Supplementary Material

Supplement to "Exiting from quantitative easing"

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Appendix B: Data description

This Appendix describes how the variables used in the paper are derived from various data sources.

B.1 *Monthly and 12-month inflation rates (p and π)*

The monthly series on the monthly inflation rate (appearing in the inflation and output reduced-form) and the 12-month inflation rate (in the Taylor rule and the excess reserve supply equation) are constructed from the CPI (consumer price index). The Japanese CPI is compiled by the Ministry of Internal Affairs and Communications of the Japanese government. The overall CPI and its various subindexes can be downloaded from the portal site of official statistics of Japan called"e-Stat". The URL for the CPI is http://www.e-stat.go.jp/SG1/estat/List.do?bid=000001033702&cycode=0. This page lists a number of links to CSV files. One of them, http://www.e-stat.go.jp/SG1/estat/Csvdl.do?sinfid=000011288575 has the "core" CPI (CPI excluding fresh food), the "core-core" CPI (CPI excluding food and energy), and other components from January 1970. They are seasonally *un*adjusted series and combine different base years from January 1970. For how the Ministry combines different base years, see Section III-6 of the document (in Japanese) downloadable from http://www.stat.go.jp/data/cpi/2010/kaisetsu/index.htm#p3. Briefly, to combine base years of 2005 and 2010, say, the Ministry multiplies one of the series by a factor called the "link factor" whose value is such that the two series agree on the average of monthly values for the year 2005.

If the 12-month inflation rates constructed from the (seasonally unadjusted) "core" CPI and the "core-core" CPI are plotted, one sees the two humps for 1989 and 1997. They are due to the increases in the consumption tax. The two inflation rates behave similarly, except for the period November 2007–May 2009.

The above URL has another CSV file, whose link is http://www.e-stat.go.jp/SG1/estat/Csvdl.do?sinfid=000011288581, has *seasonally adjusted* series for various

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subindexes (including the "core-core" CPI), but only from January 2005. As explained below, we use the "core-core" CPI between November 2007 and May 2009 that is seasonally adjusted, along with the seasonally unadjusted "core" CPI, in order to construct p (monthly inflation) and π (12-month inflation). The construction involves three steps.

Adjustment for consumption tax hikes. The consumption tax rate rose from 0% to 3% in April 1989 and to 5% in April 1997. We compute the 12-month inflation rate from the seasonally unadjusted index (as the log difference between the current value of the index and the value 12 months ago) and subtract 1.2% for $t =$ April 1989,..., March 1990 (to remove the effect of the April 1989 tax hike) and 1.5% for $t =$ April 1997,..., March 1998 (to remove the effect of the April 1997 tax hike). These two numbers (1.2% and 1.5%) are taken from *Price Report* (various years) by the Economic Planning Agency of the Japanese government (which became a part of the Cabinet Office). We then calculate the index so that its implied 12-month inflation agrees with the tax-adjusted 12-month inflation.

Seasonal adjustment. We apply the U.S. Census X12-ARIMA method to the seasonally unadjusted (but consumption tax-adjusted) "core" index from January 1987 through December 2012 (26 years). The Census's program can be downloaded from: https://www.census.gov/srd/www/winx12/winx12_down.html. The specification for the seasonal adjustment is the same as the one used by the Ministry (of Internal Affairs and Communications of the Japanese government) for seasonally adjust various CPI subindexes mentioned above. Their spec file for the Census X12-ARIMA program is available from http://www.stat.go.jp/data/cpi/2010/kaisetsu/pdf/3-7.pdf. For example, the ARIMA order is (0, 1, 1). There is no adjustment for the holiday effect.

Adjustment for the 2007–2008 Energy Price Swing. Let CPI_{1t} be the seasonally adjusted "core" CPI obtained from this operation for $t =$ January 1970,..., December 2012. Let CPI_{2t} be the seasonally adjusted "core-core" CPI for $t =$ January 2005,..., December 2012 that is directly available from the above CSV file. Our CPI measure (call it CPI) is CPI_1, except that we switch from CPI_1 to CPI_2 between November 2007 and May 2009 to remove the large movement in the energy component of the "core" CPI. More precisely,

$$CPI_t = \begin{cases} CPI_{1t} & \text{for } t = \text{January 1970}, \dots, \text{October 2007}, \\ CPI_{t-1} \times \dfrac{CPI_{2t}}{CPI_{2,t-1}} & \text{for } t = \text{November 2007}, \dots, \text{May 2009}, \\ CPI_{t-1} \times \dfrac{CPI_{1t}}{CPI_{1,t-1}} & \text{for } t = \text{June 2009}, \dots, \text{December 2012}. \end{cases} \tag{B.1}$$

