Econometrica Supplementary Material

SUPPLEMENT TO "MENU-DEPENDENT STOCHASTIC FEASIBILITY"

(*Econometrica*, Vol. 84, No. 3, May 2016, 1203–1223)

BY RICHARD L. BRADY AND JOHN REHBECK

APPENDIX B CONTAINS A DISCUSSION on an alternative form of conditioning for feasible set probabilities. Appendix C replaces IFO with a condition which restricts the support of π to sets of at most two elements. Appendix D examines differences between the regular Perception Adjusted Luce Model and an RCCSR.

APPENDIX B: ALTERNATIVE CONDITIONING

Recall that throughout our analysis, we have considered a model where the probability of obtaining a feasible set is given by

$$\Pr\bigl(F(A)=B\bigr)=\frac{\pi(B)}{\sum_{C\subseteq A}\pi(C)}.$$

However, there are other ways that one could condition to obtain a feasible set. In particular, one could consider the model given by

$$\Pr\bigl(F(A)=B\bigr)=\sum_{C\in\mathcal{D}:C\cap A=B}\pi(C),$$

where the default option is chosen if $B=\emptyset$. This alternative conditioning formula is used in Barberà and Grodal (2011) to characterize a preference for flexibility over menus.

We prefer the conditioning formula used in an RCCSR for two main reasons. First, suppose that a feasible set is generated by what items an agent considers from a menu. In this case, an RCCSR says an agent first looks at the menu, then considers a set of alternatives from the menu, and lastly makes a choice. If we use the alternative conditioning formula, it will change the timing of these actions. In particular, the alternative formulation says an agent first considers a set of alternatives, then looks at the menu and further restricts the considered objects, and finally makes a choice.[1] Therefore, in this alternative formulation, an agent could be thinking of a better/worse alternative when choosing from

[1]Using the "in the mood" interpretation, an RCCSR conditioning says a consumer sees the menu and draws a random mood which is consistent with the offered alternatives. In the alternative formulation, the consumer receives a mood before looking at the offered menu.

 DOI: 10.3982/ECTA12694

the menu. The alternative formulation also seems ill suited for the case of general feasibility. For example, it would seem surprising that the probability that an alternative is out of stock in a menu depends on alternatives not offered.

Second, we prefer the formulation used in an RCCSR for its identifiability and flexibility. The alternative formulation makes it difficult to identify π completely. In addition, this alternative conditioning formula produces choice probabilities consistent with a random utility model.

APPENDIX C: BINARY SUPPORT

We can also characterize some models which have limited support by replacing IFO with other conditions. Here, we still assume that $\mathcal{D}$ is rich.

BIFO—Binary Increasing Feasible Odds: For all distinct $a, b \in X$,

$$\Delta_a \Delta_b O_{\{a,b\}} > 0.$$

This condition restricts IFO to binary menus. In a consideration set framework, this would mean that adding acceptable alternatives draws consideration away from the default option.

CMD—Constant Marginal Differences: For all distinct $a, b \in X$ and $A, B \in \mathcal{D}$ with $a, b \in A \cap B$, then

$$\frac{P(a, A)}{P(x^*, A)} - \frac{P(a, A \setminus \{b\})}{P(x^*, A \setminus \{b\})} = \frac{P(a, B)}{P(x^*, B)} - \frac{P(a, B \setminus \{b\})}{P(x^*, B \setminus \{b\})}.$$

This condition states that the marginal effect on the odds ratio with respect to the default option of removing an alternative is constant across menus. Replacing IFO with these conditions, we get a model with $|X| + \binom{|X|}{2}$ parameters. We now define a *binary random choice set rule*.

