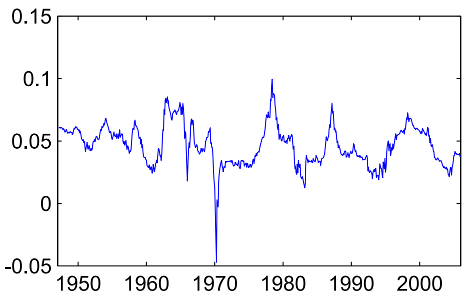

FIGURE 3.—Log earnings price ratio.

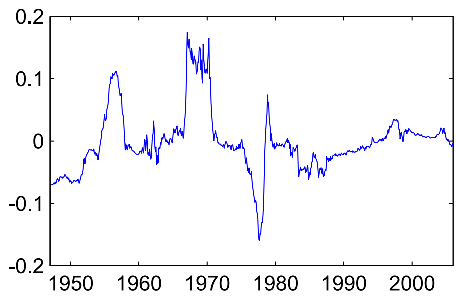

FIGURE 4.—Log dividend payout ratio.

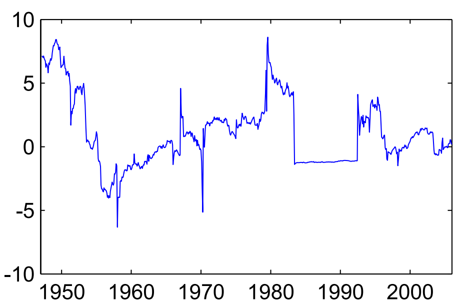

FIGURE 5.—Stock variance.

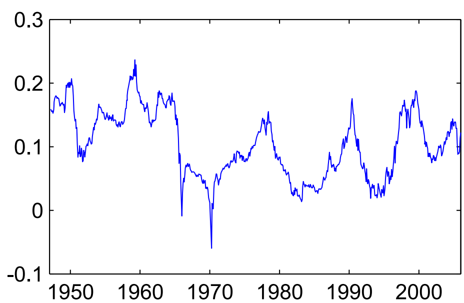

FIGURE 6.—Book-to-market ratio.

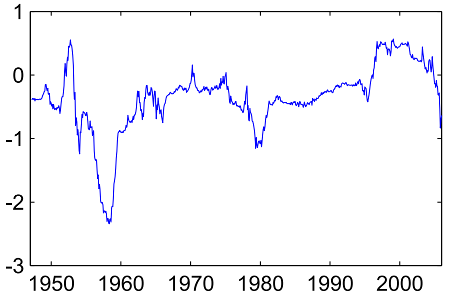

FIGURE 7.—Net equity expansion.

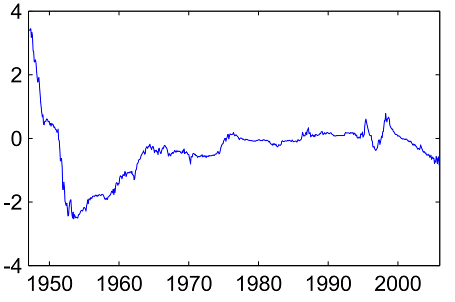

FIGURE 8.—Treasury bill rate.

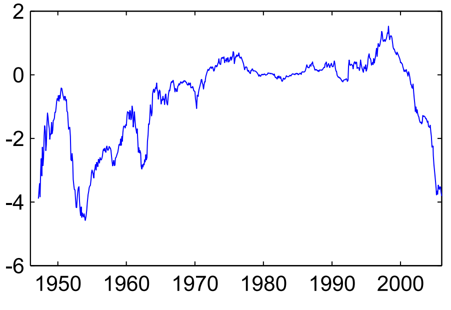

FIGURE 9.—Long-term yield.

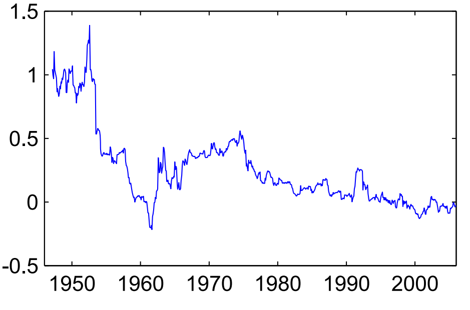

FIGURE 10.—Long-term return.

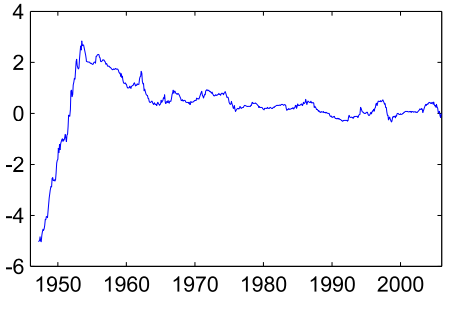

FIGURE 11.—Term spread.

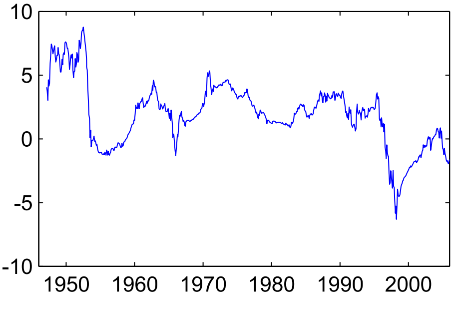

FIGURE 12.—Default yield spread.

