

SUPPLEMENT TO “LOWER BOUNDS ON APPROXIMATION ERRORS TO
NUMERICAL SOLUTIONS OF DYNAMIC ECONOMIC MODELS”:
ONLINE APPENDICES

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IN APPENDICES A AND B, we describe additional details of the lower-bound error analysis in the neoclassical stochastic growth model and in the new Keynesian model studied in the main text.

APPENDIX A: NEOCLASSICAL STOCHASTIC GROWTH MODEL

In this section, we focus on the neoclassical stochastic growth model. In Appendix A.1, we derive a lower error bound by using linearized model’s equations; in Appendix A.2, we construct a more accurate lower error bound by using nonlinear model’s equations; and in Appendix A.3, we discuss alternative implementations of the lower-bound error analysis.

A.1. Constructing Lower Error Bound by Using Linearized Model’s Equations

Euler Equation

We first linearize the Euler equation. Let us assume a CRRA utility function $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. For this utility function, Euler equation (26), expressed in terms of approximation errors, is

$$\widehat{c}_t^{-\gamma}(1 + \delta_{c_t})^{-\gamma} - \beta E_t \left\{ \widehat{c}_{t+1}^{-\gamma}(1 + \delta_{c_{t+1}})^{-\gamma} \cdot [1 - d + \alpha \exp(\theta_{t+1}) \widehat{A}k_{t+1}^{\alpha-1}(1 + \delta_{k_{t+1}})^{\alpha-1}] \right\} = 0. \tag{A.1}$$

One can view (A.1) as a function of δ ’s, that is, $f(\delta_{c_t}, \delta_{c_{t+1}}, \delta_{k_{t+1}}) = 0$. Finding a first-order Taylor expansion of f around $\delta_{c_t} \rightarrow 0$, $\delta_{c_{t+1}} \rightarrow 0$, $\delta_{k_{t+1}} \rightarrow 0$ (in particular, using $(1 + x)^\alpha \simeq 1 + \alpha x$) and omitting a second-order term $\delta_{c_{t+1}} \delta_{k_{t+1}} \approx 0$, we have

$$\begin{aligned} & \widehat{c}_t^{-\gamma} - \gamma \delta_{c_t} \widehat{c}_t^{-\gamma} - \beta E_t \left\{ \widehat{c}_{t+1}^{-\gamma} (1 - d + \alpha \exp(\theta_{t+1}) \widehat{A}k_{t+1}^{\alpha-1}) \right\} \\ & + \beta E_t \left\{ \widehat{c}_{t+1}^{-\gamma} \gamma \delta_{c_{t+1}} (1 - d + \alpha \exp(\theta_{t+1}) \widehat{A}k_{t+1}^{\alpha-1}) \right\} \\ & - \beta E_t \left\{ \widehat{c}_{t+1}^{-\gamma} (\alpha \exp(\theta_{t+1}) \widehat{A}k_{t+1}^{\alpha-1} (\alpha - 1) \delta_{k_{t+1}}) \right\} = 0. \end{aligned}$$

By discretizing the future exogenous states into J integration nodes, we replace the state-contingent functions \widehat{c}_{t+1} and $\delta_{c_{t+1}}$ by $\widehat{c}_{t+1,j}$ and $\delta_{c_{t+1,j}}$, $j = 1, \dots, J$, respectively, which yields

$$1 - \gamma \delta_{c_t} - h_1 + \gamma \sum_{j=1}^J m_j \delta_{c_{t+1,j}} - (\alpha - 1) \delta_{k_{t+1}} h_2 = 0,$$

where

$$\begin{aligned} h_1 &\equiv \beta \sum_{j=1}^J \left\{ \frac{\widehat{c}_{t+1,j}^{-\gamma}}{\widehat{c}_t^{-\gamma}} (1 - d + \alpha \exp(\theta_{t+1,j}) A \widehat{k}_{t+1}^{\alpha-1}) \right\}, \\ h_2 &\equiv \beta \sum_{j=1}^J \left\{ \frac{\widehat{c}_{t+1,j}^{-\gamma}}{\widehat{c}_t^{-\gamma}} (\alpha \exp(\theta_{t+1,j}) A \widehat{k}_{t+1}^{\alpha-1}) \right\}, \\ m_j &\equiv \beta \omega_j \frac{\widehat{c}_{t+1,j}^{-\gamma}}{\widehat{c}_t^{-\gamma}} (1 - d + \alpha \exp(\theta_{t+1,j}) A \widehat{k}_{t+1}^{\alpha-1}), \end{aligned}$$

with $\theta_{t+1,j} = \rho \theta_t + \varepsilon_j$, and ε_j , ω_j denoting a j th integration node and weight. Combining the terms yields a linear equation in δ 's,

$$a^{1,1} \delta_{c_t} + a^{1,2} \delta_{k_{t+1}} + \sum_{j=1}^J a_j^{1,3} \delta_{c_{t+1,j}} = b^1, \quad (\text{A.2})$$

where

$$a^{1,1} \equiv -\gamma, \quad a^{1,2} \equiv -(\alpha - 1)h_2, \quad a_j^{1,3} \equiv \gamma m_j, \quad b^1 \equiv h_1 - 1.$$

Budget Constraint

We next linearize the budget constraint. We rewrite the budget constraint (25) as

$$\widehat{c}_t + \delta_{c_t} \widehat{c}_t + \widehat{k}_{t+1} + \delta_{k_{t+1}} \widehat{k}_{t+1} - (1 - d)k_t - \exp(\theta_t) A k_t^\alpha = 0. \quad (\text{A.3})$$

Thus, we get

$$a^{2,1} \delta_{c_t} + a^{2,2} \delta_{k_{t+1}} = b^2, \quad (\text{A.4})$$

where

$$a^{2,1} \equiv \widehat{c}_t, \quad a^{2,2} \equiv \widehat{k}_{t+1}, \quad b^{2,1} \equiv (1 - d)k_t + \exp(\theta_t) A k_t^\alpha - \widehat{c}_t - \widehat{k}_{t+1}.$$

Minimization Problem

The minimization problem (28) in a point (period) t is given by

$$\min_{\delta_{c_t}, \delta_{k_{t+1}}, \{\delta_{c_{t+1,j}}\}_{j=1, \dots, J}} \delta_{c_t}^2 + \delta_{k_{t+1}}^2 + \sum_{j=1}^J \delta_{c_{t+1,j}}^2 \quad \text{s.t. (A.2), (A.4)}. \quad (\text{A.5})$$

To solve (A.5) numerically, we use quadratic programming software (we use a ‘‘quadprog’’ routine in MATLAB).

A.2. Constructing Lower Error Bound by Using Nonlinear Model's Equations

We now construct the lower error bound using the original nonlinear equations. Budget constraint (A.3) yields

$$\delta_{k_{t+1}} = \frac{(1 - d)k_t + \exp(\theta_t) A k_t^\alpha - \widehat{c}_t (1 + \delta_{c_t})}{\widehat{k}_{t+1}} - 1. \quad (\text{A.6})$$

From budget constraint (A.3), we also get

$$[\widehat{k}_{t+1}(1 + \delta_{k_{t+1}})]^{\alpha-1} = [(1-d)k_t + \exp(\theta_t)Ak_t^\alpha - \widehat{c}_t(1 + \delta_{c_t})]^{\alpha-1}.$$

Substituting the latter equation into Euler equation (A.1), we have

$$\begin{aligned} & (1 + \delta_{c_t})^{-\gamma} - \beta E_t \left[\frac{\widehat{c}_{t+1}^{-\gamma}}{\widehat{c}_t^{-\gamma}} (1 + \delta_{c_{t+1}})^{-\gamma} (1-d) \right] \\ & - \left[\frac{(1-d)k_t + \exp(\theta_t)Ak_t^\alpha - \widehat{c}_t(1 + \delta_{c_t})}{\widehat{k}_{t+1}} \right]^{\alpha-1} \\ & \times \beta E_t \left[\frac{\widehat{c}_{t+1}^{-\gamma}}{\widehat{c}_t^{-\gamma}} (1 + \delta_{c_{t+1}})^{-\gamma} \alpha \exp(\theta_{t+1})A\widehat{k}_{t+1}^{\alpha-1} \right] = 0. \end{aligned}$$

By discretizing the future exogenous states into J integration nodes, we replace the state-contingent functions \widehat{c}_{t+1} and $\delta_{c_{t+1}}$ by $\widehat{c}_{t+1,j}$ and $\delta_{c_{t+1,j}}$, $j = 1, \dots, J$, respectively, which yields

$$\begin{aligned} \delta_{c_t} = & \left\{ \beta \sum_{j=1}^J \omega_j \left[\frac{\widehat{c}_{t+1}^{-\gamma}}{\widehat{c}_t^{-\gamma}} (1 + \delta_{c_{t+1,j}})^{-\gamma} (1-d) \right] \right. \\ & + \left[\frac{(1-d)k_t + \exp(\theta_t)Ak_t^\alpha - \widehat{c}_t(1 + \delta_{c_t})}{\widehat{k}_{t+1}} \right]^{\alpha-1} \\ & \left. \times \beta \sum_{j=1}^J \omega_j \left[\frac{\widehat{c}_{t+1}^{-\gamma}}{\widehat{c}_t^{-\gamma}} (1 + \delta_{c_{t+1,j}})^{-\gamma} \alpha \exp(\theta_{t+1,j})A\widehat{k}_{t+1}^{\alpha-1} \right] \right\}^{-1/\gamma} - 1 \\ & = 0. \end{aligned} \tag{A.7}$$

Therefore, the least-squares problem (28) becomes

$$\min_{\delta_{c_t}, \delta_{k_{t+1}}, \{\delta_{c_{t+1,j}}\}_{j=1,\dots,J}} \delta_{c_t}^2 + \delta_{k_{t+1}}^2 + \sum_{j=1}^J \delta_{c_{t+1,j}}^2 \quad \text{s.t. (A.7), (A.6).} \tag{A.8}$$

The resulting minimization problem contains just $J + 1$ unknowns, given by δ_{c_t} and $\{\delta_{c_{t+1,j}}\}_{j=1,\dots,J}$ that are constructed using a numerical solver. Note that δ_{c_t} appears both in the left and right side of (A.7) and we need to compute it numerically. To solve problem (A.8), we use MATLAB's nonlinear optimization routine "fminsearch."

A.3. Alternative Implementations of Lower-Bound Error Analysis

There are many possible ways of defining approximation errors. First, we could consider approximation errors in model's different variables, for example, the errors in the investment or output functions instead of those in capital or consumption functions. This will affect the size of the resulting error bounds. Second, there are different ways of modeling approximation errors in conditional expectations; in particular, we can represent

TABLE SI
 APPROXIMATION ERRORS IN THE CURRENT VARIABLES AND THE EXPECTATION FUNCTIONS IN THE
 NEOCLASSICAL STOCHASTIC GROWTH MODEL^a

Norm	$\gamma = \frac{1}{10}$			$\gamma = 1$			$\gamma = 10$		
	δ_{c_t}	$\delta_{k_{t+1}}$	δ_{E_t}	δ_{c_t}	$\delta_{k_{t+1}}$	δ_{E_t}	δ_{c_t}	$\delta_{k_{t+1}}$	δ_{E_t}
PER1									
L_1	-5.41	-4.12	-4.60	-4.77	-4.12	-4.65	-4.10	-3.75	-5.14
L_∞	-4.26	-3.03	-3.55	-3.96	-3.04	-3.75	-3.52	-2.62	-4.00
PER2									
L_1	-6.61	-5.80	-6.28	-6.35	-5.69	-6.18	-5.31	-4.75	-6.18
L_∞	-5.44	-4.42	-5.19	-5.11	-4.43	-4.97	-4.06	-3.65	-4.21

^aNotes: PER1 and PER2 denote the first- and second-order perturbation solutions; δ_{c_t} , $\delta_{k_{t+1}}$, and δ_{E_t} are t-period absolute value of approximation errors in consumption, capital, and conditional expectation function, respectively; L_1 and L_∞ are, respectively, the average and maximum of absolute values of the corresponding approximation errors across optimality condition and test points (in log10 units) on a stochastic simulation of 10,000 observations; and γ is the coefficient of risk aversion.

errors in Euler equation (22) as

$$u'(\widehat{c}_t(1 + \delta_{c_t})) = \underbrace{\beta(1 + \delta_{E_t})\widehat{E}_t}_{=E_t[u'(c_{t+1})(1-d+\exp(\rho\theta_t+e_{t+1}))A_f'(k_{t+1})]} \quad (\text{A.9})$$

where δ_{E_t} is an approximation error in conditional expectation function $E_t[\cdot]$. We can use a new condition (A.9) as a restriction in the least-squares problem (28), instead of (26), by changing the objective function to $\delta_{c_t}^2 + \delta_{k_{t+1}}^2 + \delta_{E_t}^2$.

In Table SI, we show the error bounds obtained from the conditions (25), (A.9) on a stochastic simulation following the same methodology as described in Section 3.3.

The advantage of this representation is that it does not require to approximate future values of the variables and hence, it does not involve additional errors from numerical integration in the construction of lower error bound. A potential shortcoming of this alternative representation is that the error in $E_t[\cdot]$ depends on the marginal utility function, so that $\delta_{c_t}^2$, $\delta_{k_{t+1}}^2$, $\delta_{E_t}^2$ are not expressed in comparable units, and introducing a trade-off between the model's variables and marginal utility in the objective function may lead to accuracy results that are more difficult to interpret. In contrast, our baseline representation (28) contains only approximation errors in the model's variables and is not subject to this shortcoming. To deal with this issue, Kubler and Schmedders (2005) measured the error in the conditional expectation function δ_{E_t} by the average adjustment of the future consumption $\delta_{c_{t+1}}$ to satisfy the Euler equation exactly; this approach can be used in our case as well.

APPENDIX B: NEW KEYNESIAN MODEL

In this section, we implement our error bound analysis for the new Keynesian model. In Appendix B.1, we present the new Keynesian model considered; in Appendix B.2, we derive the first-order conditions (FOCs) of the studied model; in Appendix B.3, we define a lower error bound; in Appendix B.4, we derive a lower error bound by using linearized model's equations; in Appendix B.5, we define residuals in the model's equations; finally, in Appendix B.6, we describe the details of our numerical analysis and report the constructed lower error bounds.

B.1. The Model

The economy is populated by labor packers, households, final-good firms, intermediate-good firms, monetary authority, and government; see Galí (2008, Chapter 6) for a detailed description of a new Keynesian model with sticky wages and prices.

Labor Packers

Labor inputs of heterogeneous households are packed by labor packers to be sold to firms. A labor packer buys $N_t(l)$ units of labor of a household $l \in [0, 1]$ at price $W_t(l)$ and sells $N_t(l)$ units of labor at price W_t in a perfectly competitive market. The profit-maximization problem is

$$\max_{N_t(l)} W_t N_t - \int_0^1 W_t(l) N_t(l) dl \quad (\text{B.1})$$

$$\text{s.t. } N_t = \left(\int_0^1 N_t(l)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dl \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \quad (\text{B.2})$$

where (B.2) is a Dixit–Stiglitz aggregator function with $\varepsilon_w \geq 1$. Problem (B.1), (B.2) implies the demand for labor of type l :

$$N_t(l) = N_t \left(\frac{W_t(l)}{W_t} \right)^{-\varepsilon_w}. \quad (\text{B.3})$$

Households

There is a continuum of monopolistically competitive households who supply differentiated labor input to a labor packer and are indexed by $l \in [0, 1]$. Markets are complete: the households can trade state-contingent claims to insure themselves against aggregate uncertainty. As a result, in equilibrium, the households will be identical in all their choices, except of wages and hours worked (the household's index l will be suppressed elsewhere except of nominal wage $W_t(l)$ and labor $N_t(l)$).

The household of type l maximizes expected discounted lifetime-time utility subject to the capital accumulation equation, (B.5), and the period budget constraint, (B.6),

$$\max_{\{C_t, B_{t+1}, K_{t+1}, u_t, Q_{t+1}\}_{t=0, \dots, \infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - bC_{t-1}) - \psi \frac{N_t(l)^{1+\eta} - 1}{1+\eta} \right] \quad (\text{B.4})$$

$$\text{s.t. } K_{t+1} = Z_t \left(1 - \frac{\tau}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t + (1-d)K_t, \quad (\text{B.5})$$

$$\begin{aligned} C_t + I_t + \frac{B_{t+1}}{P_t} + T_t + q_{t+1,t} Q_{t+1} + \left(\chi_1(u_t - 1) + \frac{\chi_2}{2}(u_t - 1)^2 \right) \frac{K_t}{Z_t} \\ = \frac{W_t(l)}{P_t} N_t(l) + R_t u_t K_t + (1 + i_{t-1}) \frac{B_t}{P_t} + Q_t + \frac{D_t}{P_t}, \end{aligned} \quad (\text{B.6})$$

$$\ln Z_t = \rho_z \ln Z_{t-1} + \varepsilon_{z,t}, \quad \varepsilon_{z,t} \sim \mathcal{N}(0, \sigma_z^2), \quad (\text{B.7})$$

where E_t is the expectation conditional on the information of period t , and (B.7) is a process for investment shock Z_t to the efficiency of transforming investment into capital.

Here, C_t , $N_t(l)$, I_t , K_{t+1} , B_{t+1} , and Q_{t+1} are consumption, labor, investment, capital holdings, nominal-bond holdings, and a vector of state-contingent claims, respectively; P_t , $W_t(l)$, R_t , i_{t-1} , and $q_{t+1,t}$ are, respectively, the commodity price, nominal wage, real return on capital, (net) nominal interest rate, and a price vector of state-contingent claims (each of its elements is a price of a claim that pays one unit of good in a particular aggregate state of nature, x_t , in the subsequent period $t + 1$); T_t is lump-sum taxes; D_t is the profit (dividends) of intermediate-good firms; $\beta \in (0, 1)$ is the discount factor; $\psi > 0$ is the utility-function parameter; $\chi_1 \geq 0$ and $\chi_2 \geq 0$ are the parameters in the cost-of-utilization function which is quadratic in utilization relative to its normalized steady-state value, that is equal to 1; $\tau \geq 0$ is the parameter that governs the size of the adjustment cost of capital; ρ_z and σ_z are the autocorrelation coefficient and the standard deviation of disturbances, respectively.

