

Are the responses of the U.S. economy asymmetric in energy price increases and decreases?

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How much does real gross domestic product (GDP) respond to unanticipated changes in the real price of oil? Commonly used censored oil price vector autoregressive models suggest a substantial decline in real GDP in response to unexpected increases in the real price of oil, yet no response to unexpected declines. We show that these estimates are invalid. Based on a structural model that encompasses both symmetric and asymmetric models as special cases, correctly computed impulse responses are of roughly the same magnitude in either direction, consistent with formal tests for symmetric responses. We discuss implications for theoretical models and for policy responses to energy price shocks.

KEYWORDS. Asymmetry, oil price, energy prices, net increase, shocks, propagation, transmission, vector autoregression.

JEL CLASSIFICATION. C32, E37, Q43.

1. INTRODUCTION

A common view in the literature is that the effects of energy price shocks on macroeconomic aggregates such as output or employment are asymmetric. In particular, positive energy price shocks are perceived to have larger effects than negative energy price shocks. This perception has been bolstered by empirical evidence that energy price increases (obtained by censoring percentage changes in the price of energy) have more predictive power for U.S. macroeconomic aggregates than do uncensored percentage changes in the price of energy. Vector autoregressive (VAR) models relating energy price increases to macroeconomic aggregates, in particular, have shaped the discussion of the effects of energy price shocks in recent decades. In this paper, we demonstrate that the regression models and estimation methods used in this literature produce inconsistent estimates of the true effects of unanticipated energy price increases and are likely to

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have exaggerated the impact of positive energy price shocks. We show that fundamental changes are needed in how these effects are estimated in practice. In addition to addressing the problem of how to estimate asymmetric responses to energy price shocks, we propose a test of the null hypothesis of symmetric responses to positive and negative energy price shocks. Our empirical evidence suggests that there is no compelling evidence against the null of symmetric response functions.

The practical relevance of our analysis for the macroeconomic analysis of oil price shocks is best illustrated in the context of the widely studied question of how much U.S. real gross domestic product (GDP) responds to unanticipated changes in the real price of oil. Commonly used censored oil price VAR models and impulse response estimation methods suggest a decline of 1.1 percent in real GDP within 2 years in response to a 15 percent unanticipated increase in the real price of oil and a 0 percent increase in response to an unanticipated decline in the real price of oil of the same magnitude. In contrast, a linear VAR model would imply a decline in real GDP of 0.3 percent and an increase of 0.3 percent, respectively. Appropriately computed impulse responses from an unrestricted structural model that encompasses both the linear symmetric model and the asymmetric model as special cases yield a 0.47 percent decline in real GDP and a 0.39 percent increase, respectively, roughly consistent with the estimates from the linear symmetric VAR model. We discuss implications of our findings for the theoretical literature on the transmission of energy price shocks and for the debate about policy responses to energy price shocks.

1.1 *The origins of censoring changes in energy prices*

In the view of many economists, oil price shocks are perhaps the leading alternative to monetary policy shocks as the determinant of U.S. postwar recessions.¹ Increases in the price of oil preceded the recessions of 1973–1975, 1980–1982, and 1990–1991, for example. Given the striking coincidence of deteriorating macroeconomic outcomes and rising oil prices in the 1970s and early 1980s, it was natural at the time to suspect a strong link from oil price increases to recessions. Nevertheless, as discussed by Bernanke, Gertler, and Watson (1997), it has proven to be surprisingly difficult to find an indicator of oil price shocks that produces the expected responses of domestic macroeconomic variables in a VAR setting. Finding a measure of oil price shocks that “works” in a VAR context in practice is not straightforward. Simple measures of energy price shocks (such as linearly unpredictable changes in energy prices) often imply “anomalous” effects on macroeconomic outcomes relative to the conventional wisdom about the effects of oil price shocks on the economy. They also tend to have an unstable relationship with macroeconomic outcomes.²

Far from undermining the view that energy price shocks are important, these difficulties have led researchers to employ increasingly complicated specifications of the

¹For an early exposition of this idea, see Hamilton (1983).

²Kilian (2009) recently discussed some of the reasons for the apparent instability of such regressions in small samples and for the seemingly counterintuitive response estimates occasionally obtained from such regressions.

“true” relationship between oil prices and the economy. Today it is widely believed that the most appropriate specification of oil prices involves some measure of oil price increases, obtained by censoring oil price changes. This consensus dates back to the work of Mork (1989). After the sharp oil price declines of 1985–1986 failed to lead to an economic boom in oil importing economies, Mork (1989) pointed out that the effects of positive and negative oil price shocks on the economy need not be symmetric. He provided empirical evidence that positive changes in the real price of oil had far more predictive power for U.S. real GDP growth than negative changes. This was widely interpreted as evidence that only oil price increases matter for the U.S. economy (see, e.g., Bernanke, Gertler, and Watson (1997, p. 103)). Given the a priori belief that oil price shocks have quantitatively important effects on macroeconomic aggregates and given researchers’ inability to generate such responses from linear and symmetric models, VAR models of macroeconomic aggregates and oil price increases became accepted on the grounds that they produced “better looking” impulse responses (Bernanke, Gertler, and Watson (1997, p. 104)).³

The initial proposal to focus only on oil price increases was subsequently refined by Hamilton (1996, 2003), who introduced the *net oil price increase*. This oil price measure distinguishes between oil price increases that establish new highs relative to recent experience and increases that simply reverse recent decreases. Specifically, Hamilton’s net increase measure equals the maximum of (a) zero and (b) the difference between the log level of the current price of crude oil and the maximum value of the logged crude oil price over the previous year (or alternatively over the last 3 years). Hooker (2002), for example, found that the net increase measure performs “well” in the sense of having a relatively stable relationship with macroeconomic variables, and Hamilton (1996, 2003, 2009) made the case that this measure predicts declines in U.S. real GDP.

The most influential use of the net increase measure has been not for one-step-ahead prediction from single-equation regression models, but in constructing estimates of the response of macroeconomic aggregates to energy price shocks from VAR models. Many of these structural VAR estimates have become accepted in academic and policy discussions of the transmission of energy price shocks (see, e.g., Hamilton (1996), Davis and Haltiwanger (2001), Lee and Ni (2002), Jones, Leiby, and Paik (2004), Herrera (2008), Ramey and Vine (2010)). The net increase measure also plays a central role in VAR analyses of the role of monetary policy in propagating energy price shocks (see, e.g., Bernanke, Gertler, and Watson (1997, 2004), Hamilton and Herrera (2004), Herrera and Pesavento (2009)).

³This finding reinforced results based on measures of oil supply disruptions such as the quantitative dummy variable of Hamilton (1996, 2003). It also seemed consistent with evidence using an alternative VAR methodology provided by Davis and Haltiwanger (2001, p. 509), who considered “the evidence for asymmetric responses to oil price ups and downs as well established.” Related evidence includes Mork, Olsen, and Mysen (1994), Ferderer (1996), Hooker (1996a, 1996b, 2002), Hamilton (1996, 2003), Huntington (1998), and Balke, Brown, and Yücel (2002), among others. For a critical perspective on this literature, see Edelstein and Kilian (2007, 2009).

1.2 *Outline of the paper*

In this paper, we make four distinct contributions to this literature that must be viewed in conjunction. First, we establish that impulse response estimates from VAR models involving censored oil price variables are inconsistent. We demonstrate that censored energy price VAR models are fundamentally misspecified, regardless of whether the data generating process (DGP) is symmetric or asymmetric. This misspecification renders the parameter estimates inconsistent and inference invalid. Second, we show that standard approaches to the construction of structural impulse responses used in the literature are invalid even when applied to correctly specified regression models. Instead, we propose a modification of the procedure discussed in Koop, Pesaran, and Potter (1996). Third, we demonstrate that the results of standard slope-based tests for asymmetry based on single-equation models are neither necessary nor sufficient for judging the degree of asymmetry in the structural response functions, which is the question of ultimate interest to users of these models. We propose a direct test of the latter hypothesis which requires the model to be appropriately specified and the nonlinear responses to be correctly simulated, as discussed in the first two points. Fourth, using this test, we show empirically that there is no statistically significant evidence of asymmetry in the response functions for U.S. real GDP using data for 1973.II–2007.IV.

In Section 2, we establish the inconsistency of conventional estimators of the effects of energy price shocks in the context of a stylized static model. We show that estimators of static censored regressor models are consistent only in very special and theoretically implausible cases, and we study the determinants of the asymptotic bias of the estimator by simulation. In Section 3, we strengthen this result by showing that dynamic censored energy price models of the type frequently employed in the literature produce inconsistent impulse response estimates regardless of the DGP. Unlike in the static model, conventional estimators exaggerate the quantitative importance of positive energy price shocks even when energy price decreases are known to have no effect on the economy. The reason is that the underlying asymmetric DGP cannot be represented as a censored energy price VAR model. We discuss the construction of alternative regression models that allow consistent estimation of such asymmetric DGPs. Equally relevant is the result that censored energy price VAR models asymptotically overestimate the true response of macroeconomic outcomes to unanticipated energy price increases when the underlying DGP is symmetric. The latter case is of special interest because one of the reasons that these models were adopted was precisely their ability to generate larger responses to energy price shocks than symmetric VAR models.

An important problem in practice is that we may not know whether the DGP is symmetric or asymmetric, or if the DGP is known to be asymmetric, whether energy price decreases should be included in the regression. In Section 4, we propose a regression model that encompasses all these specifications and can be estimated consistently by standard methods whether the true model is symmetric and regardless of the precise form of the asymmetry.

In Section 5, we show that not only have estimates of the effects of energy price shocks typically been based on inconsistent parameter estimates from censored energy price VAR models, but the dynamic responses of macroeconomic aggregates to

unanticipated energy price increases have also been routinely computed incorrectly in a way that exaggerates the quantitative importance of these shocks. We demonstrate how asymmetric impulse responses can be computed correctly. Unlike existing methods of computing nonlinear impulse responses in the econometric literature, our approach is fully structural and avoids the ambiguities of defining a shock in nonlinear reduced-form models.

Both the regression model proposed in Section 4 and the method of computing responses to energy price shocks developed in Section 5 play a crucial role in designing tests of the symmetry of response functions with respect to positive and negative energy price shocks. In Section 6, we discuss the problem of testing the null hypothesis that the U.S. economy responds symmetrically to unanticipated energy price increases and decreases. First, we propose an alternative test of the linear symmetric structural model based on regression slopes and contrast this test with the slope-based test originally proposed by Mork (1989). Second, we observe that statistically insignificant departures from symmetry in the slopes may cause statistically significant asymmetries in the implied impulse response functions, given the nonlinearity of these functions, while significant departures from symmetry in the slopes need not imply large asymmetries in the impulse response functions. Moreover, the extent to which responses from the linear symmetric model provide a good approximation depend on the magnitude of the energy price shock. This implies that traditional slope-based tests, while informative about the degree of asymmetry in predictive regressions, do not shed light on the degree of asymmetry in the impulse response functions. Third, as an alternative, we propose a direct statistical test of the symmetry of the economy's responses to unanticipated energy price increases and decreases with reasonably accurate finite-sample size.

In Section 7, we use these tools to study the evidence against the symmetry null in three prominent empirical examples. Specifically, we model the relationship between quarterly U.S. real GDP and the real price of oil, between monthly U.S. unemployment and the real price of oil, and between monthly U.S. gasoline consumption and the U.S. real retail price of gasoline. We find no compelling evidence of asymmetric responses to positive and negative energy price shocks.

In Section 8, we extend the analysis to VAR models involving net energy price increases motivated by the analysis in Hamilton (1996, 2003). Despite the widespread use of the net oil price increase measure in VAR models, none of the symmetry test results in the literature provides a justification for the use of such models. In fact, notwithstanding the well known evidence for asymmetries in the predictive relationship between real GDP growth and net oil price increases in Hamilton (2003), no paper has adequately addressed the implications of this asymmetry for impulse response analysis. In this paper, we discuss how structural impulse responses to energy price shocks can be consistently estimated in this context, and—building on the analysis in Sections 6—we present two tests of symmetry: one based on regression slopes and the other based on the impulse response functions themselves.⁴ We apply the impulse-response-based test to the three

⁴Our analysis of the net increase model clarifies, refines, and extends the earlier analysis in Balke, Brown, and Yücel (2002), which recognized some of the problems discussed here but had no apparent impact on

empirical examples of Section 7 and find little, if any, statistically significant evidence against the null hypothesis of symmetric response functions. We also demonstrate by example that the results for the corresponding slope-based symmetry tests are misleading for the purpose of judging the degree of asymmetry in the response functions.

