Supplementary Material

Supplement to "The effect of public pensions on women's labor market participation over a full life cycle"

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Virginia Sánchez-Marcos
Departamento de Economía, Universidad de Cantabria

Carlos Bethencourt
Departamento de Economía, Facultad de Económicas, Universidad de La Laguna

Appendix A

In this Appendix, we describe the solution method for the model economy used in our analysis. Households have a finite horizon, so the model is solved numerically by backward recursion from the terminal period. At each age, we solve the value function and optimal policy rule, given the current state variables and the solution to the value function in the next period. This approach is standard. The complication in our model arises from the combination of a discrete choice (whether or not to participate) and a continuous choice (on savings). This combination means that the value function is not necessarily concave. In addition to age, there are four state variables in this problem: asset stock (a_t), the permanent component of earnings of the husband, the permanent component of earnings of the wife (v_t), and the experience level of the wife (x_t). However, after the claiming age (and before the age at which retirement is compulsory) three additional state variables are needed in order to solve the household's problem: female public pension benefit (b_t^f), the number of periods for which the female retired worker's pension is withheld, and the number of periods for which spousal benefit is withheld as a result of the Earnings Test (s_t). Due to computational restrictions, it is not feasible to keep track of all these state variables so we adopt the simplifying assumption that, after the exogenously given claiming age of 62, there is no labor market uncertainty. Under this assumption, the number of state variables needed is reduced since the permanent component of female earnings at the claiming age and the other state variables (in particular x_t and s_t) can be used both to calculate earnings and to calculate the pension benefit in each period after 62.

We discretize income variables and the experience level, leaving the asset stock as the only continuous state variable. Since both permanent components of earnings are nonstationary, we can approximate it by a stationary, discrete process only because of the finite horizon of the process. We select the nodes to match the paths of the mean shock and the unconditional variance over the life cycle. In particular, the unconditional

Virginia Sánchez-Marcos: sanchezv@unican.es
Carlos Bethencourt: cbethenc@ull.es

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variance of the permanent component must increase linearly with age, with the slope given by the conditional variance of the permanent shock.

Value functions are increasing in assets (a_t) but they are not necessarily concave, even if they are made conditional on labor market status in t. The nonconcavity arises because of changes in labor market status in future periods: the slope of the value function is given by the marginal utility of consumption, but this is not monotonic in asset stock because consumption can decline as assets increase and expected labor market status in future periods changes. By contrast, in Danforth (1979) employment is an absorbing state, so the conditional value function is concave. Under certainty, the number of kinks in the conditional value function is given by the number of periods of life remaining. If there is enough uncertainty, then changes in work status in the future will be smoothed out leaving the expected value function concave: whether or not an individual will work in $t+1$ at a given a_t depends on the realization of shocks in $t+1$. Using uncertainty to avoid nonconcavities is analogous to the use of lotteries elsewhere in the literature. This problem is also discussed in Attanasio, Low and Sánchez-Marcos (2008), Low, Meghir and Pistaferri (2010), and Low and Pistaferri (2015), among others. The choice of participation status in t is determined by the maximum of the conditional value functions in t. In solving the maximization problem at a given point in the state space, we use a simple golden search method.

Appendix B

This Appendix provides a detailed description of the approximation used for each individual AIME. As we explained in Section 2.3, we approximate the AIME as a function of earnings in the last working period and the number of years of contribution to the pension system. More specifically, for each period we calculate fictitious earnings based on the stochastic component of earnings at the claiming age (which we denote here by t^m_{cl} and t^f_{cl} for men and women, resp.).[1] There is however a drawback of relying on the last working period stochastic component of the earnings because the nature of the stochastic process that we assume means that the variance in earnings is increasing over the life cycle. In order to deal with this, we proceed in slightly different ways approximating the AIME for men and women. In the case of men, we make the $\widehat{\text{AIME}}^m$ a function of the average of the fictitious earnings over the last N working periods

$$\ln \widehat{\text{AIME}}^m = \gamma^m_1 + \gamma^m_2 \ln \sum_{k=1}^{N} \frac{\exp\bigl(\ln y^m_0 + \alpha^m_1 (t^m_{\text{cl}} - k) + \alpha^m_2 (t^m_{\text{cl}} - k)^2 + v^m_{t^m_{\text{cl}}-1}\bigr)}{N} \tag{S.1}$$

with $v^m_{t^m_{\text{cl}}-1}$ being the stochastic component of earnings in the last working period. The parameters γ^m_1 and γ^m_2 are the estimated coefficients of a linear regression of $\ln \text{AIME}^m$ (true AIME) on the average of the last N working periods fictitious earnings in the simulated data.