Wages are subject to Calvo's (1983) pricing frictions. Each period, a fraction $1 - \phi_w$ of the households sets wages optimally, $W_t(l)$ for $l \in [0, 1]$, and the fraction ϕ_w is not allowed to change the price. When the household cannot reoptimize its posted nominal wage, it will index to lagged inflation at $\zeta_w \in (0, 1)$. Let $\Pi_{t,t+s-1} \equiv \frac{P_{t+s-1}}{P_t}$ be a cumulative gross price inflation rate between periods $t - 1$ and $t + s - 1$. A non-reoptimizing household sets a $t + s$ -period nominal wage rate at

$$W_{t+s}(l) = \Pi_{t,t+s-1}^{\zeta_w} W_t(l),$$

and hence, real wage at

$$w_{t+s}(l) = w_t(l) \Pi_{t,t+s}^{-1} \Pi_{t-1,t+s-1}^{\zeta_w}, \quad (\text{B.8})$$

where $w_{t+s}(l)$ is real wage of the household of type l in period $t + s$. Note that (B.3) and (B.8) imply

$$N_{t+s}(l) = N_{t+s} \left(\frac{w_t(l) \Pi_{t,t+s}^{-1} \Pi_{t-1,t+s-1}^{\zeta_w}}{w_{t+s}} \right)^{-\varepsilon_w}, \quad (\text{B.9})$$

where w_{t+s} is real wage of packed labor. A reoptimizing household $l \in [0, 1]$ maximizes the current discounted lifetime utility over the time period when $w_t(l)$ remains effective, subject to the demand for labor (B.9) and budget constraint (B.6),

$$\max_{\{w_t(l)\}_{t=0,\dots,\infty}} E_t \sum_{s=0}^{\infty} \beta^s \phi_w^s \left[\dots - \psi \frac{N_{t+s}(l)^{1+\eta} - 1}{1+\eta} \right]$$

s.t. (B.6), (B.9).

Final-Good Firms

Perfectly competitive final-good firms produce final goods using intermediate goods. A final-good firm buys $Y_t(i)$ of an intermediate good $i \in [0, 1]$ at price $P_t(i)$ and sells Y_t of the final good at price P_t in a perfectly competitive market. The profit-maximization problem is

$$\max_{Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \quad (\text{B.10})$$

$$\text{s.t. } Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon_p-1}{\varepsilon_p}} di \right)^{\frac{\varepsilon_p}{\varepsilon_p-1}}, \quad (\text{B.11})$$

where (B.11) is a Dixit–Stiglitz aggregator function with $\varepsilon_p \geq 1$. The problem (B.10), (B.11) implies the demand for an intermediate good of type i :

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t. \quad (\text{B.12})$$

Intermediate-Good Firms

Monopolistic intermediate-good firms produce intermediate goods using capital and labor and are subject to sticky prices. A firm i produces the intermediate good i . To choose capital and labor in each period t , the firm i minimizes the nominal total cost, TC, subject to the constraint that its output is sufficient to meet demand:

$$\min_{N_t(i), K_t^\#(i)} \text{TC}(Y_t(i)) = W_t N_t(i) + R_t^n K_t^\#(i) \quad (\text{B.13})$$

$$\text{s.t. } A_t K_t^\#(i)^\alpha N_t(i)^{1-\alpha} \geq Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p}, \quad (\text{B.14})$$

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t}, \quad \varepsilon_{a,t+1} \sim \mathcal{N}(0, \sigma_a^2), \quad (\text{B.15})$$

where (B.15) is a process for the productivity level, A_t ; $N_t(i)$ is the labor input; $K_t^\#(i) \equiv u_t K_t$ is capital; A_t is the productivity level; R_t^n is the nominal rental rate; ρ_a is the auto-correlation coefficient; and σ_a is the standard deviation of the disturbance.

The firm discounts profits s periods into the future by $\tilde{M}_{t+s} \phi_p^s$, where $\tilde{M}_{t+s} = \beta^s \frac{\lambda_{t+s}}{\lambda_t}$ is a stochastic discount factor with λ_t being the marginal value of an extra unit of income (it is equal to the Lagrange multiplier on the household's budget constraint (B.6)). The firms are subject to Calvo-type price setting, namely, a fraction $1 - \phi_p$ of the firms sets prices optimally, $P_t(i)$ for $i \in [0, 1]$, and the fraction ϕ_p is not allowed to change the price. A non-reoptimizing firm indexes its price to lagged inflation at $\zeta_p \in [0, 1]$. The price charged in period $t + s$ if it is still not revised since period t is

$$P_{t+s}(i) = \Pi_{t-1, t+s-1}^{\zeta_p} P_t(i). \quad (\text{B.16})$$

A reoptimizing firm $i \in [0, 1]$ maximizes the current expected value of profit over the time period when $P_t(i)$ remains effective,

$$\max_{P_t(i)} \sum_{s=0}^{\infty} \beta^s \phi_p^s E_t \left\{ \frac{\lambda_{t+s}}{\lambda_t} \left[\frac{\Pi_{t-1, t+s-1}^{\zeta_p} P_t(i)}{P_{t+s}} Y_{t+s}(i) - \text{mc}_{t+s} Y_{t+s}(i) \right] \right\} \quad (\text{B.17})$$

$$\text{s.t. } Y_{t+s}(i) = \left(\frac{\Pi_{t-1, t+s-1}^{\zeta_p} P_t(i)}{P_{t+s}} \right)^{-\varepsilon_p} Y_{t+s}, \quad (\text{B.18})$$

where (B.18) follows from (B.12) and (B.16); P_{t+s} is the price of the final good; mc_{t+s} is the real marginal cost of output at time $t + s$ (which is identical across the firms), that is, $\text{mc}_{t+s} P_{t+s} \equiv \text{MC}_{t+s}$.

Government

Government finances a stochastic stream of public consumption by levying lump-sum taxes and by issuing nominal debt. The government budget constraint is

$$T_t + \frac{B_{t+1}}{P_t} = \omega_t^g Y_t + (1 + i_{t-1}) \frac{B_t}{P_t}, \quad (\text{B.19})$$

where $\omega_t^g Y_t = G_t$ is government spending, and ω_t^g is a government-spending shock,

$$\omega_t^g = (1 - \rho_g) \omega^g + \rho_g \omega_{t-1}^g + \varepsilon_{g,t}, \quad \varepsilon_{g,t} \sim \mathcal{N}(0, \sigma_g^2), \quad (\text{B.20})$$

where ρ_g is the autocorrelation coefficient, and σ_g is the standard deviation of disturbance.

Monetary Authority

The monetary authority follows a Taylor rule:

$$i_t = (1 - \rho_i) i + \rho_i i_{t-1} + (1 - \rho_i) [\phi_\pi (\pi_t - \pi^*) + \phi_y (\ln Y_t - \ln Y_{t-1})] + \varepsilon_{i,t}, \quad (\text{B.21})$$

where $i = 1/\beta - 1$ is the steady-state interest rate; $\phi_\pi \geq 0$ and $\phi_y \geq 0$ are the parameters; $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$ is net inflation; $\varepsilon_{i,t}$ is a monetary shock, $\varepsilon_{i,t} \sim \mathcal{N}(0, \sigma_i^2)$.

B.2. Deriving FOCs

We derive the FOCs of the studied new Keynesian model below.

Labor Packers

The FOC of the labor packer's problem (B.1), (B.2) with respect to $N_t(l)$ yields the demand for the l th type of labor, given by (B.3),

$$N_t(l) = N_t \left(\frac{W_t(l)}{W_t} \right)^{-\varepsilon_w}. \quad (\text{B.22})$$

A zero-profit condition of a labor packer implies $W_t N_t = \int_0^1 W_t(l) N_t(l) dl$. Substituting (B.22) into the latter equation gives

$$W_t = \left(\int_0^1 W_t(l)^{1-\varepsilon_w} di \right)^{\frac{1}{1-\varepsilon_w}}. \quad (\text{B.23})$$

Households

The FOCs of the household's problem (B.4)–(B.7) with respect to C_t , B_{t+1} , K_{t+1} , I_t , u_t , Q_{t+1} , respectively, are

$$\lambda_t = \frac{1}{C_t - bC_{t-1}} - \beta b E_t \left[\frac{1}{C_{t+1} - bC_t} \right], \quad (\text{B.24})$$

$$\lambda_t = \beta E_t \left[\lambda_{t+1} (1 + i_t) \frac{P_t}{P_{t+1}} \right], \quad (\text{B.25})$$

$$\mu_t = \beta E_t \left[\lambda_{t+1} \left(R_{t+1} u_{t+1} - \frac{1}{Z_{t+1}} \left(\chi_1 (u_{t+1} - 1) + \frac{\chi_2}{2} (u_{t+1} - 1)^2 \right) \right) + \mu_{t+1} (1 - d) \right], \quad (\text{B.26})$$

$$\lambda_t = \mu_t Z_t \left[1 - \frac{\tau}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \tau \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] + \beta E_t \mu_{t+1} Z_{t+1} \tau \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2, \quad (\text{B.27})$$

$$R_t = \frac{1}{Z_t} [\chi_1 + \chi_2 (u_t - 1)], \quad (\text{B.28})$$

$$\lambda_t q_{t+1,t}(x) = \beta \lambda_{t+1} \Pr\{x_{t+1} = x | x_t = x'\}, \quad (\text{B.29})$$

where λ_t and μ_t are the Lagrange multipliers associated with (B.6) and (B.5); $x_t = \{Z_t, A_t, \omega_t^g, \varepsilon_{i,t}\}$ is the economy's aggregate state; $q_{t+1,t}(x)$ is the price of a state-contingent claim, bought in period t , that pays one unit of consumption in case aggregate state x in period $t + 1$.

As for wage setting, the FOC with respect to real wage, chosen by a reoptimizing household, is

$$\begin{aligned} \varepsilon_w w_t(l)^{-\varepsilon_w(1+\eta)-1} E_t \sum_{s=0}^{\infty} \beta^s \phi_w^s \psi \Pi_{t,t+s}^{\varepsilon_w(1+\eta)} \Pi_{t-1,t+s-1}^{\zeta_w \varepsilon_w(1+\eta)} w_{t+s}^{\varepsilon_w(1+\eta)} N_{t+s}^{1+\eta} \\ + (1 - \varepsilon_w) w_t(l)^{-\varepsilon_w} \sum_{s=0}^{\infty} \beta^s \phi_w^s \lambda_{t+s} \Pi_{t,t+s}^{\varepsilon_w-1} \Pi_{t-1,t+s-1}^{\zeta_w(1-\varepsilon_w)} w_{t+s}^{\varepsilon_w} N_{t+s} = 0. \end{aligned}$$

Note that the household-specific index l enters just $w_t(l)$, so that all reoptimizers choose the same wage, that is, $w_t(l) \equiv w_t^\#$, given by

$$(w_t^\#)^{1+\varepsilon_w \eta} = \frac{\varepsilon_w}{1 - \varepsilon_w} \frac{E_t \sum_{s=0}^{\infty} \beta^s \phi_w^s \psi \Pi_{t,t+s}^{\varepsilon_w(1+\eta)} \Pi_{t-1,t+s-1}^{\zeta_w \varepsilon_w(1+\eta)} w_{t+s}^{\varepsilon_w(1+\eta)} N_{t+s}^{1+\eta}}{\sum_{s=0}^{\infty} \beta^s \phi_w^s \lambda_{t+s} \Pi_{t,t+s}^{\varepsilon_w-1} \Pi_{t-1,t+s-1}^{\zeta_w(1-\varepsilon_w)} w_{t+s}^{\varepsilon_w} N_{t+s}}.$$

We can rewrite it recursively as

$$(w_t^\#)^{1+\varepsilon_w \eta} = \frac{\varepsilon_w}{1 - \varepsilon_w} \frac{F_{1,t}}{F_{2,t}}, \quad (\text{B.30})$$

where

$$F_{1,t} = \psi w_t^{\varepsilon_w(1+\eta)} N_t^{1+\eta} + \phi_w (1 + \pi_t)^{-\zeta_w \varepsilon_w(1+\eta)} \beta E_t [(1 + \pi_{t+1})^{\varepsilon_w(1+\eta)} F_{1,t+1}], \quad (\text{B.31})$$

$$F_{2,t} = \lambda_t w_t^{\varepsilon_w} N_t + \phi_w (1 + \pi_t)^{\zeta_w(1-\varepsilon_w)} \beta E_t [(1 + \pi_{t+1})^{\varepsilon_w-1} F_{2,t+1}], \quad (\text{B.32})$$

where $1 + \pi_t \equiv \Pi_{t-1,t}$.

A power $1 + \varepsilon_w \eta$ in equation (B.30) could take very large values for empirically plausible parameterizations of the model (e.g., we calibrate $\eta = 1$ and $\varepsilon_w = 10$), which may lead to numerical problems. To deal with this issue, first, we divide both sides of (B.30) by $(w_t^\#)^{\varepsilon_w(1+\eta)}$,

$$(w_t^\#)^{1-\varepsilon_w} = \frac{\varepsilon_w}{1 - \varepsilon_w} \frac{f_{1,t}}{F_{2,t}}, \quad (\text{B.33})$$

where $f_{1,t} \equiv \frac{F_{1,t}}{(w_t^\#)^{\varepsilon_w(1+\eta)}}$. Then, equation (B.31) becomes

$$\begin{aligned} f_{1,t} &= \psi \left(\frac{w_t}{w_t^\#} \right)^{\varepsilon_w(1+\eta)} N_t^{1+\eta} + \phi_w (1 + \pi_t)^{-\zeta_w \varepsilon_w (1+\eta)} \\ &\times \beta E_t \left[(1 + \pi_{t+1})^{\varepsilon_w(1+\eta)} f_{1,t+1} \left(\frac{w_{t+1}^\#}{w_t} \right)^{\varepsilon_w(1+\eta)} \right]. \end{aligned} \quad (\text{B.34})$$

Second, we multiply both sides of (B.33) by $(w_t^\#)^{\varepsilon_w}$,

$$w_t^\# = \frac{\varepsilon_w}{1 - \varepsilon_w} \frac{f_{1,t}}{f_{2,t}}, \quad (\text{B.35})$$

where $f_{2,t} \equiv \frac{F_{2,t}}{(w_t^\#)^{\varepsilon_w}}$. Then, equation (B.32) becomes

$$\begin{aligned} f_{2,t} &= \lambda_t \left(\frac{w_t}{w_t^\#} \right)^{\varepsilon_w} N_t + \phi_w (1 + \pi_t)^{\zeta_w(1-\varepsilon_w)} \\ &\times \beta E_t \left[(1 + \pi_{t+1})^{\varepsilon_w-1} f_{2,t+1} \left(\frac{w_{t+1}^\#}{w_t} \right)^{\varepsilon_w} \right]. \end{aligned} \quad (\text{B.36})$$

Final-Good Producers

The FOC of the final-good producer's problem (B.10), (B.11) with respect to $Y_t(i)$ yields the demand for the i th intermediate good

$$Y_t(i) = Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p}. \quad (\text{B.37})$$

A zero-profit condition of a final-good producer implies $P_t Y_t = \int_0^1 P_t(i) Y_t(i) di$. Substituting (B.22) into the latter equation yields

$$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon_p} di \right)^{\frac{1}{1-\varepsilon_p}}. \quad (\text{B.38})$$

Intermediate-Good Producers

The FOCs of the cost-minimization problem (B.13)–(B.15) with respect to $N_t(i)$ and $K_t^\#(i)$ are

$$R_t^n = \Theta_t(i) \alpha A_t K_t^\#(i)^{\alpha-1} N_t(i)^{1-\alpha}, \quad (\text{B.39})$$

$$W_t = \Theta_t(i) (1 - \alpha) A_t K_t^\#(i)^\alpha N_t(i)^{-\alpha}, \quad (\text{B.40})$$

where $\Theta_t(i)$ is the Lagrange multiplier associated with (B.14). Combining (B.39) and (B.40) yields

$$\frac{W_t}{R_t^n} = \frac{1 - \alpha}{\alpha} \frac{K_t^\#(i)}{N_t(i)}.$$

This condition implies that all the firms will rent capital and hire labor in the same proportion. In real terms, the latter condition becomes

$$\frac{w_t}{R_t} = \frac{1 - \alpha}{\alpha} \left(\frac{K_t^\#}{N_t} \right),$$

where $R_t \equiv \frac{R_t^n}{P_t}$. The derivative of the total cost in (B.13) is the nominal marginal cost, $MC_t(i)$,

$$MC_t(i) \equiv \frac{dTC(Y_t(i))}{dY_t(i)} = \Theta_t(i). \quad (\text{B.41})$$

The real marginal cost is the same for all firms,

$$mc_t(i) = \frac{\Theta_t(i)}{P_t} = mc_t. \quad (\text{B.42})$$

This is because all the firms face the same factor prices, and they rent capital and hire labor in the same proportion. Conditions (B.39) and (B.40), together with (B.42), can be rewritten, respectively, as

$$R_t = mc_t \alpha A_t \left(\frac{K_t^\#}{N_t} \right)^{\alpha-1}, \quad (\text{B.43})$$

$$w_t = mc_t (1 - \alpha) A_t \left(\frac{K_t^\#}{N_t} \right)^\alpha. \quad (\text{B.44})$$

The period- t real-flow profit of the i th firm is

$$\begin{aligned} \frac{D_t(i)}{P_t} &= \frac{P_t(i)}{P_t} Y_t(i) - mc_t (1 - \alpha) A_t K_t^\#(i)^\alpha N_t(i)^{1-\alpha} - mc_t \alpha A_t K_t^\#(i)^\alpha N_t(i)^{1-\alpha} \\ &= \frac{P_t(i)}{P_t} Y_t(i) - mc_t Y_t(i). \end{aligned}$$

This result was used to derive (B.17). Substituting constraint (B.18) into the objective function yields

$$\max_{P_t(i)} \sum_{s=0}^{\infty} \beta^s \phi_p^s E_t \left\{ \frac{\lambda_{t+s}}{\lambda_t} \left(\frac{\Pi_{t-1,t+s-1}^{\zeta p} P_t(i)}{P_{t+s}} \right)^{-\varepsilon p} Y_{t+s} \left[\frac{\Pi_{t-1,t+s-1}^{\zeta p} P_t(i)}{P_{t+s}} - mc_{t+s} \right] \right\}.$$

