Econometrica Supplementary Material

SUPPLEMENT TO "BUY, KEEP, OR SELL: ECONOMIC GROWTH AND THE MARKET FOR IDEAS"

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THIS SUPPLEMENT CONTAINS TWO SECTIONS, namely Appendix A and B. Appendix A deals with theoretical aspects of the analysis. In particular, the full solution for the symmetric balanced growth is provided. Appendix B pertains to the empirical work. This section describes the databases that are used and discusses how they are cleaned and linked together. The construction of the distance metrics and patent stock measures used in the analysis are then detailed. The empirical section also repeats the panel data regression analysis reported in Table III when the licensing intensity of a sector is included. Last, the Jacobian associated with the calibration procedure is presented.

APPENDIX A: THEORY APPENDIX

A.1. *Balanced Growth*

The analysis is restricted to studying a symmetric balanced growth path. The solution to the economy along a balanced growth path will now be characterized.[21]. Suppose that mean level of productivity for firms, $\mathbf{z}$, grows at the constant gross rate $\mathbf{g}$. Specify the variables z and $\mathbf{z}$ in transformed form so that $\widetilde{\mathbf{z}} = \mathbf{z}^{\zeta/(\zeta+\lambda)}$ and $\widetilde{z} = z/\mathbf{z}^{\lambda/(\zeta+\lambda)}$. Thus, $\widetilde{\mathbf{z}}$ grows at rate $\mathbf{g}^{\zeta/(\zeta+\lambda)}$ and, on average, so will $\widetilde{z}$. It turns out that $\widetilde{\mathbf{z}}$ (or equivalently, $\mathbf{z}$) is sufficient to characterize the aggregate state of the economy along a balanced growth path. It also turns out that the form of the distribution for d-type patent buyers, or G, does not matter.

PROPOSITION 1—Balanced Growth: *There exists a symmetric balanced growth path of the following form*:

1. *The interest factor*, r, *and rental rate on capital*, $\widetilde{r}$, *are given by* (22) *and* (23).
2. *The value functions for buying*, *keeping*, *and selling firms have linear forms in the state variables* $\widetilde{z}$ *and* $\widetilde{\mathbf{z}}$. *Specifically*, $B(z;\mathbf{z}) = \mathfrak{b}_1\widetilde{z} + \mathfrak{b}_2\widetilde{\mathbf{z}}$, $K(z+\gamma_d x\mathbf{z};\mathbf{z}) = \mathfrak{k}_1\widetilde{z} + \mathfrak{k}_2(x)\widetilde{\mathbf{z}}$, *and* $S(z;\mathbf{z}) = \mathfrak{s}_1\widetilde{z} + \mathfrak{s}_2\widetilde{\mathbf{z}}$.
3. *The indicator function for an innovator specifies a threshold rule such that* $I_k(z,x;\mathbf{z}) = 1$, *whenever* $x > x_k$, *and is zero otherwise*. *That is*, *an innovating firm keeps its d-type idea when* $x > x_k$ *and sells otherwise*.

[21]A simplified version of the model with a closed-form solution was provided in Akcigit, Celik, and Greenwood (2015, Appendix 12)

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4. *The indicator function for a sale between a buyer and the patent agent for a d-type idea specifies a threshold rule such that* $I_a(z, x; \mathbf{z}) = 1$, *whenever* $x > x_a$, *and is zero otherwise. That is, a sale between a buyer and a patent agent occurs if and only if* $x > x_a$.

5. *The value function for a patent agent has the linear form* $A(\mathbf{z}) = \mathfrak{a}\widetilde{\mathbf{z}}$.

6. *The beginning-of-period value function for a firm has the linear form* $V(z; \mathbf{z}) = \mathfrak{v}_1 \widetilde{z} + \mathfrak{v}_2 \widetilde{\mathbf{z}}$. *The constant rate of innovation for a d-type idea by a firm is*

$$i = \mathbf{i} = \left\{ \frac{1}{\chi} \left[X(x_k)\mathfrak{s}_2 + \int_{x_k}^{1} \mathfrak{k}_2(x)\, dX(x) - \mathfrak{b}_2 \right] \right\}^{1/\rho}. \tag{25}$$

7. *The constant net rate of growth for aggregate productivity*, $\mathbf{g} - \mathbf{1}$, *is implicitly given by*

$$\mathbf{g} - \mathbf{1} = \gamma_d \left[\mathbf{i} \int_{x_k}^{1} x\, dX(x) + (1 - \mathbf{i}) m_b \left(\frac{n_a}{n_b} \right) \int_{x_a}^{1} x\, dx \right] + \gamma_n \mathfrak{p}, \tag{26}$$