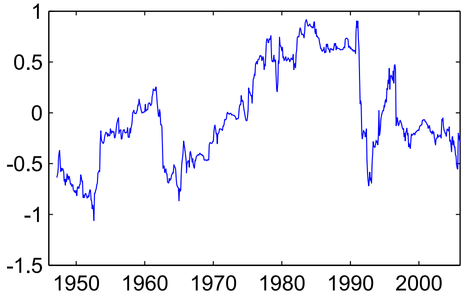

FIGURE 13.—Default return spread.

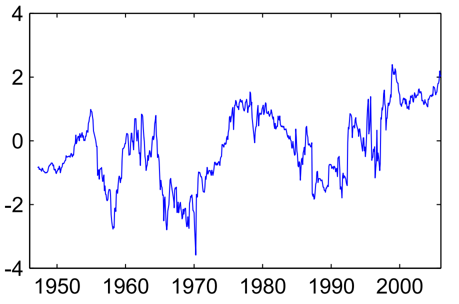

FIGURE 14.—Inflation.

TABLE A.I

EMPIRICAL LEVELS OF TESTS[a]

	$\varepsilon_t \sim$ i.i.d. $N(0,1)$			$\varepsilon_t \sim$ ARCH(1)			$\varepsilon_t \mid X_t \sim N(0, f(X_t))$		
Test[b]	100	250	500	100	250	500	100	250	500
$\hat{H}$-het	0.095	0.078	0.053	0.116	0.087	0.066	0.120	0.092	0.071
$\hat{C}$-het	0.066	0.053	0.042	0.077	0.060	0.051	0.081	0.061	0.048
$\hat{H}$	0.079	0.071	0.047	0.097	0.080	0.062	0.350	0.428	0.471
$\hat{C}$	0.052	0.047	0.035	0.067	0.050	0.048	0.243	0.288	0.328
CUSUM	0.034	0.045	0.044	0.046	0.048	0.050	0.020	0.032	0.028
MOSUM	0.036	0.044	0.050	0.045	0.048	0.057	0.033	0.039	0.046
LM1-het	0.043	0.054	0.048	0.044	0.047	0.053	0.045	0.045	0.050
LM2-het	0.040	0.049	0.047	0.036	0.043	0.047	0.032	0.042	0.045
LM3-het	0.029	0.046	0.046	0.029	0.041	0.045	0.028	0.042	0.045
LM1	0.046	0.054	0.049	0.052	0.048	0.052	0.150	0.168	0.177
LM2	0.054	0.049	0.048	0.060	0.052	0.055	0.194	0.219	0.247
LM3	0.049	0.053	0.049	0.065	0.059	0.056	0.218	0.258	0.290
Sup-LM-het	0.018	0.035	0.043	0.013	0.029	0.037	0.010	0.022	0.034
Exp-LM-het	0.033	0.049	0.049	0.030	0.041	0.050	0.026	0.036	0.043
Ave-LM-het	0.044	0.053	0.050	0.042	0.046	0.052	0.035	0.046	0.049
Sup-LM	0.029	0.043	0.045	0.048	0.050	0.055	0.205	0.282	0.330
Exp-LM	0.044	0.051	0.048	0.056	0.056	0.057	0.229	0.280	0.311
Ave-LM	0.043	0.053	0.050	0.047	0.049	0.055	0.178	0.210	0.227
UD-het	0.138	0.085	0.067	0.153	0.082	0.066	0.260	0.132	0.096
WD-het	0.225	0.125	0.089	0.214	0.109	0.079	0.374	0.191	0.120
UD	0.051	0.052	0.050	0.095	0.072	0.068	0.338	0.393	0.431
WD	0.058	0.055	0.051	0.104	0.076	0.068	0.413	0.483	0.537
qLL-het	0.060	0.050	0.051	0.081	0.067	0.055	0.060	0.058	0.054
qLL	0.065	0.055	0.052	0.088	0.074	0.061	0.454	0.503	0.514

[a] 5000 iterations; 5% significance level.

[b] $\hat{H}$ and $\hat{C}$ are the generalized Hausman and Chow tests; CUSUM and MOSUM are Brown, Dubin, and Evans' (1975) CUSUM and Hackl's (1980) MOSUM tests, respectively; LM1-3 are Lin and Teräsvirta's (1994) LM tests; Sup-LM is Andrews' (1993) supremum LM test; Exp-LM and Ave-LM are Andrews and Ploberger's (1994) exponential and average LM tests; UDMax and WDMax are Bai and Perron's (1998) double maximum tests; qLL is Elliott and Müller's (2006) efficient test based on a quasilocal level model. *-het denotes the heteroscedasticity-robust version of the corresponding * test.