This problem can be rewritten as

$$\max_{P_t(i)} \sum_{s=0}^{\infty} \beta^s \phi_p^s E_t \frac{\lambda_{t+s}}{\lambda_t} \left\{ \Pi_{t-1,t+s-1}^{\zeta p (1-\varepsilon p)} P_t(i)^{1-\varepsilon p} P_{t+s}^{\varepsilon p-1} Y_{t+s} - \Pi_{t-1,t+s-1}^{-\zeta p \varepsilon p} P_t(i)^{-\varepsilon p} mc_{t+s} P_{t+s}^{\varepsilon p} Y_{t+s} \right\}.$$

The FOC of the reoptimizing intermediate-good firm with respect to $P_t(i)$ is

$$(1 - \varepsilon_p)P_t(i)^{-\varepsilon_p} E_t \sum_{s=0}^{\infty} \beta^s \phi_p^s \lambda_{t+s} \Pi_{t-1, t+s-1}^{\zeta_p(1-\varepsilon_p)} P_{t+s}^{\varepsilon_p-1} Y_{t+s} \\ + \varepsilon_p P_t(i)^{-\varepsilon_p-1} E_t \sum_{s=0}^{\infty} \beta^s \phi_p^s \lambda_{t+s} \Pi_{t-1, t+s-1}^{-\zeta_p \varepsilon_p} \text{mc}_{t+s} P_{t+s}^{\varepsilon_p} Y_{t+s} = 0. \quad (\text{B.45})$$

Expressing $P_t(i)$, we get

$$P_t(i) = \frac{\varepsilon_p}{1 - \varepsilon_p} \frac{E_t \sum_{s=0}^{\infty} \beta^s \phi_p^s \lambda_{t+s} \Pi_{t-1, t+s-1}^{-\zeta_p \varepsilon_p} \text{mc}_{t+s} P_{t+s}^{\varepsilon_p} Y_{t+s}}{E_t \sum_{s=0}^{\infty} \beta^s \phi_p^s \lambda_{t+s} \Pi_{t-1, t+s-1}^{\zeta_p(1-\varepsilon_p)} P_{t+s}^{\varepsilon_p-1} Y_{t+s}}. \quad (\text{B.46})$$

Since nothing on the right side depends on the firm-specific index i , we have that all reoptimizing firms set the same price at t , that is, $P_t(i) = P_t^\#$,

$$P_t^\# = \frac{\varepsilon_p}{1 - \varepsilon_p} \frac{X_{1t}}{X_{2t}}, \quad (\text{B.47})$$

where

$$X_{1t} \equiv E_t \sum_{s=0}^{\infty} \beta^s \phi_p^s \lambda_{t+s} \Pi_{t-1, t+s-1}^{-\zeta_p \varepsilon_p} \text{mc}_{t+s} P_{t+s}^{\varepsilon_p} Y_{t+s}, \quad (\text{B.48})$$

$$X_{2t} \equiv E_t \sum_{s=0}^{\infty} \beta^s \phi_p^s \lambda_{t+s} \Pi_{t-1, t+s-1}^{\zeta_p(1-\varepsilon_p)} P_{t+s}^{\varepsilon_p-1} Y_{t+s}. \quad (\text{B.49})$$

For X_{1t} , a recursive formula is

$$X_{1t} = \lambda_t \text{mc}_{t+s} P_t^{\varepsilon_p} Y_t + \beta \phi_p (1 + \pi_t)^{-\zeta_p \varepsilon_p} E_t X_{1t+1}, \quad (\text{B.50})$$

while for X_{2t} , the corresponding recursive formula is

$$X_{2t} = \lambda_t P_t^{\varepsilon_p-1} Y_t + \beta \phi_p (1 + \pi_t)^{\zeta_p(1-\varepsilon_p)} E_t X_{2t+1}. \quad (\text{B.51})$$

Let us divide (B.50) and (B.51) by $P_t^{\varepsilon_p}$ and $P_t^{\varepsilon_p-1}$, respectively, so that they become

$$x_{1t} = \lambda_t \text{mc}_{t+s} Y_t + \beta \phi_p (1 + \pi_t)^{-\zeta_p \varepsilon_p} E_t [(1 + \pi_{t+1})^{\varepsilon_p} x_{1t+1}], \quad (\text{B.52})$$

$$x_{2t} = \lambda_t Y_t + \beta \phi_p (1 + \pi_t)^{\zeta_p(1-\varepsilon_p)} E_t [(1 + \pi_{t+1})^{\varepsilon_p-1} x_{2t+1}], \quad (\text{B.53})$$

where $x_{1t} \equiv \frac{X_{1t}}{P_t^{\varepsilon_p}}$ and $x_{2t} \equiv \frac{X_{2t}}{P_t^{\varepsilon_p-1}}$. In terms of the new variables x_{1t} and x_{2t} , condition (B.47) becomes

$$1 + \pi_t^\# = \frac{\varepsilon_p}{1 - \varepsilon_p} (1 + \pi_t) \frac{x_{1t}}{x_{2t}}, \quad (\text{B.54})$$

with $\pi_t^\# \equiv P_t^\# / P_{t-1} - 1$.

Aggregate Price Relationship

The condition (B.31) can be rewritten as

$$\begin{aligned}
 P_t &= \left(\int_0^1 P_t(i)^{1-\varepsilon_p} di \right)^{\frac{1}{1-\varepsilon_p}} \\
 &= \left[\int_{\text{reopt.}} P_t(i)^{1-\varepsilon_p} di + \int_{\text{non-reopt.}} P_t(i)^{1-\varepsilon_p} di \right]^{\frac{1}{1-\varepsilon_p}},
 \end{aligned} \tag{B.55}$$

where “reopt.” and “non-reopt.” denote, respectively, the firms that reoptimize and do not reoptimize their prices at t .

Note that $\int_{\text{non-reopt.}} P_t(i)^{1-\varepsilon_p} di = \int_0^1 (1 + \pi_{t-1})^{\zeta_p(1-\varepsilon_p)} P(j)^{1-\varepsilon_p} \omega_{t-1,t}(j) dj$, where $\omega_{t-1,t}(j)$ is the measure of non-reoptimizers at t that had the price $P(j)$ at $t-1$. Furthermore, $\omega_{t-1,t}(j) = \phi_p \omega_{t-1}(j)$, where $\omega_{t-1}(j)$ is the measure of firms with the price $P(j)$ in $t-1$, which implies

$$\begin{aligned}
 \int_{\text{non-reopt.}} P_t(i)^{1-\varepsilon_p} di &= \int_0^1 \phi_p (1 + \pi_{t-1})^{\zeta_p(1-\varepsilon_p)} P(j)^{1-\varepsilon_p} \omega_{t-1}(j) dj \\
 &= \phi_p (1 + \pi_{t-1})^{\zeta_p(1-\varepsilon_p)} P_{t-1}^{1-\varepsilon_p}.
 \end{aligned} \tag{B.56}$$

Substituting (B.56) into (B.55) and using the fact that all reoptimizers set $P_t^\#$, we get

$$P_t^{1-\varepsilon_p} = (1 - \phi_p) (P_t^\#)^{1-\varepsilon_p} + \phi_p (1 + \pi_{t-1})^{\zeta_p(1-\varepsilon_p)} P_{t-1}^{1-\varepsilon_p}. \tag{B.57}$$

We divide both sides of (B.57) by $P_{t-1}^{1-\varepsilon_p}$,

$$(1 + \pi_t)^{1-\varepsilon_p} = (1 - \phi_p) (1 + \pi_t^\#)^{1-\varepsilon_p} + \phi_p (1 + \pi_{t-1})^{\zeta_p(1-\varepsilon_p)}. \tag{B.58}$$

Aggregate Wage Relationship

Similarly to equation (B.57), aggregate wage index can be written as

$$W_t^{1-\varepsilon_w} = (1 - \phi_w) (W_t^\#)^{1-\varepsilon_w} + \phi_w (1 + \pi_{t-1})^{\zeta_w(1-\varepsilon_w)} W_{t-1}^{1-\varepsilon_w},$$

where the second term on the right side corresponds to aggregate wage, set by non-reoptimizing households. Dividing both sides by $P_t^{1-\varepsilon_w}$, we get

$$w_t^{1-\varepsilon_w} = (1 - \phi_w) (w_t^\#)^{1-\varepsilon_w} + \phi_w (1 + \pi_{t-1})^{\zeta_w(1-\varepsilon_w)} (1 + \pi_t)^{\varepsilon_w - 1} w_{t-1}^{1-\varepsilon_w}. \tag{B.59}$$

Aggregate Output

Since all the firms rent capital and hire labor in the same proportion, we get

$$Y_t(i) = A_t K_t^\#(i)^\alpha N_t(i)^{1-\alpha} = A_t \left(\frac{K_t^\#}{N_t} \right)^\alpha N_t(i).$$

Let us define aggregate output

$$\begin{aligned}\bar{Y}_t &\equiv \int_0^1 Y_t(i) di = \int_0^1 A_t K_t^\#(i)^\alpha N_t(i)^{1-\alpha} di \\ &= A_t \left(\frac{K_t^\#}{N_t} \right)^\alpha \int_0^1 N_t(i) di = A_t K_t^\# N_t^{1-\alpha}.\end{aligned}\tag{B.60}$$

We substitute demand for $Y_t(i)$ from (B.12) into (B.60) to get

$$\bar{Y}_t = \int_0^1 Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} di = Y_t P_t^{\varepsilon_p} \int_0^1 P_t(i)^{-\varepsilon_p} di.\tag{B.61}$$

Let us introduce a new variable \bar{P}_t ,

$$(\bar{P}_t)^{-\varepsilon_p} \equiv \int_0^1 P_t(i)^{-\varepsilon_p} di.\tag{B.62}$$

Substituting (B.60) and (B.62) into (B.61) gives us

$$Y_t \equiv \bar{Y}_t \left(\frac{\bar{P}_t}{P_t} \right)^{\varepsilon_p} = \frac{A_t (K_t^\#)^\alpha N_t^{1-\alpha}}{\Delta_t^p},\tag{B.63}$$

where Δ_t^p is a measure of price dispersion across firms, defined by

$$\Delta_t^p \equiv \left(\frac{\bar{P}_t}{P_t} \right)^{-\varepsilon_p}.\tag{B.64}$$

Note that if $P_t(i) = P_t(i')$ for all i and $i' \in [0, 1]$, then $\Delta_t^p = 1$, that is, there is no price dispersion across firms.

Law of Motion for Price Dispersion Δ_t^p

By analogy with (B.57), the variable \bar{P}_t , defined in (B.62), satisfies

$$\bar{P}_t^{-\varepsilon_p} = (1 - \phi_p) (P_t^\#)^{-\varepsilon_p} + \phi_p (1 + \pi_{t-1})^{-\zeta_p \varepsilon_p} \bar{P}_{t-1}^{-\varepsilon_p}.\tag{B.65}$$

By using (B.65) in (B.64), we get

$$\Delta_t^p = (1 - \phi_p) \left(\frac{P_t^\#}{P_t} \right)^{-\varepsilon_p} + \phi_p (1 + \pi_{t-1})^{-\zeta_p \varepsilon_p} \left(\frac{\bar{P}_{t-1}}{P_t} \right)^{-\varepsilon_p}.$$

This implies

$$\Delta_t^p = (1 - \phi_p) \left(\frac{P_t^\#}{P_t} \right)^{-\varepsilon_p} \left(\frac{P_{t-1}}{P_{t-1}} \right)^{-\varepsilon_p} + \phi_p (1 + \pi_{t-1})^{-\zeta_p \varepsilon_p} \left(\frac{\bar{P}_{t-1}}{P_t} \right)^{-\varepsilon_p} \left(\frac{P_{t-1}}{P_{t-1}} \right)^{-\varepsilon_p}.$$

Simplifying the latter expression, we obtain the law of motion for Δ_t^p ,

$$\Delta_t^p = (1 + \pi_t)^{\varepsilon_p} \cdot \left[(1 - \phi_p) (1 + \pi_t^\#)^{-\varepsilon_p} + \phi_p (1 + \pi_{t-1})^{-\zeta_p \varepsilon_p} \left(\frac{\bar{P}_t}{P_t} \right)^{-\varepsilon_p} \Delta_{t-1}^p \right].\tag{B.66}$$

Aggregate Resource Constraint

Summing up the household's budget constraint (B.6) across all agents eliminates the state-contingent claims as they are in a zero net supply. Combining the resulting household's budget constraint (B.6) with the government budget constraint (B.19), we have the aggregate resource constraint

$$C_t + I_t + \omega_t^s Y_t = \frac{W_t N_t}{P_t} + R_t u_t K_t - \left(\chi_1 (u_t - 1) + \frac{\chi_2}{2} (u_t - 1)^2 \right) \frac{K_t}{Z_t} + \frac{D_t}{P_t}, \quad (\text{B.67})$$

where $W_t N_t = \int_0^1 W_t(l) N_t(l) dl$. Note that the i th intermediate-good firm's profit at t is $D_t(i) \equiv P_t(i) Y_t(i) - W_t N_t(i) - R_t^n K_t^\#$. Consequently,

$$\begin{aligned} D_t &= \int_0^1 D_t(i) di = \int_0^1 P_t(i) Y_t(i) di - W_t \int_0^1 N_t(i) di + R_t^n \int_0^1 K_t^\#(i) di \\ &= P_t Y_t - W_t N_t - R_t^n K_t^\#, \end{aligned}$$

where $P_t Y_t = \int_0^1 P_t(i) Y_t(i) di$ follows by a zero-profit condition of the final-good firms. Hence, (B.67) can be rewritten as

$$C_t + I_t + G_t + \left(\chi_1 (u_t - 1) + \frac{\chi_2}{2} (u_t - 1)^2 \right) \frac{K_t}{Z_t} = Y_t. \quad (\text{B.68})$$

Full Set of Optimality Conditions

Below, we summarize the full set of the equilibrium conditions in the studied new Keynesian model (B.1)–(B.21):

$$\lambda_t = \frac{1}{C_t - bC_{t-1}} - \beta b E_t \frac{1}{C_{t+1} - bC_t}, \quad (\text{B.69})$$

$$R_t = \frac{1}{Z_t} [\chi_1 + \chi_2 (u_t - 1)], \quad (\text{B.70})$$

$$\lambda_t = \beta E_t \lambda_{t+1} (1 + i_t) (1 + \pi_{t+1})^{-1}, \quad (\text{B.71})$$

$$\begin{aligned} \lambda_t &= \mu_t Z_t \left[1 - \frac{\tau}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \tau \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] \\ &\quad + \beta E_t \mu_{t+1} Z_{t+1} \tau \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2, \end{aligned} \quad (\text{B.72})$$

$$\begin{aligned} \mu_t &= \beta E_t \left[\lambda_{t+1} \left(R_{t+1} u_{t+1} - \frac{1}{Z_{t+1}} \left[\chi_1 (u_{t+1} - 1) + \frac{\chi_2}{2} (u_{t+1} - 1)^2 \right] \right) \right. \\ &\quad \left. + \mu_{t+1} (1 - d) \right], \end{aligned} \quad (\text{B.73})$$

$$w_t^\# = \frac{\varepsilon_w \widehat{f}_{1,t}}{\varepsilon_w - 1 \widehat{f}_{2,t}}, \quad (\text{B.74})$$

$$\begin{aligned} \widehat{f}_{1,t} &= \psi \left(\frac{w_t}{w_t^\#} \right)^{\varepsilon_w(1+\eta)} N_t^{1+\eta} \\ &\quad + \phi_w \beta (1 + \pi_t)^{-\zeta_w \varepsilon_w(1+\eta)} E_t \left[(1 + \pi_{t+1})^{\varepsilon_w(1+\eta)} \left(\frac{w_{t+1}^\#}{w_t^\#} \right)^{\varepsilon_w(1+\eta)} \widehat{f}_{1,t+1} \right], \end{aligned} \quad (\text{B.75})$$

$$\begin{aligned} \widehat{f}_{2,t} &= \lambda_t \left(\frac{w_t}{w_t^\#} \right)^{\varepsilon_w} N_t \\ &\quad + \phi_w \beta (1 + \pi_t)^{\zeta_w(1-\varepsilon_w)} E_t \left[(1 + \pi_{t+1})^{\varepsilon_w-1} \left(\frac{w_{t+1}^\#}{w_t^\#} \right)^{\varepsilon_w} \widehat{f}_{2,t+1} \right], \end{aligned} \quad (\text{B.76})$$

$$w_t^{1-\varepsilon_w} = (1 - \phi_w)(w_t^\#)^{1-\varepsilon_w} + (1 + \pi_{t-1})^{\zeta_w(1-\varepsilon_w)} (1 + \pi_t)^{\varepsilon_w-1} \phi_w w_{t-1}^{1-\varepsilon_w}, \quad (\text{B.77})$$

$$Y_t = \frac{A_t (K_t^\#)^\alpha N_t^{1-\alpha}}{\Delta_t^p}, \quad (\text{B.78})$$

$$\Delta_t^p = (1 + \pi_t)^{\varepsilon_p} [(1 - \phi_w)(1 + \pi_t^\#)^{-\varepsilon_p} + (1 + \pi_{t-1})^{-\zeta_p \varepsilon_p} \phi_p \Delta_{t-1}^p], \quad (\text{B.79})$$

$$(1 + \pi_t)^{1-\varepsilon_p} = (1 - \phi_p)(1 + \pi_t^\#)^{1-\varepsilon_p} + \phi_p (1 + \pi_{t-1})^{\zeta_p(1-\varepsilon_p)}, \quad (\text{B.80})$$

$$1 + \pi_t^\# = \frac{\varepsilon_p}{\varepsilon_p - 1} (1 + \pi_t) \frac{x_{1,t}}{x_{2,t}}, \quad (\text{B.81})$$

$$x_{1,t} = \lambda_t \text{mc}_t Y_t + \phi_p \beta (1 + \pi_t)^{-\zeta_p \varepsilon_p} E_t [(1 + \pi_{t+1})^{\varepsilon_p} x_{1,t+1}], \quad (\text{B.82})$$

$$x_{2,t} = \lambda_t Y_t + \phi_p \beta (1 + \pi_t)^{\zeta_p(1-\varepsilon_p)} E_t [(1 + \pi_{t+1})^{\varepsilon_p-1} x_{2,t+1}], \quad (\text{B.83})$$

$$\frac{w_t}{R_t} = \frac{1 - \alpha}{\alpha} \cdot \frac{K_t^\#}{N_t}, \quad (\text{B.84})$$

$$w_t = \text{mc}_t (1 - \alpha) A_t \left(\frac{K_t^\#}{N_t} \right)^\alpha, \quad (\text{B.85})$$

$$\begin{aligned} i_t &= (1 - \rho_i) i_t + \rho_i i_{t-1} \\ &\quad + (1 - \rho_i) [\phi_\pi (\pi_t - \pi^*) + \phi_y (\ln Y_t - \ln Y_{t-1})] + \varepsilon_{i,t}, \end{aligned} \quad (\text{B.86})$$

$$Y_t = C_t + I_t + G_t + (\chi_1 (u_t - 1) + \chi_2 (u_t - 1)^2) \frac{K_t}{Z_t}, \quad (\text{B.87})$$

$$K_{t+1} = Z_t \left[1 - \frac{\tau}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t + (1 - d) K_t, \quad (\text{B.88})$$

where $K_t^\# = u_t K_t$, $G_t = \omega_t^g Y_t$, and exogenous shocks A_t , Z_t , and ω_t^g follow (B.15), (B.7), (B.20), respectively; and $f_{1,t}$, $f_{2,t}$ and $x_{1,t}$, $x_{2,t}$ are supplementary variables introduced for writing the problem in a recursive form; and Δ_t^p is a measure of price dispersion across firms. In total, there are 25 equations in 25 variables:

$$\begin{aligned} &\{ \lambda_t, C_t, R_t, Z_t, u_t, i_t, \pi_t, I_t, \mu_t, w_t^\#, \widehat{f}_{1,t}, \widehat{f}_{2,t}, w_t, Y_t, \\ &\quad A_t, N_t, \Delta_t^p, \pi_t^\#, x_{1,t}, x_{2,t}, \text{mc}_t, K_t, K_t^\#, G_t, \omega_t^g \}. \end{aligned}$$