Our results highlight the dangers of incorrectly imposing asymmetry in estimation and are consistent with the view that linear impulse response analysis is adequate for many applications. This finding has important implications for the theoretical literature on the transmission of energy price shocks and for the debate about monetary policy responses to oil price shocks. Finally, to the extent that there is evidence of asymmetries, our analysis suggests that important changes are needed in the way these asymmetries are modeled in the VAR literature. In Section 9, we relate our findings to the analysis in Balke, Brown, and Yücel (2002). Section 10 provides sensitivity analysis for several alternative model specifications. We show that our results hold even for the model specification favored by Hamilton (2003, *forthcoming*). Section 11 contains concluding remarks.

2. A STYLIZED MODEL

It is well known that censoring dependent variables causes ordinary least-squares (OLS) estimates of the coefficients of linear models to be biased. The problem of censoring endogenous variables in VAR models, in contrast, is not well understood. To build intuition, we first consider a purely static model and defer discussion of the dynamic models in which we are most interested. Because we do not know whether the DGP is symmetric or asymmetric, we discuss each case in turn, starting with the linear and symmetric case.

2.1 Asymptotic biases from using censored regressors

Consider the symmetric DGP

$$\begin{aligned}x_t &= \alpha_1 + \varepsilon_{1,t}, \\y_t &= \alpha_2 + x_t\beta + \varepsilon_{2,t},\end{aligned}\tag{1}$$

where α_1 , α_2 , and β are constants, $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are mean zero independent and identically distributed (i.i.d.) Gaussian random variables with variances σ_1^2 and σ_2^2 , and $t = 1, \dots, T$. It is straightforward to show that the OLS estimator of a and b in the regression model

$$y_t = a + x_t b + u_t\tag{2}$$

is a consistent estimator of α and β . To illustrate the effect of replacing negative values of x_t with zero in this regression, define the censored variable x_t^+ as

$$x_t^+ = \begin{cases} x_t, & \text{if } x_t > 0, \\ 0, & \text{if } x_t \leq 0, \end{cases}\tag{3}$$

the empirical practice in this literature. A partial exception is Herrera (2008), who conducted a sensitivity analysis based on Balke, Brown, and Yücel's methodology in a not-for-publication appendix, as a complement to results from censored oil price VAR models.

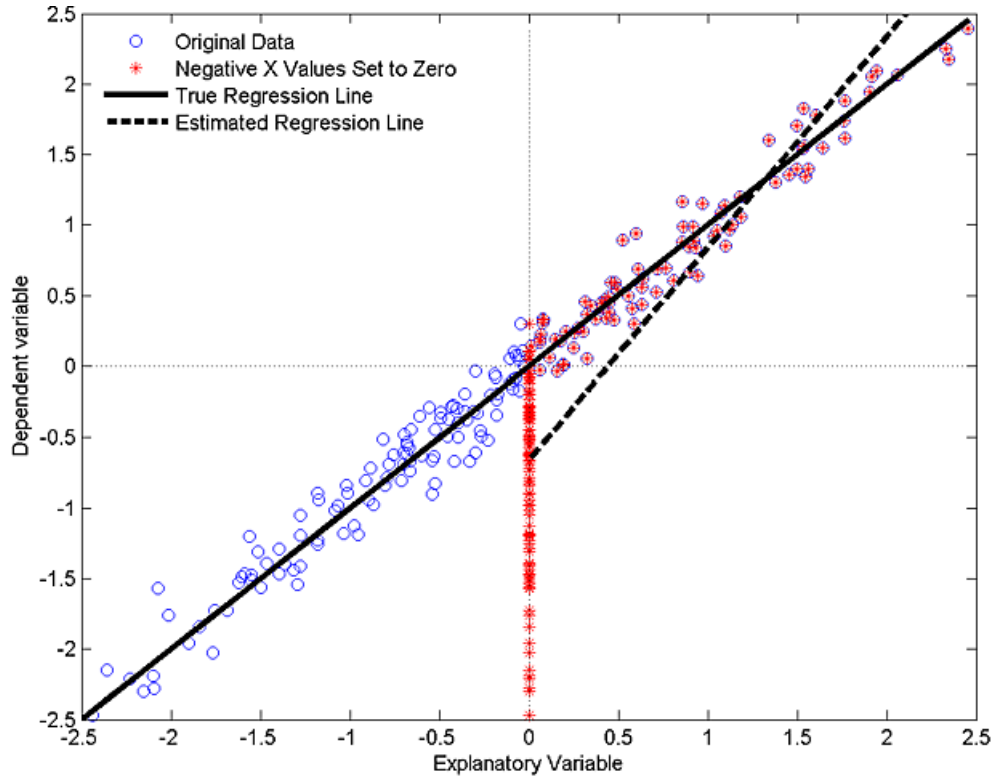


FIGURE 1. The effect of censoring negative values of the explanatory variable.

and consider estimating the regression model

$$y_t = a + x_t^+ b + u_t \quad (4)$$

rather than model (2). Censoring the explanatory variable renders the estimator of b inconsistent for β . Figure 1 illustrates the problem. Censoring amounts to replacing negative x_t -values in the original data set with zeros. Fitting the transformed data points requires a steeper regression line than fitting the original data set. The upward bias in the estimated effect of x_t on y_t is not a small-sample problem. For the case in which $\alpha = 0$, and in which x_t has a symmetric distribution with mean zero and variance 1 and is uncorrelated with $\varepsilon_{2,t}$, it is straightforward to derive the limit of \hat{b} . Observe that $E(x_t^+) = 0.5\mu$, where $\mu \equiv E(x_t | x_t > 0)$. Hence

$$\hat{b} \xrightarrow{p} \beta \frac{1}{1 - 0.5\mu^2}. \quad (5)$$

If the variable x_t has a standard normal distribution, for example, the effect of x_t on y_t is overestimated by almost 50 percent.

TABLE 1. Asymptotic bias from censoring in the asymmetric static model.^a

Population Slope Parameters			Average Estimated Slope b	
β	γ	$\beta + \gamma$	Standard Model	Censored Model
0.00	1	1	0.50	1.00
0.25	0.75	1	0.62	1.12
0.50	0.50	1	0.75	1.23
0.60	0.40	1	0.80	1.28
0.70	0.30	1	0.85	1.32
0.80	0.20	1	0.90	1.37
1.00	0.00	1	1.00	1.47
1.25	-0.25	1	1.12	1.58
1.50	-0.50	1	1.25	1.70

^aAsymmetric DGP: $\alpha_1 = \alpha_2 = 0$, $\sigma_1 = \sigma_2 = 1$. Average results for 2000 samples of 100,000 observations each.

2.2 Further illustrations of the asymptotic bias from censoring

The same inconsistency problem may arise even when the true response of y_t to x_t is asymmetric in positive and negative values. Consider the asymmetric DGP

$$\begin{aligned} x_t &= \alpha_1 + \varepsilon_{1,t}, \\ y_t &= \alpha_2 + x_t\beta + x_t^+\gamma + \varepsilon_{2,t}, \end{aligned} \tag{6}$$

where γ captures the asymmetric response that is of interest to many economic researchers. This process allows for both positive and negative values of x_t to affect y_t , but with different coefficients. Equivalently, we could have specified the second equation of model (6) as a regression in x_t^+ and x_t^- with potentially different coefficients.

Given this DGP, if one estimated the regression model

$$y_t = a + x_t^+b + u_t, \tag{7}$$

one would want the value of b to equal $\beta + \gamma$ in the limit. However, as shown in Table 1, this is not the case unless β equals zero, in which case only positive x_t have an effect on y_t . For all other values of β , the estimator of the slope coefficient is biased upward. More generally, Table 1 demonstrates that the estimate of an increase in x_t on y_t is less biased when using the original sample than when using the censored sample if the slope for negative x_t is at least half of the slope for positive x_t . In other words, even in the presence of asymmetries, the linear and symmetric model may provide a better approximation to the DGP than the censored regressor model.⁵

Table 1 highlights that only when β is *known* to be zero will the static censored regressor model consistently estimate the effect of an energy price increase. This point is

⁵Additional simulation exercises (not shown to conserve space) confirm that the asymptotic biases reported in Tables 2 and 3 that arise from the misspecification of the regression model carry over to small samples. Relative to models with Gaussian errors, small sample biases may increase substantially when the errors are fat-tailed.

important because Mork (1989) merely failed to reject the null hypothesis that energy price decreases have no predictive power for real GDP growth. He did not establish that $\beta = 0$ and, indeed, was careful only to suggest that these coefficients are “perhaps zero.” In addition, economic theory does not predict that $\beta = 0$ (see, e.g., Edelstein and Kilian (2007), Kilian (2008a)). If, in fact, both energy price increases and decreases matter for real GDP but to a different extent, as suggested by economic theory, then the censored regressor model is likely to overestimate the effect of an energy price increase even in this simplest possible model. The only way to protect from this inconsistency is to include both energy price increases and decreases in the regression.

3. HOW EMPIRICALLY RELEVANT IS THE ASYMPTOTIC BIAS OF VAR MODELS OF ENERGY PRICE INCREASES?

The static model is useful for building intuition, but extending the analysis to dynamic regression models leads to additional insights. In many cases, researchers are specifically interested in the response of the economy over time to an unexpected energy price increase:

$$I_y(h, \varepsilon_{1,t}, \Omega_{t-1}) = E(y_{t+h} | \varepsilon_{1,t}, \Omega_{t-1}) - E(y_{t+h} | \Omega_{t-1}),$$

where Ω_{t-1} is the information set at the time of the shock. In this section, we show that the censored energy price VAR models routinely employed in the literature produce inconsistent impulse response estimates not only in the empirically plausible case of nonzero coefficients on both energy price increases and decreases, but even when all coefficients on current and lagged x_t are zero in population. In other words, even if oil price decreases had no effect on real GDP growth in population, as postulated in much of the existing empirical literature, censored VAR models would yield invalid impulse response estimates. This result is in sharp contrast to the static model.

It is common practice in the literature to compute responses to unanticipated energy price increases from censored oil price VAR models. For example, Bernanke, Gertler, and Watson (1997, p. 103) observed that “Knut Mork provided evidence that only positive changes in the relative price of oil have important effects on output. Accordingly, in our VARs we employ an indicator that equals the log difference of the relative price of oil when that change is positive and otherwise is zero.” Similarly, Leduc and Sill (2004, p. 790) stated that “to get an empirical estimate of the output response to positive oil-price shocks, we run a VAR using... oil-price increases [...] constructed by taking the first difference of the log of oil prices, then setting negative values to zero. Thus, only oil-price increases affect the other variables in the system.” The censored oil price VAR models in question are recursively identified such that the energy price increase variable is ordered above the macroeconomic aggregate of interest. The prototypical example is a linear bivariate autoregression for $(x_t^+ \ y_t)'$.⁶

⁶The inclusion of additional macroeconomic variables in the VAR does not affect the econometric points of interest in this paper and, indeed, is not required for consistently estimating the response of y_t to an unanticipated increase in energy prices under the maintained assumption of predetermined (or contem-

Whereas impulse responses in linear models are independent of the history of the observations, impulse responses in nonlinear models such as the censored oil price VAR model are dependent on the history of the observations and on the magnitude of the shock (see, e.g., Gallant, Rossi, and Tauchen (1993), Koop, Pesaran, and Potter (1996)). Their construction requires Monte Carlo integration over all possible paths of the data. This point continues to be routinely ignored in the applied literature to this day. In practice, researchers instead typically present impulse response estimates computed exactly as in linear VAR models. For now we follow that convention because we wish to illustrate the asymptotic biases in the results reported in the literature. Discussing one problem at a time facilitates the exposition. We return to this point in Section 5, however, and show how impulse responses can be computed correctly and what difference this computation makes.

3.1 Linear and symmetric VAR data generating processes

As in the static case, we consider both symmetric and asymmetric DGPs in turn, starting with the symmetric DGP. The intuition we developed in Section 2 carries over to DGPs based on linear and symmetric VAR models. Consider the bivariate VAR(p) DGP

$$\begin{aligned}x_t &= b_{10} + \sum_{i=1}^p b_{11,i}x_{t-i} + \sum_{i=1}^p b_{12,i}y_{t-i} + \varepsilon_{1,t}, \\y_t &= b_{20} + \sum_{i=0}^p b_{21,i}x_{t-i} + \sum_{i=1}^p b_{22,i}y_{t-i} + \varepsilon_{2,t},\end{aligned}\tag{8}$$

where x_t denotes the percent change in energy prices, y_t denotes the percent change in the macroeconomic aggregate of interest, and $\varepsilon_t \sim (0, \Sigma)$ is uncorrelated white noise. We focus on three illustrative examples that are representative of models employed in the literature:

EXAMPLE 1. A quarterly VAR for the percent change in the real price of crude oil and the growth rate of U.S. real GDP. The sample period is 1973.II–2007.IV. The oil price series is based on an index of U.S. refiners' acquisition cost, extrapolated as in Kilian (2009), and deflated by the U.S. consumer price index (CPI). The real GDP data are from the Bureau of Economic Analysis (BEA).