TABLE A.II

POWERS OF TESTS UNDER DGP P.1: SINGLE STRUCTURAL BREAK[a]

	$t_1 = 0.1T$			$t = 0.3T$			$t = 0.5T$		
Test[b]	100	250	500	100	250	500	100	250	500
$\hat{H}$-het	0.181	0.350	0.669	0.416	0.857	0.990	0.498	0.916	1.00
$\hat{C}$-het	0.143	0.302	0.575	0.314	0.728	0.972	0.402	0.820	0.997
$\hat{H}$	0.173	0.354	0.676	0.431	0.867	0.991	0.529	0.917	1.00
$\hat{C}$	0.141	0.302	0.583	0.352	0.731	0.972	0.414	0.838	0.998
CUSUM	0.099	0.137	0.218	0.087	0.141	0.245	0.072	0.099	0.176
MOSUM	0.080	0.124	0.185	0.124	0.236	0.382	0.144	0.249	0.415
LM1-het	0.104	0.220	0.384	0.404	0.814	0.993	0.610	0.954	1.00
LM2-het	0.089	0.236	0.504	0.349	0.833	0.995	0.480	0.915	0.998
LM3-het	0.105	0.220	0.546	0.290	0.763	0.986	0.423	0.901	0.999
LM1	0.144	0.271	0.470	0.458	0.853	0.993	0.647	0.958	1.00
LM2	0.169	0.392	0.643	0.429	0.878	0.998	0.504	0.927	0.999
LM3	0.181	0.417	0.714	0.354	0.819	0.989	0.495	0.923	1.00
Sup-LM-het	0.083	0.195	0.454	0.427	0.888	0.999	0.583	0.949	0.999
Exp-LM-het	0.093	0.208	0.436	0.443	0.898	0.999	0.633	0.958	1.00
Ave-LM-het	0.093	0.211	0.369	0.409	0.861	0.997	0.616	0.956	1.00
Sup-LM	0.150	0.327	0.608	0.501	0.923	1.00	0.622	0.951	1.00
Exp-LM	0.148	0.306	0.563	0.517	0.932	1.00	0.656	0.960	1.00
Ave-LM	0.126	0.262	0.437	0.498	0.887	0.998	0.668	0.963	1.00
UD-het	0.118	0.269	0.528	0.393	0.871	0.996	0.488	0.932	0.999
WD-het	0.107	0.235	0.527	0.325	0.803	0.994	0.404	0.908	0.999
UD	0.138	0.328	0.607	0.494	0.922	1.00	0.608	0.949	1.00
WD	0.117	0.297	0.568	0.438	0.893	0.998	0.532	0.939	0.999
qLL-het	0.151	0.365	0.615	0.376	0.865	0.996	0.492	0.937	1.00
qLL	0.172	0.382	0.637	0.428	0.873	0.996	0.554	0.950	1.00

[a] 1000 iterations; 5% significance level.

[b] $\hat{H}$ and $\hat{C}$are the generalized Hausman and Chow tests; CUSUM and MOSUM are Brown, Durbin, and Evans' (1975) CUSUM and Hackl's (1980) MOSUM tests, respectively; LM1-3 are Lin and Teräsvirta's (1994) LM tests; Sup-LM is Andrews' (1993) supremum LM test; Exp-LM and Ave-LM are Andrews and Ploberger's (1994) exponential and average LM tests; UDMax and WDMax are Bai and Perron's (1998) double maximum tests; qLL is Elliott and Müller's (2006) efficient test based on a quasilocal level model. *-het denotes the heteroscedasticity-robust version of the corresponding * test.

TABLE A.III

POWERS OF TESTS UNDER DGP P.2: MULTIPLE STRUCTURAL BREAKS[a]

	$(\alpha_{01}, \alpha_{11}, \alpha_{02}, \alpha_{12}) =$ (1.5, 1, 0.6, 0.3)			$(\alpha_{01}, \alpha_{11}, \alpha_{02}, \alpha_{12}) =$ (0.6, 0.3, 1.5, 1)			$(\alpha_{01}, \alpha_{11}, \alpha_{02}, \alpha_{12}) =$ (1.5, 1, 1.5, 1)		
Test[b]	100	250	500	100	250	500	100	250	500
$\hat{H}$-het	0.296	0.688	0.988	0.308	0.646	0.954	0.147	0.431	0.889
$\hat{C}$-het	0.280	0.702	0.989	0.276	0.615	0.942	0.168	0.568	0.950
$\hat{H}$	0.301	0.712	0.989	0.306	0.665	0.957	0.150	0.472	0.906
$\hat{C}$	0.302	0.728	0.991	0.296	0.621	0.949	0.182	0.595	0.956
CUSUM	0.112	0.180	0.340	0.090	0.174	0.322	0.066	0.089	0.197
MOSUM	0.157	0.295	0.512	0.143	0.277	0.509	0.074	0.115	0.185
LM1-het	0.074	0.059	0.065	0.052	0.054	0.061	0.079	0.085	0.111
LM2-het	0.117	0.119	0.247	0.108	0.168	0.349	0.065	0.116	0.187
LM3-het	0.157	0.206	0.351	0.139	0.221	0.419	0.079	0.111	0.184
LM1	0.082	0.071	0.078	0.053	0.045	0.061	0.085	0.105	0.141
LM2	0.105	0.127	0.249	0.119	0.203	0.374	0.085	0.148	0.228
LM3	0.153	0.191	0.363	0.132	0.230	0.443	0.090	0.135	0.209
Sup-LM-het	0.123	0.218	0.512	0.115	0.231	0.478	0.085	0.157	0.372
Exp-LM-het	0.115	0.213	0.500	0.107	0.219	0.513	0.081	0.141	0.324
Ave-LM-het	0.095	0.150	0.355	0.070	0.144	0.393	0.078	0.108	0.162
Sup-LM	0.191	0.336	0.638	0.145	0.286	0.554	0.120	0.213	0.449
Exp-LM	0.187	0.319	0.645	0.142	0.279	0.591	0.107	0.202	0.407
Ave-LM	0.130	0.207	0.475	0.095	0.180	0.449	0.095	0.145	0.207
UD-het	0.166	0.343	0.738	0.183	0.400	0.773	0.099	0.174	0.385
WD-het	0.206	0.490	0.917	0.218	0.494	0.890	0.149	0.371	0.809
UD	0.242	0.525	0.917	0.240	0.556	0.914	0.135	0.299	0.663
WD	0.300	0.679	0.977	0.290	0.658	0.962	0.189	0.563	0.931
qLL-het	0.237	0.668	0.957	0.217	0.637	0.928	0.119	0.363	0.768
qLL	0.330	0.723	0.975	0.297	0.657	0.942	0.192	0.464	0.845