with the aggregate law of motion (3) *taking the simple form*

$$\mathbf{z}' = \mathbf{g}\mathbf{z}.$$

8. *The prices for selling and buying d-type patents are*

$$q = \mathfrak{a}\widetilde{\mathbf{z}},$$

and

$$P(z, x; \mathbf{z}) = \left[(1 - \omega)\sigma r \mathbf{g}^{\zeta/(\zeta+\lambda)} \mathfrak{a} + \omega \left(\pi + r \mathfrak{v}_1 / \mathbf{g}^{\lambda/(\zeta+\lambda)} \right) \gamma_d x \right] \widetilde{\mathbf{z}},$$

where π *is a constant.*

9. *The matching probabilities for sellers and buyers of d-type patents are constant and implicitly defined by*

$$m_a \left(\frac{n_a}{n_b} \right) = \eta \left\{ \frac{ \left\{ 1 - \sigma \left[1 - m_a \left(\frac{n_a}{n_b} \right) (1 - x_a) \right] \right\} (1 - \mathbf{i}) }{ \sigma \mathbf{i} X(x_k) } \right\}^{1-\mu}, \tag{27}$$

and

$$m_b \left(\frac{n_a}{n_b} \right) = \eta \left\{ \frac{ \sigma \mathbf{i} X(x_k) }{ \left\{ 1 - \sigma \left[1 - m_a \left(\frac{n_a}{n_b} \right) (1 - x_a) \right] \right\} (1 - \mathbf{i}) } \right\}^{\mu}. \tag{28}$$

10. *The constants* $\mathfrak{a}$, $\mathfrak{b}_1$, $\mathfrak{b}_2$, $\mathfrak{k}_1$, π, $\mathfrak{s}_1$, $\mathfrak{s}_2$, $\mathfrak{v}_1$, $\mathfrak{v}_2$, x_a, *and* x_k, *as well as the linear term* $\mathfrak{k}_2(x)$, *are determined by a nonlinear equation system, in conjunction with*

the five equations (22), (25), (26), (27), *and* (28) *that determine the five variables* $\mathbf{g}$, $\mathbf{i}$, r, $m_a(n_a/n_b)$, *and* $m_b(n_a/n_b)$. *This system of nonlinear equations does not involve either* $\widetilde{z}$ *or* $\widetilde{\mathbf{z}}$.

Along a balanced growth path, wages grow at the constant gross rate $\mathbf{g}^{\zeta/(\zeta+\lambda)}$, a fact evident from equation (20). So will aggregate output and profits, as can be seen from (7). The gross interest rate, $1/r$, will remain constant along a balanced growth path. Point (2) implies that, on average, the values of the firm at the buying, selling, and keeping stages also grow at the rate of growth of output. So, the relative values of a firm at these stages remain constant in a balanced growth equilibrium. Thus, it is not surprising then that the decisions to buy, sell, or keep d-type patents in terms of propinquity, x, do not change over time. Hence, the function $I_k(z, x; \mathbf{z})$ does not depend on $\mathbf{z}$. It may seem surprising that the decision does not depend on z, either. This transpires because a firm's profits are linear in z, as equation (7) shows. It turns out that $\mathfrak{k}_1 = \mathfrak{s}_1$, which implies that only x is relevant (when comparing $\mathfrak{k}_1\widetilde{z} + \mathfrak{k}_2(x)\widetilde{\mathbf{z}}$ with $\mathfrak{s}_1\widetilde{z} + \mathfrak{s}_2\widetilde{\mathbf{z}}$). Likewise, the value of a patent agent also increases at rate $\mathbf{g}^{\zeta/(\zeta+\lambda)}$—point (3). Hence, equation (21) dictates that the price, q, at which a firm can sell a d-type patent must also grow at this rate. Additionally, it is easy to see from (16) that the price at which the agent sells a d-type patent to firms, p, will appreciate at this rate, too. Note that this price does not depend on z, because given the linear form of the value function, V, only x will be relevant (when comparing $\mathfrak{v}_1 z'$ with $\mathfrak{v}_1 z$). Additionally, using (17), it should now not be too difficult to see that the function $I_a(z, x; \mathbf{z})$ will only depend on x. It is easy to deduce from equation (14) that the rate of innovation, $\mathbf{i}$, will be constant over time if B, K, and S grow at the same rate as aggregate output. Since the decisions to buy and sell patents only depend on x, it is straightforward that the number of buyers and sellers on the patent market are fixed along a balanced growth path. To see that the form for the distribution function over buyers, $G(z)$, does not matter, note that this function only enters the value function for the patent agent (15). But, by points (4) and (8), the functions $I_a(z, x; \mathbf{z})$ and $P(z, x; \mathbf{z})$ do not depend on z. Thus, $G(z)$ is irrelevant in (15). Last, the evolution of shape of the distribution function Z over time does not matter for the analysis. Its mean grows at the gross rate $\mathbf{g}$, independently of any transformation in shape.

PROOF OF THE EXISTENCE OF A BALANCED GROWTH PATH: The proof proceeds using a guess and verify procedure (or the method of undetermined coefficients).