EXAMPLE 2. A monthly VAR for the percent change in the real price of crude oil and the change in the U.S. unemployment rate. The sample period is 1973.2–2007.12. The unemployment rate data are from the Bureau of Labor Statistics (BLS).

EXAMPLE 3. A monthly VAR in the percent change in real gasoline prices and the percent change in U.S. real gasoline consumption, as constructed by the BEA. The sample period is 1973.2–2007.12.

poraneously exogenous) energy prices (see Kilian (2008a)). The identifying assumption of predetermined energy prices with respect to the macroeconomic aggregate of interest is not only standard in the literature, but is consistent with empirical evidence presented in Kilian and Vega (2011).

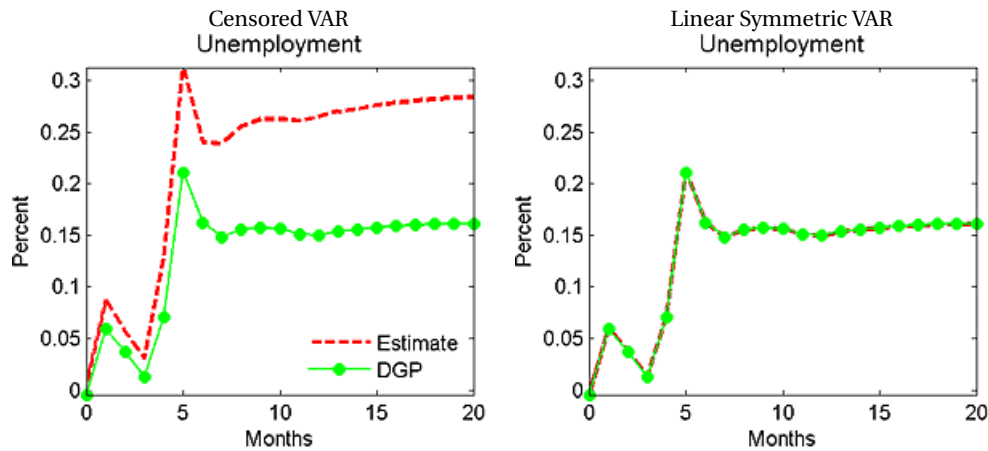


FIGURE 2. Inconsistency of the estimated effect of positive oil price shocks. Simulations from symmetric VAR DGP based on U.S. data; $T = 1,000,000$.

The lag order p is set to 6 for expository purposes. For each data set, we construct a DGP by replacing the model parameters by their least-squares estimates obtained from fitting this model to the data set in question and by treating the structural errors as Gaussian white noise. For each DGP, we generate a sample of length T , fit a VAR(6) model for $(x_t^+ \ y_t)'$, and construct the cumulative response of y_{t+h} , $h = 0, 1, \dots$, to an unanticipated unit increase in energy prices. We compare that response to the response of y_{t+h} to an unanticipated unit increase in energy prices in the DGP. Since we are interested in evaluating the asymptotic bias of the responses implied by the censored VAR model, all results in Figure 2 are based on $T = 1,000,000$. To conserve space, we focus on the response of unemployment, but note that analogous results hold in the other two applications.

The left panel of Figure 2 quantifies the asymptotic bias induced by censoring energy price decreases. The impulse response implied by the DGP is shown as the solid line. The estimated impulse response from the censored oil price VAR model is shown as the dashed line. As expected, the censored oil price VAR response tends to overestimate the true response. The response of unemployment after 12 months is overestimated by more than one-third. The reason for the inconsistency of these estimates is the same as in the static model. The right panel of Figure 2 demonstrates that our results are not driven by sampling uncertainty. It shows that the true response and the estimated response lie exactly on top of one another if we fit a linear symmetric VAR model to the same data.

3.2 Asymmetric data generating processes

In the preceding example, applied researchers choosing between symmetric and asymmetric models merely on the basis of the magnitude of the response estimates, as has been common in applied work, would have mistakenly selected the censored energy

price model. Now suppose instead that the DGP is known to be asymmetric and consider a researcher trying to quantify the asymmetric effects of energy price shocks based on a censored energy price VAR model. Here we follow the bulk of the empirical literature on energy price shocks and focus on the extreme example of a model in which only energy price increases matter for macroeconomic aggregates.

For expository purposes, first consider the simplest possible dynamic model, in which energy prices are exogenous and energy price decreases have no effect on macroeconomic outcomes at all,

$$\begin{aligned}x_t &= \alpha_1 + \rho x_{t-1} + \varepsilon_{1,t}, \\y_t &= \alpha_2 + x_t^+ \gamma + \varepsilon_{2,t},\end{aligned}\tag{9}$$

where x_t^+ is defined as above. Setting the initial conditions to zero, in this system the impact response of y_t to a positive shock to x_t would be γ . In the next period, the response would be $\rho\gamma$, provided that ρ is positive. If this system is estimated, then, as expected, estimates of both γ and ρ are consistent. If, instead, a researcher estimated the censored system

$$\begin{aligned}x_t^+ &= \alpha_1 + \rho x_{t-1}^+ + \varepsilon_{1,t}, \\y_t &= \alpha_2 + x_t^+ \gamma + \varepsilon_{2,t},\end{aligned}\tag{10}$$

the estimate of ρ would be inconsistent and so would be the impulse response estimate, even though the estimate of γ is consistent. This inconsistency arises because the DGP cannot be represented as a bivariate VAR for $(x_t^+ \ y_t)'$. The same problem arises more generally.

One telltale sign of this problem is that a censored energy price VAR DGP with positive probability generates realizations for x_t^+ that are negative. It may seem that this contradiction could be avoided by censoring the realizations much like researchers have censored percentage changes in actual energy prices, but in that case, the same inconsistencies would arise that we already documented for the linear symmetric model. This point is illustrated in Figure 3. Based on a censored VAR DGP, the censored VAR run on censored realizations of x_t^+ generates responses to energy price increases that are systematically higher than the pseudo-true response even in the limit.

The source of the problem in Figure 3 is that the censored oil price VAR regression model is not a valid description of the DGP. This problem can be avoided only by fully specifying the underlying structural model of the form

$$\begin{aligned}x_t &= \alpha_1 x_{t-1} + \alpha_2 y_{t-1} + \cdots + \varepsilon_{1,t}, \\y_t &= \beta_1 x_t^+ + \beta_2 x_{t-1}^+ + \beta_3 y_{t-1} + \cdots + \varepsilon_{2,t},\end{aligned}\tag{11}$$

where the structural shocks $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are uncorrelated and where, for expository purposes, we have omitted the definition of x_t^+ as a function of x_t .⁷

⁷Although the slope parameters of model (11) can be estimated consistently by OLS, the resulting residuals are not uncorrelated. To impose the latter restriction requires the use of a restricted maximum likelihood estimator.

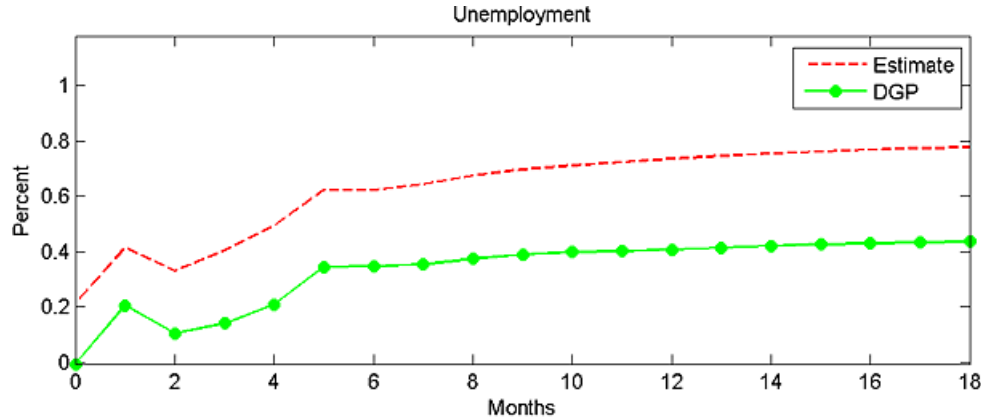


FIGURE 3. Inconsistency of the estimated effect of positive oil price shocks. Simulations from censored oil price VAR DGP based on U.S. data. $T = 1,000,000$.

The DGP model (11) postulates that percentage changes in energy prices evolve in an unconstrained fashion; only the feedback from energy prices to the macroeconomic aggregates is constrained. This model is easily recognizable as a generalization of model (6) with $\beta = 0$ to the VAR context. Note that in this model a negative shock to x_t may have a nonzero effect on y_{t+h} if the negative shock over time induces positive values in x_{t+h} .

Also note that model (11) is not equivalent to the model

$$\begin{aligned} x_t &= \alpha_1 x_{t-1} + \alpha_2 y_{t-1} + \cdots + \varepsilon_{1,t}, \\ y_t &= \beta_2 x_{t-1}^+ + \beta_3 y_{t-1} + \cdots + \beta_1 \varepsilon_{1,t} + \varepsilon_{2,t}. \end{aligned} \quad (12)$$

The key difference between models (11) and (12) is that the impact effect of a negative value of $\varepsilon_{1,t}$ is zero in model (12) and is β_1 in model (11). Furthermore, model (11) cannot be estimated from

$$\begin{aligned} x_t &= \alpha_1 x_{t-1} + \alpha_2 y_{t-1} + \cdots + \varepsilon_{1,t}, \\ y_t &= \beta_2 x_{t-1}^+ + \beta_3 y_{t-1} + \cdots + u_{2,t}, \end{aligned} \quad (13)$$

where $u_{2,t} = \beta_1 \varepsilon_{1,t} + \varepsilon_{2,t}$, and applying a Cholesky decomposition to the variance-covariance matrix of the two error terms $\varepsilon_{1,t}$ and $u_{2,t}$. The key difference is that the Cholesky decomposition does not discriminate between positive and negative shocks.

Figure 4a confirms that even when the data are generated from model (11), asymptotic biases arise when estimating the response to energy price increases from a censored energy price VAR model. We focus on the same illustrative example as in Figure 2, except that we now construct the DGP under the working hypothesis that the data are generated by the asymmetric model (11) in which there is no effect from current or lagged x_t on y_t . We treat the least-squares estimates of the slope parameters and innovation variances obtained on the actual data as the population parameters in the simulation, and we impose the zero correlation of the innovation variances. All results

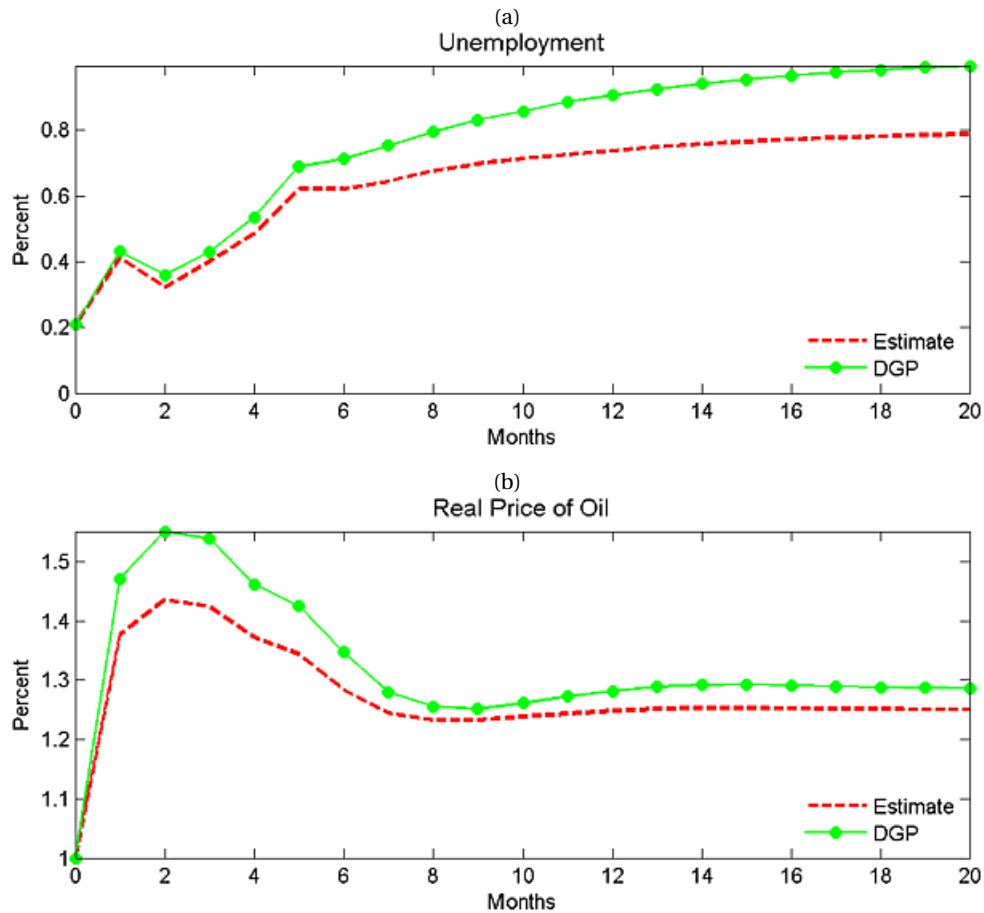


FIGURE 4. Inconsistency of the estimated effect of unanticipated oil price increases for the asymmetric structural model DGP: Fitting the censored VAR model. Simulations based on model (11) estimated on U.S. data. $T = 1,000,000$.

are based on $T = 1,000,000$. Figure 4a shows that even in this case, the response estimates implied by the censored oil price VAR model are inconsistent. The direction of the asymptotic bias is ambiguous in general. For the unemployment rate, after 20 months, the estimated response to an unanticipated oil price increase is about 80 percent of the true response.