That is, the "core" CPI (the CPI excluding fresh food) monthly inflation rate is set equal to that given by the "core-core" CPI (the CPI excluding food and energy) for those months. This is the only period during which the two CPI measures give substantially different inflation rates. It appears that this deviation were discounted as temporary by the BOJ. The monetary policy announcement of August 19, 2008 (http://www.boj.or.jp/en/announcements/release_2008/k080819.pdf), which stated that the policy rate would remain at around 50 basis points, has the following passage: "The CPI inflation rate (excluding fresh food) is currently around 2 percent, highest since the first half of 1990s, due to increased prices of petroleum products and food."

Finally, the monthly inflation rate for month t, p_t, is calculated as

$$p_t \equiv 1200 \times \left[\log(CPI_t) - \log(CPI_{t-1})\right]. \tag{B.2}$$

The 12-month inflation rate for month t, π_t, is

$$\pi_t \equiv 100 \times \left[\log(CPI_t) - \log(CPI_{t-12})\right]. \tag{B.3}$$

B.2 *Excess reserve rate (m)*

Monthly series on actual and required reserves are available from September 1959. The source is the BOJ's portal site http://www.stat-search.boj.or.jp/index_en.html. The value for month t is defined as the average of daily balances over the reserve maintenance period of the 16th day of month t to the 15th day of month $t+1$. We define the excess reserve rate for month t (m_t) as

$$\begin{aligned} m_t \equiv \big[&\log(\text{actual reserve balance for month } t) \\ &- \log(\text{required reserve balance for month } t)\big]. \end{aligned} \tag{B.4}$$

We make three changes on the series. First, as was argued in Section 3, observed reserves after the second ELB (effective lower bound) spell (which ends June 2006) and before the Lehman crisis of September 2008 do not seem to represent demand. For this reason, we set $m_t = 0$ for $t =$ July 2006,..., August 2008. Second, there is a Y2K spike in m for $t =$ December 1999 (which is for the reserve maintenance period of December 16, 1999, through January 15, 2000). We remove this spike by the average of m over the first ELB spell (March 1999–July 2000) excluding December 1999. Third, m is substantially above zero for February 1991–April 1991 ($m = 0.01932, 0.05227, 0.01076$). This would be due to some technical factors having to do with the remittance by the Japanese government to the U.S. meant to cover a share of the cost of the Gulf war. m for those 3 months is set to the average over the 8 months surrounding the 3 months (October 1990–January 1991 and May 1991–August 1991). The average is 0.00070.

B.3 *Interest rate paid on reserves ($\bar{r}$)*

$\bar{r}_t$ is 0% until October 2008 and 0.1% since November 2008.

B.4 *The policy rate (r)*

We obtained daily data on the uncollateralized overnight "Call" rate (the Japanese equivalent of the U.S. Federal Funds rate) since the inception of the market (which is July 1985) from *Nikkei* (a data vendor maintained by a subsidiary of *Nihon Keizai Shinbun* (the Japan Economic Daily)). The policy rate for month t, r_t, for $t =$ August 1985,...,December 2012 is the average of the daily values over the reserve maintenance period of the 16th of month t to the 15th of month $t+1$.

In Section 3 of the text, we defined the ELB spell as months for which the net policy rate $r_t - \bar{r}_t$ is less than 5 basis points. We ignore variations within the 5 basis points by setting $r_t - \bar{r}_t = 0$ for the ELB spells.

B.5 *Monthly output gap (x)*

The three series. Three quarterly series go into our monthly output gap construction: (i) quarterly seasonally adjusted real GDP (from the National Income Accounts (NIA), compiled by the Cabinet Office of the Japanese government), (ii) the monthly "all-industry activity index" (compiled by the Ministry of Economy, Trade, and Industry of the Japanese government (METI) available from January 1988), and (iii) the quarterly GDP gap estimate by the Cabinet Office of the Japanese government. We first provide a description of those series along with their sources.

(i) *Quarterly NIA GDP*

Japanese NIA in general. The Japanese national accounts adopted the chain-linking method in 2004. Quarterly chain-linked real GDP series (seasonally-adjusted) are available from the Cabinet Office. The relevant homepage is http://www.esri.cao.go.jp/en/sna/sokuhou/sokuhou_top.html.