Point (1). To derive the interest factor and rental rate, r and $\widetilde{r}$, imagine the problem of a consumer/worker who can invest in one period bonds that pay a gross interest rate of $1/r$. The Euler equation for asset accumulation will read

$$c^{-\varepsilon} = (\beta/r)\left(c'\right)^{-\varepsilon}.$$

Along a balanced growth path, if the mean level of productivity grows at rate $\mathbf{g}$, then consumption, the capital stock, and output must grow at rate $\mathbf{g}^{\zeta/(\zeta+\lambda)}$. This fact can be gleaned from the production function (1), by assuming z grows at rate $\mathbf{g}$, that capital and output grow at another common rate, and that labor remains constant. Therefore, $r = \beta/\mathbf{g}^{\varepsilon\zeta/(\zeta+\lambda)}$. In standard fashion, the rental rate on capital is given by $\widetilde{r} = 1/r - 1 + \delta = \mathbf{g}^{\varepsilon\zeta/(\zeta+\lambda)}/\beta - 1 + \delta$.

Point (4). The form of the threshold rule for buying a d-type patent follows from the fact that the sum of the surplus (sans price) accruing to a firm that buys a patent and the surplus (sans price) to the patent agent must be greater than zero; otherwise, a nonnegative sale price, p, for the d-type patent would not exist. First, plug the solutions for w and $\widetilde{r}$, or (20) and (23), into the profit function (7) to obtain

$$e'\Pi(z, \mathbf{z}) = \pi \frac{e'z}{\mathbf{z}^{\lambda/(\zeta+\lambda)}} = \pi e'\widetilde{z}, \tag{29}$$

and

$$E\big[e'\Pi(z, \mathbf{z})\big] = \pi\widetilde{z}, \quad \text{since} \quad E\big[e'\big] = 1,$$

with

$$\pi \equiv \frac{\zeta}{\mathbf{g}^{\lambda/(\zeta+\lambda)}}\left(\frac{\kappa}{\mathbf{g}^{\varepsilon\zeta/(\zeta+\lambda)}/\beta + \delta - 1}\right)^{\kappa/(\zeta+\lambda)}. \tag{30}$$

Second, conjecture that the value functions $V(z; \mathbf{z})$ and $A(s)$ have the forms $V(z; \mathbf{z}) = \mathfrak{v}_1\widetilde{z} + \mathfrak{v}_2\widetilde{\mathbf{z}}$ and $A(s) = \mathfrak{a}\widetilde{\mathbf{z}}$. Third, given the above, note that the (sans price) surpluses for a buying firm and the patent agent are given by

$$\begin{aligned} &\pi(\widetilde{z} + \gamma_d x\widetilde{\mathbf{z}}) - \pi\widetilde{z} + rE\big[V\big(z + \gamma_d x\mathbf{z}, \mathbf{z}'\big)\big] - rE\big[V\big(z, \mathbf{z}'\big)\big] \\ &\quad = \left(\pi + \frac{r\mathfrak{v}_1}{\mathbf{g}^{\lambda/(\zeta+\lambda)}}\right)\gamma_d x\widetilde{\mathbf{z}}, \end{aligned}$$

and

$$-\sigma rA\big(\mathbf{z}'\big) = -\sigma r\mathbf{g}^{\zeta/(\zeta+\lambda)}\mathfrak{a}\widetilde{\mathbf{z}} \quad \text{(cf. (17))}.$$

It is easy to deduce from (16) and (17) that the sum of these two quantities must be positive for a trade to take place. Note that whether or not the sum of the above two equations is nonnegative does not depend on $\widetilde{\mathbf{z}}$. This sum is also increasing in x. Solving for the value of x that sets the sum to zero yields

$$x_a = \frac{\sigma r\mathbf{g}^{\zeta/(\zeta+\lambda)}\mathfrak{a}}{\big(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\big)\gamma_d}. \tag{31}$$

Thus, x_a is a constant.

Point (8). The solutions for d-type patent prices, q and $P(z, x; \mathbf{z})$, are easy to obtain. Insert the above formulae for the (sans price) surplus for a buying firm and the (sans price) surplus for a patent agent into expression (16) to get

$$P(z, x; \mathbf{z}) = \left[\omega\left(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\right)\gamma_d x + (1-\omega)\sigma r \mathbf{g}^{\zeta/(\zeta+\lambda)}\mathfrak{a}\right]\widetilde{\mathbf{z}}.$$

It is immediate from (21) that $q = \mathfrak{a}\widetilde{\mathbf{z}}$, predicated upon the guess $A(\mathbf{z}) = \mathfrak{a}\widetilde{\mathbf{z}}$.