Figure 4b illustrates the source of the problem by plotting the corresponding responses of the real price of oil. Although there is no problem in consistently estimating the second equation of the system that includes the censored regressors (and, indeed, the impact response of y_t is correctly estimated in Figure 4a), the fact that the first equation in the censored oil price VAR model is misspecified causes both response estimates to be inconsistent as the energy price shock is propagated over time. The results in Figure 4a and b represent the best possible scenario in that we postulated that only energy price increases matter in the DGP. Additional asymptotic biases would arise if the asym-

metric DGP allowed for nonzero effects from energy price decreases, and those biases would affect even the impact responses.⁸

4. ELIMINATING THE INCONSISTENCY: A GENERAL MODEL OF THE OIL PRICE–ECONOMY LINK

Until now, we have imposed the strongest form of asymmetry in which energy price declines have no effect on the macroeconomic aggregate of interest. In the interest of full generality, we now relax this assumption by allowing for both energy price increases and decreases to have an effect, but to different extents.⁹ The first equation of the resulting encompassing model is identical to the first equation of a standard linear VAR in x_t and y_t , but the second equation now includes both x_t and x_t^+ and, as such, both energy price increases and decreases affect y_t :

$$\begin{aligned} x_t &= b_{10} + \sum_{i=1}^p b_{11,i} x_{t-i} + \sum_{i=1}^p b_{12,i} y_{t-i} + \varepsilon_{1,t}, \\ y_t &= b_{20} + \sum_{i=0}^p b_{21,i} x_{t-i} + \sum_{i=1}^p b_{22,i} y_{t-i} + \sum_{i=0}^p g_{21,i} x_{t-i}^+ + \varepsilon_{2,t}. \end{aligned} \tag{14}$$

Given estimates of these coefficients, one can calculate the dynamic responses to unanticipated positive and negative energy price changes.¹⁰ Note that the OLS residuals of model (14) are uncorrelated, whereas the OLS residuals of model (11) may be correlated. This means that model (14) can be estimated by standard regression methods.

As demonstrated in Kilian and Vigfusson (2009), the key advantage of model (14) is that the dynamic responses are consistently estimated regardless of whether the true DGP is symmetric or asymmetric. In short, the advantage of the encompassing model (14) is that it can be used without knowing the nature of the DGP. Its only disadvantage is that the parameter estimates are not efficient asymptotically.

5. COMPUTING RESPONSES TO ENERGY PRICE SHOCKS IN NONLINEAR MODELS

In Section 4, we followed the convention in the empirical literature on energy price shocks of computing impulse responses as one would for linear VAR models. While this approach simplifies the computation of the responses from asymmetric models, it can be misleading in that the effect of a given shock in asymmetric models depends on the recent history of the series in question and on the magnitude of the shock. This point

⁸Although we focused on bivariate VAR models for $(x_t^+, y_t)'$, the same impulse response inconsistency problems arise when fitting trivariate VAR models for $(x_t^+, x_t^-, y_t)'$. Similar problems also arise if we are fitting a VAR model involving $(x_t, |x_t|, y_t)'$.

⁹Theoretical models of asymmetry do not imply the strong form of asymmetry, but do allow for nontrivial effects of both energy price increases and decreases (see, e.g., Edelstein and Kilian (2007, 2009), Kilian (2008a)).

¹⁰If energy prices never declined, this model would suffer from collinearity, but in the data we observe both energy price increases and declines.

is well known (see, e.g., Gallant, Rossi, and Tauchen (1993), Koop, Pesaran, and Potter (1996)), but has been typically ignored in the literature on estimating the effects of energy price shocks.

In this section, we propose an adaption of these methods for computing structural impulse responses from nonlinear models specifically designed for model (14). Having estimated the encompassing model (14), we proceed as follows:

Step 1. Take a block of p consecutive values of x_t and y_t . This defines a history Ω^i .

Note that the choice of history does not affect the coefficients of the model. For all histories, the model coefficients are fixed at their estimated values.

Step 2. Given Ω^i , simulate two time paths for x_{t+h} and y_{t+h} for $h = 0, 1, \dots, H$. When generating the first time path, the value of $\varepsilon_{1,t}$ is set equal to a prespecified value δ . The realizations of $\varepsilon_{1,t+h}$ for $h = 1, \dots, H$ are drawn from the marginal empirical distribution of $\varepsilon_{1,t}$. The realizations of $\varepsilon_{2,t+h}$ for $h = 0, \dots, H$ are drawn independently from the marginal distribution of $\varepsilon_{2,t}$. When generating the second time path, all $\varepsilon_{1,t+h}$ and $\varepsilon_{2,t+h}$ for $h = 0, \dots, H$ are drawn from their respective marginal distributions.

Step 3. Calculate the difference between the time paths for y_{t+h} , $h = 0, \dots, H$.

Step 4. Average this difference across $m = 500$ repetitions of Steps 2 and 3.

This average is the response of y_{t+h} at horizon $h = 0, \dots, H$ to a shock of size δ conditional on Ω^i :

$$I_y(h, \delta, \Omega^i). \quad (15)$$

The unconditional response $I_y(h, \delta)$ is defined as the value of $I_y(h, \delta, \Omega^i)$ averaged across all histories:

$$I_y(h, \delta) = \int I_y(h, \delta, \Omega^i) d\Omega^i. \quad (16)$$

Whereas the response conditional on current history is the relevant statistic for forecasting and policy purposes, the unconditional response is a better measure of the overall importance of oil price shocks as a source of economic fluctuations. In the remainder of the paper, we focus on the unconditional response (16).¹¹

The impulse response typically computed in the literature on the transmission of oil price shocks instead has been $I_y^*(h, \delta, \underline{Q})$. The latter response conditions on a hypothetical historical path involving $x_{t-i} = y_{t-i} = 0$ for all i , and incorrectly evaluates future

¹¹Our approach is related to, but distinct from, the algorithm for nonlinear reduced-form VAR models discussed in Koop, Pesaran, and Potter (1996), who suggested that responses can be based on draws from the joint distribution of reduced-form errors. Such responses may be useful in characterizing the persistence of the data, but they are devoid of any economic interpretation because reduced-form errors are inevitably mutually correlated. Moreover, they are not unique: different random draws generate different response functions. In contrast, the nonlinear structural impulse responses (15) and (16) relate to an energy price shock that is orthogonal to other shocks and uniquely defined (up to scale).

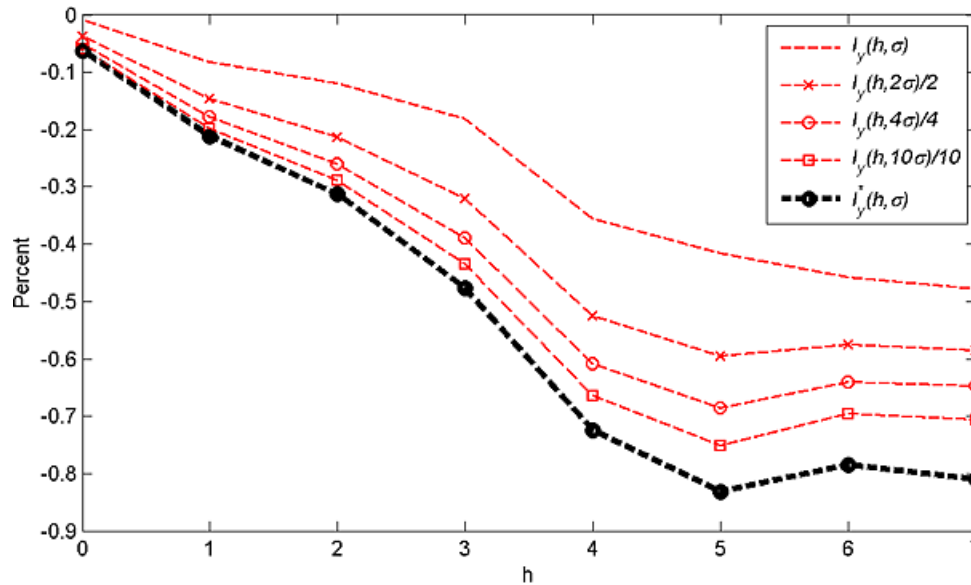


FIGURE 5. The response of GDP to a positive energy price shock by shock size. $I_y(h, \sigma)$ is calculated by Monte Carlo integration over 300 histories with 10,000 paths each. h denotes the horizon.

shocks at their expected value of zero, ignoring the nonlinearity of the impulse response function. It can be shown that the impulse response $I_y^*(h, \delta, \underline{0})$ corresponds to a scaled version of the correctly computed response of the form $I_y(h, n\delta, \Omega^i)/n$ as n approaches infinity (see the [Appendix](#)). The example in Figure 5 illustrates that $I_y(h, \delta)$, in general, is much smaller in absolute value than $I_y^*(h, \delta, \underline{0})$. Only for a price shock in excess of 10 standard deviations does the value of $I_y(h, \delta)$ approximate (up to scale) that of the correctly computed responses. We conclude that even if the dynamic regression model is correctly specified (and hence the parameter estimates are consistent), the use of traditional impulse response functions exaggerates the effect of a positive oil price shock. This finding reinforces our earlier concern with the methods underlying the existing literature.

6. TESTING SYMMETRY IN ENERGY PRICE INCREASES AND DECREASES

The existing empirical literature has taken for granted that the responses of U.S. macroeconomic aggregates are asymmetric in positive and negative energy price shocks. This raises the question of how to test the null hypothesis of symmetric response functions. We first discuss the problem of testing models in which energy price increases matter more than energy price decreases before adapting these tests to the problem of testing models of net energy price increases.

6.1 Slope-based tests

If energy price increases and decreases received exactly the same weight in regressions of y_t on lagged y_t and current and lagged x_t^- and x_t^+ , it would follow immediately that the

dynamic responses to energy price shocks must be symmetric in positive and negative shocks. This line of reasoning has motivated the development of slope-based tests of symmetry. Such tests are attractive in that they do not require the complete specification of the system to be estimated nor do they require the computation of impulse responses.

The traditional approach to testing for symmetry in the transmission of energy price shocks involves tests of the symmetry of the slope coefficients in predictive regressions of y_t on lagged x_t^- and x_t^+ (see, e.g., Mork (1989)). This is equivalent to testing

$$H_0 : g_{21,1} = \dots = g_{21,p} = 0$$

in model (14) by means of a Wald test with an asymptotic χ_p^2 distribution. Closer inspection of model (14) reveals that this test does not exploit all restrictions implied by the null hypothesis of symmetry in the structural model. Specifically, Mork's test omits the contemporaneous regressor because he works with a predictive model. An alternative test of all symmetry restrictions on the slopes involves the null hypothesis

$$H_0 : g_{21,0} = \dots = g_{21,p} = 0.$$

In that case, one estimates the second equation of model (14) by least squares and uses a Wald test to determine whether including $\{x_{t-i}^+\}_{i=0}^p$ improves the fit of the model. This modified slope-based test has an asymptotic χ_{p+1}^2 distribution. It can be shown that this test has similarly accurate size, but may have higher power than Mork's test, making it useful to consider both types of slope-based tests (see Kilian and Vigfusson (2009)).

While slope-based tests are useful in assessing the symmetry of the slope parameters of single-equation regression models, they are not informative about the degree of symmetry of the impulse response function obtained from a fully specified dynamic structural model. There are two possible outcomes when conducting slope-based tests.

If the test rejects symmetry, that result is sufficient for concluding that the impulse responses are asymmetric, but it does not tell us whether the departures from symmetry are economically or statistically significant. Given that impulse response functions are highly nonlinear functions of the slope parameters and innovation variances, it is quite conceivable that the degree of asymmetry in the impulse responses to positive and negative energy price shocks could be quite small, making responses based on the linear model a good approximation, despite the statistical rejection of symmetric slopes. Moreover, the quality of the linear approximation will differ depending on the magnitude of the shock. For that reason, the applied user will want to plot the point estimates of the impulse response functions and inspect them. If the test fails to reject symmetry, on the other hand, we again learn little, because statistically insignificant departures from symmetry in the slopes may cause large and statistically significant asymmetries in the implied impulse response functions, given the nonlinearity of these functions.