Quarterly GDP from 1994:Q1 ($GDP1$). The current quarterly estimates are continuously revised by the Cabinet Office. We used the "Quarterly Estimates of GDP January–March 2014 (The Second Preliminary)(Benchmark year = 2005)", released on June 9, 2014, and available from the above homepage. The CSV file holding this series is: https://www.esri.cao.go.jp/jp/sna/content/20140604_gaku-jk1412.csv. The latest quarter is 2014:Q1 (the first quarter of 2014). For later reference, call this series "$GDP1$." The series goes back only to 1994:Q1.

Quarterly GDP from 1980:Q1 ($GDP2$). Recently, the Cabinet Office released the chain-linked GDP series (for the same benchmark year of 2005) since 1980. The homepage from which this series can be downloaded is http://www.esri.cao.go.jp/jp/sna/sonota/kan-i/kan-i_top.html, which unfortunately is in Japanese. The URL for the Excel file holding this series is http://www.esri.cao.go.jp/jp/sna/data/data_list/kan-i/files/pdf/gaku-jk_kan-i.xls. The URL for the documentation (in Japanese) is http://www.esri.cao.go.jp/jp/sna/data/data_list/kan-i/files/pdf/gaiyou.pdf. This series, call it "$GDP2$", is from 1980:Q1 to the 1995:Q1.

Linking $GDP1$ and $GDP2$. Because the seasonal adjustment underlying the continuously revised current GDP series, whose first quarter is 1994:Q1, is retroactive and alters the whole series at each release, there is a slight difference between $GDP1_t$ (at 447,159.1 trillion yen) and $GDP2_t$ (at 447,168.3 trillion yen) for t = first quarter of 1994. We link the two series at 1994:Q1 as follows:

$$GDP_t = \begin{cases} GDP2_t \times \lambda & \text{for } t = \text{1980:Q1–1993:Q4}, \\ GDP1_t & \text{for } t = \text{1994:Q1–2014:Q1}, \end{cases} \tag{B.5}$$

where λ is the ratio of $GDP1_t$ for $t =$ 1994:Q1 to $GDP2_t$ for $t =$ 1994:Q1.

(ii) *METI's monthly all-industry activity index*. This index is a Laspeyres index combining four subindexes: a construction industry index, the IP (the Index of Industrial Production), a services industry index, and a government services index. It therefore excludes agriculture. The latest base year is 2010. The seasonally adjusted series, along with a very brief documentation, can be downloaded from http://www.meti.go.jp/statistics/tyo/zenkatu/result-2.html. In particular, the seasonally adjusted series for 1988–2014 can be downloaded from http://www.meti.go.jp/statistics/tyo/zenkatu/result-2/xls/b2010_IAA_linkj.xls. More recent data can be downloaded from http://www.meti.go.jp/statistics/tyo/zenkatu/result-2/xls/b2010_zsmj.xls.

(iii) *GDP gap estimate by the Cabinet Office*. In constructing potential quarterly GDP underlying their GDP gap estimate, the Cabinet Office uses a production function approach. A documentation (in Japanese) can be found in: http://www5.cao.go.jp/j-j/wp/wp-je07/07f61020.html. To summarize the document, the production function is Cobb–Douglas with 0.33 as capital's share. Capital input is defined as an estimate of the capital stock (available from the National Income Accounts) times capacity utilization. Labor input is the number of persons employed times hours worked per person. The TFP (total factor productivity) level implied by this production function and actual quarterly, real, seasonally adjusted GDP is smoothed by the HP (Hodrick–Prescott) filter. Potential GDP is defined as the value implied by the production function with the smoothed TFP level. The capital and labor in this potential GDP calculation is also HP smoothed. The (quarterly) GDP gap is defined as: $100\times$(actual GDP—potential GDP)/potential GDP.

The Cabinet Office does not release their potential GDP series, but they provide their current GDP gap series upon request. The GDP gap series we obtained is for 1980:Q1–2014:Q1. We verified, through email correspondences with them, that this series is to be paired with the quarterly GDP series released on June 9, 2014 (the GDP series described above). The GDP gap series is reproduced here (137 numbers):