Point (5). It will now be shown that the value function for the patent agent, $A(\mathbf{z})$, has the conjectured linear form. Focus on equation (15), which specifies the solution for A. The price for a d-type patent does not depend on z, given point (8). Additionally, $D(x) = U[0, 1]$. Furthermore, $I_a = 1$ for $x > x_a$ and is zero otherwise. Thus,

$$\begin{aligned} A(\mathbf{z}) &= \mathfrak{a}\widetilde{\mathbf{z}} \\ &= m_a(n_a/n_b)\int_{x_a}^{1} P(z, x; \mathbf{z})\,dx \\ &\quad + \left[1 - m_a(n_a/n_b)Pr(x \geq x_a)\right]\sigma r A\left(\mathbf{z}'\right), \end{aligned}$$

from which it follows that

$$\begin{aligned} \mathfrak{a} &= \sigma r \mathbf{g}^{\zeta/(\zeta+\lambda)}\mathfrak{a} - m_a(n_a/n_b)(1-x_a)\omega\sigma r \mathbf{g}^{\zeta/(\zeta+\lambda)}\mathfrak{a} \\ &\quad + m_a(n_a/n_b)\omega\left(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\right)\gamma_d(1-x_a)(1+x_a)/2. \end{aligned} \tag{32}$$

Point (2). The value function for a buying firm, $B(z; \mathbf{z})$, can be determined in a manner similar to that for A in point (5). Here

$$B(z; \mathbf{z}) = \mathfrak{b}_1\widetilde{z} + \mathfrak{b}_2\widetilde{\mathbf{z}},$$

with

$$\mathfrak{b}_1 = \pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}, \tag{33}$$

and

$$\begin{aligned} \mathfrak{b}_2 &= -m_b(n_a/n_b)(1-x_a)(1-\omega)\sigma r \mathbf{g}^{\zeta/(\zeta+\lambda)}\mathfrak{a} + r\mathfrak{v}_2\mathbf{g}^{\zeta/(\zeta+\lambda)} \\ &\quad + m_b(n_a/n_b)(1-\omega)\left(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\right)\gamma_d(1-x_a)(1+x_a)/2 \\ &\quad + \left(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\right)\gamma_n\mathfrak{p}. \end{aligned} \tag{34}$$

To derive this solution, the results in points (4) and (8), along with the conjectured solution for V, are used in equation (8). Similarly, using equation (11), it can be shown that the value function for a seller, $S(z; \mathbf{z})$, is given by

$$S(z; \mathbf{z}) = \mathfrak{s}_1\widetilde{z} + \mathfrak{s}_2\widetilde{\mathbf{z}},$$

with

$$\mathfrak{s}_1 = \pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}, \tag{35}$$

and

$$\mathfrak{s}_2 = \sigma\mathfrak{a} + r\mathfrak{v}_2\mathbf{g}^{\zeta/(\zeta+\lambda)} + \left(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\right)\gamma_n\mathfrak{p}. \tag{36}$$

Last, following from (10), a keeper's value function can be written as

$$K(z + \gamma_d x\mathbf{z}; \mathbf{z}) = \mathfrak{k}_1\widetilde{z} + \mathfrak{k}_2(x)\widetilde{\mathbf{z}},$$

with

$$\mathfrak{k}_1 = \pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}, \tag{37}$$

and

$$\mathfrak{k}_2(x) = \left(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\right)\gamma_d x + r\mathfrak{v}_2\mathbf{g}^{\zeta/(\zeta+\lambda)} + \left(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\right)\gamma_n\mathfrak{p}. \tag{38}$$

Point (3). The threshold rule for keeping or selling a d-type patent is determined by the condition

$$\mathfrak{k}_1\widetilde{z} + \mathfrak{k}_2(x_k)\widetilde{\mathbf{z}} = \mathfrak{s}_1\widetilde{z} + \mathfrak{s}_2\widetilde{\mathbf{z}};$$

that is, at the threshold, a firm is indifferent between keeping or selling the patent. Now, $\mathfrak{s}_1 = \mathfrak{k}_1$ so that

$$\begin{aligned}&\left(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\right)\gamma_d x_k + r\mathfrak{v}_2\mathbf{g}^{\zeta/(\zeta+\lambda)} + \left(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\right)\gamma_n\mathfrak{p}\\ &\quad = \sigma\mathfrak{a} + r\mathfrak{v}_2\mathbf{g}^{\zeta/(\zeta+\lambda)} + \left(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\right)\gamma_n\mathfrak{p}.\end{aligned}$$

Hence,

$$x_k = \frac{\sigma\mathfrak{a}}{\left[\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\right]\gamma_d}, \tag{39}$$

a constant.