This observation suggests that a more useful approach is to test the symmetry of the economy's dynamic responses to unanticipated energy price increases and decreases directly based on the impulse response functions (also see Edelstein and Kilian (2007, 2009)). This alternative approach to testing the null of symmetry is discussed next. Note that what is at issue in conducting this impulse-response-based test is not the

existence of asymmetries in the reduced-form parameters, but the question of whether possible asymmetries in the reduced-form imply significant asymmetries in the impulse response function. While any asymmetry in the reduced-form representation (whether statistically significant or not) implies some degree of asymmetry in the impulse response function, the question is whether the impulse responses constructed from linear symmetric VAR models still provide a good approximation.

6.2 Impulse-response-based tests

We first estimate the unrestricted encompassing model (14) and calculate the unconditional impulse responses to both positive and negative energy price shocks. We then construct a Wald test of the joint null hypothesis of symmetric responses to positive and negative energy price shocks up to a prespecified horizon H . Symmetry means that

$$I_y(h, \delta) = -I_y(h, -\delta) \quad \text{for } h = 0, 1, 2, \dots, H$$

or, equivalently,

$$I_y(h, \delta) + I_y(h, -\delta) = 0 \quad \text{for } h = 0, 1, 2, \dots, H.$$

The variance-covariance matrix of the vector sum of response coefficients can be estimated by bootstrap simulation. Given the asymptotic normality of the parameter estimators of model (14), the test has an asymptotic χ_{H+1}^2 distribution.

Unlike the slope-based test, this test depends on the magnitude of δ , so the evidence against symmetry depends on the magnitude of the shock considered. For small shocks, a symmetric model provides a better approximation than for large shocks. How accurate and powerful the impulse-response-based test is relative to the slope-based test is an empirical question. It can be shown that, in our three applications, this test has acceptable size properties, despite a slight tendency to overreject, as the horizon increases (see Kilian and Vigfusson (2009)).

7. EMPIRICAL TESTS OF SYMMETRY IN ENERGY PRICE INCREASES AND DECREASES

In this section, we apply both types of symmetry tests to the three empirical examples introduced in Section 3. The p -values in Table 2 for the modified slope-based test are based on the baseline model with six lags. There is no evidence against the symmetry null at the 10% significance level in monthly U.S. unemployment rates, in quarterly U.S. real GDP, and in U.S. gasoline consumption. The same qualitative result, but with higher p -values, holds using Mork's test.¹²

¹²An important question is whether these empirical results are sensitive to the choice of lag order. Our baseline results rely on six lags. For sensitivity analysis, we considered a grid of lag orders $p \in \{2, 4, 6, 8, 10, 12\}$. For lag orders larger than six, we find rejections of the symmetry null hypothesis at the 5% level for gasoline consumption, but not for the other examples. The implied impulse response functions, however, are strikingly symmetric even in that case, illustrating the drawbacks of slope-based tests.

TABLE 2. Empirical symmetry tests: Baseline model.

Variable	Modified Test of Symmetric		Mork's Test of Symmetric	
	Slope Coefficients	Marginal Significance Level	Slope Coefficients	Marginal Significance Level
Unemployment	7.7224	0.358	3.1317	0.792
Gas consumption	11.3755	0.123	9.2366	0.161
Real GDP	10.4722	0.163	9.7565	0.135

Table 3 reports the corresponding tests of the symmetry of the impulse response functions. Neither for U.S. real GDP nor for unemployment is there statistically significant evidence against the symmetry of the functions.¹³ For gasoline consumption, the results are mixed. Whereas there is no evidence against symmetry based on 2 standard deviation shocks, based on 1 standard deviation shocks the test rejects the null hypothesis of symmetry at the 5% level at one horizon and at the 10% level at several additional horizons. The evidence against symmetry appears stronger than that based on the slope-based test. However, as shown in Figure 6, which reports $I_y(h, \sigma)$ and $-I_y(h, -\sigma)$, the actual difference between these two responses is fairly small, and one would be hard pressed to make the case for using the asymmetric model on economic grounds. In particular, the response to a negative shock is clearly not zero (as implied by the standard censored energy price VAR model), but about as large in absolute terms as the response to a positive shock. Thus, we may rule out the presence of the type of asymmetric response envisioned by Mork (1989) in all three empirical examples.

8. TESTING SYMMETRY BASED ON MODELS OF NET ENERGY PRICE INCREASES

As noted in the [Introduction](#), much of the recent empirical work on the transmission of oil price shocks has focused on the net increase in the price of oil as defined in Hamilton

TABLE 3. Testing the symmetry of the response $I_y(h, \delta) = -I_y(h, -\delta)$ for $h = 0, 1, 2, \dots, H$.^a

H	Gas Consumption		GDP		Unemployment	
	1 Std.	2 Std.	1 Std.	2 Std.	1 Std.	2 Std.
	Deviation Shock	Deviation Shock	Deviation Shock	Deviation Shock	Deviation Shock	Deviation Shock
0	0.45	0.47	0.40	0.47	0.36	0.43
1	0.13	0.28	0.44	0.54	0.65	0.73
2	0.05	0.15	0.59	0.69	0.83	0.88
3	0.09	0.25	0.56	0.68	0.92	0.96
4	0.07	0.21	0.66	0.78	0.97	0.98
5	0.04	0.15	0.78	0.87	0.99	1.00
6	0.06	0.18	0.48	0.59	1.00	1.00
7	0.09	0.26	0.58	0.69	1.00	1.00

^aBased on 20,000 simulations of model (14). p -values are based on the χ^2_{H+1} distribution.

¹³These results are qualitatively consistent with the findings in Edelstein and Kilian (2007, 2009) based on a somewhat different methodology.

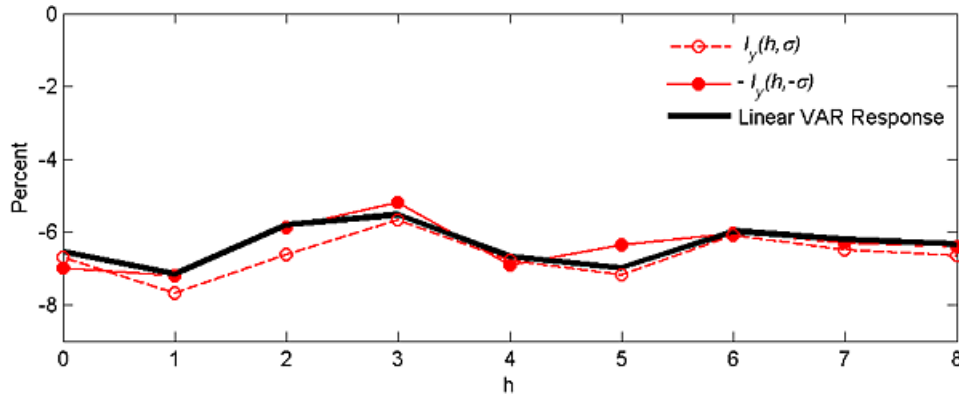


FIGURE 6. The response of gas consumption to a 1 standard deviation energy price shock: Base-line model with 6 lags.

(1996, 2003) rather than Mork's (1989) oil price increase measure.¹⁴ Given its widespread use in applied VAR work, it is important to assess the empirical support for the net increase model. Both Hamilton (2003) and Balke, Brown, and Yücel (2002) fitted predictive regressions of the form

$$y_t = b_{20} + \sum_{i=1}^p b_{21,i} x_{t-i} + \sum_{i=1}^p b_{22,i} y_{t-i} + \sum_{i=1}^p g_{21,i} x_{t-i}^{+,net} + v_t$$

by least squares and reported strong rejections of the joint null hypothesis $H_0: g_{21,i} = 0 \forall i$. This evidence traditionally has been taken as sufficient reason for constructing VAR models involving the net oil price increase variable.

It is important to keep in mind, however, that, even if we take these results about the existence of an asymmetric predictive relationship involving net oil price increases at face value, the rejection of the null hypothesis of symmetric slopes does not justify the use of censored oil price VAR models of the form

$$x_t^{+,net} = b_{10} + \sum_{i=1}^p b_{11,i} x_{t-i}^{+,net} + \sum_{i=1}^p b_{12,i} y_{t-i} + \varepsilon_{1,t},$$

$$y_t = b_{20} + \sum_{i=1}^p b_{22,i} y_{t-i} + \sum_{i=1}^p g_{21,i} x_{t-i}^{+,net} + a\varepsilon_{1,t} + \varepsilon_{2,t},$$

¹⁴For example, Lee and Ni (2002, p. 834) noted that the "oil price variable [in their VAR] is Hamilton's (1996) 'net oil price increase,' defined as the percentage change of oil price over the maximum value of the preceding year if the price of the current month exceeds the previous year's maximum, and zero otherwise." Likewise, Bernanke, Gertler, and Watson's (1997, p. 104) VAR analysis relies, as the main measure of oil price shocks, on Hamilton's measure which "equals the maximum of (a) zero and (b) the difference between the log-level of the crude oil price for the current month and the maximum value of the logged crude oil price achieved in the previous twelve months." Similar net oil price increase measures also were used by Davis and Haltiwanger (2001), Lee and Ni (2002), and Hamilton and Herrera (2004), among others.

where a is estimated by the Cholesky factorization of the variance–covariance matrix of the reduced-form residuals. For the reasons discussed earlier, the structural DGPs that give rise to the asymmetries documented in Hamilton (2003) cannot be represented as censored oil price VAR models and there is no way to construct valid structural impulse response functions from such models.

Thus, Hamilton's work leaves unanswered the question of how much the response of real GDP to an exogenous oil price innovation is affected by the nonlinearity of the DGP relative to the linear case. Even if there is an asymmetry in the slope parameters of the reduced form, that asymmetry need not have large effects on the implied impulse response function. We illustrate this point below. Moreover, the extent to which responses from a linear symmetric VAR model provide a good approximation is a function of the magnitude of the energy price shock. Answering that question requires a fully specified multivariate structural model.

In this section, we outline two tests of the net increase model, building on the analysis in Sections 4 and 5. Rather than testing the null hypothesis of symmetry between net oil price decreases and net oil price increases, as in Edelstein and Kilian (2009), we nest the net increase model in the standard linear symmetric VAR model. In essence, we ask whether there is incremental explanatory power in including net oil price increases in the baseline model. This results in a model structure similar to model (14) with x_t^+ replaced by $x_t^{+,net}$, where $x_t^{+,net} = \max[0, x_t - x_t^*]$ and x_t^* is the maximum of x_t over the preceding year (or 3 years, alternatively), following Hamilton (1996, 2003). We follow Kilian (2008b) in specifying the net increase in the real price of oil rather than the nominal price as in Hamilton (1996, 2003), because the real price is the relevant measure of the price of oil in theoretical models of the transmission of oil price shocks.

The problems with the use of net oil price increase measures in VAR models are fundamentally the same as with the use of oil price increase measures and can be addressed along similar lines. By analogy to the discussion in Section 4, the structural model

$$\begin{aligned} x_t &= b_{10} + \sum_{i=1}^p b_{11,i} x_{t-i} + \sum_{i=1}^p b_{12,i} y_{t-i} + \varepsilon_{1,t}, \\ y_t &= b_{20} + \sum_{i=0}^p b_{21,i} x_{t-i} + \sum_{i=1}^p b_{22,i} y_{t-i} + \sum_{i=0}^p g_{21,i} x_{t-i}^{+,net} + \varepsilon_{2,t} \end{aligned} \tag{17}$$

can be estimated consistently by least squares. Note that model (17) allows us to compute impulse response functions that take into account the magnitude and direction of the innovation $\varepsilon_{1,t}$ as well as the history of observations, whereas in the censored oil price VAR model, an oil price shock is not well defined.

8.1 Slope-based symmetry tests for the net increase model

In assessing the evidence for this structural net increase model, a natural starting point is the slope-based test

$$H_0 : g_{21,0} = \dots = g_{21,p} = 0$$

TABLE 4. Slope-based test of the linear symmetric VAR model against the net increase VAR model: Baseline model with 6 lags.

Variable	1-Year Net Increase		3-Year Net Increase	
	Test of Linear Symmetric Model	Marginal Significance Level	Test of Linear Symmetric Model	Marginal Significance Level
Unemployment	10.5099	0.162	9.6332	0.210
Gas consumption	9.8879	0.195	14.5307	0.043
Real GDP	7.2617	0.402	14.2965	0.046

based on (17). This test relates to the conventional test conducted in Balke, Brown, and Yücel (2002) like the modified slope-based test in Section 6 relates to Mork's (1989) test of symmetry. The only difference is the additional inclusion of the contemporaneous regressor. Table 4 suggests that there is no evidence of asymmetries using the 1-year net increase measure, but for the 3-year net increase measure, the symmetry test rejects at the 5% level for gasoline consumption and real GDP.