[a] 1000 iterations; 5% significancel level.

[b] $\hat{H}$ and $\hat{C}$are the generalized Hausman and Chow tests; CUSUM and MOSUM are Brown, Durbin, and Evans' (1975) CUSUM and Hackl's (1980) MOSUM tests, respectively; LM1-3 are Lin and Teräsvirta's (1994) LM tests; Sup-LM is Andrews' (1993) supremum LM test; Exp-LM and Ave-LM are Andrews and Ploberger's (1994) exponential and average LM tests; UDMax and WDMax are Bai and Perron's (1998) double maximum tests; qLL is Elliott and Müller's (2006) efficient test based on a quasilocal level model. *-het denotes the heteroscedasticity-robust version of the corresponding * test.

TABLE A.IV

POWERS OF TESTS UNDER DGP P.3: NONPERSISTENT TEMPORAL STRUCTURAL BREAKS[a]

	$t_1 = 0.2T$, $t_2 = 0.4T$			$t_1 = 0.4T$, $t_2 = 0.6T$			$t_1 = 0.6T$, $t_2 = 0.8T$		
Test[b]	100	250	500	100	250	500	100	250	500
$\hat{H}$-het	0.304	0.735	0.988	0.319	0.748	0.990	0.300	0.728	0.988
$\hat{C}$-het	0.295	0.703	0.977	0.306	0.693	0.971	0.283	0.696	0.979
$\hat{H}$	0.316	0.740	0.988	0.325	0.765	0.987	0.316	0.749	0.989
$\hat{C}$	0.305	0.704	0.978	0.330	0.700	0.977	0.296	0.703	0.980
CUSUM	0.116	0.244	0.462	0.096	0.198	0.399	0.078	0.136	0.337
MOSUM	0.152	0.296	0.580	0.138	0.292	0.519	0.152	0.321	0.596
LM1-het	0.142	0.253	0.521	0.059	0.058	0.059	0.123	0.266	0.479
LM2-het	0.148	0.300	0.590	0.164	0.435	0.764	0.132	0.292	0.598
LM3-het	0.218	0.514	0.903	0.164	0.364	0.701	0.194	0.499	0.881
LM1	0.139	0.263	0.532	0.046	0.050	0.046	0.129	0.270	0.492
LM2	0.139	0.325	0.604	0.183	0.488	0.791	0.129	0.298	0.614
LM3	0.261	0.591	0.938	0.174	0.413	0.731	0.252	0.593	0.918
Sup-LM-het	0.183	0.415	0.811	0.129	0.330	0.665	0.175	0.422	0.795
Exp-LM-het	0.184	0.421	0.832	0.127	0.329	0.702	0.175	0.436	0.813
Ave-LM-het	0.162	0.395	0.784	0.088	0.229	0.609	0.153	0.395	0.749
Sup-LM	0.206	0.469	0.868	0.143	0.348	0.708	0.203	0.486	0.843
Exp-LM	0.215	0.467	0.883	0.145	0.362	0.733	0.219	0.494	0.849
Ave-LM	0.192	0.433	0.813	0.096	0.256	0.633	0.187	0.439	0.787
UD-het	0.255	0.660	0.976	0.236	0.634	0.973	0.241	0.651	0.970
WD-het	0.271	0.687	0.985	0.255	0.667	0.987	0.258	0.689	0.984
UD	0.289	0.745	0.990	0.262	0.736	0.993	0.295	0.735	0.989
WD	0.317	0.810	0.993	0.303	0.784	1.00	0.308	0.784	0.995
qLL-het	0.235	0.713	0.966	0.222	0.709	0.968	0.247	0.696	0.964
qLL	0.295	0.749	0.970	0.324	0.734	0.976	0.310	0.733	0.976

[a] 1000 iterations; 5% significance level.

[b] $\hat{H}$ and $\hat{C}$are the generalized Hausman and Chow tests; CUSUM and MOSUM are Brown, Durbin, and Evans' (1975) CUSUM and Hackl's (1980) MOSUM tests, respectively; LM1-3 are Lin and Teräsvirta's (1994) LM tests; Sup-LM is Andrews' (1993) supremum LM test; Exp-LM and Ave-LM are Andrews and Ploberger's (1994) exponential and average LM tests; UDMax and WDMax are Bai and Perron's (1998) double maximum tests; qLL is Elliott and Müller's (2006) efficient test based on a quasilocal level model. *-het denotes the heteroscedasticity-robust version of the corresponding * test.