B.3. Defining a Lower Error Bound

Defining Approximation Errors in Variables

The approximation errors in the model's variables are defined by the following twenty equations that correspond to the optimality conditions (B.69)–(B.88), respectively:

$$\widehat{\lambda}_t(1 + \delta_{\lambda_t}) = \frac{1}{\widehat{C}_t(1 + \delta_{C_t}) - bC_{t-1}} - E_t \left[\frac{\beta b}{\widehat{C}_{t+1}(1 + \delta_{C_{t+1}}) - b\widehat{C}_t(1 + \delta_{C_t})} \right], \quad (\text{B.89})$$

$$\widehat{R}_t(1 + \delta_{R_t}) = \frac{1}{Z_t} [\chi_1 + \chi_2(\widehat{u}_t(1 + \delta_{u_t}) - 1)], \quad (\text{B.90})$$

$$\widehat{\lambda}_t(1 + \delta_{\lambda_t}) = \beta E_t \left[\frac{\widehat{\lambda}_{t+1}(1 + \delta_{\lambda_{t+1}})(1 + \widehat{i}_t(1 + \delta_{i_t}))}{1 + \widehat{\pi}_{t+1}(1 + \delta_{\pi_{t+1}})} \right], \quad (\text{B.91})$$

$$\begin{aligned} \widehat{\lambda}_t(1 + \delta_{\lambda_t}) &= \widehat{\mu}_t(1 + \delta_{\mu_t}) \\ &\times Z_t \left[1 - \frac{\tau}{2} \left(\frac{\widehat{I}_t(1 + \delta_{I_t})}{I_{t-1}} - 1 \right)^2 \right. \\ &\left. - \tau \left(\frac{\widehat{I}_t(1 + \delta_{I_t})}{I_{t-1}} - 1 \right) \frac{\widehat{I}_t(1 + \delta_{I_t})}{I_{t-1}} \right] \\ &+ \beta E_t \left[\widehat{\mu}_{t+1}(1 + \delta_{\mu_{t+1}}) Z_{t+1} \tau \left(\frac{\widehat{I}_{t+1}(1 + \delta_{I_{t+1}})}{I_t(1 + \delta_{I_t})} - 1 \right) \right. \\ &\left. \times \left(\frac{\widehat{I}_{t+1}(1 + \delta_{I_{t+1}})}{I_t(1 + \delta_{I_t})} \right)^2 \right], \end{aligned} \quad (\text{B.92})$$

$$\begin{aligned} \widehat{\mu}_t(1 + \delta_{\mu_t}) &= \beta E_t \left[\widehat{\lambda}_{t+1}(1 + \delta_{\lambda_{t+1}}) \times \left\{ \widehat{R}_{t+1}(1 + \delta_{R_{t+1}}) \cdot \widehat{u}_{t+1}(1 + \delta_{u_{t+1}}) \right. \right. \\ &\left. \left. - \frac{1}{Z_{t+1}} \left[\chi_1(\widehat{u}_{t+1}(1 + \delta_{u_{t+1}}) - 1) \right. \right. \right. \\ &\left. \left. \left. + \frac{\chi_2}{2}(\widehat{u}_{t+1}(1 + \delta_{u_{t+1}}) - 1)^2 \right] \right\} \right. \\ &\left. + \widehat{\mu}_{t+1}(1 + \delta_{\mu_{t+1}})(1 - d) \right], \end{aligned} \quad (\text{B.93})$$

$$\widehat{w}_t^\#(1 + \delta_{w_t^\#}) = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\widehat{f}_{1,t}(1 + \delta_{f_{1t}})}{\widehat{f}_{2,t}(1 + \delta_{f_{2t}})}, \quad (\text{B.94})$$

$$\begin{aligned} \widehat{f}_{1,t}(1 + \delta_{f_{1t}}) &= \psi \left(\frac{\widehat{w}_t(1 + \delta_{w_t})}{\widehat{w}_t^\#(1 + \delta_{w_t^\#})} \right)^{\varepsilon_w(1+\eta)} [\widehat{N}_t(1 + \delta_{N_t})]^{1+\eta} \\ &+ \phi_w \beta (1 + \widehat{\pi}_t(1 + \delta_{\pi_t}))^{-\xi_w \varepsilon_w(1+\eta)} \end{aligned} \quad (\text{B.95})$$

$$\begin{aligned}
& \times E_t \left[\left[1 + \widehat{\pi}_{t+1}(1 + \delta_{\pi_{t+1}}) \right]^{\varepsilon w(1+\eta)} \right. \\
& \quad \times \left. \left(\frac{\widehat{w}_{t+1}^\#(1 + \delta_{w_{t+1}^\#})}{\widehat{w}_t^\#(1 + \delta_{w_t^\#})} \right)^{\varepsilon w(1+\eta)} \widehat{f}_{1,t+1}(1 + \delta_{f_{1,t+1}}) \right], \\
\widehat{f}_{2,t}(1 + \delta_{f_{2t}}) &= \widehat{\lambda}_t \left(\frac{\widehat{w}_t(1 + \delta_{w_t})}{\widehat{w}_t^\#(1 + \delta_{w_t^\#})} \right)^{\varepsilon w} \widehat{N}_t(1 + \delta_{N_t}) \\
& \quad + \phi_w \beta (1 + \widehat{\pi}_t(1 + \delta_{\pi_t}))^{\zeta w(1-\varepsilon w)} \\
& \quad \times E_t \left[(1 + \widehat{\pi}_{t+1}(1 + \delta_{\pi_{t+1}}))^{\varepsilon w-1} \right. \\
& \quad \times \left. \left(\frac{\widehat{w}_{t+1}(1 + \delta_{w_{t+1}})}{\widehat{w}_{t+1}^\#(1 + \delta_{w_{t+1}^\#})} \right)^{\varepsilon w} \widehat{f}_{2,t+1}(1 + \delta_{f_{2,t+1}}) \right],
\end{aligned} \tag{B.96}$$

$$\begin{aligned}
\widehat{w}_t^{1-\varepsilon w}(1 + \delta_{w_t})^{1-\varepsilon w} &= (1 - \phi_w)(\widehat{w}_t^\#)^{1-\varepsilon w}(1 + \delta_{w_t^\#})^{1-\varepsilon w} \\
& \quad + (1 + \pi_{t-1})^{\zeta w(1-\varepsilon w)}(1 + \widehat{\pi}_t(1 + \delta_{\pi_t}))^{\varepsilon w-1} \phi_w w_{t-1}^{1-\varepsilon w},
\end{aligned} \tag{B.97}$$

$$\widehat{Y}_t(1 + \delta_{Y_t}) = A_t (\widehat{K}_t^\#)^\alpha (1 + \delta_{K_t^\#})^\alpha \widehat{N}_t^{1-\alpha} (1 + \delta_{N_t})^{1-\alpha} [\widehat{\Delta}_t^p]^{-1} (1 + \delta_{\Delta_t^p})^{-1}, \tag{B.98}$$

$$\begin{aligned}
\widehat{\Delta}_t^p(1 + \delta_{\Delta_t^p}) &= (1 + \widehat{\pi}_t(1 + \delta_{\pi_t}))^{\varepsilon p} \\
& \quad \times [(1 - \phi_w)(1 + \widehat{\pi}_t^\#(1 + \delta_{\pi_t^\#}))^{-\varepsilon p} \\
& \quad + (1 + \pi_{t-1})^{-\zeta_p \varepsilon p} \phi_p \Delta_{t-1}^p],
\end{aligned} \tag{B.99}$$

$$(1 + \widehat{\pi}_t(1 + \delta_{\pi_t}))^{1-\varepsilon p} = (1 - \phi_p)(1 + \widehat{\pi}_t^\#(1 + \delta_{\pi_t^\#}))^{1-\varepsilon p} + \phi_p(1 + \pi_{t-1})^{\zeta_p(1-\varepsilon p)}, \tag{B.100}$$

$$1 + \widehat{\pi}_t^\#(1 + \delta_{\pi_t^\#}) = \frac{\varepsilon_p}{\varepsilon_p - 1} (1 + \widehat{\pi}_t(1 + \delta_{\pi_t})) \frac{\widehat{x}_{1,t}}{\widehat{x}_{2,t}} (1 + \delta_{x_{1t}})(1 + \delta_{x_{2t}})^{-1}, \tag{B.101}$$

$$\begin{aligned}
\widehat{x}_{1,t}(1 + \delta_{x_{1t}}) &= \widehat{\lambda}_t(1 + \delta_{\lambda_t}) \widehat{\text{mc}}_t(1 + \delta_{\text{mc}_t}) \widehat{Y}_t(1 + \delta_{Y_t}) \\
& \quad + \phi_p \beta (1 + \widehat{\pi}_t(1 + \delta_{\pi_t}))^{-\zeta_p \varepsilon p} \\
& \quad \times E_t \left[(1 + \widehat{\pi}_{t+1}(1 + \delta_{\pi_{t+1}}))^{\varepsilon p} \widehat{x}_{1,t+1}(1 + \delta_{x_{1,t+1}}) \right],
\end{aligned} \tag{B.102}$$

$$\begin{aligned}
\widehat{x}_{2,t}(1 + \delta_{x_{2t}}) &= \widehat{\lambda}_t(1 + \delta_{\lambda_t}) \widehat{Y}_t + \phi_p \beta (1 + \widehat{\pi}_t(1 + \delta_{\pi_t}))^{\zeta_p(1-\varepsilon p)} \\
& \quad \times E_t \left[(1 + \widehat{\pi}_{t+1}(1 + \delta_{\pi_{t+1}}))^{\varepsilon p-1} \widehat{x}_{2,t+1}(1 + \delta_{x_{2,t+1}}) \right],
\end{aligned} \tag{B.103}$$

$$\frac{\widehat{w}_t(1 + \delta_{w_t})}{\widehat{R}_t(1 + \delta_{R_t})} = \frac{1 - \alpha}{\alpha} \cdot \widehat{K}_t^\#(1 + \delta_{K_t^\#}) \widehat{N}_t^{-1}(1 + \delta_{N_t})^{-1}, \tag{B.104}$$

$$\widehat{w}_t(1 + \delta_{w_t}) = \widehat{\text{mc}}_t(1 + \delta_{\text{mc}_t})(1 - \alpha) A_t (\widehat{K}_t^\#)^\alpha (1 + \delta_{K_t^\#})^\alpha N_t^{-\alpha} (1 + \delta_{N_t})^{-\alpha}, \tag{B.105}$$

$$\begin{aligned}
\widehat{i}_t(1 + \delta_{i_t}) &= (1 - \rho_i) i + \rho_i i_{t-1} \\
& \quad + (1 - \rho_i) [\phi_\pi (\widehat{\pi}_t(1 + \delta_{\pi_t}) - \pi^*)]
\end{aligned} \tag{B.106}$$

$$\begin{aligned}
 & + \phi_y (\ln \widehat{Y}_t + \ln(1 + \delta_{Y_t}) - \ln Y_{t-1}) + \varepsilon_{i,t}, \\
 \widehat{Y}_t(1 + \delta_{Y_t}) &= \widehat{C}_t(1 + \delta_{C_t}) + \widehat{I}_t(1 + \delta_{I_t}) \\
 & + \widehat{G}_t(1 + \delta_{G_t}) + (\chi_1(\widehat{u}_t(1 + \delta_{u_t}) - 1) \\
 & + \chi_2(\widehat{u}_t(1 + \delta_{u_t}) - 1)^2) \frac{K_t}{Z_t},
 \end{aligned} \tag{B.107}$$

$$K_{t+1}(1 + \delta_{K_{t+1}}) = Z_t \left[1 - \frac{\tau}{2} \left(\frac{\widehat{I}_t(1 + \delta_{I_t})}{I_{t-1}} - 1 \right)^2 \right] \widehat{I}_t(1 + \delta_{I_t}) + (1 - d)K_t, \tag{B.108}$$

where hats on the variables denote their approximated values; $f_{1,t}$, $f_{2,t}$ and $x_{1,t}$, $x_{2,t}$ are supplementary variables; Δ_t^p is a measure of price dispersion across firms.

Setting up a Minimization Problem

To construct the lower bound on approximation errors, we minimize the least-squares criterion for each t :

$$\begin{aligned}
 \min_{x_t} & \delta_{\lambda_t}^2 + \delta_{C_t}^2 + \delta_{\mu_t}^2 + \delta_{R_t}^2 + \delta_{u_t}^2 + \delta_{\pi_t}^2 + \delta_{I_t}^2 + \delta_{i_t}^2 + \delta_{f_{1t}}^2 + \delta_{f_{2t}}^2 + \delta_{w_t}^2 + \delta_{w_t^\#}^2 \\
 & + \delta_{N_t}^2 + \delta_{Y_t}^2 + \delta_{K_t^\#}^2 + \delta_{\Delta_t^p}^2 + \delta_{\pi_t^\#}^2 + \delta_{x_{1t}}^2 + \delta_{x_{2t}}^2 + \delta_{mc_t}^2 + \delta_{K_{t+1}}^2 + \delta_{G_t}^2 \\
 & + \sum_{j=1}^J [\delta_{\lambda_{t+1,j}}^2 + \delta_{C_{t+1,j}}^2 + \delta_{\mu_{t+1,j}}^2 + \delta_{\pi_{t+1,j}}^2 + \delta_{I_{t+1,j}}^2 + \delta_{R_{t+1,j}}^2 + \delta_{f_{1t+1,j}}^2 \\
 & + \delta_{f_{2t+1,j}}^2 + \delta_{w_{t+1,j}^\#}^2 + \delta_{x_{1t+1,j}}^2 + \delta_{x_{2t+1,j}}^2 + \delta_{u_{t+1}}^2]
 \end{aligned} \tag{B.109}$$

s.t. (B.89)–(B.108),

where $x_t \equiv \{\delta_{\lambda_t}, \delta_{\lambda_{t+1,j}}, \dots\}$ is a list of all approximation errors to the corresponding model's variables $\{\lambda_t, \lambda_{t+1,j}, \dots\}$ that appear in the objective function (B.109). Similarly to the optimal growth model, approximation errors in the current period variables are defined in a given point of the state space, while approximation errors in future variables are defined in J integration nodes. Restrictions (B.89)–(B.108) are the optimality conditions (B.69)–(B.88) written in terms of an approximation solution and the corresponding approximation errors; they are provided in Appendix B.2. Again, using linearized optimality conditions in place of nonlinear optimality conditions leads to a linear-quadratic programming problem that is more simple to solve numerically and that produces a good initial guess for the problem with the nonlinear restrictions. A linearization of the optimality conditions (B.69)–(B.88) is shown in Appendix B.4.

B.4. Constructing Approximation Errors Using Linearized Model's Equations

We construct approximation errors satisfying linearized model's equations (B.89)–(B.108).

Condition (B.89)

Finding a first-order Taylor expansion of equation (B.89) and omitting second-order terms, we have

$$0 = -\delta_{\lambda_t} \cdot \widehat{\lambda}_t - \delta_{C_t} \cdot \{(\widehat{C}_t - b\widehat{C}_{t-1})^{-2}\widehat{C}_t + b\widehat{C}_t \cdot \beta b E_t(\widehat{C}_{t+1} - b\widehat{C}_t)^{-2}\} \\ + \beta b \sum_{j=1}^J \omega_j [(\widehat{C}_{t+1,j} - b\widehat{C}_t)^{-2}\widehat{C}_{t+1,j} \delta_{C_{t+1,j}}] + \lambda_t \mathcal{R}_t^1.$$

For convenience, we introduce the following compact notation:

$$h^1 \equiv \beta b E_t(\widehat{C}_{t+1} - b\widehat{C}_t)^{-2}.$$

Introducing compact notation, we get

$$a^{1,1} \cdot \delta_{\lambda_t} + a^{1,3} \cdot \delta_{C_t} + \sum_{j=1}^J \omega_j a_j^{1,4} \cdot \delta_{C_{t+1,j}} + b^1 = 0,$$

where

$$a^{1,1} \equiv -\widehat{\lambda}_t, \\ a^{1,3} \equiv -(\widehat{C}_t - b\widehat{C}_{t-1})^{-2}\widehat{C}_t - b\widehat{C}_t \cdot h^1, \\ a_j^{1,4} \equiv \beta b (\widehat{C}_{t+1,j} - b\widehat{C}_t)^{-2}\widehat{C}_{t+1,j}, \\ b^1 \equiv \widehat{\lambda}_t \mathcal{R}_t^1,$$

with \mathcal{R}_t^1 being the residual of this FOC, given by (B.110).