0.3 −1.3 0.0 1.2 0.9 1.0 −0.2 −0.5 0.4 −0.2 −0.7 −0.4 −1.4 −1.4 −1.0 −1.2 −1.1 −0.5 −0.5 −1.4 −0.1 0.2 1.1 1.4 0.6 −0.8 −1.2 −1.3 −2.8 −2.0 −1.2 0.1 1.2 0.1 0.9 0.9 2.5 0.0 0.6 2.6 0.8 2.8 3.7 2.5 2.5 2.8 2.0 1.9 1.3 0.6 0.5 −0.7 −0.2 −1.4 −2.3 −2.2 −1.8 −3.2 −1.7 −3.1 −2.9 −1.8 −1.5 −1.7 −1.3 −0.5 −0.8 0.5 1.0 −0.2 0.0 −0.3 −2.4 −3.1 −3.0 −2.6 −3.6 −3.4 −3.8 −3.4 −2.0 −2.0 −2.5 −2.1 −1.6 −2.1 −3.4 −3.7 −4.1 −3.4 −3.0 −2.9 −3.7 −2.7 −2.6 −1.8 −1.2 −1.4 −1.5 −2.0 −2.0 −1.0 −0.9 −0.9 −0.7 −0.5 −0.8 0.3 1.1 1.0 0.5 1.2 1.8 0.5 −0.7 −4.0 −7.9 −6.5 −6.5 −5.0 −3.7 −2.8 −1.5 −2.1 −3.2 −3.6 −2.4 −2.4 −1.6 −2.4 −3.3 −3.4 −2.3 −1.7 −1.6 −1.7 −0.2.

Construction of potential quarterly GDP. We can back out the Cabinet Office's estimate of potential quarterly GDP by combining this series with the actual GDP series. For quarter t, let GDP_t be (real, seasonally adjusted) GDP described in (i) above and let v_t be the GDP gap shown in (iii) above. The implied potential GDP for quarter t, GDP_t^*, satisfies the relation

$$v_t = 100 \times \frac{GDP_t - GDP_t^*}{GDP_t^*}. \tag{B.6}$$

Construction of monthly series. Given the two quarterly series, GDP_t (actual GDP) and GDP_t^* (potential GDP), we create the monthly output gap series x_t for January 1988–December 2012 as follows.

(i) *Monthly interpolation of GDP_t*. Using the METI all-industry activity index described in (ii) above, the allocation of quarterly GDP between the three months constituting the quarter is done by the method of Chow and Lin ("Best Linear Unbiased Interpolation, Distribution, and Extrapolation of Time Series by Related Series," *Review of Economics and Statistics*, Vol. 53, pp. 372–375, 1971). Quarterly GDP at annual rate for 1988:Q1–2012:Q4 is treated as the low frequency data, and the METI all-industry activity index for January 1988–December 2012 as the high frequency (monthly) indicator. The quarterly averages of interpolated series are constrained to be equal to the corresponding quarterly series. The estimation method is weighted least squares. Actual computation is done using Mr. Enrique M. Quilis's Matlab code available from: https://jp.mathworks.com/matlabcentral/profile/authors/1008800-enrique-m-quilis.

(ii) *Monthly interpolation of GDP^*_t*. We used the spline method. A spline is fitted to GDP^*_t for $t = 1980{:}Q1$ to 2012:Q4. The value of the interpolated monthly series for the middle month of the quarter is constrained to be equal to the quarterly series. We used the Matlab function "spline" for this operation.

(iii) *Calculation of x_t for January 1988–December 2012*. Finally, using this smoothed monthly potential GDP and the monthly actual GDP, we define the monthly output gap for month t, x_t, as

$$x_t \equiv 100 \times \big[\log(\text{actual GDP for month } t) \\ - \log(\text{potential GDP for month} t)\big]. \tag{B.7}$$

HP-filtered GDP as Measure of Potential GDP. In the other GDP gap series used in the paper, potential GDP is the HP-filtered actual GDP. To construct this GDP gap series, we first apply the HP (Hodrick–Prescott) filter to the log of actual *quarterly* GDP for 1980:Q1–2012:Q4. The smoothness parameter is the customary 1600. The exponent of this HP-filtered series is the potential quarterly GDP series. We then apply the same spline method to this series for 1980:Q1–2012:Q4, to obtain the monthly potential GDP series. Output gap for 1988:Q1–2012:Q4 is then calculated by the formula (B.7).

Appendix C: Impulse responses in terms of shocks

This section is a derivation of the shock-based translation of the three effects—the policy-rate effect (6.1), the QE effect (6.2), and the transitional effect (the first component in (6.4))—defined in Section 5 for our nonlinear model.