Point (6). Turn now to the beginning-of-period value function for the firm, $V(z; \mathbf{z})$, and the rate of innovation, i, that it will choose. By using the linear forms for the value functions $B(z; \mathbf{z})$, $K(z + \gamma_d x\mathbf{z}; \mathbf{z})$, and $S(z; \mathbf{z})$, the fact that $\mathfrak{b}_1 = \mathfrak{k}_1 = \mathfrak{s}_1$, and the property that the threshold rule takes the form $I_k = 1$ for $x > x_k$ and $I_k = 0$ otherwise, the firm's dynamic programming problem (13) can be rewritten as

$$\begin{aligned}V(z; \mathbf{z}) = \widetilde{\mathbf{z}}\max_{i\in[0,1]}&\left\{\left[X(x_k)\mathfrak{s}_2 + \int_{x_k}^1 \mathfrak{k}_2(x)\,dX(x) - \mathfrak{b}_2\right]i - \frac{\chi}{1+\rho}i^{1+\rho}\right\}\\ &+ \left(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\right)\widetilde{z} + \mathfrak{b}_2\widetilde{\mathbf{z}}.\end{aligned}$$

Differentiating with respect to i then gives

$$X(x_k)\mathfrak{s}_2 + \int_{x_k}^{1} \mathfrak{k}_2(x)\,dX(x) - \mathfrak{b}_2 = \chi i^{\rho},$$

from which (25) follows. Using the solution for i, as given by (25), in the above programming problem yields

$$V(z;\mathbf{z}) = \frac{\rho}{(1+\rho)\chi^{1/\rho}}\left[X(x_k)\mathfrak{s}_2 + \int_{x_k}^{1} \mathfrak{k}_2(x)\,dX(x) - \mathfrak{b}_2\right]^{1+1/\rho}\widetilde{\mathbf{z}} + \left(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\right)\widetilde{z} + \mathfrak{b}_2\widetilde{\mathbf{z}}.$$

It then follows that

$$\text{(40)} \qquad \mathfrak{v}_1 = \frac{\mathbf{g}^{\lambda/(\zeta+\lambda)}}{\mathbf{g}^{\lambda/(\zeta+\lambda)} - r}\pi,$$

and

$$\text{(41)} \qquad \mathfrak{v}_2 = \mathfrak{b}_2 + \frac{\rho}{(1+\rho)\chi^{1/\rho}}\left[X(x_k)\mathfrak{s}_2 + \int_{x_k}^{1} \mathfrak{k}_2(x)\,dX(x) - \mathfrak{b}_2\right]^{1+1/\rho}.$$

Point (7). The gross rate of growth for aggregate productivity, $\mathbf{g}$, will now be calculated. Suppose that aggregate productivity is currently $\mathbf{z}$. A fraction $\mathbf{i}$ of firms will innovate today. Those firms that draw $x > x_k$ will keep their good patent. The productivity for these firms will increase. The fraction $1-\mathbf{i}$ of firms will fail to innovate. Out of these firms, the proportion $m_b(n_a/n_b)$ will find a seller on the market for d-type patents. They will buy a d-type patent when $x > x_a$. Thus, $\mathbf{z}$ will evolve according to

$$\mathbf{z}' = \mathbf{z} + \mathbf{i}\int_{x_k}^{1} \gamma_d x\mathbf{z}\,dX(x) + m_b(n_a/n_b)(1-\mathbf{i})\int_{x_a}^{1} \gamma_d x\mathbf{z}\,dx + \gamma_n\mathfrak{p}\mathbf{z}.$$

This implies (26).

Point (9). The number of buyers on the market for d-type patents is given by $n_b = 1-\mathbf{i}$; all firms that fail to innovate will try to buy a d-type patent. Along a symmetric balanced growth path, the number of patent agents, n_a, must satisfy the equation

$$n_a = \sigma n_a\left[1 - m_a(n_a/n_b)(1-x_a)\right] + \sigma\mathbf{i}X(x_k).$$

Focus on the right-hand side. Take the first term. Suppose that there are n_a patent agents at the beginning of the period. A fraction σ of these agents will survive into next period. Out of these, $m_b(n_a/n_b)(1-x_a)$ will find a buyer.

Thus, they will not be around these next period. Move to the second term. A mass of $\mathbf{i}X(x_k)$ new firms will decide to sell their patents. Out of this, the fraction σ will survive. The sum of these two terms equals the new stock of patent for sale, n_a. Solving yields

$$n_a = \frac{\sigma \mathbf{i} X(x_k)}{1-\sigma\big[1-m_a(n_a/n_b)(1-x_a)\big]}, \quad \text{and}$$
$$\frac{n_a}{n_b} = \frac{\sigma \mathbf{i} X(x_k)}{(1-\mathbf{i})\big\{1-\sigma\big[1-m_a(n_a/n_b)(1-x_a)\big]\big\}}.$$

Equations (27) and (28) follow immediately.