DEFINITION C.1: A *binary random choice set rule* (BRCSR) is a random choice rule $P_{\succ,\alpha}$ for which there exists a pair $(\succ, \alpha)$, where $\succ$ is a strict preference ordering on X and $\alpha : \mathcal{D} \to [0, 1)$ a distribution with $\alpha(A) > 0$ for sets $A \in \mathcal{D}$ with $|A| \leq 2$ and zero otherwise, such that for all $A \in \mathcal{D}$ and for all $a \in A$,

$$P_{\succ,\alpha}(a, A) = \frac{\alpha(\{a\}) + \sum_{b \in A | a \succ b} \alpha(\{a, b\})}{\sum_{C \subseteq A : |C| \leq 2} \alpha(C)}.$$

THEOREM C.1: *A random choice rule satisfies ASI, TSI, ESI, BIFO, and CMD if and only if it is a BRCSR* $P_{\succ,\alpha}$. *Moreover, both* $\succ$ *and* α *are unique, that is, for any BRCSR with* $P_{\succ,\alpha} = P_{\succ',\alpha'}$, *we have that* $(\succ, \alpha) = (\succ', \alpha')$.

PROOF: That a BRCSR satisfies ASI, TSI, ESI, BIFO, and CMD is simple to check and is omitted here.

Now, suppose $|X| = N \geq 1$ and P is a random choice rule that satisfies ASI, TSI, ESI, BIFO, and CMD. By Lemma A.1 and $\mathcal{D}$ rich, we can define an ordering $\succ$ on X which is a total order. Let $M = \max_{A\in\mathcal{D}} |A|$ be the largest order of sets in $\mathcal{D}$. Let $D_M = \arg\max_{A\in\mathcal{D}} |A|$ be the elements of $\mathcal{D}$ with maximal order. We want to show that the $P(\cdot, \cdot)$ has the BRCSR representation. We prove the representation inductively on menu size.

First, define $\lambda : \mathcal{D} \to \mathbb{R}$ such that, for $A \in \mathcal{D}$, we have that

$$\lambda_A = \lambda(A) = \sum_{B\subseteq A} (-1)^{|A\setminus B|} \frac{1}{P(x^*, B)}.$$

This is related to a Möbius inversion formula. Theorem A.1 gives us that

$$\frac{1}{P(x^*, A)} = \sum_{B\subseteq A} \lambda_B.$$

First, note that for singleton menus $\{a\} \in \mathcal{D}$,

$$\lambda_{\{a\}} = \frac{1}{P(x^*, \{a\})} - 1 > 0,$$

since $P(x^*, \{a\}) < 1$ by definition of a random choice rule. Next, for binary menus $\{a, b\} \in \mathcal{D}$, assume without loss of generality that $a \succ b$. Then BIFO implies

$$\begin{aligned}
\Delta_a \Delta_b O_{\{a,b\}} &= \sum_{B\subseteq\{a,b\}:B\neq\emptyset} (-1)^{|\{a,b\}\setminus B|} \frac{P(B, B)}{P(x^*, B)} \\
&= \sum_{B\subseteq\{a,b\}} (-1)^{|\{a,b\}\setminus B|} + \sum_{B\subseteq\{a,b\}:B\neq\emptyset} (-1)^{|\{a,b\}\setminus B|} \frac{P(B, B)}{P(x^*, B)} \\
&= \sum_{B\subseteq\{a,b\}} (-1)^{|\{a,b\}\setminus B|} \left(1 + \frac{P(B, B)}{P(x^*, B)}\right) \\
&= \sum_{B\subseteq\{a,b\}} (-1)^{|\{a,b\}\setminus B|} \frac{1}{P(x^*, B)} > 0,
\end{aligned}$$

where we used that $\sum_{B\subseteq\{a,b\}} (-1)^{|\{a,b\}\setminus B|} = \sum_{i=0}^{2} (-1)^i \binom{2}{i} = 0$.

Therefore, we have

$$P\big(x^*, \{a, b\}\big)^{-1} - P\big(x^*, \{b\}\big)^{-1} - P\big(x^*, \{a\}\big)^{-1} + 1 > 0.$$

Thus we have

$$\lambda_{\{a,b\}} = \sum_{B \subseteq \{a,b\}} (-1)^{|\{a,b\} \setminus B|} \frac{1}{P\big(x^*, B\big)} > 0.$$