C.1 *The mapping and the correspondence*

The model, described in Section 4, is summarized by the mapping (4.8), reproduced here as

$$(s_t, \mathbf{y}_t) = f_t\big((\boldsymbol{\varepsilon}_t, \mathbf{v}_t), I_{t-1}\big), \tag{C.1}$$

where $\mathbf{v}_t \equiv (v_{rt}, v_{\overline{\pi}t}, v_{st})$, $I_{t-1} \equiv (s_{t-1}, \mathbf{y}_{t-1}, \ldots)$ is the lagged information set, and the parameter vector has been suppressed. The discrete variable of the model is s_t representing the monetary policy regime.

Conditional on lagged information I_{t-1}, the mapping (C.1) is from the shock vector $(\boldsymbol{\varepsilon}_t, \mathbf{v}_t)$ to the variables $(s_t, \mathbf{y}_t)$. The inverse mapping is a correspondence ϕ from the variables to the shock vector defined by the set

$$\phi(s_t, \mathbf{y}_t; I_{t-1}) \equiv \{(\boldsymbol{\varepsilon}, \mathbf{v}) | (s_t, \mathbf{y}_t) = f_t((\boldsymbol{\varepsilon}, \mathbf{v}), I_{t-1})\}. \tag{C.2}$$

The correspondence is conditional on *lagged* information I_{t-1}. Thus the set ϕ can—and will —depend on the lagged regime s_{t-1} which is part of I_{t-1}.

The conditional expectations entering the definition of the three effects are conditional on the history of the variables. They can be written as conditional on shocks by an identity. That is, for $y = p, x, r, m$ and by the definition of ϕ,

$$\mathrm{E}(y_{t+k} | (s_t, \mathbf{y}_t), I_{t-1}) = \mathrm{E}(y_{t+k} | (\boldsymbol{\varepsilon}_t, \mathbf{v}_t) \in \phi(s_t, \mathbf{y}_t; I_{t-1}), I_{t-1}). \tag{C.3}$$

The rest of this Appendix is to describe this set ϕ for several relevant configurations of $(s_t, \mathbf{y}_t)$.

C.2 *Conditioning expectations by equalities and inequalities on shocks*

The mapping (C.1) can be broken into two stages. In the first stage, the bivariate reduced form determines (p_t, x_t) given I_{t-1} and $\boldsymbol{\varepsilon}_t$. Let $(\widehat{p}_t, \widehat{x}_t)$ be the systematic component of (p_t, x_t), so $(\widehat{p}_t, \widehat{x}_t)$ is a function of I_{t-1} and $(p_t, x_t)' = (\widehat{p}_t, \widehat{x}_t)' + \boldsymbol{\varepsilon}_t$. The mapping from $\boldsymbol{\varepsilon}_t$ to (p_t, x_t) is one-to-one. In the second stage, given (p_t, x_t, I_{t-1}), the three monetary policy shocks $(v_{rt}, v_{\overline{\pi}t}, v_{st})$ determine the regime s_t, the policy rate (r_t), and the excess reserve rate (m_t). How (s_t, r_t, m_t) is determined is described by (3.4)–(3.7) of the text. Since the mapping from $\boldsymbol{\varepsilon}_t$ to (p_t, x_t) in the first stage is one-to-one, the set ϕ can be written as

$$\phi(s_t, \mathbf{y}_t; I_{t-1}) = \left\{ (\boldsymbol{\varepsilon}, \mathbf{v}) | \boldsymbol{\varepsilon} = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, \mathbf{v} \in \mathcal{V}(s_t, \mathbf{y}_t, I_{t-1}) \right\}, \tag{C.4}$$

where the set $\mathcal{V}$ is determined by (4.4)–(4.7). Recalling that $I_{t-1} \equiv (s_{t-1}, \mathbf{y}_{t-1}, \ldots)$,

$$\begin{aligned} &\mathrm{E}(y_{t+k} | (s_t, \mathbf{y}_t), I_{t-1}) \\ &\quad = \mathrm{E}(y_{t+k} | (\boldsymbol{\varepsilon}_t, \mathbf{v}_t) \in \phi(s_t, \mathbf{y}_t; I_{t-1}), I_{t-1}) \quad (\text{by (C.3)}) \\ &\quad = \mathrm{E}\left(y_{t+k} | \boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, \mathbf{v}_t \in \mathcal{V}(s_t, \mathbf{y}_t, I_{t-1}), I_{t-1} \right) \quad (\text{by (C.4)}). \end{aligned} \tag{C.5}$$