TABLE A.V

POWERS OF TESTS UNDER DGP P.4: SMOOTH STRUCTURAL CHANGES[a]

	Monotonic Smooth Structural Changes			Nonmonotonic Smooth Structural Changes			High-Frequency Smooth Structural Changes		
Test[b]	100	250	500	100	250	500	100	250	500
$\hat{H}$-het	0.400	0.824	0.988	0.443	0.884	0.998	0.293	0.678	0.964
$\hat{C}$-het	0.370	0.782	0.980	0.378	0.786	0.992	0.238	0.550	0.904
$\hat{H}$	0.401	0.824	0.988	0.452	0.886	0.998	0.287	0.701	0.964
$\hat{C}$	0.381	0.780	0.979	0.401	0.793	0.992	0.251	0.555	0.910
CUSUM	0.059	0.086	0.187	0.320	0.681	0.943	0.243	0.481	0.807
MOSUM	0.105	0.243	0.442	0.374	0.780	0.966	0.393	0.764	0.971
LM1-het	0.229	0.513	0.817	0.065	0.055	0.059	0.194	0.379	0.705
LM2-het	0.235	0.631	0.949	0.520	0.946	0.999	0.315	0.708	0.957
LM3-het	0.214	0.647	0.970	0.427	0.903	0.996	0.316	0.720	0.976
LM1	0.318	0.612	0.861	0.074	0.072	0.063	0.195	0.384	0.706
LM2	0.424	0.804	0.978	0.548	0.964	0.999	0.312	0.735	0.965
LM3	0.474	0.884	0.993	0.492	0.919	0.997	0.317	0.737	0.979
Sup-LM-het	0.248	0.584	0.935	0.215	0.598	0.942	0.267	0.614	0.926
Exp-LM-het	0.242	0.558	0.922	0.193	0.572	0.940	0.279	0.627	0.941
Ave-LM-het	0.223	0.482	0.799	0.132	0.422	0.891	0.241	0.578	0.930
Sup-LM	0.404	0.783	0.981	0.281	0.675	0.964	0.263	0.608	0.925
Exp-LM	0.389	0.746	0.973	0.259	0.650	0.969	0.282	0.633	0.950
Ave-LM	0.305	0.575	0.862	0.177	0.473	0.916	0.267	0.588	0.931
UD-het	0.293	0.680	0.954	0.258	0.683	0.978	0.212	0.567	0.938
WD-het	0.261	0.591	0.944	0.269	0.708	0.985	0.178	0.545	0.944
UD	0.391	0.789	0.981	0.345	0.781	0.989	0.283	0.640	0.954
WD	0.356	0.746	0.969	0.361	0.804	0.990	0.258	0.643	0.955
qLL-het	0.388	0.798	0.988	0.418	0.892	0.997	0.282	0.740	0.977
qLL	0.408	0.792	0.987	0.440	0.897	0.997	0.292	0.748	0.981

[a] 1000 iterations; 5% significance level.

[b] $\hat{H}$ and $\hat{C}$are the generalized Hausman and Chow tests; CUSUM and MOSUM are Brown, Durbin, and Evans' (1975) CUSUM and Hackl's (1980) MOSUM tests, respectively; LM1-3 are Lin and Teräsvirta's (1994) LM tests; Sup-LM is Andrews' (1993) supremum LM test; Exp-LM and Ave-LM are Andrews and Ploberger's (1994) exponential and average LM tests; UDMax and WDMax are Bai and Perron's (1998) double maximum tests; qLL is Elliott and Müller's (2006) efficient test based on a quasilocal level model. *-het denotes the heteroscedasticity-robust version of the corresponding * test.

TABLE A.VI

POWERS OF TESTS UNDER DGP P.5: UNIT ROOT IN PARAMETER[a]