Figure 7 illustrates how misleading slope-based tests of symmetry can be. We focus on the 3-year net increase. Results for the 1-year net increase are very similar. For example, Table 4 indicates the presence of significant asymmetries for gasoline consumption and for real GDP. Despite these rejections of the hypothesis of symmetric slopes, however, the implied responses in Figure 7 to shocks of typical magnitude are almost perfectly symmetric, as evidenced by the fact that the two lines shown are nearly identical.¹⁵ Broadly similar results hold for 2 standard deviation shocks with the exception of the unemployment rate model. Ironically, the unemployment rate model is the one model that passes the slope-based tests of symmetry in Table 4, highlighting the importance of actually computing the impulse response functions. In contrast, the other point estimates look fairly symmetric. Although the response of real GDP to a positive 2 standard deviation shock is somewhat larger in absolute terms than the response to a negative shock of this magnitude, both responses are clearly negative and have a similar pattern. In the gasoline consumption model, the symmetry of the two response functions is even more pronounced.

8.2 An impulse-response-based symmetry test for the net increase model

It may be tempting to decide the question of symmetry based on the estimates of the impulse response functions in Figure 7. Figure 7 underscores that there is no reason to question the symmetry assumption for shocks of typical magnitude. For 2 standard deviation shocks, the evidence is less clear, however, especially in the unemployment example. Because the point estimates in Figure 7 are subject to considerable sampling uncertainty, especially when considering large energy price shocks, it is useful to con-

¹⁵By construction, a 1 standard deviation shock is a typical shock in that about two-thirds of energy price shocks in historical data are no larger than 1 standard deviation.

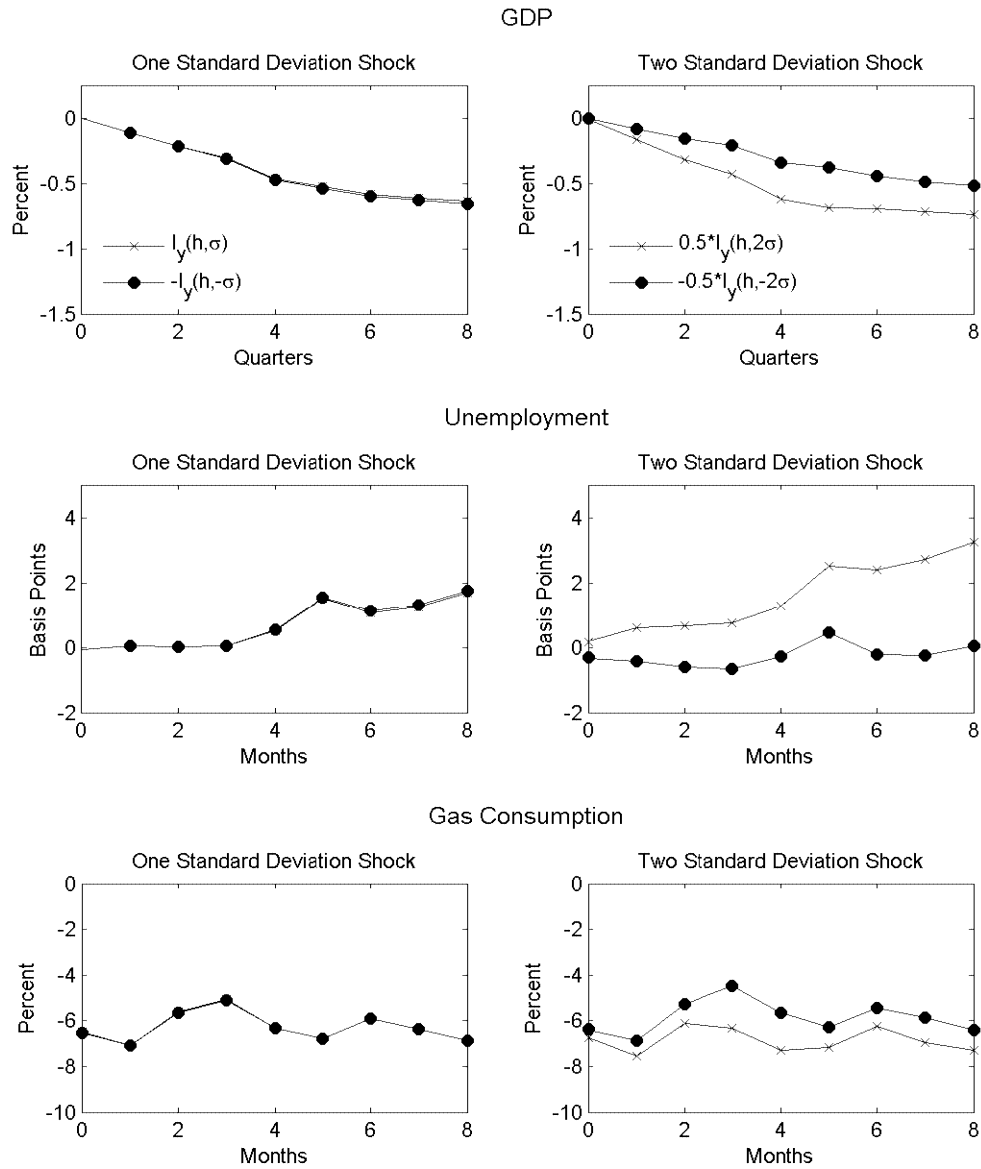


FIGURE 7. Empirical responses to 1 and 2 standard deviation positive and negative energy price shocks in baseline model with 6 lags: 3-year net increase. The responses to negative shocks are shown as mirror images to facilitate the comparison. Some of the responses in the left panel are nearly invisible because the responses are almost perfectly symmetric.

duct a formal test of the symmetry of the response functions based on model (17). As in Section 7, we test the null of $I_y(h, \delta) = -I_y(h, -\delta)$ for $h = 0, 1, 2, \dots, H$.¹⁶

¹⁶An alternative and asymptotically equivalent approach is to test the equality of the impulse responses obtained from the linear model, on the one hand, and either the response to an energy price increase or a decrease, on the other. We do not pursue that approach because it is not clear how to use the bootstrap to

TABLE 5. p -values of the test of $H_0: I_y(h, \delta) = -I_y(h, -\delta)$ for $h = 0, 1, 2, \dots, H$.^a

H	Gasoline Consumption		GDP		Unemployment	
	1 Std. Deviation Shock	2 Std. Deviation Shock	1 Std. Deviation Shock	2 Std. Deviation Shock	1 Std. Deviation Shock	2 Std. Deviation Shock
	a. 1-Year Net Increase					
0	0.95	0.82	0.96	0.50	0.99	0.01
1	1.00	0.91	0.96	0.72	0.83	0.03
2	1.00	0.34	0.98	0.58	0.94	0.07
3	0.99	0.11	0.99	0.65	0.98	0.12
4	0.98	0.18	0.99	0.64	0.99	0.20
5	0.98	0.26	0.99	0.74	1.00	0.21
6	0.99	0.34	0.99	0.60	1.00	0.19
7	1.00	0.43	1.00	0.66	1.00	0.24
	b. 3-Year Net Increase					
0	0.94	0.43	0.98	0.79	0.95	0.13
1	0.98	0.56	0.99	0.30	0.99	0.24
2	1.00	0.66	1.00	0.26	1.00	0.40
3	1.00	0.23	1.00	0.38	1.00	0.57
4	1.00	0.32	1.00	0.41	1.00	0.71
5	1.00	0.44	1.00	0.53	1.00	0.69
6	1.00	0.55	1.00	0.57	1.00	0.69
7	1.00	0.59	1.00	0.67	1.00	0.77

^aBased on 20,000 simulations of model (30). p -values are based on the χ^2_{H+1} distribution.

Table 5 shows that, as expected, the p -values decline with the magnitude of the shock. At conventional significance levels, there is no evidence against the symmetry null hypothesis in any of the three empirical examples in response to a 1 standard deviation shock, whether we focus on the 1-year or the 3-year net changes. The same results hold in response to a 2 standard deviation shock when using the 3-year net changes. Similar results also hold for the 1-year net changes with the partial exception of the unemployment rate at very short horizons. We conclude that there is very little, if any, evidence of asymmetric responses to energy price increases and decreases. In particular, there is no such evidence for U.S. real GDP.

These empirical results are important in light of the consensus view, exemplified by Davis and Haltiwanger (2001, p. 509), that the evidence for asymmetric responses to oil price ups and downs is well established. Our analysis suggests that the evidence against the symmetry hypothesis has been overstated. It is of course possible that impulse-response-based tests lack the power to detect asymmetries in the data, especially if those asymmetries are relatively weak for shocks of typical size, but the rejections found in some of our empirical examples suggest that lack of power is not a concern. The same point has been demonstrated in Herrera, Lagalo, and Wada (forthcoming), who showed

evaluate the variance of the Wald test statistic in that case. In contrast, the symmetry test is straightforward to implement.

that our test has sufficient power to reject the null of symmetric response functions in disaggregate data on samples of comparable length.

We certainly would not want to rule out the existence of asymmetries in all possible applications on the basis of our empirical evidence. Part of our objective has been to provide tools to detect asymmetries and deal with asymmetries on a case-by-case basis. What our empirical evidence does suggest, however, is that asymmetry in the responses to energy price shocks is clearly not a pervasive and robust feature of the U.S. data. This point is important because a large literature has developed that aims to explain the perceived asymmetry of responses to energy price shocks from a theoretical point of view (see, e.g., Bernanke (1983), Hamilton (1988), Pindyck (1991)). Our evidence casts doubt on the empirical relevance of these theoretical models.

9. RELATIONSHIP WITH THE RELATED LITERATURE

It may seem at first sight that the central ideas of our paper are already contained in Balke, Brown, and Yücel (2002). Although they certainly deserve credit for being the first researchers to recognize that censored oil price VAR models are inherently misspecified, this is not the case. In fact, their approach to this problem is different from ours in several dimensions. It is important to make these differences explicit. First, Balke, Brown, and Yücel did not explain why impulse response estimates from censored oil price models are invalid and they did not establish that these estimates are inconsistent, which helps explain why the use of censored oil price VAR models has remained standard to this day.¹⁷

Second, the structure and the identifying assumptions of Balke, Brown, and Yücel's model differ from the rest of the literature. Abstracting from nonessential variables, the model they used can be written as:¹⁸

$$x_t = \alpha_1 + \sum_{i=1}^p b_{11,i} x_{t-i} + \sum_{i=0}^p b_{12,i} y_{t-i} + \sum_{i=1}^p g_{12,i} x_{t-i}^{\text{net},+,3\text{yr}} + e_{1,t},$$

$$y_t = \alpha_2 + \sum_{i=1}^p b_{21,i} x_{t-i} + \sum_{i=1}^p b_{22,i} y_{t-i} + \sum_{i=1}^p g_{21,i} x_{t-i}^{\text{net},+,3\text{yr}} + e_{2,t},$$

where x_t is the percentage change in the price of oil and y_t denotes U.S. real GDP growth. The standard view in the literature is that the price of oil is predetermined with respect

¹⁷The full extent of their analysis of the problems with censored oil price VAR models is a statement that censored oil price VAR models “are not completely suitable for an examination of asymmetry” and that “it is not at all clear how to interpret a negative Hamilton innovation.”

¹⁸The original specification in Balke, Brown, and Yücel included additional macroeconomic aggregates, given their focus on separately identifying monetary policy reactions to the price of oil. For further discussion of this approach, see Kilian and Lewis (forthcoming) and the references therein. Under standard identifying assumptions, the inclusion of additional variables in the VAR model does not affect the asymptotic properties of the response of real GDP to oil price innovations, but it may affect the accuracy of the response estimates in small samples. Here we abstract from these small-sample issues and focus on the more fundamental differences between the analysis in Balke, Brown, and Yücel (2002) and our analysis.

to U.S. real output, which implies that $b_{12,0} = 0$. This view is consistent with recent empirical evidence in Kilian and Vega ([forthcoming](#)). The model used by Balke, Brown, and Yücel, however, imposes a recursive ordering that treats real output rather than the price of oil as predetermined. Their key identifying assumption is that there is feedback within the impact period from innovations in real output to the price of oil ($b_{12,0} \neq 0$), but no feedback within the impact period from innovations in the price of oil to real output ($b_{21,0} = 0$). Our analysis, in contrast, imposes the standard identifying assumption familiar from structural VAR models of the relationship between oil prices and real output that $b_{12,0} = 0$ and $b_{21,0} \neq 0$:

$$x_t = \alpha_1 + \sum_{i=1}^p b_{11,i} x_{t-i} + \sum_{i=1}^p b_{12,i} y_{t-i} + e_{1,t},$$

$$y_t = \alpha_2 + \sum_{i=0}^p b_{21,i} x_{t-i} + \sum_{i=1}^p b_{22,i} y_{t-i} + \sum_{i=0}^p g_{21,i} x_{t-i}^{\text{net},+,3\text{yr}} + e_{2,t}.$$

Another important difference between these models is that we postulate that the price of oil is a linear function of past data, similar to the specification in Hamilton (2003), for example. Balke, Brown, and Yücel instead added an additional nonlinearity in the first equation of their model which is not implied by economic theory and which makes it difficult to compare their results to other models in the literature. Moreover, their model is specified in terms of the nominal price of oil rather than the real price of oil and is estimated on data starting in January 1965, which is not valid, given that the process generating the price of oil prior to 1973 cannot be represented by standard dynamic models (see, e.g., Kilian and Vigfusson ([forthcoming](#))).