Now, we show a result on how the λ terms relate to $P(\cdot, \cdot)$ under CMD and then show, for all $A \in \mathcal{D}$ such that $|A| \geq 3$, that $\lambda_A = 0$. Note, for $A = \{a, b, c\}$ such that $a \succ b$ and $a \succ c$, then we have by CMD that

$$\frac{P(a, A)}{P\big(x^*, A\big)} - \frac{P\big(a, \{a, c\}\big)}{P\big(x^*, \{a, c\}\big)} = \frac{P\big(a, \{a, b\}\big)}{P\big(x^*, \{a, b\}\big)} - \frac{P\big(a, \{a\}\big)}{P\big(x^*, \{a\}\big)}.$$

First, looking at the left side of the equality and using Lemma 3.1,

$$\begin{aligned}
&\frac{P(a, A)}{P\big(x^*, A\big)} - \frac{P\big(a, \{a, c\}\big)}{P\big(x^*, \{a, c\}\big)} \\
&\quad = \frac{1 - P\big((A \setminus \{a\})^*, A\big)}{P\big(x^*, A\big)} - \frac{1 - P\big(\{c, x^*\}, \{a, c\}\big)}{P\big(x^*, \{a, c\}\big)} \\
&\quad = P\big(x^*, A\big)^{-1} - P\big(x^*, \{b, c\}\big)^{-1} - P\big(x^*, \{a, c\}\big)^{-1} + P\big(x^*, \{c\}\big)^{-1}.
\end{aligned}$$

Similarly, looking at the right side of the equality and using Lemma 3.1,

$$\begin{aligned}
&\frac{P\big(a, \{a, b\}\big)}{P\big(x^*, \{a, b\}\big)} - \frac{P\big(a, \{a\}\big)}{P\big(x^*, \{a\}\big)} \\
&\quad = P\big(x^*, \{a, b\}\big)^{-1} - P\big(x^*, \{b\}\big)^{-1} - P\big(x^*, \{a\}\big)^{-1} + 1 \\
&\quad = \lambda_{\{a,b\}}.
\end{aligned}$$

Rearranging the equality, we see that

$$\begin{aligned}
&P\big(x^*, A\big)^{-1} \\
&\quad = \lambda_{\{a,b\}} + P\big(x^*, \{b, c\}\big)^{-1} + P\big(x^*, \{a, c\}\big)^{-1} - P\big(x^*, \{c\}\big)^{-1} \\
&\quad = \lambda_{\{a,b\}} + \big(P\big(x^*, \{b, c\}\big)^{-1} - P\big(x^*, \{b\}\big)^{-1} - P\big(x^*, \{c\}\big)^{-1} + 1\big) \\
&\qquad + P\big(x^*, \{a, c\}\big)^{-1} + \big(P\big(x^*, \{b\}\big)^{-1} - 1\big) \\
&\quad = \lambda_{\{a,b\}} + \lambda_{\{b,c\}} + \lambda_{\{b\}} + P\big(x^*, \{a, c\}\big)^{-1}
\end{aligned}$$

$$\begin{aligned}
&= \lambda_{\{a,b\}} + \lambda_{\{b,c\}} + \lambda_{\{b\}} + \big(P\big(x^*, \{a, c\}\big)^{-1} \\
&\quad - P\big(x^*, \{a\}\big)^{-1} - P\big(x^*, \{c\}\big)^{-1} + 1\big) \\
&\quad + \big(P\big(x^*, \{a\}\big)^{-1} - 1\big) + \big(P\big(x^*, \{c\}\big)^{-1} - 1\big) + 1 \\
&= \sum_{B \subsetneq A} \lambda_B.
\end{aligned}$$

Therefore, we have that $\frac{1}{P(x^*, A)} = \sum_{B \subsetneq A} \lambda_B$ for all $|A| = 3$. However, using the Möbius inversion result, we know that

$$\sum_{B \subsetneq A} \lambda_B = \frac{1}{P\big(x^*, A\big)} = \sum_{B \subseteq A} \lambda_B \quad \Rightarrow \quad \lambda_A = 0.$$