Condition (B.90)

By finding a first-order Taylor expansion in errors of condition (B.90), we obtain

$$-\delta_{R_t} + \mathcal{R}_t^{39} + \frac{1}{\widehat{R}_t Z_t} \chi_2 \widehat{u}_t \delta_{u_t} = 0.$$

Introducing compact notation, we get

$$a^{2,7} \cdot \delta_{R_t} + a^{2,8} \cdot \delta_{u_t} + b^2 = 0,$$

where

$$a^{2,7} \equiv -1, \quad a^{2,8} = \frac{1}{\widehat{R}_t Z_t} \chi_2 \widehat{u}_t, \quad b^2 = \mathcal{R}_t^2,$$

where \mathcal{R}_t^2 is the residual in equation (B.111).

Condition (B.91)

A first-order Taylor expansion of (B.91) yields

$$\delta_{\lambda_t} = \ln(1 + \mathcal{R}_t^3) + E_t \delta_{\lambda_{t+1}} + \frac{\widehat{i}_t}{1 + \widehat{i}_t} \delta_{i_t} - E_t \frac{\widehat{\pi}_{t+1}}{1 + \widehat{\pi}_{t+1}} \delta_{\pi_{t+1}}.$$

The latter condition can be rewritten as

$$a^{3,1} \cdot \delta_{\lambda_t} + a^{3,2} \cdot \sum_{j=1}^J \omega_j \delta_{\lambda_{t+1,j}} + \sum_{j=1}^J \omega_j a_j^{3,11} \cdot \delta_{\pi_{t+1,j}} + a^{3,15} \cdot \delta_{i_t} + b^3 = 0,$$

where

$$\begin{aligned} a^{3,1} &\equiv -1, & a^{3,2} &\equiv 1, & a^{3,15} &\equiv \frac{\widehat{i}_t}{1 + \widehat{i}_t}, \\ a_j^{3,11} &\equiv - \sum_{j=1}^J \omega_j \frac{\widehat{\pi}_{t+1,j}}{1 + \widehat{\pi}_{t+1,j}}, & b^3 &\equiv \ln(1 + \mathcal{R}_t^3), \end{aligned}$$

with \mathcal{R}_t^3 being a residual, defined in (B.112).

Condition (B.92)

A first-order Taylor expansion of (B.92) yields

$$\begin{aligned} 0 = & -\widehat{\lambda}_t + \widehat{\mu}_t Z_t + \frac{3}{2} \widehat{\mu}_t Z_t \tau \left(\frac{\widehat{I}_t}{I_{t-1}} \right)^2 + 2 \widehat{\mu}_t Z_t \tau \frac{\widehat{I}_t}{I_{t-1}} + \frac{1}{2} \widehat{\mu}_t Z_t \tau + h_{1t}^4 - h_{2t}^4 \\ & - \widehat{\lambda}_t \delta_{\lambda_t} \\ & + \widehat{\mu}_t Z_t \left[2\tau \frac{\widehat{I}_t}{I_{t-1}} - \frac{3}{2} \tau \left(\frac{\widehat{I}_t}{I_{t-1}} \right)^2 + \frac{1}{2} \tau + 1 \right] \delta_{\mu_t} \\ & + \beta E_t \left[\left(\widehat{\mu}_{t+1} Z_{t+1} \tau \left(\frac{\widehat{I}_{t+1}}{I_t} \right)^3 - \delta_{\mu_{t+1}} \widehat{\mu}_{t+1} Z_{t+1} \tau \left(\frac{\widehat{I}_{t+1}}{I_t} \right)^2 \right) \cdot \delta_{\mu_{t+1}} \right] \\ & + \widehat{\mu}_t Z_t \left[2\tau \frac{\widehat{I}_t}{I_{t-1}} - 3\tau \left(\frac{\widehat{I}_t}{I_{t-1}} \right)^2 + 2h_{1t}^4 - 3h_{2t}^4 \right] \delta_{I_t} \\ & + \beta E_t \left[3\widehat{\mu}_{t+1} Z_{t+1} \tau \left(\frac{\widehat{I}_{t+1}}{I_t} \right)^3 - 2\widehat{\mu}_{t+1} Z_{t+1} \tau \left(\frac{\widehat{I}_{t+1}}{I_t} \right)^2 \delta_{I_{t+1}} \right], \end{aligned}$$

where the following compact notation is used:

$$\begin{aligned} h_1^4 &\equiv \beta \widehat{\mu}_{t+1} Z_{t+1} \tau \left(\frac{\widehat{I}_{t+1}}{I_t} \right)^3, \\ h_2^4 &\equiv \beta \widehat{\mu}_{t+1} Z_{t+1} \tau \left(\frac{\widehat{I}_{t+1}}{I_t} \right)^2. \end{aligned}$$

Introducing compact notation, we have

$$a^{4,1} \cdot \delta_{\lambda_t} + a^{4,5} \cdot \delta_{\mu_t} + \sum_{j=1}^J \omega_j a_j^{4,6} \cdot \delta_{\mu_{t+1,j}} + a^{4,12} \cdot \delta_{I_t} + \sum_{j=1}^J \omega_j a_j^{4,13} \cdot \delta_{I_{t+1,j}} + b^4 = 0,$$

where

$$a^{4,1} \equiv -\widehat{\lambda}_t,$$

$$\begin{aligned}
a^{4,5} &\equiv \widehat{\mu}_t Z_t \left[2\tau \frac{\widehat{I}_t}{I_{t-1}} - \frac{3}{2}\tau \left(\frac{\widehat{I}_t}{I_{t-1}} \right)^2 + \frac{1}{2}\tau + 1 \right], \\
a_j^{4,6} &\equiv m_{1,j}^4 - m_{2,j}^4, \\
a^{4,12} &\equiv \widehat{\mu}_t Z_t \left[2\tau \frac{\widehat{I}_t}{I_{t-1}} - 3\tau \left(\frac{\widehat{I}_t}{I_{t-1}} \right)^2 \right] - 3h_1^4 + 2h_2^4, \\
a_j^{4,13} &\equiv 3m_{1,j}^4 - 2m_{2,j}^4, \\
b^4 &\equiv -\widehat{\lambda}_t + \widehat{\mu}_t Z_t + \frac{3}{2}\widehat{\mu}_t Z_t \tau \left(\frac{\widehat{I}_t}{I_{t-1}} \right)^2 + 2\widehat{\mu}_t Z_t \tau \frac{\widehat{I}_t}{I_{t-1}} + \frac{1}{2}\widehat{\mu}_t Z_t \tau + h_1^4 - h_2^4,
\end{aligned}$$

with $m_{1,j}^4 \equiv \beta \widehat{\mu}_{t+1,j} Z_{t+1,j} \tau \left(\frac{\widehat{I}_{t+1,j}}{I_t} \right)^3$ and $m_{2,j}^4 \equiv \beta \widehat{\mu}_{t+1,j} Z_{t+1,j} \tau \left(\frac{\widehat{I}_{t+1,j}}{I_t} \right)^2$.

Condition (B.93)

A first-order Taylor expansion of (B.93) implies

$$\begin{aligned}
0 &= \mathcal{R}_t^5 \\
&- \sum_{j=1}^J \omega_j m_{1,j}^5 \widehat{\lambda}_{t+1,j} \left[\left(-\frac{1}{Z_{t+1,j}} \right) \chi_2 (\widehat{u}_{t+1,j} - 1) - \widehat{R}_{t+1,j} \widehat{u}_{t+1,j} \right] \cdot \delta_{\lambda_{t+1,j}} \\
&- \delta_{\mu_t} \\
&+ \sum_{j=1}^J \omega_j [\widehat{\mu}_{t+1} (1-d) \cdot \delta_{\mu_{t+1}}] \\
&+ \sum_{j=1}^J \omega_j m_{1,j}^5 \widehat{\lambda}_{t+1,j} \widehat{R}_{t+1,j} \widehat{u}_{t+1,j} \cdot \delta_{R_{t+1,j}} \\
&- \sum_{j=1}^J \omega_j m_{1,j}^5 \widehat{\lambda}_{t+1,j} \left(\left(-\frac{1}{Z_{t+1,j}} \right) [\chi_1 (2\widehat{u}_{t+1} - 1) + \chi_2 \widehat{u}_{t+1,j} (\widehat{u}_{t+1,j} - 1)] \right. \\
&\quad \left. - \widehat{R}_{t+1,j} \widehat{u}_{t+1,j} \right) \cdot \delta_{u_{t+1,j}},
\end{aligned}$$

where \mathcal{R}_t^5 is a residual defined in (B.114), and

$$m_{1,j}^5 \equiv \left[\widehat{\lambda}_{t+1,j} \left(\widehat{R}_{t+1,j} \widehat{u}_{t+1,j} - \frac{1}{Z_{t+1,j}} \left[\chi_1 (\widehat{u}_{t+1,j} - 1) + \frac{\chi_2}{2} (\widehat{u}_{t+1,j} - 1)^2 \right] \right) + \widehat{\mu}_{t+1,j} (1-d) \right]^{-1}.$$

Introducing further more compact notation, we have

$$\sum_{j=1}^J a_j^{5,2} \cdot \delta_{\lambda_{t+1,j}} + a^{5,5} \cdot \delta_{\mu_t} + \sum_{j=1}^J a_j^{5,6} \cdot \delta_{\mu_{t+1,j}} + \sum_{j=1}^J a_j^{5,14} \cdot \delta_{R_{t+1,j}} + \sum_{j=1}^J a_j^{5,33} \cdot \delta_{u_{t+1,j}} + b^5 = 0,$$

where

$$\begin{aligned}
 a_j^{5,2} &\equiv -\omega_j m_{1,j}^5 \widehat{\lambda}_{t+1,j} \left[\left(-\frac{1}{Z_{t+1,j}} \right) \chi_2 (\widehat{u}_{t+1,j} - 1) - \widehat{R}_{t+1,j} \widehat{u}_{t+1,j} \right], \\
 a_j^{5,5} &\equiv -1, \\
 a_j^{5,6} &\equiv \omega_j \widehat{\mu}_{t+1,j} (1-d), \\
 a_j^{5,14} &\equiv m_{1,j}^5 \widehat{\lambda}_{t+1,j} \widehat{R}_{t+1,j} \widehat{u}_{t+1,j}, \\
 a_j^{5,33} &\equiv -\omega_j m_{1,j}^5 \widehat{\lambda}_{t+1,j} \left(\left(-\frac{1}{Z_{t+1,j}} \right) [\chi_1 (2\widehat{u}_{t+1} - 1) + \chi_2 \widehat{u}_{t+1,j} (\widehat{u}_{t+1,j} - 1)] - \widehat{R}_{t+1,j} \widehat{u}_{t+1,j} \right), \\
 b^5 &\equiv \mathcal{R}_t^5.
 \end{aligned}$$

Condition (B.94)

A first-order Taylor expansion of (B.94) leads us to

$$\delta_{w_t^\#} = \mathcal{R}_t^6 + \delta_{f_{1t}} - \delta_{f_{2t}}.$$

Introducing compact notation, we get

$$a^{6,16} \cdot \delta_{f_{1t}} + a^{6,18} \cdot \delta_{f_{2t}} + a^{6,21} \cdot \delta_{w_t^\#} + b^6 = 0,$$

where

$$a^{6,16} \equiv 1, \quad a^{6,18} \equiv -1, \quad a^{6,21} \equiv -1, \quad b_t^6 \equiv \mathcal{R}_t^6.$$

Condition (B.95)

A first-order Taylor expansion of (B.95) implies

$$\begin{aligned}
 0 &= \widehat{f}_{1,t} \mathcal{R}_t^7 \\
 &\quad + h_{1t}^7 \delta_{\pi_t} \\
 &\quad + \phi_w \beta (1 + \widehat{\pi}_t)^{-\zeta_w \varepsilon_w (1+\eta)} \cdot \varepsilon_w (1 + \eta) \\
 &\quad \cdot E_t \left[[1 + \widehat{\pi}_{t+1}]^{\varepsilon_w (1+\eta) - 1} \left(\frac{\widehat{w}_{t+1}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w (1+\eta)} \widehat{f}_{1,t+1} \cdot \widehat{\pi}_{t+1} \cdot \delta_{\pi_{t+1}} \right] \\
 &\quad - \widehat{f}_{1,t} \delta_{f_{1t}} \\
 &\quad + \phi_w \beta (1 + \widehat{\pi}_t)^{-\zeta_w \varepsilon_w (1+\eta)} E_t \left[[1 + \widehat{\pi}_{t+1}]^{\varepsilon_w (1+\eta)} \left(\frac{\widehat{w}_{t+1}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w (1+\eta)} \widehat{f}_{1,t+1} \cdot \delta_{f_{1t+1}} \right] \\
 &\quad + \psi \left[\frac{\widehat{w}_t^{\varepsilon_w}}{\widehat{w}_t^{\# \varepsilon_w}} \widehat{N}_t \right]^{(1+\eta)} (1 + \eta) \varepsilon_w \delta_{w_t} \\
 &\quad - \left[\psi \left(\frac{\widehat{w}_t^{\varepsilon_w}}{\widehat{w}_t^{\# \varepsilon_w}} \widehat{N}_t \right)^{(1+\eta)} (1 + \eta) \varepsilon_w + h_{2t}^7 \right] \delta_{w_t^\#} \\
 &\quad + \varepsilon_w (1 + \eta) \phi_w \beta (1 + \widehat{\pi}_t)^{-\zeta_w \varepsilon_w (1+\eta)}
 \end{aligned}$$

$$\begin{aligned} & \times E_t \left[[1 + \widehat{\pi}_{t+1}]^{\varepsilon_w(1+\eta)} (\widehat{w}_{t+1}^\#)^{\varepsilon_w(1+\eta)} (\widehat{w}_t^\#)^{-\varepsilon_w(1+\eta)} \widehat{f}_{1,t+1} \cdot \delta_{w_t^\#} \right] \\ & + \psi \left[\frac{\widehat{w}_t^{\varepsilon_w}}{\widehat{w}_t^{\# \varepsilon_w}} \widehat{N}_t \right]^{(1+\eta)} (1+\eta) \delta_{N_t}, \end{aligned}$$

where \mathcal{R}_t^7 denotes a residual (B.116), and where

$$h_{1t}^7 \equiv -\zeta_w \varepsilon_w (1+\eta) \widehat{\pi}_t \cdot \phi_w \beta (1 + \widehat{\pi}_t)^{-\zeta_w \varepsilon_w (1+\eta) - 1} \cdot E_t \left[[1 + \widehat{\pi}_{t+1}]^{\varepsilon_w(1+\eta)} \left(\frac{\widehat{w}_{t+1}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w(1+\eta)} \widehat{f}_{1,t+1} \right],$$

$$h_{2t}^7 \equiv -\varepsilon_w (1+\eta) \phi_w \beta (1 + \widehat{\pi}_t)^{-\zeta_w \varepsilon_w (1+\eta)} E_t \left[[1 + \widehat{\pi}_{t+1}]^{\varepsilon_w(1+\eta)} \left(\frac{\widehat{w}_{t+1}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w(1+\eta)} \widehat{f}_{1,t+1} \right].$$

Using compact notation, we get

$$\begin{aligned} & a^{7,10} \cdot \delta_{\pi_t} + \sum_{j=1}^J a_j^{7,11} \cdot \delta_{\pi_{t+1,j}} + a^{7,16} \cdot \delta_{f_{1t}} + \sum_{j=1}^J a_j^{7,17} \cdot \delta_{f_{1t+1,j}} + a^{7,20} \cdot \delta_{w_t} \\ & + a^{7,21} \cdot \delta_{w_t^\#} + \sum_{j=1}^J a_j^{7,22} \cdot \delta_{w_{t+1,j}^\#} + a^{7,23} \cdot \delta_{N_t} + b^7 = 0, \end{aligned}$$

where

$$\begin{aligned} a^{7,10} & \equiv -\zeta_w \varepsilon_w (1+\eta) \widehat{\pi}_t \cdot \phi_w (1 + \widehat{\pi}_t)^{-\zeta_w \varepsilon_w (1+\eta) - 1} \\ & \quad \cdot \beta E_t \left[[1 + \widehat{\pi}_{t+1}]^{\varepsilon_w(1+\eta)} \left(\frac{\widehat{w}_{t+1}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w(1+\eta)} \widehat{f}_{1,t+1} \right], \\ a_j^{7,11} & \equiv \phi_w \beta (1 + \widehat{\pi}_t)^{-\zeta_w \varepsilon_w (1+\eta)} \cdot \varepsilon_w (1+\eta) \\ & \quad \cdot \omega_j \left[[1 + \widehat{\pi}_{t+1,j}]^{\varepsilon_w(1+\eta) - 1} \left(\frac{\widehat{w}_{t+1,j}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w(1+\eta)} \widehat{f}_{1,t+1,j} \cdot \widehat{\pi}_{t+1,j} \right], \\ a^{7,16} & \equiv -\widehat{f}_{1,t}, \\ m_{1,j}^7 & \equiv \omega_j \phi_w (1 + \widehat{\pi}_t)^{-\zeta_w \varepsilon_w (1+\eta)} \beta \left[[1 + \widehat{\pi}_{t+1,j}]^{\varepsilon_w(1+\eta)} \left(\frac{\widehat{w}_{t+1,j}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w(1+\eta)} \widehat{f}_{1,t+1,j} \right], \\ a_j^{7,17} & \equiv m_{1,j}^7, \quad h_1^7 \equiv \psi \left[\frac{\widehat{w}_t^{\varepsilon_w}}{\widehat{w}_t^{\# \varepsilon_w}} \widehat{N}_t \right]^{(1+\eta)} (1+\eta), \quad a^{7,20} \equiv h_1^7 \cdot \varepsilon_w, \\ h_2^7 & \equiv \phi_w (1 + \widehat{\pi}_t)^{-\zeta_w \varepsilon_w (1+\eta)} \beta E_t \left[[1 + \widehat{\pi}_{t+1}]^{\varepsilon_w(1+\eta)} \left(\frac{\widehat{w}_{t+1}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w(1+\eta)} \widehat{f}_{1,t+1} \right], \\ a^{7,21} & \equiv -h_1^7 \cdot \varepsilon_w - h_2^7 \cdot \varepsilon_w (1+\eta), \quad a_j^{7,22} \equiv \omega_j m_{1,j}^7 \cdot \varepsilon_w (1+\eta), \\ a^{7,23} & \equiv h_1^7, \quad b^7 \equiv \widehat{f}_{1,t} \mathcal{R}_t^{65}. \end{aligned}$$