Third, Balke, Brown, and Yücel do not formally test the null of symmetric response functions. Neither the traditional slope-based test nor the additional t -tests for pointwise symmetry of the real output responses that they report are informative about the degree of asymmetry of the response functions. Their approach of conducting pointwise t -tests at all horizons would be valid if and only if the t -tests were independent across horizons, which they are not, necessitating a joint test of these restrictions that takes into account the covariance terms. Moreover, a joint test also eliminates the size distortions that arise from the repeated application of t -tests across multiple horizons which cause spurious rejections of the symmetry null (see, e.g., Kilian and Vega (2011)).

For these three reasons, the evidence in Balke, Brown, and Yücel cannot be compared directly with our evidence in this paper and is not dispositive about the degree of asymmetry in the response functions of U.S. real economic activity to oil price innovations.

10. SENSITIVITY ANALYSIS

An important concern is whether our results are sensitive to minor changes in the model specification such as changes in the lag specification or in the oil price measure used. The robustness of our findings is illustrated in Table 6, which focuses on testing the null

TABLE 6. p -values for test of the null of symmetric slopes: Regression model for U.S. real GDP growth.^a

			Three-Year Net Oil Price Increase	
			Traditional Test	Modified Test
$p = 4$	PPI	Real	0.005	0.011
		Nominal	0.013	0.027
	Import RAC	Real	0.007	0.014
		Nominal	0.004	0.008
$p = 6$	PPI	Real	0.020	0.037
		Nominal	0.049	0.080
	Import RAC	Real	0.027	0.046
		Nominal	0.016	0.028

^aPPI stands for the U.S. producer price index for crude oil; Import RAC stands for the U.S. refiners' acquisition cost for imported crude oil. Boldface indicates statistical significance at the 5% level.

of symmetric slopes in the regression model for U.S. real GDP growth. The alternative model allows for added predictive power from the 3-year net oil price increase, as in Hamilton (2003). Recall that in Table 4 we rejected the null of symmetric slopes for this type of model at the 5% level. Table 6 shows that the same conclusion is reached across a wide range of alternative specifications, including the specification favored by Hamilton (2003) that involves four lags and the nominal producer price index (PPI). The corresponding sensitivity analysis for the impulse-response-based test is shown in Table 7. The test results are robust across all specifications and substantively identical to our results for the baseline model. In no case do we reject the null of symmetric slopes at horizons up to 1 year, in contrast to the results from the slope-based test.¹⁹

TABLE 7. p -values of tests of the null of symmetric response functions: Real GDP responses to oil price innovations at horizons up to 1 year.^a

			Three-Year Net Oil Price Increase Impulse-Response-Based Test	
			1 Std. Dev. Shock	2 Std. Dev. Shock
$p = 4$	PPI	Real	1.000	0.346
		Nominal	1.000	0.451
	Import RAC	Real	1.000	0.430
		Nominal	1.000	0.510
$p = 6$	PPI	Real	1.000	0.322
		Nominal	1.000	0.428
	Import RAC	Real	1.000	0.384
		Nominal	1.000	0.577

^aBased on model (14). Boldface indicates statistical significance at the 5% level.

¹⁹Hamilton (forthcoming), in a comment on this paper, recently questioned the robustness of our findings. The premise of his analysis was the assertion that our modified slope-based test fails to reject the null of symmetric slopes at conventional significance levels. As shown in Table 6, this is not the case and our

11. CONCLUSIONS

Our empirical results have important implications for studies of the transmission of energy price shocks. First, one reason that researchers were eager to accept the apparent finding of asymmetry in the 1990s was that it seemed consistent with theoretical models of the transmission of energy price shocks that emphasized asymmetries arising from shifts in uncertainty or frictions impeding the reallocation of factors of production within and across sectors (see, e.g., Bernanke (1983), Hamilton (1988), Pindyck (1991)). The latter models are required to rationalize large effects from oil price shocks that are difficult to obtain in conventional models based on cost shocks or aggregate demand shocks. Our evidence provides no support for theoretical models with built-in asymmetries. If such asymmetric effects exist, they appear to be too weak to be detected in aggregate data.

Second, in the absence of asymmetries, the responses of the U.S. economy to positive energy price shocks appear much more modest, which is fully consistent with conventional macroeconomic models of the transmission of energy price shocks that do not predict large fluctuations in U.S. output in response to energy price shocks (see, e.g., Kilian (2008b)). Thus, the absence of larger effects is not a puzzle. Our results suggest that oil price shocks are only one of many factors that contribute to recessions, not the key factor. Furthermore, the response to unexpected energy price declines is not negligible, a finding that matters especially for the interpretation of the data since mid-2008. Our findings also lend credence to recent linear models of how the oil demand and oil supply shocks that drive oil price shocks affect the U.S. economy (see, e.g., Kilian (2009)), and support the common practice of linearizing dynamic stochastic general equilibrium (DSGE) models with an oil sector about their steady state.

Finally, our analysis calls into question several empirical findings reported in the literature about the channels of the transmission of energy price shocks. To the extent that these studies used censored VAR models and/or computed impulse responses to energy price shocks incorrectly, they are invalid. For example, much of the consensus on how monetary policy responds to oil price shocks is based on the censored VAR model introduced by Bernanke, Gertler, and Watson (1997). That study and subsequent papers using the same type of model will have to be reexamined in light of our findings. For example, Kilian and Lewis (forthcoming) show that without censoring, there is little evidence that the Federal Reserve caused large effects on real output or inflation by responding to oil price shocks. Similarly, influential studies of sectoral responses to oil price shocks such as Lee and Ni (2002) or employment responses at the plant level as in Davis and Haltiwanger (2001) will have to be reexamined.

APPENDIX

This appendix demonstrates that as δ increases, the importance of accounting for the history Ω^i and of accounting for the variability of ε_{1t} declines such that

$$\lim_{n \rightarrow \infty} \frac{1}{n} (I_y(0, n\delta)) = I_y^*(0, \delta, \underline{Q}).$$

results are robust. For further discussion of this point and additional sensitivity analysis, see our reply to Hamilton in Kilian and Vigfusson (forthcoming).

To keep the analysis tractable, we focus on the impact responses.

Impact response of x_t

Because of the linear nature of the first equation in model (14), the impact effect on x_t of a shock of size δ is a constant

$$I_x(0, \delta, \Omega^i) = \delta. \quad (18)$$

Impact response of y_t

The impact effect of ε_{1t} on y_t for a given history Ω^i is

$$I_y(0, \delta, \Omega^i) = b_{21,0}\delta + g_{21,0}(E(x_t^+|\delta, \Omega^i) - E(x_t^+|\Omega^i)). \quad (19)$$

The term $E(x_t^+|\delta, \Omega^i) - E(x_t^+|\Omega^i)$ plays a central role in the construction of nonlinear impulse responses for y_{t+h} , $h = 1, \dots, H$. Absent uncertainty about the value of $\varepsilon_{1,t}$, the value of $E(x_t^+|\delta, \Omega^i) - E(x_t^+|\Omega^i)$ is easy to calculate. In particular, consider the value of $E(x_t^+|\delta, \Omega^i) - E(x_t^+|\varepsilon_{1,t} = 0, \Omega^i)$, where, for ease of notation, we define

$$\tilde{x}_t \equiv E(x_t|\varepsilon_{1,t} = 0, \Omega^i). \quad (20)$$

Here \tilde{x}_t summarizes the effect of the history Ω^i on x_t . For $\delta > 0$, then

$$E(x_t^+|\delta, \Omega^i) - E(x_t^+|\varepsilon_{1,t} = 0, \Omega^i) = \begin{cases} \delta, & \text{if } \tilde{x} > 0, \\ \tilde{x} + \delta, & \text{if } -\delta < \tilde{x} < 0, \\ 0, & \text{if } \tilde{x} < -\delta. \end{cases} \quad (21)$$

Computing the value of $E(x_t^+|\delta, \Omega^i) - E(x_t^+|\Omega^i)$ is more of a challenge because we need to account for uncertainty about $\varepsilon_{1,t}$. With uncertainty, we have that

$$\begin{aligned} E(x_t^+|\delta, \Omega^i) &= E(\max(\tilde{x}_t + \delta, 0)|\delta, \Omega^i) = \max(\tilde{x}_t + \delta, 0), \\ E(x_t^+|\Omega^i) &= E(\max(\tilde{x}_t + \varepsilon_{1,t}, 0)|\Omega^i). \end{aligned} \quad (22)$$

The value of $E(x_t^+|\Omega^i)$ depends on the variance of the shocks. Note that $E(x_t^+|\Omega^i)$ can be positive even if \tilde{x}_t is negative. In fact, by Jensen's inequality, $E(x_t^+|\Omega^i) \geq \tilde{x}_t$ for all values of \tilde{x}_t . In particular, if \tilde{x}_t equals zero and $\varepsilon_{1,t}$ has a standard normal distribution, then $E(x_t^+|\Omega^i)$ has a value of 0.4. Hence, when $\tilde{x}_t = 0$ and $\varepsilon_{1,t}$ has a standard normal distribution, we have that

$$E(x_t^+|\delta, \Omega^i) - E(x_t^+|\Omega^i) = E(x_t^+|\delta, \tilde{x}_t = 0) - E(x_t^+|\tilde{x}_t = 0) = \delta - 0.4. \quad (23)$$

This first result implies that the larger is δ , the smaller is the effect of incorrectly treating ε_{1t} as equal to zero under the counterfactual path, relative to magnitude of the impulse response. In other words, all else equal, the larger is δ , the more similar are the traditional incorrectly computed impulse response and the correctly computed uncon-

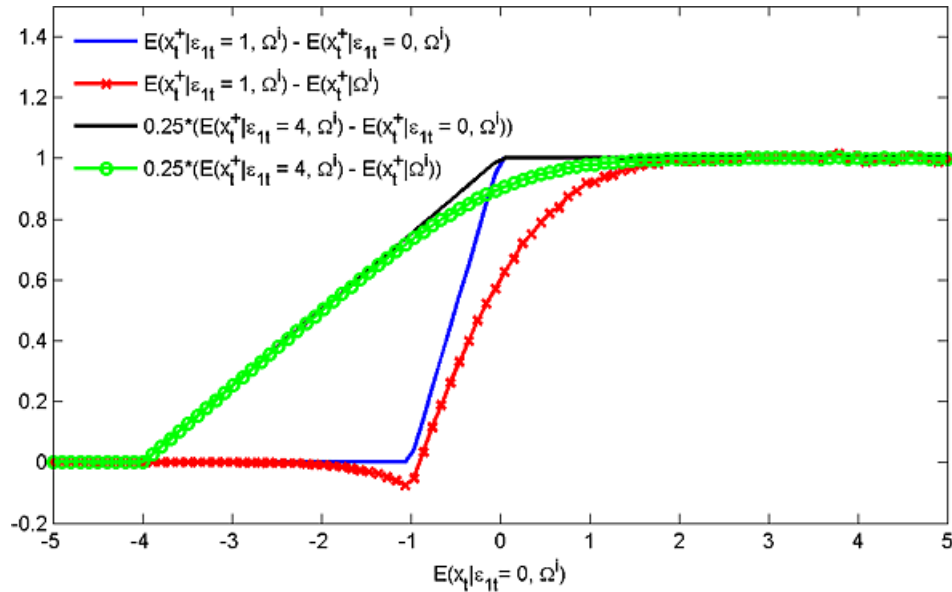


FIGURE A.1A. $E(x_t^+ | \delta, \Omega^i) - E(x_t^+ | \Omega^i)$.

ditional response. This point is important because most energy price innovations measured at the monthly or quarterly frequency tend to be quite small (e.g., Edelstein and Kilian (2009)). We conclude that traditional, incorrectly computed impulse responses tend to exaggerate the effect of an unanticipated energy price increase. Figure A.1a illustrates this point under the assumption that

$$\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \right).$$

The horizontal axis shows alternative representations of \tilde{x}_t representing alternative histories.