Now, suppose that $\frac{1}{P(x^*, A)} = \sum_{B \subsetneq A} \lambda_B$ holds for sets $A \in \mathcal{D}$ with $|A| = m - 1$ and $3 \leq m - 1 < M$. For $A \in \mathcal{D}$ such that $|A| = m$ and $\forall c \in A \setminus \{a, b\}$ such that $a \succ b \succ c$, we have

$$\frac{P(a, A)}{P\big(x^*, A\big)} - \frac{P\big(a, A \setminus \{b\}\big)}{P\big(x^*, A \setminus \{b\}\big)} = \frac{P\big(a, \{a, b\}\big)}{P\big(x^*, \{a, b\}\big)} - \frac{P\big(a, \{a\}\big)}{P\big(x^*, \{a\}\big)}.$$

We can perform the same substitutions using Lemma 3.1 as in the three element case so

$$\begin{aligned}
&P\big(x^*, A\big)^{-1} - P\big(x^*, A \setminus \{a\}\big)^{-1} \\
&\quad - P\big(x^*, A \setminus \{b\}\big)^{-1} + P\big(x^*, A \setminus \{a, b\}\big)^{-1} = \lambda_{\{a,b\}}.
\end{aligned}$$

Since $A \setminus \{a\}$ and $A \setminus \{b\}$ are $m - 1$ element sets, we can use our induction step and then rearrange so

$$\begin{aligned}
P\big(x^*, A\big)^{-1} &= \lambda_{\{a,b\}} + \sum_{B \subseteq A \setminus \{a\}: |B| \leq 2} \lambda_B + \sum_{B \subseteq A \setminus \{b\}: |B| \leq 2} \lambda_B - \sum_{B \subseteq A \setminus \{a,b\}: |B| \leq 2} \lambda_B \\
&= \lambda_{\{a,b\}} + \sum_{B \subseteq A \setminus \{a\}: |B| \leq 2} \lambda_B + \sum_{B \subseteq A \setminus \{b\}: |B| = 2 \text{ and } a \in B} \lambda_B + \lambda_{\{a\}} \\
&= \sum_{B \subseteq A: |B| \leq 2} \lambda_B.
\end{aligned}$$

We restrict looking at weights λ_B with $|B| \leq 2$ since the inductive step makes other λ terms zero. Performing subtraction of the rightmost terms leads to the second equality. The third equality comes by collecting all terms. Thus, we have that $\frac{1}{P(x^*, A)} = \sum_{B \subseteq A: |B| \leq 2} \lambda_B = \sum_{B \subsetneq A} \lambda_B$ since $\lambda_B = 0$ for all $B \subsetneq A$ with $|B| \geq$

3 by induction. Using the Möbius inversion formula, $\sum_{B\subsetneq A} \lambda_B = \sum_{B\subseteq A} \lambda_B$ so that $\lambda_A = 0$. Therefore, we have shown by induction that $\lambda_A = 0$ for all $A \in \mathcal{D}$ with $|A| \geq 3$. The representation now holds immediately from the proof of Theorem 3.1 and letting $\alpha = \tilde{\lambda}$. *Q.E.D.*

APPENDIX D: COMPARISON TO PALM

The regular perception-adjusted Luce model (rPALM) of Echenique, Saito, and Tserenjigmid (2014) is described by a pair $(\succsim_P, u)$, where $\succsim_P$ is a weak order on X and $u : 2^X \to \mathbb{R}$ is a function such that

$$P_{\succsim_P,u}(a, A) = \mu(a, A) \prod_{\alpha \in A/\succsim_P : \alpha \succ_P a} \left(1 - \sum_{c \in A : c \in \alpha} \mu(c, A)\right),$$

where

$$\mu(a, A) = \frac{u(a)}{\displaystyle\sum_{b \in A} u(b) + u(A)}$$

and

$$u(c) \geq u(\{a, b\}) - u(\{a, b, c\})$$

for all $a, b, c \in X$ with strict inequality if $b \nsim_P c$.

The notation $A/\succsim_P$ is for the set of equivalence classes according to $\succsim_P$ that partition A, so the product is over all classes of alternatives that are ordered ahead of a. In rPALM, $\succsim_P$ is interpreted as a perception priority relation, and the authors attribute all violations of IIA to perception priority. More specifically, when $a, b \in X$ do not violate IIA, then we have $a \sim_P b$.