Point (10). The 12 constants, viz. $\mathfrak{a}$, $\mathfrak{b}_1$, $\mathfrak{b}_2$, $\mathfrak{k}_1$, π, $\mathfrak{s}_1$, $\mathfrak{s}_2$, $\mathfrak{v}_1$, $\mathfrak{v}_2$, x_a, and x_k, in conjunction with the linear term $\mathfrak{k}_2(x)$, are specified by the 12 nonlinear equations (30) to (41). The equations include the variables $\mathbf{g}$, $\mathbf{i}$, r, $m_a(n_a/n_b)$, and $m_b(n_a/n_b)$. So, equations (22), (25), (26), (27), and (28) must be appended to the system to obtain a system of 17 equations in 17 unknowns. This system does not depend on either $\widetilde{z}$ or $\widetilde{\mathbf{z}}$. *Q.E.D.*

A.2. *More on Tacking on a Market for n-Type Patents*

The discussion in Section 3.4 is completed here. An n-type idea is worth $(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)})\gamma_n\widetilde{\mathbf{z}}$ in production value to a firm.[22] Specifically, it will increase z' by $\gamma_n\mathbf{z}$. This will lead to increase in current profits in the amount $\pi\gamma_n\widetilde{\mathbf{z}}$ and discounted expected future profits by $r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)}\gamma_n\widetilde{\mathbf{z}}$. Any price, q_b, in the interval $[0, (\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)})\gamma_n\widetilde{\mathbf{z}}]$ can be an equilibrium market price on the market for n-type patents. The exact value for q_b does not matter, though. At the time of all decision making, the expected discounted present value of profits arising from an n-type patent is $\mathfrak{p}[(1-\mathfrak{p}_s)(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)})\gamma_n\widetilde{\mathbf{z}} + \mathfrak{p}_s q_b] + (1-\mathfrak{p})\mathfrak{p}_b[(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)})\gamma_n\widetilde{\mathbf{z}} - q_b]$, which takes into account the keeping, selling, and buying events, respectively. This expression reduces to $\mathfrak{p}(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)})\gamma_n\widetilde{\mathbf{z}}$, using the fact that $\mathfrak{p}\mathfrak{p}_s = (1-\mathfrak{p})\mathfrak{p}_b$. Thus, the expected discounted present value of profits associated with an n-type patent does not involve the equilibrium market price, q_b, or the buying and selling probabilities, $\mathfrak{p}_b$ and $\mathfrak{p}_s$. Therefore, adding a market for n-type patents does not alter the solution for the balanced growth path presented in Proposition 1.

[22]In Section A.1, it is shown that the value functions for buying, keeping, selling, and innovating firms are linear in the expected value of a new n-type idea, as can be seen by examining the coefficients, $\mathfrak{b}_2$, $\mathfrak{k}_2(x)$, $\mathfrak{s}_2$, and $\mathfrak{v}_2$. The terms in question all have the form $(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)})\gamma_n\mathfrak{p}$, implying that the production value of an n-type idea is $(\pi + r\mathfrak{v}_1/\mathbf{g}^{\lambda/(\zeta+\lambda)})\gamma_n\widetilde{\mathbf{z}}$—see (34), (36), (38), and (41).

APPENDIX B: EMPIRICAL APPENDIX

The brunt of the analysis relies on data from three sources: the USPTO, the NBER Patent Database Project (PDP), and Compustat. The first source contains information on patents that are reassigned across firms. The second is used to retrieve information on the technology classes for patents and the stocks of patents for firms. Facts about the employments and stock market values for publicly traded U.S. firms are obtained from the third source.

B.1. *Patent Reassignment Data (PRD)*

The patent reassignment data are obtained from the publicly available U.S. Patent and Trademark Office (USPTO) patent assignment files hosted by Google Patents Beta. These files contain all records of changes made to U.S. patents for the years 1980–2011. The files are parsed and combined to create the data set. The following variables are kept:

- Patent number: The unique patent number assigned to each patent granted by the USPTO.
- Record date: The date of creation for the record.
- Execution date: The date for the legal execution of the record.
- Conveyance text: A text variable describing the reason for the creation of the record. Examples are: "Assignment of assignor's interest," "Security Agreement," "Merger," etc.
- Assignee: The name of the entity assigning the patent (i.e., the seller if the patent is being sold).
- Assignor: The name of the entity to which the patent is being assigned (i.e., the buyer if the patent is being sold).
- Patent application date: The date of application for the patent.
- Patent grant date: The date of grant for the patent.
- Patent technology class: The technology class assigned to the patent by the USPTO according to its internal classification system.[23]

The entries for which this information is inaccessible are dropped from the sample.

During the parsing process, the following are done:

- Only transfer agreements between companies are kept.
- Only utility patents are kept; entries regarding design patents are dropped.

This cleaning process leaves 966,427 observations. Using the string variable that states the reason for the record, all reassignments that are not directly related to sales are dropped (for instance, mergers, license grants, splits, mortgages, court orders, etc.).

In order to create an even more conservative indicator of patent sales, a company name-matching algorithm is employed, so that marking internal

[23]This variable is not used, however, to represent the technology class for a patent, as is discussed below.

transfers as sales can be avoided, where patents are moved within the same firm, or between the subsidiaries of the firm. The idea behind the company name-matching algorithm is to clean the string variables for the assignor and the assignee of all unnecessary indicators and company type abbreviations. If the cleaned assignor and assignee strings are equal, the type of the record is changed to internal transfer, provided that it was identified as a reassignment before.

The pseudo-code for the algorithm, an enhanced version of Kerr and Fu (2006), is as follows:

(i) All letters are made upper case.