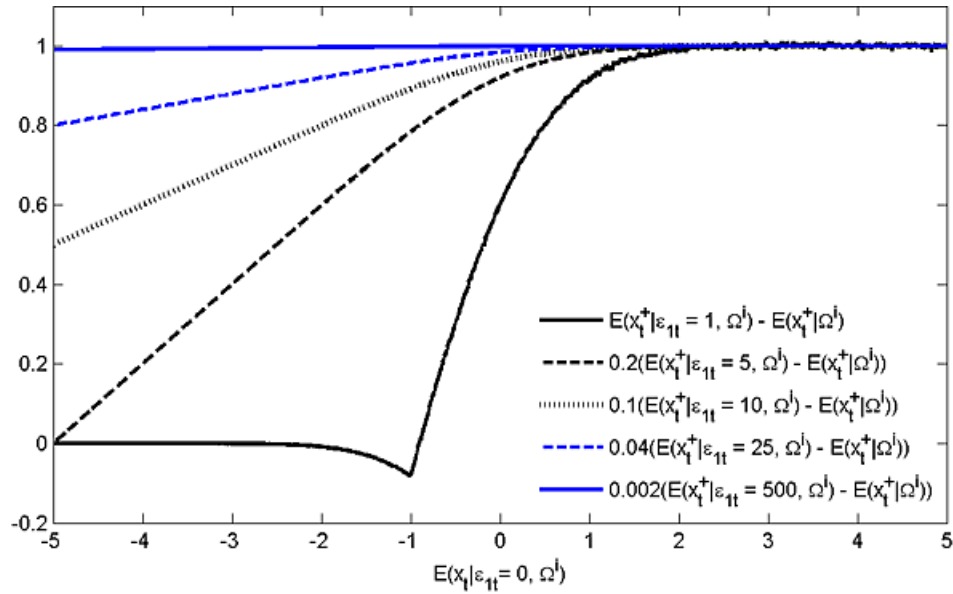
Note that the difference between the response computed by correctly accounting for the uncertainty of $\varepsilon_{1,t}$ and the incorrectly computed response obtained from treating $\varepsilon_{1,t}$ as fixed declines, as δ increases, for all possible histories \tilde{x}_t .

Our second result is that $E(x_t^+ | \delta, \Omega^i) - E(x_t^+ | \Omega^i)$ becomes less sensitive to \tilde{x}_t , as δ increases. Figure A.1b illustrates this relationship. As δ increases, the importance of the history \tilde{x}_t declines and the magnitude of the impulse responses becomes constant across alternative histories. The reduced importance of the histories can be explained by the limit argument

$$\lim_{n \rightarrow \infty} \frac{1}{n} (I_y(0, n\delta)) = I_y^*(0, \delta, \underline{0}).$$

This result relies on three observations. First, note that $I_y^*(0, \delta, \underline{0}) = b_{21,0}\delta + g_{21,0}\delta$. Second,

$$\lim_{n \rightarrow \infty} \frac{1}{n} (I_y(0, n\delta)) = \lim_{n \rightarrow \infty} \frac{1}{n} \left(\int I_y(0, n\delta, \Omega^i) d\Omega^i \right).$$

FIGURE A.1B. $E(x_t^+ | \delta, \Omega^i) - E(x_t^+ | \Omega^i)$.

In addition, observe that

$$\begin{aligned} & \lim_{n \rightarrow \infty} \frac{1}{n} \left(\int I_y(0, n\delta, \Omega^i) d\Omega^i \right) \\ &= b_{21,0}\delta + g_{21,0} \left(\frac{1}{n} \int (E(x_t^+ | n\delta, \Omega^i) - E(x_t^+ | \Omega^i)) d\Omega^i \right). \end{aligned}$$

To complete the proof we need to show that

$$\lim_{n \rightarrow \infty} \frac{1}{n} \int (E(x_t^+ | n\delta, \Omega^i) - E(x_t^+ | \Omega^i)) d\Omega^i = \delta.$$

Recall that, by definition, $\tilde{x}_t \equiv E(x_t^+ | \varepsilon_{1,t} = 0, \Omega^i)$. Therefore, $E(x_t^+ | n\delta, \Omega^i) = n\delta + \tilde{x}_t$ if $\tilde{x}_t > -n\delta$ and

$$\begin{aligned} & \frac{1}{n} \left(\int E(x_t^+ | n\delta, \Omega^i) - E(x_t^+ | \varepsilon_{1,t}, \Omega^i) d\Omega^i \right) \\ &= \left(\delta P(\tilde{x}_t > -n\delta) + \frac{1}{n} \int_{-n\delta}^{\infty} \tilde{x}_t d\Omega^i - \frac{1}{n} \int_{-\infty}^{\infty} E(x_t^+ | \Omega^i) d\Omega^i \right). \end{aligned}$$

As long as \tilde{x}_t does not have too much mass in the left tail and the variance of $\varepsilon_{1,t}$ is small enough such that

$$\int_{-n\delta}^{\infty} \tilde{x}_t d\Omega^i - \int_{-\infty}^{\infty} E(x_t^+ | \Omega^i) d\Omega^i$$

remains finite, it follows that

$$\lim_{n \rightarrow \infty} \frac{1}{n} \int (E(x_t^+ | n\delta, \Omega^i) - E(x_t^+ | \Omega^i)) d\Omega^i = \delta,$$

because the value of $P(\tilde{x}_t > -n\delta)$ converges toward 1, as n increases, and $E(\tilde{x}_t | \tilde{x}_t < 0)$ is finite.

We conclude that for sufficiently large energy price shocks, conventional impulse response estimates are expected to become a good approximation to the correctly constructed estimate. For energy price shocks of more typical magnitude, however, such as a 1-standard deviation shock to $\varepsilon_{1,t}$, the interaction of the innovation with the history Ω^i is quantitatively important.

REFERENCES

- Balke, N. S., S. P. A. Brown, and M. K. Yücel (2002), "Oil price shocks and the U.S. economy: Where does the asymmetry originate?" *Energy Journal*, 23, 27–52. [421, 423, 424, 439, 441, 444]
- Bernanke, B. S. (1983), "Irreversibility, uncertainty, and cyclical investment." *Quarterly Journal of Economics*, 98, 85–106. [444, 447]
- Bernanke, B. S., M. Gertler, and M. Watson (1997), "Systematic monetary policy and the effects of oil price shocks." *Brookings Papers on Economic Activity*, 1, 91–142. [420, 421, 427, 439, 447]
- Bernanke, B. S., M. Gertler, and M. Watson (2004), "Oil shocks and aggregate macroeconomic behavior: The role of monetary policy: A reply." *Journal of Money, Credit, and Banking*, 36, 287–291. [421]
- Davis, S. J. and J. Haltiwanger (2001), "Sectoral job creation and destruction responses to oil price changes." *Journal of Monetary Economics*, 48, 465–512. [421, 439, 443, 447]
- Edelstein, P. and L. Kilian (2007), "The response of business fixed investment to energy price changes: A test of some hypotheses about the transmission of energy price shocks." *B.E. Journal of Macroeconomics*, 7 (1) (Contributions), article 35. [421, 427, 433, 436, 438]
- Edelstein, P. and L. Kilian (2009), "How sensitive are consumer expenditures to retail energy prices." *Journal of Monetary Economics*, 56, 766–779. [421, 433, 436, 438, 440, 449]
- Ferderer, J. P. (1996), "Oil price volatility and the macroeconomy." *Journal of Macroeconomics*, 18, 1–26. [421]
- Gallant, A. R., P. Rossi, and G. Tauchen (1993), "Nonlinear dynamic structures." *Econometrica*, 61, 871–907. [428, 434]
- Hamilton, J. D. (1983), "Oil and the macroeconomy since World War II." *Journal of Political Economy*, 91, 228–248. [420]
- Hamilton, J. D. (1988), "A neoclassical model of unemployment and the business cycle." *Journal of Political Economy*, 96, 593–617. [444, 447]

- Hamilton, J. D. (1996), "This is what happened to the oil price-macro-economy relationship." *Journal of Monetary Economics*, 38, 215–220. [421, 423, 439, 440]
- Hamilton, J. D. (2003), "What is an oil shock?" *Journal of Econometrics*, 113, 363–398. [421, 423, 424, 439, 440, 445, 446]
- Hamilton, J. D. (2009), "Causes and consequences of the oil shock of 2007-08." *Brookings Papers on Economic Activity*, 1 (Spring), 215–261. [421]
- Hamilton, J. D. (forthcoming), "Nonlinearities and the macroeconomic effects of oil prices." *Macroeconomic Dynamics*. [424, 446]
- Hamilton, J. D. and A. M. Herrera (2004), "Oil shocks and aggregate macroeconomic behavior: The role of monetary policy." *Journal of Money, Credit, and Banking*, 36, 265–286. [421, 439]
- Herrera, A. M. (2008), "Oil price shocks, inventories, and macroeconomic dynamics." Mimeo, Department of Economics, Michigan State University. [421, 424]
- Herrera, A. M., L. G. Lagalo, and T. Wada (forthcoming), "Oil price shocks and industrial production: Is the relationship linear?" *Macroeconomic Dynamics*. [443]
- Herrera, A. M. and E. Pesavento (2009), "Oil price shocks, systematic monetary policy and the 'great moderation'." *Macroeconomic Dynamics*, 13, 107–137. [421]
- Hooker, M. A. (1996a), "What happened to the oil price-macro-economy relationship?" *Journal of Monetary Economics*, 38, 195–213. [421]
- Hooker, M. A. (1996b), "This is what happened to the oil price-macro-economy relationship: Reply." *Journal of Monetary Economics*, 38, 221–222. [421]
- Hooker, M. A. (2002), "Are oil shocks inflationary? Asymmetric and nonlinear specifications versus changes in regime." *Journal of Money, Credit, and Banking*, 34, 540–561. [421]
- Huntington, H. G. (1998), "Crude oil prices and U.S. economic performance: Where does the asymmetry reside?" *Energy Journal*, 19, 107–148. [421]
- Jones, D. W., P. N. Leiby, and I. K. Paik (2004), "Oil price shocks and the macroeconomy: What has been learned since 1996?" *Energy Journal*, 25, 1–32. [421]
- Kilian, L. (2008a), "The economic effects of energy price shocks." *Journal of Economic Literature*, 46, 871–909. [427, 428, 433]
- Kilian, L. (2008b), "Exogenous oil supply shocks: How big are they and how much do they matter for the U.S. economy?" *Review of Economics and Statistics*, 90, 216–240. [440, 447]
- Kilian, L. (2009), "Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market." *American Economic Review*, 99, 1053–1069. [420, 428, 433, 447]
- Kilian, L. and L. T. Lewis (forthcoming), "Does the Fed respond to oil price shocks?" *Economic Journal*. [444, 447]

Kilian, L. and C. Vega (2011), “Do energy prices respond to U.S. macroeconomic news? A test of the hypothesis of predetermined energy prices.” *Review of Economics and Statistics*, 93, 660–671. [428, 445]

Kilian, L. and R. Vigfusson (2009), “Pitfalls in estimating asymmetric effects of energy price shocks.” Working Paper 7284, CEPR. [436, 437]

Kilian, L. and R. Vigfusson (forthcoming), “Nonlinearities in the oil price-output relationship.” *Macroeconomic Dynamics*. [445, 447]

Koop, G., M. H. Pesaran, and S. M. Potter (1996), “Impulse response analysis in nonlinear multivariate models.” *Journal of Econometrics*, 74, 119–147. [422, 428, 434]

Leduc, S. and K. Sill (2004), “A quantitative analysis of oil price shocks, systematic monetary policy and downturns.” *Journal of Monetary Economics*, 51, 781–808. [427]

Lee, K. and S. Ni (2002), “On the dynamic effects of oil price shocks: A study using industry level data.” *Journal of Monetary Economics*, 49, 823–852. [421, 439, 447]

Mork, K. A. (1989), “Oil and the macroeconomy. When prices go up and down: An extension of Hamilton’s results.” *Journal of Political Economy*, 97, 740–744. [421, 423, 427, 436, 438, 439, 441]

Mork, K. A., O. Olsen, and H. T. Mysen (1994), “Macroeconomic responses to oil price increases and decreases in seven OECD countries.” *Energy Journal*, 15, 19–35. [421]

Pindyck, R. S. (1991), “Irreversibility, uncertainty and investment.” *Journal of Economic Literature*, 29, 1110–1148. [444, 447]

Ramey, V. A. and D. J. Vine (2010), “Oil, automobiles, and the U.S. economy: How much have things really changed?” *NBER Macroeconomics Annual*, 25, 333–368. [421]

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