Condition (B.96)

A first-order Taylor expansion of (B.96) is

$$\begin{aligned}
 0 = & \widehat{f}_{2,t} \mathcal{R}_t^8 \\
 & + \widehat{\lambda}_t \frac{\widehat{w}_t^{\varepsilon_w}}{\widehat{w}_t^{\#\varepsilon_w}} \widehat{N}_t \cdot \delta_{\lambda_t} \\
 & + \zeta_w (1 - \varepsilon_w) \widehat{\pi}_t \cdot \phi_w \beta (1 + \widehat{\pi}_t)^{\zeta_w(1-\varepsilon_w)-1} \cdot E_t \left[[1 + \widehat{\pi}_{t+1}]^{\varepsilon_w-1} \left(\frac{\widehat{w}_{t+1}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w} \widehat{f}_{2,t+1} \right] \cdot \delta_{\pi_t} \\
 & + \phi_w \beta (1 + \widehat{\pi}_t)^{\zeta_w(1-\varepsilon_w)} \cdot (\varepsilon_w - 1) \cdot E_t \left[[1 + \widehat{\pi}_{t+1}]^{\varepsilon_w-2} \left(\frac{\widehat{w}_{t+1}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w} \widehat{f}_{2,t+1} \cdot \widehat{\pi}_{t+1} \cdot \delta_{\pi_{t+1}} \right] \\
 & - \widehat{f}_{2,t} \delta_{f_{2t}} \\
 & + \phi_w \beta (1 + \widehat{\pi}_t)^{\zeta_w(1-\varepsilon_w)} E_t \left[[1 + \widehat{\pi}_{t+1}]^{\varepsilon_w-1} \left(\frac{\widehat{w}_{t+1}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w} \widehat{f}_{2,t+1} \cdot \delta_{f_{2t+1}} \right] \\
 & + \widehat{\lambda}_t \frac{\widehat{w}_t^{\varepsilon_w}}{\widehat{w}_t^{\#\varepsilon_w}} \widehat{N}_t \varepsilon_w \cdot \delta_{w_t} \\
 & - \left[\varepsilon_w \widehat{\lambda}_t \frac{\widehat{w}_t^{\varepsilon_w}}{\widehat{w}_t^{\#\varepsilon_w}} \widehat{N}_t + \varepsilon_w \phi_w \beta (1 + \widehat{\pi}_t)^{\zeta_w(1-\varepsilon_w)} E_t \left([1 + \widehat{\pi}_{t+1}]^{\varepsilon_w-1} \left(\frac{\widehat{w}_{t+1}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w} \widehat{f}_{2,t+1} \right) \right] \cdot \delta_{w_t^\#} \\
 & + \varepsilon_w \phi_w \beta (1 + \widehat{\pi}_t)^{\zeta_w(1-\varepsilon_w)} E_t \left[[1 + \widehat{\pi}_{t+1}]^{\varepsilon_w-1} \left(\widehat{w}_{t+1}^\# \right)^{\varepsilon_w} \left(\widehat{w}_t^\# \right)^{-\varepsilon_w} \widehat{f}_{2,t+1} \cdot \delta_{w_{t+1}^\#} \right] \\
 & + \widehat{\lambda}_t \frac{\widehat{w}_t^{\varepsilon_w}}{\widehat{w}_t^{\#\varepsilon_w}} \widehat{N}_t \cdot \delta_{N_t},
 \end{aligned}$$

where \mathcal{R}_t^8 denotes a residual (B.117). Introducing new notation, we can rewrite the last equations as

$$\begin{aligned}
 & a^{8,1} \cdot \delta_{\lambda_t} + a^{8,10} \cdot \delta_{\pi_t} + \sum_{j=1}^J a_j^{8,11} \cdot \delta_{\pi_{t+1,j}} + a^{8,18} \cdot \delta_{f_{2t}} + \sum_{j=1}^J a_j^{8,19} \cdot \delta_{f_{2t+1,j}} + a^{8,20} \cdot \delta_{w_t} \\
 & + a^{8,21} \cdot \delta_{w_t^\#} + \sum_{j=1}^J a_j^{8,22} \cdot \delta_{w_{t+1,j}^\#} + a^{8,23} \cdot \delta_{N_t} + b^8 = 0,
 \end{aligned}$$

where

$$\begin{aligned}
 h_1^8 & \equiv \widehat{\lambda}_t \frac{\widehat{w}_t^{\varepsilon_w}}{\widehat{w}_t^{\#\varepsilon_w}} \widehat{N}_t, & a^{8,1} & = h_1^8, & a^{8,18} & = -\widehat{f}_{2,t}, & a^{8,20} & = h_1^8 \cdot \varepsilon_w, \\
 a^{8,10} & \equiv \zeta_w (1 - \varepsilon_w) \widehat{\pi}_t \cdot \phi_w (1 + \widehat{\pi}_t)^{\zeta_w(1-\varepsilon_w)-1} \cdot \beta E_t \left[[1 + \widehat{\pi}_{t+1}]^{\varepsilon_w-1} \left(\frac{\widehat{w}_{t+1}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w} \widehat{f}_{2,t+1} \right], \\
 a_j^{8,11} & \equiv \omega_j \phi_w (1 + \widehat{\pi}_t)^{\zeta_w(1-\varepsilon_w)} \cdot (\varepsilon_w - 1) \cdot \beta E_t \left[[1 + \widehat{\pi}_{t+1,j}]^{\varepsilon_w-2} \left(\frac{\widehat{w}_{t+1}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w} \widehat{f}_{2,t+1,j} \cdot \widehat{\pi}_{t+1,j} \right],
 \end{aligned}$$

$$\begin{aligned}
m_j^8 &\equiv \omega_j \phi_w (1 + \widehat{\pi}_t)^{\zeta_w(1-\varepsilon_w)} \beta \left[[1 + \widehat{\pi}_{t+1,j}]^{\varepsilon_w-1} \left(\frac{\widehat{w}_{t+1,j}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w} \widehat{f}_{2,t+1,j} \right], \\
a^{8,21} &\equiv -h_{1t}^8 \cdot \varepsilon_w - h_2^8 \cdot \varepsilon_w, \\
h_2^8 &\equiv \phi_w (1 + \widehat{\pi}_t)^{\zeta_w(1-\varepsilon_w)} \cdot \beta E_t \left([1 + \widehat{\pi}_{t+1}]^{\varepsilon_w-1} \left(\frac{\widehat{w}_{t+1}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w} \widehat{f}_{2,t+1} \right), \\
a_j^{8,22} &\equiv \omega_j m_j^8 \cdot \varepsilon_w, \quad a^{8,23} = h_1^8, \quad b^8 = \widehat{f}_{2,t} \mathcal{R}_t^8.
\end{aligned}$$

Condition (B.97)

A first-order Taylor expansion of (B.97) leads to

$$\begin{aligned}
\widehat{w}_t^{1-\varepsilon_w} \mathcal{R}_t^9 + (1 - \varepsilon_w) \widehat{w}_t^{1-\varepsilon_w} \cdot \delta_{w_t} &= (1 - \phi_w)(1 - \varepsilon_w) (\widehat{w}_t^\#)^{1-\varepsilon_w} \delta_{w_t^\#} \\
&\quad + (1 + \pi_{t-1})^{\zeta_w(1-\varepsilon_w)} \phi_w w_{t-1}^{1-\varepsilon_w} (\varepsilon_w - 1) \widehat{\pi}_t (1 + \widehat{\pi}_t)^{\varepsilon_w-2} \cdot \delta_{\pi_t},
\end{aligned}$$

where \mathcal{R}_t^9 is a residual of this equation, defined in (B.118). After introducing more compact notation, we obtain

$$a^{9,10} \cdot \delta_{\pi_t} + a^{9,20} \cdot \delta_{w_t} + a^{9,21} \cdot \delta_{w_t^\#} + b^9 = 0,$$

where

$$\begin{aligned}
a^{9,20} &\equiv -(1 - \varepsilon_w) \widehat{w}_t^{1-\varepsilon_w}, \\
a^{9,20} &\equiv -(1 - \varepsilon_w) \widehat{w}_t^{1-\varepsilon_w}, \\
a^{9,21} &\equiv (1 - \phi_w)(1 - \varepsilon_w) (\widehat{w}_t^\#)^{1-\varepsilon_w}, \\
a^{9,10} &\equiv (1 + \pi_{t-1})^{\zeta_w(1-\varepsilon_w)} \phi_w w_{t-1}^{1-\varepsilon_w} (\varepsilon_w - 1) \widehat{\pi}_t (1 + \widehat{\pi}_t)^{\varepsilon_w-2}, \\
b^9 &\equiv \widehat{w}_t^{1-\varepsilon_w} \mathcal{R}_t^9.
\end{aligned}$$

Condition (B.98)

A first-order Taylor expansion of (B.98) is

$$(1 - \alpha) \delta_{N_t} - \delta_{Y_t} + \alpha \delta_{K_t^\#} - \delta_{\Delta_t^p} + \mathcal{R}_t^{10} = 0,$$

where the residual of this equation, \mathcal{R}_t^{10} , is defined in (B.119). We rewrite it as

$$a^{10,23} \cdot \delta_{N_t} + a^{10,24} \cdot \delta_{Y_t} + a^{10,25} \cdot \delta_{K_t^\#} + a^{10,26} \cdot \delta_{\Delta_t^p} + b^{10} = 0,$$

where

$$a^{10,24} \equiv -1, \quad a^{10,25} \equiv \alpha, \quad a^{10,23} \equiv 1 - \alpha, \quad a^{10,26} \equiv -1, \quad b^{10} \equiv \ln(1 + \mathcal{R}_t^{10}).$$

Condition (B.99)

A Taylor expansion of equation (B.99) is

$$\begin{aligned}
\varepsilon_p \frac{1}{1 + \widehat{\pi}_t} \widehat{\pi}_t \cdot \delta_{\pi_t} - \delta_{\Delta_t^p} - \left[(1 - \phi_w)(1 + \widehat{\pi}_t^\#)^{-\varepsilon_p} + (1 + \pi_{t-1})^{-\varepsilon_p \varepsilon_p} \phi_p \Delta_{t-1}^p \right]^{-1} \\
\times \varepsilon_p (1 - \phi_w)(1 + \widehat{\pi}_t^\#)^{-\varepsilon_p-1} \widehat{\pi}_t^\# \cdot \delta_{\pi_t^\#} + \mathcal{R}_t^{11} = 0,
\end{aligned}$$

where \mathcal{R}_t^{11} is the residual (B.120) of this equation. In terms of new notation, this becomes

$$a^{11,10} \cdot \delta_{\pi_t} + a^{11,26} \cdot \delta_{\Delta_t^p} + a^{11,27} \cdot \delta_{\pi_t^\#} + b^{11} = 0,$$

where

$$a^{11,10} \equiv \varepsilon_p \frac{1}{1 + \widehat{\pi}_t} \widehat{\pi}_t, \quad a^{11,26} = -1,$$

$$a^{11,27} \equiv -[(1 - \phi_w)(1 + \widehat{\pi}_t^\#)^{-\varepsilon_p} + (1 + \pi_{t-1})^{-\xi_p \varepsilon_p} \phi_p \Delta_{t-1}^p]^{-1} \varepsilon_p (1 - \phi_w)(1 + \widehat{\pi}_t^\#)^{-\varepsilon_p - 1} \widehat{\pi}_t^\#, \\ b^{11} \equiv \mathcal{R}_t^{11}.$$

Condition (B.100)

An expansion of (B.100) is

$$-(1 - \varepsilon_p)(1 + \widehat{\pi}_t)^{-\varepsilon_p} \widehat{\pi}_t \cdot \delta_{\pi_t} + (1 + \widehat{\pi}_t)^{1 - \varepsilon_p} \mathcal{R}_t^{12} + (1 - \phi_p)(1 - \varepsilon_p)(1 + \widehat{\pi}_t^\#)^{-\varepsilon_p} \widehat{\pi}_t^\# \cdot \delta_{\pi_t^\#} = 0,$$

with \mathcal{R}_t^{12} being a residual (B.121). We rewrite this equation as follows:

$$a^{12,10} \cdot \delta_{\pi_t} + a^{12,27} \cdot \delta_{\pi_t^\#} + b^{12} = 0,$$

where

$$a^{12,10} \equiv -(1 - \varepsilon_p)(1 + \widehat{\pi}_t)^{-\varepsilon_p} \widehat{\pi}_t, \\ a^{12,27} \equiv (1 - \phi_p)(1 - \varepsilon_p)(1 + \widehat{\pi}_t^\#)^{-\varepsilon_p} \widehat{\pi}_t^\#, \\ b^{12} \equiv (1 + \widehat{\pi}_t)^{1 - \varepsilon_p} \mathcal{R}_t^{12}.$$

Condition (B.101)

A first-order Taylor expansion of (B.101) implies

$$\frac{\widehat{\pi}_t}{1 + \widehat{\pi}_t} \delta_{\pi_t} - \frac{\widehat{\pi}_t^\#}{1 + \widehat{\pi}_t^\#} \delta_{\pi_t^\#} + \delta_{x_{1t}} - \delta_{x_{2t}} + \mathcal{R}_t^{13} = 0,$$

with \mathcal{R}_t^{13} being this equation's residual that is defined in (B.122); this yields

$$a^{13,10} \cdot \delta_{\pi_t} + a^{13,27} \cdot \delta_{\pi_t^\#} + a^{13,28} \cdot \delta_{x_{1t}} + a^{13,29} \cdot \delta_{x_{2t}} + b^{13} = 0,$$

where

$$a^{13,10} \equiv \frac{\widehat{\pi}_t}{1 + \widehat{\pi}_t}, \quad a^{13,27} \equiv -\frac{\widehat{\pi}_t^\#}{1 + \widehat{\pi}_t^\#}, \quad a^{13,28} \equiv 1, \\ a^{13,29} \equiv -1, \quad b^{13} \equiv \ln(1 + \mathcal{R}_t^{13}).$$

Condition (B.102)

A Taylor expansion of (B.102) is

$$\widehat{x}_{1,t} \delta_{x_{1t}} = \mathcal{R}_t^{51} \widehat{x}_{1,t} + \widehat{\lambda}_t \widehat{mc}_t \widehat{Y}_t [\delta_{\lambda_t} + \delta_{mc_t} + \delta_{Y_t}]$$

$$\begin{aligned}
& - \zeta_p \varepsilon_p \phi_p \beta (1 + \widehat{\pi}_t)^{-\zeta_p \varepsilon_p - 1} \widehat{\pi}_t E_t [(1 + \widehat{\pi}_{t+1})^{\varepsilon_p} \widehat{x}_{1,t+1}] \cdot \delta_{\pi_t} \\
& + \phi_p \beta (1 + \widehat{\pi}_t)^{-\zeta_p \varepsilon_p} \varepsilon_p E_t [(1 + \widehat{\pi}_{t+1})^{\varepsilon_p - 1} \widehat{\pi}_{t+1} \widehat{x}_{1,t+1} \cdot \delta_{\pi_{t+1}}] \\
& + \phi_p \beta (1 + \widehat{\pi}_t)^{-\zeta_p \varepsilon_p} E_t [(1 + \widehat{\pi}_{t+1})^{\varepsilon_p} \widehat{x}_{1,t+1} \cdot \delta_{x_{1,t+1}}],
\end{aligned}$$

with \mathcal{R}_t^{14} being a residual (B.123). In compact notation, it becomes

$$\begin{aligned}
& a^{14,1} \cdot \delta_{\lambda_t} + a^{14,10} \cdot \delta_{\pi_t} + \sum_{j=1}^J a_j^{14,11} \cdot \delta_{\pi_{t+1,j}} + a^{14,24} \cdot \delta_{Y_t} + a^{14,28} \cdot \delta_{x_{1t}} \\
& + \sum_{j=1}^J a_j^{14,30} \cdot \delta_{x_{1t+1,j}} + a^{14,34} \cdot \delta_{m_{ct}} + b^{14} = 0,
\end{aligned}$$

where

$$\begin{aligned}
h^{14} & \equiv \widehat{\lambda}_t \widehat{m}_{ct} \widehat{Y}_t, & a^{14,1} & \equiv h^{14}, \\
a^{14,10} & \equiv -\zeta_p \varepsilon_p \phi_p (1 + \widehat{\pi}_t)^{-\zeta_p \varepsilon_p - 1} \widehat{\pi}_t \beta E_t [(1 + \widehat{\pi}_{t+1})^{\varepsilon_p} \widehat{x}_{1,t+1}], \\
a_j^{14,11} & \equiv \omega_j \phi_p (1 + \widehat{\pi}_t)^{-\zeta_p \varepsilon_p} \varepsilon_p \beta [(1 + \widehat{\pi}_{t+1,j})^{\varepsilon_p - 1} \widehat{\pi}_{t+1,j} \widehat{x}_{1,t+1,j}], \\
a^{14,24} & \equiv h^{14}, & a^{14,34} & \equiv h^{14}, & a^{14,28} & \equiv -\widehat{x}_{1,t}, \\
a^{14,30} & \equiv \omega_j \phi_p (1 + \widehat{\pi}_t)^{-\zeta_p \varepsilon_p} \beta [(1 + \widehat{\pi}_{t+1,j})^{\varepsilon_p} \widehat{x}_{1,t+1,j}], \\
b^{14} & \equiv \mathcal{R}_t^{14} \widehat{x}_{1,t}.
\end{aligned}$$

Condition (B.103)