(ii) The portion of the string after the first comma is deleted. (e.g., AMF INCORPORATED, A CORP OF N.J. becomes AMF INCORPORATED).

(iii) If the string starts with "THE ," the first four characters are deleted.

(iv) All non-alphanumeric characters are removed.

(v) Trailing company identifiers are deleted if found. The string goes through this process five times. The company identifiers are the following: B, AG, BV, CENTER, CO, COMPANY, COMPANIES, CORP, CORPORATION, DIV, GMBH, GROUP, INC, INCORPORATED, KG, LC, LIMITED, LIMITED PARTNERSHIP, LLC, LP, LTD NV, PLC, SA, SARL, SNC, SPA, SRL, TRUST, USA, KABUSHIKI, KAISHA, AKTIENGESELLSCHAFT, AKTIEBOLAG, SE, CORPORATIN, CORPORATON, TRUST, GROUP, GRP, HLDGS, HOLDINGS, COMM, INDS, HLDG, TECH, and GAISHA.

(vi) If the resulting string has length zero, that string is declared as needing protection. Some examples that are protected by this procedure: "CORPORATION, ORACLE," "KAISHA, ASAHI KAISEI KABUSHIKI," "LIMITED, ZELLWEGER ANALYTICS."

(vii) The algorithm is re-run from the beginning on the original strings with one difference: The strings that are declared as needing protection skip the second step.

B.2. *USPTO Utility Patents Grant Data (PDP)*

The patent grant data come from the NBER Patent Data Project (PDP), and contain data for all the utility patents granted between the years 1976 and 2006. How the PDP and PRD are linked to each other is discussed later on.

B.3. *Compustat North American Fundamentals (Annual)*

The Compustat data for publicly traded firms in North America between the years 1974 and 2006 are retrieved from Wharton Research Data Services. The Compustat database and the NBER PDP database are connected using the matching procedure provided alongside the PDP data. Extensive information on how the matching is done can be found on the project website.

B.4. *Connecting PRD and PDP Data*

There are two different questions of interest, which require combining the Patent Database Project data with the Patent Reassignment Data. The first question concerns whether a patent is ever sold over its entire lifetime, and what determines the likelihood of this event. For this purpose, it is only necessary to connect the information from the PRD to the firm that applied for the patent. This is easily done by using the unique patent number each patent is given by the USPTO.

The second question involves the change in the match quality of the patent when it is traded between two firms. In this case, one needs to know the characteristics of both the assignor and the assignee firms for each reassignment record in the PRD data set. However, there is no existing connection established between the PRD and PDP data sets. To connect these data sets, the company name-matching algorithm described earlier is employed.

B.5. *Variable Construction*

B.5.1. *Patent-to-Patent Distance Metric*

In order to construct a topology on the technology space empirically, it is necessary to create a distance metric between different technology classes. Such a metric enables one to speak about the distance between two patents in the technology space, and leads to the construction of more advanced metrics.

The first two digits of the IPC (International Patent Classification) codes for a patent are chosen to indicate its technology class. The IPC code used for a patent is taken from the PDP data and differs from the classification scheme employed in the PRD data. It should be noted that the PDP data set actually contains more than a single IPC class for a patent in some cases, since the IPC codes were assigned using a concordance between the IPC and the internal classification system of the USPTO. The IPC code provided in the PDP file with the assignees is used in such cases, which is unique for each patent.

As discussed in the main text, a plausible distance metric between patent classes can be generated by looking at how often two different technology classes are cited together. Formally,

$$d(X,Y) \equiv 1 - \frac{\#(X \cap Y)}{\#(X \cup Y)}, \quad \text{with} \quad 0 \le d(X,Y) \le 1,$$

where $\#(X \cap Y)$ denotes the number of patents that cite technology classes X and Y simultaneously, whereas $\#(X \cup Y)$ denotes the number of patents that cite X or Y or both.

B.5.2. *Definition of a Firm in the Data*

There are four different entity identifiers in the NBER PDP data set. The USPTO assignee number is the identifier provided by the USPTO itself, but the authors of the PDP data set have found that it is not very accurate. A single assignee might have many different USPTO assignee numbers. The PDP uses some matching algorithms on the names of the assignees to create a more accurate assignee identifier, called PDPASS. The authors also link the patent data to the Compustat data. Compustat has an identifier called GVKEY. However, these refer to securities rather than firms. So a single firm might be represented by many GVKEYs. For this reason, they use a dynamic matching algorithm to link all GVKEYs to certain PDPCOs, where the latter are unique firm identifiers that are created for the NBER PDP data set. The NBER project creates this identifier in order to be able to account for name changes, mergers & acquisitions, etc. The current work follows the same procedure for the market value regressions.