	Slope			Intercept			Both		
Test[b]	100	250	500	100	250	500	100	250	500
$\hat{H}$-het	0.414	0.874	0.994	0.342	0.828	0.990	0.617	0.983	1.00
$\hat{C}$-het	0.359	0.851	0.991	0.305	0.789	0.972	0.555	0.982	1.00
$\hat{H}$	0.417	0.885	0.996	0.333	0.832	0.991	0.626	0.987	1.00
$\hat{C}$	0.369	0.853	0.992	0.292	0.786	0.972	0.564	0.985	1.00
CUSUM	0.063	0.051	0.082	0.260	0.691	0.931	0.277	0.622	0.897
MOSUM	0.080	0.104	0.139	0.387	0.813	0.948	0.395	0.745	0.929
LM1-het	0.349	0.692	0.831	0.344	0.674	0.826	0.557	0.889	0.971
LM2-het	0.326	0.769	0.939	0.342	0.777	0.926	0.567	0.946	0.995
LM3-het	0.304	0.786	0.966	0.325	0.808	0.963	0.558	0.956	0.998
LM1	0.390	0.722	0.851	0.354	0.675	0.832	0.585	0.896	0.972
LM2	0.397	0.808	0.950	0.334	0.781	0.925	0.609	0.960	0.995
LM3	0.399	0.844	0.973	0.332	0.811	0.960	0.616	0.971	0.999
Sup-LM-het	0.334	0.791	0.957	0.366	0.779	0.955	0.563	0.945	0.999
Exp-LM-het	0.350	0.796	0.955	0.376	0.787	0.953	0.595	0.950	0.999
Ave-LM-het	0.355	0.757	0.939	0.362	0.747	0.935	0.576	0.943	0.999
Sup-LM	0.408	0.841	0.977	0.362	0.781	0.956	0.623	0.964	0.999
Exp-LM	0.418	0.840	0.974	0.377	0.792	0.957	0.635	0.967	0.999
Ave-LM	0.409	0.798	0.954	0.379	0.754	0.936	0.625	0.952	0.999
UD-het	0.345	0.814	0.977	0.284	0.959	0.999	0.524	0.785	0.966
WD-het	0.336	0.812	0.988	0.240	0.963	1.00	0.483	0.777	0.981
UD	0.421	0.868	0.987	0.368	0.972	0.999	0.631	0.815	0.978
WD	0.394	0.873	0.994	0.328	0.976	1.00	0.596	0.822	0.985
qLL-het	0.378	0.865	0.991	0.371	0.849	0.986	0.613	0.979	1.00
qLL	0.438	0.879	0.992	0.362	0.849	0.985	0.645	0.983	1.00

[a] 1000 iterations; 5% significance level.

[b] $\hat{H}$ and $\hat{C}$are the generalized Hausman and Chow tests; CUSUM and MOSUM are Brown, Durbin, and Evans' (1975) CUSUM and Hackl's (1980) MOSUM tests, respectively; LM1-3 are Lin and Teräsvirta's (1994) LM tests; Sup-LM is Andrews' (1993) supremum LM test; Exp-LM and Ave-LM are Andrews and Ploberger's (1994) exponential and average LM tests; UDMax and WDMax are Bai and Perron's (1998) double maximum tests; qLL is Elliott and Müller's (2006) efficient test based on a quasilocal level model. *-het denotes the heteroscedasticity-robust version of the corresponding * test.

TABLE A.VII

EMPIRICAL LEVELS OF TESTS WITH DIFFERENT BANDWIDTHS[a]

		DGP S.1: $\varepsilon_t \sim$ i.i.d. $N(0,1)$						DGP S.2: $\varepsilon_t \sim$ ARCH(1)						DGP S.3: $\varepsilon_t \mid X_t \sim N(0, f(X_t))$					
		100		250		500		100		250		500		100		250		500	
	Test[b]	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV
$c = 1/1.5$	$\hat{H}$-het	0.075	0.196	0.061	0.126	0.070	0.079	0.080	0.225	0.067	0.140	0.048	0.097	0.087	0.215	0.068	0.149	0.068	0.107
	$\hat{C}$-het	0.071	0.118	0.057	0.069	0.062	0.054	0.080	0.136	0.063	0.085	0.052	0.066	0.080	0.125	0.066	0.088	0.070	0.065
	$\hat{H}$	0.084	0.159	0.071	0.106	0.078	0.068	0.086	0.188	0.074	0.124	0.054	0.085	0.131	0.508	0.105	0.579	0.094	0.625
	$\hat{C}$	0.080	0.085	0.065	0.051	0.066	0.045	0.086	0.109	0.070	0.070	0.054	0.061	0.129	0.331	0.100	0.377	0.096	0.406
$c = 1$	$\hat{H}$-het	0.065	0.095	0.059	0.078	0.038	0.053	0.071	0.116	0.059	0.087	0.048	0.066	0.083	0.120	0.061	0.092	0.062	0.071
	$\hat{C}$-het	0.069	0.066	0.060	0.053	0.046	0.042	0.071	0.077	0.063	0.060	0.056	0.051	0.081	0.081	0.056	0.061	0.062	0.048
	$\hat{H}$	0.070	0.079	0.066	0.071	0.040	0.047	0.079	0.097	0.063	0.080	0.050	0.062	0.117	0.350	0.084	0.428	0.084	0.471
	$\hat{C}$	0.073	0.052	0.066	0.047	0.046	0.035	0.079	0.067	0.069	0.050	0.060	0.048	0.118	0.243	0.080	0.288	0.078	0.328
$c = 1.5$	$\hat{H}$-het	0.065	0.057	0.059	0.052	0.066	0.039	0.065	0.062	0.051	0.055	0.054	0.047	0.071	0.074	0.054	0.063	0.062	0.053
	$\hat{C}$-het	0.065	0.040	0.058	0.034	0.054	0.030	0.060	0.049	0.050	0.041	0.064	0.039	0.066	0.052	0.051	0.040	0.058	0.036
	$\hat{H}$	0.068	0.048	0.062	0.049	0.074	0.037	0.068	0.059	0.056	0.054	0.050	0.046	0.095	0.241	0.066	0.308	0.070	0.346
	$\hat{C}$	0.072	0.033	0.061	0.031	0.052	0.028	0.065	0.043	0.054	0.036	0.064	0.037	0.089	0.168	0.064	0.221	0.074	0.261
$c = 2$	$\hat{H}$-het	0.061	0.050	0.048	0.040	0.040	0.035	0.063	0.059	0.053	0.041	0.082	0.038	0.066	0.061	0.056	0.047	0.060	0.042
	$\hat{C}$-het	0.061	0.038	0.050	0.032	0.044	0.205	0.060	0.044	0.056	0.035	0.048	0.030	0.065	0.046	0.057	0.035	0.068	0.034
	$\hat{H}$	0.065	0.044	0.049	0.038	0.046	0.032	0.066	0.053	0.056	0.038	0.074	0.038	0.087	0.214	0.065	0.230	0.066	0.266
	$\hat{C}$	0.063	0.032	0.053	0.029	0.044	0.025	0.064	0.039	0.056	0.032	0.058	0.028	0.085	0.159	0.066	0.180	0.078	0.212
CV	$\hat{H}$-het	0.071	0.079	0.051	0.061	0.063	0.047	0.076	0.094	0.055	0.070	0.044	0.057	0.070	0.127	0.057	0.090	0.055	0.064
	$\hat{C}$-het	0.070	0.121	0.046	0.111	0.059	0.099	0.083	0.137	0.054	0.116	0.041	0.113	0.066	0.156	0.057	0.130	0.051	0.110
	$\hat{H}$	0.070	0.070	0.055	0.055	0.064	0.042	0.078	0.083	0.055	0.065	0.048	0.054	0.089	0.232	0.068	0.247	0.064	0.234
	$\hat{C}$	0.077	0.108	0.051	0.101	0.061	0.094	0.087	0.123	0.062	0.110	0.043	0.109	0.089	0.311	0.081	0.363	0.065	0.374