The question then boils down to characterizing $\mathcal{V}$ by (4.4)–(4.7). To reproduce (4.4)–(4.7) compactly here, we need to introduce additional notation. Let (r_t^e, m_{st}^e) be the systematic components of (r_t, m_{st}). So

$$\begin{aligned} &r_t^e \equiv (1 - \gamma_r) r_t^* + \gamma_r r_{t-1}, \qquad m_{st}^e \equiv \alpha_s + \underset{(1\times 2)}{\boldsymbol{\beta}_s{}'} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \gamma_s m_{t-1} \\ &\quad (\text{so } m_{st} = m_{st}^e + v_{st}). \end{aligned} \tag{C.6}$$

(See (3.1) for the definition of r_t^*, and (3.6) for m_{st}.) (r_t^e, m_{st}^e) are functions of (p_t, x_t, I_{t-1}). Equations (4.4)–(4.7) can be written as

$$\begin{cases} \text{If } s_{t-1} = \mathbf{P}, \quad s_t = \begin{cases} \mathbf{P} & \text{if } \underbrace{r_t^e + v_{rt}}_{\text{Taylor rate}} > \bar{r}_t, \\ \mathbf{Z} & \text{otherwise}, \end{cases} \\ \text{If } s_{t-1} = \mathbf{Z}, \quad s_t = \begin{cases} \mathbf{P} & \text{if } \underbrace{r_t^e + v_{rt}}_{\text{Taylor rate}} > \bar{r}_t \text{ and } \pi_t \geq \underbrace{\overline{\pi} + v_{\overline{\pi}t}}_{\text{period } t \text{ threshold}}, \\ \mathbf{Z} & \text{otherwise}, \end{cases} \end{cases} \tag{C.7}$$

$$r_t = \begin{cases} \underbrace{r_t^e + v_{rt}}_{\text{Taylor rate}} & \text{if } s_t = \mathbf{P}, \\ \bar{r}_t & \text{if } s_t = \mathbf{Z}, \end{cases} \tag{C.8}$$

$$m_t = \begin{cases} 0 & \text{if } s_t = \mathbf{P}, \\ \max\left[m_{st}^e + v_{st}, 0\right] & \text{if } s_t = \mathbf{Z}. \end{cases} \tag{C.9}$$

We now describe the set $\mathcal{V}(s_t, \mathbf{y}_t, I_{t-1})$ for $\mathbf{v} \equiv (v_r, v_{\overline{\pi}}, v_s)$ by equalities and inequalities on $\mathbf{v}$, for several configurations of $(s_t, \mathbf{y}_t)$.

(a) $s_t = \mathbf{P}$, $\mathbf{y}_t = (p_t, x_t, r_t, 0)$, $r_t > \bar{r}_t$. Thanks to the exit condition, the set depends on the previous regime $s_{t-1} \in I_{t-1}$.

- Suppose first that $s_{t-1} = \mathbf{P}$. By (C.7), we have $s_t = \mathbf{P}$ if and only if $r_t^e + v_{rt} > \bar{r}_t$. Because the exit condition is mute, the threshold inflation $\overline{\pi} + v_{\overline{\pi}t}$ is irrelevant. Given $s_t = \mathbf{P}$, we have $r_t = r_t^e + v_{rt}$ from (C.8). For $r_t > \bar{r}_t$, the inequality condition "$r_t^e + v_{rt} > \bar{r}_t$" is redundant. Because $m_t = 0$ regardless of v_{mt} under $s_t = \mathbf{P}$ by (C.9), the money supply shock v_{st} can be any value. Thus, if $s_{t-1} = \mathbf{P}$ and $r_t > \bar{r}_t$,

$$\mathcal{V}\big(s_t = \mathbf{P}, \underbrace{(p_t, x_t, r_t, 0)}_{\mathbf{y}_t}, I_{t-1}\big) = \big\{\mathbf{v} | v_r = r_t - r_t^e, v_{\overline{\pi}} \in \mathbb{R}, v_s \in \mathbb{R}\big\}. \tag{C.10}$$

With this $\mathcal{V}$, the rewriting of the conditional expectation (C.5) for the current configuration of $(s_t, \mathbf{y}_t)$ is

$$\begin{aligned} &\mathrm{E}\big(y_{t+k} | s_t = \mathbf{P}, \underbrace{(p_t, x_t, r_t, 0)}_{\mathbf{y}_t}, I_{t-1}\big) \\ &= \mathrm{E}\left(y_{t+k} | \boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, v_{rt} = r_t - r_t^e, \underbrace{s_{t-1} = \mathbf{P}, \mathbf{y}_{t-1}, \ldots}_{I_{t-1}}\right). \end{aligned} \tag{C.11}$$