In the case of women, who may have a number of periods of contribution of $x_{t^f_{\text{cl}}} < N$, we use a different formula to prevent the effect of periods with zero earnings on the ap-

[1]This is similar to the approach in Low and Pistaferri (2015).

proximated AIME being smoothed out (as would happen if we used the formula above). In fact, it is very important for our analysis to capture incentives to work through the AIME. Therefore, we calculate the $\widehat{\text{AIME}}^f$ as follows:

$$\ln \widehat{\text{AIME}}^f = \frac{\sum_{k=1}^{\min(N, x_{t_{\text{cl}}^f})} \exp\big(\ln y_0^f + \alpha_1^f (x_{t_{\text{cl}}^f} - k) + \alpha_2^f (x_{t_{\text{cl}}^f} - k)^2 + \exp\big(\gamma_1^f + \gamma_2^f \ln v_{t_{\text{cl}}^f - 1}^f\big)\big)}{N}, \tag{S.2}$$

where in this case γ_1^f and γ_2^f are the estimated coefficients of a linear regression of the log of the average of the stochastic component of earnings over the working career on the log of its value in the last working period $v_{t_{\text{cl}}^f - 1}^f$ in the simulated data.

The parameter values that we estimate for the approximation are $\gamma_1^m = 2.47$, $\gamma_2^m = 0.78$, $\gamma_1^f = 0.93$, and $\gamma_2^f = 0.67$. Note that we need to solve the model and iterate in these parameters so that individual decisions are based on the formula that uses parameter values that are consistent with the simulated data.

To assess the accuracy of our approximations in Table S.1, we compare the distribution of the true AIME^g and the distribution of $\widehat{\text{AIME}}^g$, with $g = \{f, m\}$, in the simulated data. We believe that the approximation is satisfactory.

TABLE S.1. Accuracy of AIME approximation.

	True	Approximation
Men's percentiles:		
1%	16,122	16,775
5%	20,089	20,589
10%	23,729	24,929
25%	32,583	35,773
50%	48,089	44,510
75%	70,618	64,866
90%	96,950	93,083
95%	116,558	112,702
100%	146,817	137,514
Women's percentiles:		
1%	4996	4714
5%	7736	7411
10%	9942	9559
25%	15,244	15,680
50%	23,530	23,369
75%	35,778	35,281
90%	48,916	45,687
95%	57,935	55,162
100%	70,957	68,975

Table S.2. Change in employment rate with respect to the benchmark (percentage points). Claiming age at 66.

	Reform 1	Reform 2	Reform 3
25–29	1.28	3.36	0.62
30–34	2.38	5.79	0.07
35–39	3.35	7.32	−0.08
40–44	4.83	10.68	0.17
45–49	7.26	14.39	0.28
50–54	7.76	14.96	0.15
55–59	5.43	13.56	0.45
60–65	0.62	10.60	0.88

Note: Reform 1: removing spousal benefit, reform 2: removing both spousal and survivor pension benefits, and reform 3: increasing the number of periods used in calculating the AIME from 35 to 40.

Table S.3. Change in employment rate with respect to the benchmark (percentage points). Cohort 1964–1968.

	Reform 1	Reform 2	Reform 3
25–29	1.98	4.65	0.80
30–34	2.42	5.28	0.08
35–39	3.12	6.63	0.00
40–44	3.73	8.78	0.24
45–49	5.11	10.52	0.17
50–54	5.35	11.88	0.12
55–59	4.08	10.97	0.59
60–65	0.53	9.90	0.42

Note: Reform 1: removing spousal benefit, reform 2: removing both spousal and survivor pension benefits, and reform 3: increasing the number of periods used in calculating the AIME from 35 to 40.

Appendix C

In this Appendix, we report the effect of the policy reforms in two alternative scenarios to our benchmark economy that we discussed in Section 4.4. First, in Table S.2 we show the changes in employment rates by age under the assumption that claiming age is 66 instead of 62. Second, in Table S.3, we present what the variation in employment rates would be for a younger cohort of women with higher attachment to the labor market.

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