A first-order Taylor expansion of (B.103) implies

$$\begin{aligned}
\widehat{x}_{2,t} \delta_{x_{2t}} & \equiv \mathcal{R}_t^{15} \widehat{x}_{2,t} + \widehat{\lambda}_t \widehat{Y}_t \delta_{\lambda_t} + \widehat{\lambda}_t \widehat{Y}_t \delta_{Y_t} \\
& + \zeta_p (1 - \varepsilon_p) \phi_p \beta (1 + \widehat{\pi}_t)^{\zeta_p (1 - \varepsilon_p) - 1} \widehat{\pi}_t E_t [(1 + \widehat{\pi}_{t+1})^{\varepsilon_p - 1} \widehat{x}_{2,t+1}] \cdot \delta_{\pi_t} \\
& + \phi_p \beta (1 + \widehat{\pi}_t)^{\zeta_p (1 - \varepsilon_p)} (\varepsilon_p - 1) E_t [(1 + \widehat{\pi}_{t+1})^{\varepsilon_p - 2} \widehat{\pi}_{t+1} \widehat{x}_{2,t+1} \cdot \delta_{\pi_{t+1}}] \\
& + \phi_p \beta (1 + \widehat{\pi}_t)^{\zeta_p (1 - \varepsilon_p)} E_t [(1 + \widehat{\pi}_{t+1})^{\varepsilon_p - 1} \widehat{x}_{2,t+1} \cdot \delta_{x_{2t+1}}],
\end{aligned}$$

where \mathcal{R}_t^{15} is the residual introduced in (B.124). Rearranging the terms and using new notation, we have

$$a^{15,1} \cdot \delta_{\lambda_t} + a^{15,10} \cdot \delta_{\pi_t} + \sum_{j=1}^J a_j^{15,11} \cdot \delta_{\pi_{t+1,j}} + a^{15,29} \cdot \delta_{x_{2t}} + \sum_{j=1}^J a_j^{15,31} \cdot \delta_{x_{2t+1,j}} + b^{15} = 0,$$

where

$$\begin{aligned}
a^{15,1} & \equiv \widehat{\lambda}_t \widehat{Y}_t, \\
a^{15,10} & \equiv \zeta_p (1 - \varepsilon_p) \phi_p (1 + \widehat{\pi}_t)^{\zeta_p (1 - \varepsilon_p) - 1} \widehat{\pi}_t \beta E_t [(1 + \widehat{\pi}_{t+1})^{\varepsilon_p - 1} \widehat{x}_{2,t+1}], \\
a_j^{15,11} & \equiv \omega_j \phi_p (1 + \widehat{\pi}_t)^{\zeta_p (1 - \varepsilon_p)} (\varepsilon_p - 1) \beta [(1 + \widehat{\pi}_{t+1,j})^{\varepsilon_p - 2} \widehat{\pi}_{t+1,j} \widehat{x}_{2,t+1,j}],
\end{aligned}$$

$$\begin{aligned}
 a^{15,24} &\equiv \widehat{\lambda}_t \widehat{Y}_t, & a^{15,29} &\equiv -\widehat{x}_{2,t}, \\
 a_j^{15,31} &\equiv \omega_j \phi_\rho (1 + \widehat{\pi}_t)^{\xi \rho (1 - \varepsilon \rho)} \beta [(1 + \widehat{\pi}_{t+1,j})^{\varepsilon \rho - 1} \widehat{x}_{2,t+1,j}], \\
 b^{15} &\equiv \mathcal{R}_t^{15} \widehat{x}_{2,t}.
 \end{aligned}$$

Condition (B.104)

A first-order Taylor expansion of (B.104) leads to

$$\delta_{R_t} - \delta_{w_t} - \delta_{N_t} + \delta_{K_t^\#} + \mathcal{R}_t^{16} = 0,$$

where \mathcal{R}_t^{16} is the residual of the equation; see (B.125). In terms of coefficients, we get

$$a^{16,7} \cdot \delta_{R_t} + a^{16,20} \cdot \delta_{w_t} + a^{16,23} \cdot \delta_{N_t} + a^{16,25} \cdot \delta_{K_t^\#} + b^{16} = 0,$$

where

$$a^{16,7} = 1, \quad a^{16,20} = -1, \quad a^{16,23} = -1, \quad a^{16,25} = 1, \quad b^{16} = \mathcal{R}_t^{16}.$$

Condition (B.105)

A first-order Taylor expansion of (B.105) is

$$a^{17,20} \cdot \delta_{w_t} + a^{17,23} \delta_{\widehat{N}_t} + a^{17,25} \cdot \delta_{K_t^\#} + a^{17,34} \cdot \delta_{mct} + b^{17} = 0,$$

where \mathcal{R}_t^{17} is the residual in (B.126), and

$$a^{17,20} = -1, \quad a^{17,23} = -\alpha, \quad a^{17,25} = \alpha, \quad a^{17,34} = 1, \quad b^{17} = \ln(1 + \mathcal{R}_t^{17}).$$

Condition (B.106)

A Taylor expansion of (B.106) is

$$\widehat{i}_t \delta_{i_t} = (1 - \rho_i) \phi_\pi \widehat{\pi}_t \cdot \delta_{\pi_t} + (1 - \rho_i) \phi_y \cdot \delta_{Y_t} + \mathcal{R}_t^{18} \widehat{i}_t,$$

where \mathcal{R}_t^{18} is the residual in (B.127). In compact notation, we get

$$a^{18,10} \cdot \delta_{\pi_t} + a^{18,15} \cdot \delta_{i_t} + a^{18,24} \cdot \delta_{Y_t} + b^{18} = 0,$$

where

$$\begin{aligned}
 a^{18,10} &\equiv (1 - \rho_i) \phi_\pi \widehat{\pi}_t, \\
 a^{18,15} &\equiv -\widehat{i}_t, \\
 a^{18,24} &\equiv (1 - \rho_i) \phi_y, \\
 b^{18} &\equiv \mathcal{R}_t^{18} \widehat{i}_t.
 \end{aligned}$$

Condition (B.107)

A first-order Taylor expansion of (B.107) leads to

$$\widehat{Y}_t \delta_{Y_t} = \mathcal{R}_t^{56} \widehat{Y}_t + \widehat{C}_t \delta_{C_t} + \widehat{I}_t \delta_{I_t} + \widehat{G}_t \delta_{G_t} + \chi_1 \widehat{u}_t \delta_{u_t} \frac{K_t}{Z_t} + 2\chi_2 (\widehat{u}_t - 1) \frac{K_t}{Z_t} \widehat{u}_t \cdot \delta_{u_t},$$

Introducing compact notation, we get

$$a^{19,3} \cdot \delta_{C_t} + a^{19,8} \cdot \delta_{u_t} + a^{19,12} \cdot \delta_{I_t} + a^{19,24} \cdot \delta_{Y_t} + a^{19,36} \delta_{G_t} + b^{19} = 0,$$

where

$$\begin{aligned} a^{19,3} &= \widehat{C}_t, \\ a^{19,8} &= \chi_1 \widehat{u}_t \frac{K_t}{Z_t} + 2\chi_2 (\widehat{u}_t - 1) \frac{K_t}{Z_t} \widehat{u}_t, \\ a^{19,12} &= \widehat{I}_t, \quad a^{19,24} = -\widehat{Y}_t, \quad a^{19,36} = \widehat{G}_t, \quad b^{19} = \mathcal{R}_t^{19} \widehat{Y}_t, \end{aligned}$$

with \mathcal{R}_t^{19} being the residual defined in (B.128).

Condition (B.108)

An expansion of (B.108) is

$$\begin{aligned} \delta_{K_{t+1}} \widehat{K}_{t+1} &= \mathcal{R}_t^{20} \widehat{K}_{t+1} \\ &+ Z_t \left\{ \left[-\tau \left(\frac{\widehat{I}_t}{I_{t-1}} - 1 \right) \frac{\widehat{I}_t}{I_{t-1}} \right] \widehat{I}_t + \left[1 - \frac{\tau}{2} \left(\frac{\widehat{I}_t}{I_{t-1}} - 1 \right)^2 \right] \widehat{I}_t \right\} \cdot \delta_{I_t}, \end{aligned}$$

where \mathcal{R}_t^{20} is the residual (B.129); introducing compact notation, we get

$$a^{20,12} \cdot \delta_{I_t} + a^{20,35} \cdot \delta_{K_{t+1}} + b^{20} = 0,$$

where

$$\begin{aligned} a^{20,12} &= Z_t \left\{ \left[-\tau \left(\frac{\widehat{I}_t}{I_{t-1}} - 1 \right) \frac{\widehat{I}_t}{I_{t-1}} \right] \widehat{I}_t + \left[1 - \frac{\tau}{2} \left(\frac{\widehat{I}_t}{I_{t-1}} - 1 \right)^2 \right] \widehat{I}_t \right\}, \\ a^{20,35} &\equiv -\widehat{K}_{t+1}, \quad b^{20} \equiv \mathcal{R}_t^{20} \widehat{K}_{t+1}. \end{aligned}$$

B.5. Defining Residuals in Equations

The unit-free residuals are defined by the following twenty equations that correspond to the optimality conditions (B.69)–(B.88), respectively:

$$\mathcal{R}_t^1 = \frac{1}{\widehat{\lambda}_t} \left[\frac{1}{\widehat{C}_t - b\widehat{C}_{t-1}} - \beta b E_t \left(\frac{1}{\widehat{C}_{t+1} - b\widehat{C}_t} \right) \right] - 1, \quad (\text{B.110})$$

$$\mathcal{R}_t^2 = \frac{1}{\widehat{R}_t Z_t} [\chi_1 + \chi_2 (\widehat{u}_t - 1) - 1] - 1, \quad (\text{B.111})$$

$$\mathcal{R}_t^3 = \frac{1}{\widehat{\lambda}_t} \beta E_t [\widehat{\lambda}_{t+1} \cdot (1 + \widehat{i}_t) \cdot (1 + \widehat{\pi}_{t+1})^{-1}] - 1, \quad (\text{B.112})$$

$$\begin{aligned} \mathcal{R}_t^4 = & \frac{1}{\widehat{\lambda}_t} \left\{ \widehat{\mu}_t Z_t \left[1 - \frac{\tau}{2} \left(\frac{\widehat{I}_t}{\widehat{I}_{t-1}} - 1 \right)^2 - \tau \left(\frac{\widehat{I}_t}{\widehat{I}_{t-1}} - 1 \right) \frac{\widehat{I}_t}{\widehat{I}_{t-1}} \right] \right. \\ & \left. + \beta E_t \left[\widehat{\mu}_{t+1} Z_{t+1} \tau \left(\frac{\widehat{I}_{t+1}}{\widehat{I}_t} - 1 \right) \left(\frac{\widehat{I}_{t+1}}{\widehat{I}_t} \right)^2 \right] \right\} - 1, \end{aligned} \quad (\text{B.113})$$

$$\begin{aligned} \mathcal{R}_t^5 = & \frac{1}{\widehat{\mu}_t} \beta E_t \left[\widehat{\lambda}_{t+1} \left(\widehat{R}_{t+1} \widehat{u}_{t+1} - \frac{1}{Z_{t+1}} \left[\chi_1 (\widehat{u}_{t+1} - 1) + \frac{\chi_2}{2} (\widehat{u}_{t+1} - 1)^2 \right] \right) \right. \\ & \left. + \widehat{\mu}_{t+1} (1 - d) \right] - 1, \end{aligned} \quad (\text{B.114})$$

$$\mathcal{R}_t^6 = \frac{1}{\widehat{w}_t^\#} \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\widehat{f}_{1,t}}{\widehat{f}_{2,t}} - 1, \quad (\text{B.115})$$

$$\begin{aligned} \mathcal{R}_t^7 = & \frac{1}{\widehat{f}_{1,t}} \left\{ \psi \left(\frac{\widehat{w}_t}{\widehat{w}_t^\#} \right)^{\varepsilon_w (1+\eta)} \widehat{N}_t^{1+\eta} \right. \\ & \left. + \phi_w \beta (1 + \widehat{\pi}_t)^{-\zeta_w \varepsilon_w (1+\eta)} E_t \left[[1 + \widehat{\pi}_{t+1}]^{\varepsilon_w (1+\eta)} \left(\frac{\widehat{w}_{t+1}^\#}{\widehat{w}_t^\#} \right)^{\varepsilon_w (1+\eta)} \widehat{f}_{1,t+1} \right] \right\} - 1, \end{aligned} \quad (\text{B.116})$$

$$\begin{aligned} \mathcal{R}_t^8 = & \frac{1}{\widehat{f}_{2,t}} \left\{ \widehat{\lambda}_t \left(\frac{\widehat{w}_t}{\widehat{w}_t^\#} \right)^{\varepsilon_w} \widehat{N}_t \right. \\ & \left. + \phi_w \beta (1 + \widehat{\pi}_t)^{\zeta_w (1-\varepsilon_w)} E_t \left[(1 + \widehat{\pi}_{t+1})^{\varepsilon_w - 1} \left(\frac{\widehat{w}_{t+1}}{\widehat{w}_{t+1}^\#} \right)^{\varepsilon_w} \widehat{f}_{2,t+1} \right] \right\} - 1, \end{aligned} \quad (\text{B.117})$$

$$\begin{aligned} \mathcal{R}_t^9 = & \frac{1}{\widehat{w}_t^{1-\varepsilon_w}} \{ (1 - \phi_w) (\widehat{w}_t^\#)^{1-\varepsilon_w} \\ & + (1 + \pi_{t-1})^{\zeta_w (1-\varepsilon_w)} (1 + \widehat{\pi}_t)^{\varepsilon_w - 1} \phi_w w_{t-1}^{1-\varepsilon_w} \} - 1, \end{aligned} \quad (\text{B.118})$$

$$\mathcal{R}_t^{10} = \frac{1}{\widehat{Y}_t} A_t (\widehat{K}_t^\#)^\alpha \widehat{N}_t^{1-\alpha} [\widehat{\Delta}_t^p]^{-1} - 1, \quad (\text{B.119})$$

$$\mathcal{R}_t^{11} = \frac{1}{\widehat{\Delta}_t^p} (1 + \widehat{\pi}_t)^{\varepsilon_p} \left[(1 - \phi_w) (1 + \widehat{\pi}_t^\#)^{-\varepsilon_p} + (1 + \pi_{t-1})^{-\zeta_p \varepsilon_p} \phi_p \Delta_{t-1}^p \right] - 1, \quad (\text{B.120})$$

$$\mathcal{R}_t^{12} = \frac{1}{(1 + \widehat{\pi}_t)^{1-\varepsilon_p}} \left[(1 - \phi_p) (1 + \widehat{\pi}_t^\#)^{1-\varepsilon_p} + \phi_p (1 + \pi_{t-1})^{\zeta_p (1-\varepsilon_p)} \right] - 1, \quad (\text{B.121})$$

$$\mathcal{R}_t^{13} = \frac{1}{1 + \widehat{\pi}_t^\#} \frac{\varepsilon_p}{\varepsilon_p - 1} (1 + \widehat{\pi}_t) \frac{\widehat{x}_{1,t}}{\widehat{x}_{2,t}} - 1, \quad (\text{B.122})$$

$$\mathcal{R}_t^{14} = \frac{1}{\widehat{x}_{1,t}} \left\{ \widehat{\lambda}_t \widehat{m} \widehat{c}_t \widehat{Y}_t + \phi_p \beta (1 + \widehat{\pi}_t)^{-\zeta_p \varepsilon_p} E_t \left[(1 + \widehat{\pi}_{t+1})^{\varepsilon_p} \widehat{x}_{1,t+1} \right] \right\} - 1, \quad (\text{B.123})$$

$$\mathcal{R}_t^{15} = \frac{1}{\widehat{x}_{2,t}} \left\{ \widehat{\lambda}_t \widehat{Y}_t + \phi_p \beta (1 + \widehat{\pi}_t)^{\zeta_p (1-\varepsilon_p)} E_t \left[(1 + \widehat{\pi}_{t+1})^{\varepsilon_p - 1} \widehat{x}_{2,t+1} \right] \right\} - 1, \quad (\text{B.124})$$

$$\mathcal{R}_t^{16} = \frac{\widehat{R}_t}{\widehat{w}_t} \frac{1 - \alpha}{\alpha} \cdot \widehat{K}_t^\# \widehat{N}_t^{-1} - 1, \quad (\text{B.125})$$

$$\mathcal{R}_t^{17} = \frac{1}{\widehat{w}_t} \widehat{\text{mc}}_t (1 - \alpha) A_t (\widehat{K}_t^\#)^\alpha N_t^{-\alpha} - 1, \quad (\text{B.126})$$

$$\begin{aligned} \mathcal{R}_t^{18} = & \frac{1}{\widehat{i}_t} \left\{ (1 - \rho_i) i + \rho_i i_{t-1} \right. \\ & \left. + (1 - \rho_i) [\phi_\pi (\widehat{\pi}_t - \pi^*) + \phi_y (\ln \widehat{Y}_t - \ln Y_{t-1})] + \varepsilon_{i,t} \right\} - 1, \end{aligned} \quad (\text{B.127})$$

$$\mathcal{R}_t^{19} = \frac{1}{\widehat{Y}_t} \left[\widehat{C}_t + \widehat{I}_t + \widehat{G}_t + (\chi_1 (\widehat{u}_t - 1) + \chi_2 (\widehat{u}_t - 1)^2) \frac{K_t}{Z_t} \right] - 1, \quad (\text{B.128})$$

$$\mathcal{R}_t^{20} = \frac{1}{\widehat{K}_{t+1}} \left\{ Z_t \left[1 - \frac{\tau}{2} \left(\frac{\widehat{I}_t}{I_{t-1}} - 1 \right)^2 \right] \widehat{I}_t + (1 - d) K_t \right\} - 1. \quad (\text{B.129})$$

B.6. Details of Numerical Analysis

We describe the calibration and solution procedures, and we outline the numerical results.

B.6.1. Calibration and Solution Procedures

We split the parameters of the model into two sets: we calibrate the parameters

$$\{\varepsilon_w, \varepsilon_p, \omega^g, \pi^*, \alpha, \chi_2, \psi, \beta, d\}$$

to the standard values in the literature, and we fix the remaining parameters

$$\{\rho_i, \rho_a, \rho_z, \rho_g, \sigma_{\varepsilon_a}, \sigma_{\varepsilon_i}, \sigma_{\varepsilon_z}, \sigma_{\varepsilon_g}, b, \phi_w, \phi_p, \zeta_w, \zeta_p, \eta, \tau, \phi_\pi, \phi_y\}$$

in line with the estimates obtained in Sims (2014) for the U.S. economy. Finally, parameter χ_1 is calculated as $1/\beta - (1 - \delta)$ (which is obtained under a normalization of u_t to unity in the steady state). Table SII summarizes our benchmark parameter choice.