- Suppose next that $s_{t-1} = \mathbf{Z}$. Now the exit condition kicks in and requires that the actual inflation exceed the threshold by (C.7). Thus there should be an additional condition $\pi_t \geq \overline{\pi} + v_{\overline{\pi}t}$. Thus, if $s_{t-1} = \mathbf{Z}$ and $r_t > \bar{r}_t$,

$$\mathcal{V}\big(s_t = \mathbf{P}, \underbrace{(p_t, x_t, r_t, 0)}_{\mathbf{y}_t}, I_{t-1}\big) = \big\{\mathbf{v} | v_r = r_t - r_t^e, v_{\overline{\pi}} \leq \pi_t - \overline{\pi}, v_s \in \mathbb{R}\big\}, \tag{C.12}$$

so the same conditional expectation can be written as

$$\begin{aligned} &\mathrm{E}\big(y_{t+k} | s_t = \mathbf{P}, \underbrace{(p_t, x_t, r_t, 0)}_{\mathbf{y}_t}, I_{t-1}\big) \\ &= \mathrm{E}\left(y_{t+k} | \boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, v_{rt} = r_t - r_t^e, \right. \\ &\qquad \left. v_{\overline{\pi}t} \leq \pi_t - \overline{\pi}, \underbrace{s_{t-1} = \mathbf{Z}, \mathbf{y}_{t-1}, \dots}_{I_{t-1}} \right). \end{aligned} \tag{C.13}$$

(b) $s_t = \mathbf{Z}, \mathbf{y}_t = (p_t, x_t, \bar{r}_t, m_t), m_t > 0$.

- Case: $s_{t-1} = \mathbf{P}$. By (C.7), we have $s_t = \mathbf{Z}$ if and only if $r_t^e + v_{rt} \leq \bar{r}_t$. Because the exit condition is mute, the threshold inflation $\overline{\pi} + v_{\overline{\pi}t}$ is irrelevant. Given $s_t = \mathbf{Z}$, there is no further restriction on v_{rt} because by (C.8) $r_t = \bar{r}_t$ regardless of v_{rt}. By (C.9), we have $m_t = \max[m_{st}^e + v_{st}, 0]$. For $m_t > 0$, it must be that $m_{st}^e + v_{st} = m_t$. Thus, if $s_{t-1} = \mathbf{P}$ and $m_t > 0$,

$$\mathcal{V}\big(s_t = \mathbf{Z}, \underbrace{(p_t, x_t, \bar{r}_t, m_t)}_{\mathbf{y}_t}, I_{t-1}\big) = \big\{\mathbf{v} | v_r \leq \bar{r}_t - r_t^e, v_{\overline{\pi}} \in \mathbb{R}, v_s = m_t - m_{st}^e\big\}, \tag{C.14}$$

so

$$\begin{aligned} &\mathrm{E}\big(y_{t+k} | s_t = \mathbf{Z}, \underbrace{(p_t, x_t, \bar{r}_t, m_t)}_{\mathbf{y}_t}, I_{t-1}\big) \\ &= \mathrm{E}\left(y_{t+k} | \boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, v_{rt} \leq \bar{r}_t - r_t^e, v_{st} = m_t - m_{st}^e, \right. \\ &\qquad \left. \underbrace{s_{t-1} = \mathbf{P}, \mathbf{y}_{t-1}, \dots}_{I_{t-1}} \right). \end{aligned} \tag{C.15}$$

- Case: $s_{t-1} = \mathbf{Z}$. The exit condition becomes relevant. The regime $\mathbf{Z}$ continues if *either* the rate is below the lower bound *or* inflation is below the threshold. So $s_t = \mathbf{Z}$ if and only if $r_t^e + v_{rt} \leq \bar{r}_t$ or $\pi_t \leq \overline{\pi} + v_{\overline{\pi}t}$. Thus, if $s_{t-1} = \mathbf{Z}$ and $m_t > 0$,

$$\begin{aligned} &\mathcal{V}\big(s_t = \mathbf{Z}, \underbrace{(p_t, x_t, \bar{r}_t, m_t)}_{\mathbf{y}_t}, I_{t-1}\big) \\ &= \big\{\mathbf{v} | \big(v_r \leq \bar{r}_t - r_t^e \text{ or } v_{\overline{\pi}} > \pi_t - \overline{\pi}\big), v_s = m_t - m_{st}^e\big\}, \end{aligned} \tag{C.16}$$