One of the distinguishing features of an RCCSR relative to an rPALM is that the choice frequency of the default alternative must obey monotonicity with respect to set inclusion under an RCCSR: $B \subset A \Rightarrow P(x^*, B) > P(x^*, A)$. In the context of availability, this restriction is logical in that larger menus are more likely to have an alternative available. An rPALM places no such consistency restrictions on choice frequency of the default alternative. This is one potential way in which the two models can be distinguished from choice data.

Another feature of rPALM is the hazard rate the authors define as

$$q(a, A) = \frac{P_{\succsim_P,u}(a, A)}{1 - P_{\succsim_P,u}(A^a, A)},$$

where $A^a = \{b \in A : b \succ_P a\}$. The authors impose that the hazard rate obeys both IIA ($\frac{q(a,\{a,b\})}{q(b,\{a,b\})} = \frac{q(a,A)}{q(b,A)}$ for all $a, b \in X$ and $A \subseteq X$ such that $a, b \in A$) and regularity ($q(a, \{a, b\}) \geq q(a, \{a, b, c\})$ for all $a, b, c \in X$ and with strict inequality only when $b \nsim_P c$). We will use this to show that an RCCSR is not a special case of rPALM. It is easy to see that an RCCSR can violate hazard rate IIA (in Example 1, it is violated for a, b). Now consider the choice frequencies in Example 1 and note that we have $P(a, \{a, b, c\}) > P(a, \{a, b\})$ and $P(b, \{a, b, c\}) > P(b, \{a, b\})$. An rPALM is unable to generate these choice frequencies. In what follows, let $A = \{a, b, c\}$.

Case 1: $a \succsim_P b, a \succsim_P c, b \nsim_P c$. By regularity, we have $P_{\succsim_P,u}(a, A) = q(a, A) < q(a, \{a, b\}) = P_{\succsim_P,u}(a, \{a, b\})$.

Case 2: $a \succ_P b \sim_P c$. By regularity, we have $P_{\succsim_P,u}(a, A) = q(a, A) = q(a, \{a, b\}) = P_{\succsim_P,u}(a, \{a, b\})$.

Case 3: $b \succsim_P a, b \succsim_P c, a \nsim_P c$. By regularity, we have $P_{\succsim_P,u}(b, A) = q(b, A) < q(b, \{a, b\}) = P_{\succsim_P,u}(b, \{a, b\})$.

Case 4: $b \succ_P a \sim_P c$. By regularity, we have $P_{\succsim_P,u}(b, A) = q(b, A) = q(b, \{a, b\}) = P_{\succsim_P,u}(b, \{a, b\})$.

Case 5: $c \succ_P a \succsim_P b$. By regularity, we have $P_{\succsim_P,u}(a, A) = q(a, A)(1 - P_{\succsim_P,u}(c, A)) < q(a, A) \leq q(a, \{a, b\}) = P_{\succsim_P,u}(a, \{a, b\})$.

Case 6: $c \succ_P b \succsim_P a$. By regularity, we have $P_{\succsim_P,u}(b, A) = q(b, A)(1 - P_{\succsim_P,u}(c, A)) < q(b, A) \leq q(b, \{a, b\}) = P_{\succsim_P,u}(b, \{a, b\})$.

Case 7: $a \sim_P b \sim_P c$. rPALM cannot violate IIA in this case, but

$$\frac{P(a, A)}{P(b, A)} = \frac{7}{11} \neq \frac{2}{3} = \frac{P(a, \{a, b\})}{P(b, \{a, b\})}$$

in Example 1.

REFERENCES

BARBERÀ, S., AND B. GRODAL (2011): "Preference for Flexibility and the Opportunities of Choice," *Journal of Mathematical Economics*, 47 (3), 272–278. [1]

ECHENIQUE, F., K. SAITO, AND G. TSERENJIGMID (2014): "The Perception-Adjusted Luce Model," Caltech Discussion Paper. [6]

Dept. of Economics, University of California, San Diego, La Jolla, CA 92093, U.S.A.; rlbrady@ucsd.edu

and

Dept. of Economics, University of California, San Diego, La Jolla, CA 92093, U.S.A.; jrehbeck@ucsd.edu.

Co-editor Itzhak Gilboa handled this manuscript.

Manuscript received July, 2014; final revision received January, 2016.