TABLE SII
BENCHMARK PARAMETERIZATION OF THE NEW KEYNESIAN MODEL

Parameters in the Processes for Shocks Estimated From the U.S. Economy Data									
	ρ_a	ρ_i	ρ_z	ρ_g	σ_a	σ_i	σ_z	σ_g	ω^g
Values	0.99	0.79	0.90	0.96	0.0074	0.0013	0.0091	0.0038	0.2
Other Parameters Estimated From the U.S. Economy Data									
	ϕ_π	ϕ_y	ϕ_w	ϕ_p	ζ_w	ζ_p	η	b	τ
Values	1.35	0.32	0.43	0.71	0.38	0.03	1.23	0.72	1.87
Parameters Calibrated to the U.S. Economy Data									
	ε_w	ε_p	π^*	α	χ_1	χ_2	ψ	β	d
Values	10	10	0	1/3	0.0351	0.01	2	0.99	0.025

We evaluate the accuracy of perturbation solutions on a stochastic simulation of 10,200 observations (the first 200 observations were discarded to eliminate the effect of the initial conditions). The Dynare's representation of the state space includes the current endogenous state variables $\{\pi_{t-1}, w_{t-1}, C_{t-1}, I_{t-1}, N_{t-1}, Y_{t-1}, \Delta_{t-1}^p, i_{t-1}, K_t\}$, the past exogenous state variables $\{A_{t-1}, Z_{t-1}, \omega_{t-1}^g\}$, and the current disturbances $\{\varepsilon_{a,t}, \varepsilon_{i,t}, \varepsilon_{z,t}, \varepsilon_{g,t}\}$. We use a Dynare's option of pruning for simulating a second-order perturbation solution. To compute conditional expectations, we use a monomial integration rule with $J = 2N$ nodes, where $N = 4$ is the number of exogenous shocks. This rule delivers very accurate approximation to expectation functions (up to six accuracy digits) in the context of real business-cycle models (see Judd, Maliar, and Maliar (2011), for a detailed description of this rule).

B.6.2. Numerical Results on the Lower Error Bound

We report the size of approximation errors in Table SIII. For a future variable $x_{t+1,j} \in \{\delta_{\lambda_{t+1,j}}, \delta_{C_{t+1,j}}, \dots\}$, statistics reported in columns L_1 and L_∞ are the mean and maximum of t -period absolute values of approximation errors in that variable across $J = 8$ integration nodes, that is, $\frac{1}{J} \sum_{j=1}^J \delta_{x_{t+1,j}}$ and $\max_{j \in J} \delta_{x_{t+1,j}}$, respectively. We consider three alternative parameterizations. The first parameterization corresponds to the benchmark values of the parameters in Table SII. The second parameterization considers the benchmark values for all the parameters, except of π^* , which is set to 0.02. In the final parameterization, we decrease the values of ε_w and ε_p relative to the benchmark parameterization; namely, ε_w and ε_p are set to 5.

Under Parameterization 1, we got a lower bound on approximation errors of order $10^{0.11} \approx 129\%$, which corresponds to an approximation error in π_t . Parameterization 2 produces a similar size of approximation errors (but the biggest approximation error is obtained in variable $\pi_t^\#$). Under Parameterization 3, the lower error bound for PER2 reaches $10^{-0.43} \approx 37\%$, which corresponds to an approximation error in N_t . Overall, as it follows from Table SII, for the studied new Keynesian model, such variables are inflation variables π_t and $\pi_t^\#$, investment variables I_t, i_t , and u_t , as well as price dispersion Δ_t^p and labor variable N_t .

B.6.3. Analysis of Residuals in the New Keynesian Model

In Appendix B.3, we listed twenty equations (B.110)–(B.129) that define unit-free residuals $\mathcal{R}_t^1, \dots, \mathcal{R}_t^{20}$ corresponding to the twenty FOCs (B.69)–(B.88) of the new Keynesian model. We evaluate the accuracy of perturbation solutions on the same set of simulated points as the one used for constructing the approximation errors.

We report the residuals in Table SIV. If we exclude from consideration residuals \mathcal{R}_t^{11} , \mathcal{R}_t^{12} , \mathcal{R}_t^{13} , and \mathcal{R}_t^{18} in equations (B.79)–(B.81) and (B.86), the remaining residuals are quite low; for example, under Parameterization 1, the maximum residuals would be $10^{-4.55} \approx 0.0028\%$ for a PER2 solution. However, the residuals are enormous if we take into account these four residuals \mathcal{R}_t^{11} , \mathcal{R}_t^{12} , \mathcal{R}_t^{13} , and \mathcal{R}_t^{18} , namely, the maximum residual is $10^{-0.27} \approx 54\%$.

The analysis of residuals also provides us with some insight into which variables are approximated inaccurately. For example, equation (B.80) contains only current and past inflation measures and definition (B.120) of residual \mathcal{R}_t^{12} indicates that the inflation variables $\hat{\pi}_t$ and $\hat{\pi}_t^\#$ are approximated poorly (either one or the other or both):

$$\mathcal{R}_t^{12} = \frac{1}{(1 + \hat{\pi}_t)^{1-\varepsilon_p}} \left[(1 - \phi_p)(1 + \hat{\pi}_t^\#)^{1-\varepsilon_p} + \phi_p(1 + \pi_{t-1})^{\varepsilon_p(1-\varepsilon_p)} \right] - 1. \quad (\text{B.130})$$

TABLE III

APPROXIMATION ERRORS IN THE CURRENT AND FUTURE VARIABLES IN THE NEW KEYNESIAN MODEL^a

	Parameterization 1				Parameterization 2				Parameterization 3			
	PER1		PER2		PER1		PER2		PER1		PER2	
	L_1	L_∞	L_1	L_∞	L_1	L_∞	L_1	L_∞	L_1	L_∞	L_1	L_∞
δ_{λ_t}	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00
$\delta_{\lambda_{t+1,j}}$	-3.15	-2.79	-3.19	-2.81	-2.31	-2.09	-2.31	-2.09	-3.21	-2.79	-3.25	-2.81
δ_{C_t}	-2.25	-1.99	-2.25	-1.98	-3.32	-2.63	-3.25	-2.58	-2.13	-1.84	-2.13	-1.83
$\delta_{C_{t+1,j}}$	-1.85	-1.56	-1.84	-1.56	-2.95	-2.24	-2.89	-2.20	-1.78	-1.47	1.78	-1.47
δ_{μ_t}	-1.47	-1.41	-1.47	-1.41	-1.50	-1.47	-1.49	-1.46	-1.38	-1.31	-1.37	-1.31
$\delta_{\mu_{t+1,j}}$	-1.38	-1.32	-1.38	-1.32	-1.44	-1.41	-1.44	-1.40	-1.35	-1.28	-1.35	-1.28
δ_{R_t}	-1.33	-1.09	-1.33	-1.09	-2.16	-1.69	-2.12	-1.67	-1.40	-1.13	-1.40	-1.13
δ_{u_t}	-0.79	-0.55	-0.78	-0.54	-1.62	-1.14	-1.57	-1.12	-0.85	-0.59	-0.85	-0.58
δ_{π_t}	0.07	0.30	-0.05	0.11	-1.87	-1.87	-1.86	-1.86	-0.04	-0.03	-0.04	-0.04
$\delta_{\pi_{t+1,j}}$	-5.07	-4.04	-5.12	-4.04	-3.33	-3.05	-3.40	-3.10	-5.41	-4.37	-5.40	-4.38
δ_{I_t}	-0.34	-0.33	-0.34	-0.33	-0.33	-0.33	-0.33	-0.33	-0.34	-0.34	-0.34	-0.34
$\delta_{I_{t+1,j}}$	-1.45	-1.35	-1.45	-1.35	-1.54	-1.47	-1.54	-1.47	-1.47	-1.36	-1.47	-1.36
$\delta_{R_{t+1,j}}$	-3.66	-3.59	-3.66	-3.59	-3.75	-3.71	-3.76	-3.72	-3.69	-3.62	-3.69	-3.62
δ_{i_t}	-0.78	-0.59	-0.78	-0.59	-0.70	-0.51	-0.69	-0.50	-0.80	-0.59	-0.80	-0.59
$\delta_{f_{1t}}$	-1.58	-1.50	-1.58	-1.49	-1.36	-1.31	-1.36	-1.32	-1.32	-1.23	-1.32	-1.23
$\delta_{f_{1t+1,j}}$	-2.91	-2.78	-2.91	-2.78	-2.44	-2.32	-2.45	-2.32	-2.77	-2.65	-2.77	-2.65
$\delta_{f_{2t}}$	-2.04	-1.95	-2.03	-1.94	-1.51	-1.47	-1.53	-1.48	-1.75	-1.65	-1.75	-1.65
$\delta_{f_{2t+1,j}}$	-2.51	-2.40	-2.51	-2.40	-2.13	-2.05	-2.14	-2.05	-2.28	-2.18	-2.28	-2.18
δ_{w_t}	-2.97	-2.88	-2.96	-2.87	-3.05	-3.01	-3.04	-3.00	-2.37	-2.28	-2.36	-2.28
$\delta_{w_t^\#}$	-1.77	-1.69	-1.77	-1.69	-1.88	-1.83	-1.87	-1.82	-1.52	-1.43	-1.52	-1.43
$\delta_{w_{t+1,j}^\#}$	-2.42	-2.20	-2.42	-2.20	-2.17	-1.77	-2.17	-1.75	-2.15	-2.03	-2.15	-2.03
δ_{N_t}	-0.48	-0.40	-0.48	-0.40	-0.59	-0.54	-0.59	-0.54	-0.52	-0.43	-0.52	-0.43
δ_{Y_t}	-1.59	-1.05	-1.59	-1.05	-1.04	-0.93	-1.04	-0.93	-1.61	-1.06	-1.61	-1.06
$\delta_{K_t^\#}$	-0.54	-0.49	-0.54	-0.49	-0.60	-0.57	-0.60	-0.57	-0.57	-0.52	-0.57	-0.52
$\delta_{\Delta_t^p}$	-0.47	-0.47	-0.47	-0.47	-0.46	-0.46	-0.46	-0.46	-0.50	-0.50	-0.50	-0.50
$\delta_{\pi_t^\#}$	-0.66	-0.21	-0.31	-0.14	0.10	0.10	0.10	0.10	-0.33	-0.32	-0.32	-0.31
$\delta_{x_{1t}}$	-2.29	-1.60	-2.33	-1.61	-1.82	-1.72	-1.91	-1.78	-2.33	-1.62	-2.34	-1.62
$\delta_{x_{2t}}$	-3.02	-3.01	-3.57	-3.55	-2.73	-2.72	-3.84	-3.79	-3.38	-3.37	-3.63	-3.61
$\delta_{x_{1t+1,j}}$	-3.34	-2.63	-3.38	-2.65	-2.88	-2.73	-2.96	-2.79	-3.39	-2.66	-3.40	-2.67
$\delta_{x_{2t+1,j}}$	-4.12	-4.09	-4.67	-4.63	-3.80	-3.76	-4.91	-4.82	-4.53	-4.51	-4.78	-4.75
$\delta_{u_{t+1,j}}$	-5.64	-5.27	-5.63	-5.17	-5.62	-5.36	-5.50	-5.13	-5.69	-5.32	-5.68	-5.23
$\delta_{m_{ct}}$	-1.79	-1.56	-1.79	-1.55	-2.54	-2.12	-2.50	-2.11	-1.79	-1.55	-1.79	-1.55
$\delta_{K_{t+1}}$	-1.94	-1.93	-1.94	-1.93	-1.94	-1.93	-1.94	-1.93	-1.95	-1.94	-1.95	-1.94
δ_{G_t}	-0.99	-0.72	-0.98	-0.71	-2.12	-1.43	-2.07	-1.39	-1.00	-0.70	-1.00	-0.70

^aNotes: Parameterization 1 corresponds to our benchmark parameter choice summarized in Table SII. Parameterization 2 changes the inflation target parameter π^* to 0.02. Parameterization 3 assumes that $\varepsilon_w, \varepsilon_p$ are equal to 5. PER1 and PER2 denote the first- and second-order perturbation solutions. L_1 and L_∞ are, respectively, the average and maximum of absolute values of the corresponding approximation errors across test points (in log10 units) on a stochastic simulation of 10,000 observations. For a future variable $x_{t+1,j}$, statistics reported in columns L_1 and L_∞ are the mean and maximum of t -period absolute values of approximation errors in that variable across $J = 8$ integration nodes, that is, $\frac{1}{J} \sum_{j=1}^J \delta_{x_{t+1,j}}$ and $\max_{j \in J} \delta_{x_{t+1,j}}$, respectively.

However, in general, the model's equations are complex and it is difficult to see which variables are approximated poorly by looking at the size of residuals. In this respect, our lower-bound error analysis has more sharp implications.

Finally, our analysis of residuals shows that for more nonlinear models, like our new Keynesian model, a specific way of constructing the residuals might be critical for the results. For example, consider the residual \mathcal{R}_t^{12} given in (B.130); the mean of \mathcal{R}_t^{12} for

TABLE SIV
RESIDUALS IN THE EQUILIBRIUM CONDITIONS OF THE NEW KEYNESIAN MODEL^a

	Parameterization 1				Parameterization 2				Parameterization 3			
	PER1		PER2		PER1		PER2		PER1		PER2	
	L_1	L_∞	L_1	L_∞	L_1	L_∞	L_1	L_∞	L_1	L_∞	L_1	L_∞
R^1	-3.86	-3.86	-5.58	-5.43	-3.88	-3.88	-4.49	-4.45	-3.88	-3.88	-5.75	-5.58
R^2	-7.53	-6.37	-6.11	-5.96	-7.61	-6.41	-5.10	-5.05	-7.56	-6.44	-6.20	-6.06
R^3	-4.53	-4.53	-7.12	-6.87	-4.54	-4.54	-5.30	-5.26	-4.54	-4.54	-7.34	-7.07
R^4	-3.29	-3.29	-6.26	-5.59	-3.49	-3.49	-5.16	-5.01	-3.35	-3.35	-6.43	-5.76
R^5	-4.60	-4.60	-6.67	-6.58	-4.64	-4.64	-5.02	-4.97	-4.59	-4.59	-6.77	-6.65
R^6	-16.04	-15.35	-15.98	-15.35	-15.97	-15.35	-15.90	-15.18	-15.88	-15.35	-15.90	-15.18
R^7	-2.90	-2.90	-4.67	-4.55	-2.33	-2.33	-4.03	-3.98	-3.18	-3.18	-5.37	-5.18
R^8	-3.65	-3.65	-5.36	-5.28	-3.17	-3.17	-4.27	-4.23	-3.94	-3.94	-6.03	-5.81
R^9	-7.57	-6.37	-6.37	-5.76	-7.36	-6.12	-4.68	-4.50	-8.24	-7.03	-6.27	-5.96
R^{10}	-15.76	-15.05	-15.78	-15.05	-15.54	-15.05	-15.70	-15.05	-15.86	-15.18	-15.79	-15.05
R^{11}	-0.54	-0.54	-0.54	-0.54	-0.53	-0.53	-0.53	-0.53	-0.55	-0.55	-0.55	-0.55
R^{12}	-0.54	-0.54	-0.54	-0.54	-0.81	-0.81	-0.80	-0.80	-0.66	-0.66	-0.66	-0.66
R^{13}	-0.30	-0.30	-0.30	-0.30	-0.32	-0.32	-0.32	-0.32	-0.32	-0.32	-0.32	-0.32
R^{14}	-3.01	-3.01	-5.63	-5.20	-2.59	-2.59	-5.11	-4.65	-3.38	-3.38	-5.86	-5.55
R^{15}	-3.13	-3.13	-5.78	-5.34	-2.74	-2.73	-5.60	-4.95	-3.60	-3.60	-5.98	-5.70
R^{16}	-15.61	-14.95	-15.41	-14.78	-15.64	-15.00	-15.42	-14.75	-15.59	-14.91	-15.46	-14.81
R^{17}	-15.86	-15.18	-15.80	-15.05	-15.67	-15.05	-15.78	-15.05	-15.85	-15.18	-15.05	-15.05
R^{18}	-1.00	-0.28	-0.99	-0.27	-1.48	-0.75	-1.47	-0.75	-1.48	-0.75	-0.75	-0.75
R^{19}	-6.83	-6.12	-6.30	-5.96	-7.34	-6.45	-5.75	-5.50	-5.66	-5.56	-5.51	-5.51
R^{20}	-8.04	-6.88	-8.18	-7.51	-8.31	-7.12	-8.62	-7.92	-8.06	-6.91	-7.38	-7.38

^aNotes: Parameterization 1 corresponds to our benchmark parameter choice summarized in Table SII. Parameterization 2 changes the inflation target parameter π^* to 0.02. Parameterization 3 assumes that $\varepsilon_w, \varepsilon_p$ are equal to 5. PER1 and PER2 denote the first- and second-order perturbation solutions; L_1 and L_∞ are, respectively, the average and maximum of absolute values of residuals in the model's equations across optimality condition and test points (in log10 units) on a stochastic simulation of 10,000 observations.

the PER1 method is equal to -0.5385 ; see Table SIV. Consider another expression for a unit-free residual in the same equation (B.80):

$$\overline{\mathcal{R}}_t^{12} = \frac{[(1 - \phi_p)(1 + \widehat{\pi}_t^\#)^{1-\varepsilon_p} + \phi_p(1 + \pi_{t-1})^{\varepsilon_p(1-\varepsilon_p)}]^{1/(1-\varepsilon_p)} - 1}{\widehat{\pi}_t} - 1. \quad (\text{B.131})$$

The mean residual $\overline{\mathcal{R}}_t^{12}$ of PER1 is now equal to 4.145, which is huge (and the maximum residual is even larger)! It is easy to see why our benchmark representation \mathcal{R}_t^{12} leads to much smaller residuals than the alternative representation $\overline{\mathcal{R}}_t^{12}$: in the former case, the residual is evaluated relative to the denominator $(1 + \widehat{\pi}_t)^{1-\varepsilon_p} \approx 1$, while in the latter case, it is evaluated relative to $\widehat{\pi}_t \approx 0$. We find that the residuals \mathcal{R}_t^{11} and \mathcal{R}_t^{13} in equations (B.79) and (B.81), respectively, also significantly depend on a specific way in which they are represented. Hence, to make meaningful qualitative inferences about accuracy from the analysis of residuals, it is important to take into account the size of variables with respect to which residuals are evaluated. In turn, our lower error bounds are not subject to this shortcoming: they are independent of the way in which the model's equations are written.

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