[a] 5000 iterations; 5% significance level. BCV and ECV denote results based on bootstrap critical values and empirical critical values, respectively; c denotes the scaling parameter in the rule-of-thumb bandwidth selection, $h = c(1/\sqrt{12})T^{-1/5}$; CV denotes the cross-validation method.

[b] $\hat{H}$ and $\hat{C}$ are the generalized Hausman and Chow tests; *-het denotes the heteroscedasticity-robust version of the corresponding * test.

TABLE A.VIII

EMPIRICAL POWERS OF TESTS WITH DIFFERENT BANDWIDTHS[a]

		DGP P.1 $t = 0.3T$						DGP P.2 $(\alpha_{01}, \alpha_{11}, \alpha_{02}, \alpha_{12}) = (0.6, 0.3, 1.5, 1)$						DGP P.3 $t_1 = 0.4T$, $t_2 = 0.6T$					
		100		250		500		100		250		500		100		250		500	
	Test[b]	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV
$c = 1/1.5$	$\hat{H}$-het	0.370	0.347	0.783	0.779	0.981	0.982	0.344	0.334	0.726	0.706	0.972	0.972	0.345	0.319	0.769	0.753	0.989	0.985
	$\hat{C}$-het	0.300	0.267	0.637	0.664	0.947	0.961	0.302	0.309	0.676	0.675	0.966	0.965	0.293	0.270	0.656	0.684	0.969	0.969
	$\hat{H}$	0.398	0.371	0.797	0.793	0.984	0.988	0.376	0.354	0.746	0.707	0.975	0.973	0.365	0.333	0.791	0.774	0.990	0.990
	$\hat{C}$	0.322	0.293	0.670	0.678	0.951	0.969	0.332	0.318	0.711	0.700	0.969	0.969	0.317	0.275	0.690	0.709	0.973	0.968
$c = 1$	$\hat{H}$-het	0.399	0.416	0.847	0.857	0.998	0.990	0.293	0.308	0.673	0.646	0.951	0.954	0.324	0.319	0.770	0.748	0.977	0.990
	$\hat{C}$-het	0.320	0.314	0.751	0.728	0.975	0.972	0.269	0.276	0.676	0.615	0.955	0.942	0.304	0.306	0.729	0.693	0.968	0.971
	$\hat{H}$	0.418	0.431	0.855	0.867	0.998	0.991	0.319	0.306	0.694	0.665	0.953	0.957	0.343	0.325	0.788	0.765	0.977	0.987
	$\hat{C}$	0.337	0.352	0.761	0.731	0.979	0.972	0.286	0.296	0.690	0.621	0.961	0.949	0.321	0.330	0.746	0.700	0.970	0.977
$c = 1.5$	$\hat{H}$-het	0.463	0.467	0.890	0.896	0.997	0.994	0.210	0.209	0.588	0.564	0.901	0.916	0.278	0.273	0.674	0.674	0.969	0.963
	$\hat{C}$-het	0.386	0.397	0.822	0.814	0.984	0.983	0.239	0.254	0.552	0.521	0.838	0.851	0.282	0.275	0.721	0.715	0.967	0.972
	$\hat{H}$	0.474	0.478	0.893	0.905	0.997	0.996	0.220	0.224	0.606	0.574	0.906	0.920	0.298	0.277	0.689	0.692	0.971	0.967
	$\hat{C}$	0.396	0.404	0.829	0.817	0.984	0.983	0.255	0.263	0.564	0.539	0.842	0.863	0.301	0.289	0.735	0.725	0.968	0.977
$c = 2$	$\hat{H}$-het	0.463	0.486	0.888	0.908	0.998	0.997	0.152	0.157	0.424	0.422	0.830	0.817	0.227	0.213	0.604	0.582	0.924	0.932
	$\hat{C}$-het	0.399	0.403	0.832	0.847	0.994	0.984	0.215	0.208	0.510	0.520	0.824	0.826	0.234	0.225	0.638	0.634	0.950	0.958
	$\hat{H}$	0.485	0.499	0.891	0.912	0.998	0.996	0.164	0.172	0.437	0.431	0.844	0.833	0.239	0.230	0.615	0.609	0.930	0.932
	$\hat{C}$	0.416	0.431	0.838	0.865	0.994	0.985	0.225	0.219	0.521	0.549	0.830	0.831	0.243	0.228	0.649	0.669	0.950	0.957
CV	$\hat{H}$-het	0.406	0.423	0.862	0.871	0.992	0.995	0.283	0.255	0.637	0.604	0.946	0.943	0.290	0.255	0.717	0.692	0.970	0.957
	$\hat{C}$-het	0.373	0.378	0.837	0.857	0.991	0.993	0.289	0.259	0.657	0.642	0.958	0.952	0.302	0.256	0.737	0.730	0.983	0.971
	$\hat{H}$	0.426	0.431	0.871	0.879	0.992	0.996	0.301	0.261	0.643	0.615	0.946	0.944	0.305	0.261	0.724	0.707	0.971	0.964
	$\hat{C}$	0.396	0.404	0.845	0.867	0.991	0.995	0.307	0.273	0.678	0.641	0.961	0.955	0.310	0.273	0.750	0.735	0.984	0.972