B.5.3. *Patent-to-Firm Distance Metric*

In order to measure how close a patent is to a firm in the technology space, a metric is necessary. However, throughout their lifetimes firms register patents in multiple technology classes. Hence the patent-to-patent distance metric is insufficient for this purpose. One possible way to construct a patent-to-firm distance metric is to compare a patent to the past patent portfolio of the firm. The distance measure between each patent a firm registered in the past, and the new patent in question can be calculated using the patent-to-patent distance metric described earlier. The distance between the firm and the patent should be a function of these separate distances. Equation (24) defines a parametric family of distance measures indexed by ι. The value for ι used in the baseline analysis is 2/3.

B.5.4. *Creating the Patent Stock Variable for Compustat Firms*

As argued in Hall, Jaffe, and Trajtenberg (2005), the citation-weighted patent portfolio of a firm is a plausible indicator of its intangible knowledge. The authors demonstrated that this measure has additional explanatory power for the market value of a firm above and beyond the conventional discounted sum of R&D spending, since R&D is a stochastic process that can succeed or fail, whereas patents are quantifiable products of this process when it is successful. Furthermore, it is revealed that the number of citations a patent receives is a fine indicator of the patent's worth, increasing the market value of a firm at an increasing rate as the number of citations go higher.

Since all the future citations to a patent cannot be observed at any given date, the citations variable suffers from a truncation problem. There are also technology class and year fixed effects to consider. All of these issues were thoroughly investigated by Hall, Jaffe, and Trajtenberg (2005); they provided a variable called hjtwt in order to correct the citation number of each patent in the PDP data set. This study uses their correction method. In the end, a corrected citation number for each patent is obtained. In order to create the patent stock variable for a firm (PDPCO), the corrected citation numbers for all of the patents of the firm are added together for each year. This variable is called the patent stock of a firm.

In addition to the patent stock, the corrected citation numbers across all of the patents for a firm, multiplied by the patent-to-firm distance generated at the date of the patent's inclusion into the portfolio, are also added together to create a new variable. This variable quantifies the overall waste in the patent stock caused by the mismatch between the technology classes of the patents and the firm. This variable has a negative effect on the market value of equity for a firm. The variable is called the distance-adjusted patent stock.

B.6. *Patent Sale Decision With Licensing Intensity*

Table VIII introduces the licensing intensity of the sector. This variable is available only for Compustat firms. Therefore, the sample is reduced by half. Because of this sizable change, columns 1–3 repeat the same exercises as their counterparts in Table III. One major difference to note is that the association between the distance and sale indicators becomes more pronounced, almost double. Column 4 introduces licensing intensity and column 5 includes the litigation and licensing controls simultaneously. The last column repeats the regression in column 1 while purging the patents that were not renewed once.

B.7. *The Impact of Parameter Values on the Data Targets*

Table IX presents the Jacobian associated with the calibration/estimation. This Jacobian provides useful information about how the parameters influence

TABLE VIII

PATENT SALE DECISION (COMPUSTAT SAMPLE WITH LICENSING INTENSITY)[a]

	Dependent Variable (= 1 if Sold, = 0 Otherwise)					
	(1)	(2)	(3)	(4)	(5)	(6)
Distance	3.737***	3.728***	3.741***	4.125***	4.123***	4.413***
	(0.138)	(0.138)	(0.138)	(0.142)	(0.142)	(0.157)
Tech-class litigation intensity	no	yes	no	no	yes	no
Patent litigation dummy	no	no	yes	no	yes	no
Sector licensing intensity	no	no	no	yes	yes	no
Only renewed patents	no	no	no	no	no	yes
Observations	1,151,348	1,151,348	1,151,348	1,078,735	1,078,735	919,421
R^2	0.32	0.32	0.32	0.32	0.32	0.34

[a]See the notes for Table III.

TABLE IX

CALIBRATION/ESTIMATION JACOBIAN (ELASTICITIES, %)[a]

Param	Growth	R&D/GDP	Frac. Sold	Avg. Dur.	Dur. c.v.	daps/ps	ps/emp	dist red, All
γ_d	74.39	47.39	−8.64	0.36	−0.18	15.71	−4.64	−4.12
χ	−17.18	−10.55	23.06	−4.81	2.42	−7.68	−23.37	4.67
μ	0.86	−1.41	9.63	−4.22	2.12	−4.27	−2.03	18.87
η	3.32	−5.45	37.25	−16.34	8.21	13.00	−8.60	73.06
γ_n	22.46	−24.81	5.66	−1.75	0.88	−17.79	−98.84	0.21
$\mathfrak{p}$	22.46	−24.81	47.46	−1.75	0.88	−3.74	−51.07	−71.53
$\mathfrak{p}_s$	0	0	64.57	0	0	18.55	−36.70	−71.74
STD($\ln e'$)	0	0	0	0	0	−3.84	225.95	0

[a]The data targets in the Jacobian follow the order in which they are presented in Table V.

the model's ability to hit the data targets. By moving along a row, one can see how a parameter in question influences the various data targets. Alternatively, by going down a column, one can gauge what parameters are important for hitting the data target of concern.

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