[a] 1000 iterations; 5% significance level. BCV and ECV denote results based on bootstrap critical values and empirical critical values, respectively; c denotes the scaling parameter in the rule-of-thumb bandwidth selection, $h = c(1/\sqrt{12})T^{-1/5}$; CV denotes the cross-validation method.

[b] $\hat{H}$ and $\hat{C}$ are the generalized Hausman and Chow tests; *-het denotes the heteroscedasticity-robust version of the corresponding * test.

TABLE VIII—*Continued*

		DGP P.4 Nonmonotonic Smooth Changes						DGP P.5 Intercept and Slope					
		100		250		500		100		250		500	
	Test[b]	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV	BCV	ECV
$c = 1/1.5$	$\hat{H}$-het	0.405	0.358	0.823	0.834	0.993	0.994	0.599	0.553	0.986	0.987	1.00	1.00
	$\hat{C}$-het	0.331	0.302	0.706	0.723	0.962	0.973	0.538	0.495	0.979	0.975	1.00	1.00
	$\hat{H}$	0.444	0.380	0.838	0.841	0.993	0.995	0.627	0.569	0.989	0.989	1.00	1.00
	$\hat{C}$	0.364	0.310	0.734	0.746	0.966	0.980	0.573	0.520	0.979	0.979	1.00	1.00
$c = 1$	$\hat{H}$-het	0.484	0.443	0.889	0.884	0.997	0.998	0.636	0.617	0.988	0.983	1.00	1.00
	$\hat{C}$-het	0.406	0.378	0.799	0.786	0.984	0.992	0.560	0.555	0.983	0.982	1.00	1.00
	$\hat{H}$	0.512	0.452	0.900	0.886	0.997	0.998	0.654	0.626	0.989	0.987	1.00	1.00
	$\hat{C}$	0.428	0.401	0.809	0.793	0.985	0.992	0.589	0.564	0.985	0.985	1.00	1.00
$c = 1.5$	$\hat{H}$-het	0.499	0.437	0.901	0.910	0.998	0.999	0.658	0.615	0.983	0.983	1.00	1.00
	$\hat{C}$-het	0.453	0.437	0.851	0.867	0.994	0.998	0.602	0.597	0.979	0.971	1.00	1.00
	$\hat{H}$	0.511	0.449	0.905	0.909	0.999	0.999	0.669	0.631	0.984	0.987	1.00	1.00
	$\hat{C}$	0.478	0.445	0.861	0.869	0.994	0.997	0.618	0.601	0.983	0.972	1.00	1.00
$c = 2$	$\hat{H}$-het	0.440	0.395	0.885	0.891	1.00	0.999	0.619	0.609	0.974	0.977	1.00	0.999
	$\hat{C}$-het	0.470	0.425	0.879	0.900	0.996	0.999	0.582	0.578	0.967	0.964	1.00	0.998
	$\hat{H}$	0.459	0.407	0.882	0.894	0.998	0.999	0.626	0.631	0.977	0.982	1.00	0.999
	$\hat{C}$	0.476	0.440	0.882	0.901	0.996	0.998	0.593	0.598	0.968	0.968	1.00	0.998
CV	$\hat{H}$-het	0.384	0.348	0.824	0.838	0.987	0.995	0.605	0.620	0.982	0.989	1.00	1.00
	$\hat{C}$-het	0.408	0.387	0.848	0.875	0.996	0.998	0.594	0.626	0.981	0.990	1.00	1.00
	$\hat{H}$	0.405	0.357	0.830	0.840	0.989	0.995	0.626	0.640	0.983	0.990	1.00	1.00
	$\hat{C}$	0.437	0.402	0.858	0.883	0.996	0.998	0.615	0.640	0.982	0.990	1.00	1.00