so

$$\begin{aligned} &\mathrm{E}\big(y_{t+k}|s_t = \mathbf{Z}, \underbrace{(p_t, x_t, \bar{r}_t, m_t)}_{\mathbf{y}_t}, I_{t-1}\big) \\ &= \mathrm{E}\left(y_{t+k}|\boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, \big(v_{rt} \le \bar{r}_t - r_t^e \text{ or } v_{\overline{\pi}t} > \pi_t - \overline{\pi}\big), \right. \\ &\qquad \left. v_{st} = m_t - m_{st}^e, \underbrace{s_{t-1} = \mathbf{Z}, \mathbf{y}_{t-1}, \dots}_{I_{t-1}}\right). \end{aligned} \tag{C.17}$$

(c) $s_t = \mathbf{Z}, \mathbf{y}_t = (p_t, x_t, \bar{r}_t, 0)$. Here, the only difference from the previous configuration is that $m_t = 0$. The restriction on v_{st} implied by the excess reserve supply equation $m_t = \max[m_{st}^e + v_{st}, 0]$ is that $m_{st}^e + v_{st} \le 0$. Thus,

- Case: $s_{t-1} = \mathbf{P}$.

$$\mathcal{V}\big(s_t = \mathbf{Z}, \underbrace{(p_t, x_t, \bar{r}_t, 0)}_{\mathbf{y}_t}, I_{t-1}\big) = \big\{\mathbf{v}|v_r \le \bar{r}_t - r_t^e, v_{\overline{\pi}} \in \mathbb{R}, v_s \le -m_{st}^e\big\}, \tag{C.18}$$

so

$$\begin{aligned} &\mathrm{E}\big(y_{t+k}|s_t = \mathbf{Z}, \underbrace{(p_t, x_t, \bar{r}_t, 0)}_{\mathbf{y}_t}, I_{t-1}, \dots\big) \\ &= \mathrm{E}\left(y_{t+k}|\boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, v_{rt} \le \bar{r}_t - r_t^e, \right. \\ &\qquad \left. v_{st} \le -m_{st}^e, \underbrace{s_{t-1} = \mathbf{P}, \mathbf{y}_{t-1}, \dots}_{I_{t-1}}\right). \end{aligned} \tag{C.19}$$

- Case: $s_{t-1} = \mathbf{Z}$.

$$\mathcal{V}\big(s_t = \mathbf{Z}, \underbrace{(p_t, x_t, \bar{r}_t, 0)}_{\mathbf{y}_t}, I_{t-1}\big) = \big\{\mathbf{v}|\big(v_r \le \bar{r}_t - r_t^e \text{ or } v_{\overline{\pi}} > \pi_t - \overline{\pi}\big), v_s \le -m_{st}^e\big\}, \tag{C.20}$$

so

$$\begin{aligned} &\mathrm{E}\big(y_{t+k}|s_t = \mathbf{Z}, \underbrace{(p_t, x_t, \bar{r}_t, 0)}_{\mathbf{y}_t}, I_{t-1}\big) \\ &= \mathrm{E}\left(y_{t+k}|\boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, \big(v_{rt} \le \bar{r}_t - r_t^e \text{ or } v_{\overline{\pi}t} > \pi_t - \overline{\pi}\big), \right. \\ &\qquad \left. v_{st} \le -m_{st}^e, \underbrace{s_{t-1} = \mathbf{Z}, \mathbf{y}_{t-1}, \dots}_{I_{t-1}}\right). \end{aligned} \tag{C.21}$$

C.3 *Equivalent statements in terms of shocks*

With the expectations conditioned on equalities and inequalities on the shocks, rather than on the variables, it is now straightforward to translate the responses in terms of shocks. As before, in the expressions below, "y" is either "p", "x", "r," or "m."

The policy-rate effect (6.1). By rewriting the two conditional expectations in (6.1) using (C.11) and (C.13), we obtain the translation: for $r_t > \overline{r}_t$ and $r_t + \delta_r > \overline{r}_t$,

$$
\begin{aligned}
&\mathrm{E}\big(y_{t+k}|s_t = \mathbf{P}, \underbrace{(p_t, x_t, r_t + \delta_r, 0),}_{\mathbf{y}_t \equiv (p_t, x_t, r_t, m_t) \text{ in the alternative history}} I_{t-1}\big) \\
&\quad - \mathrm{E}\big(y_{t+k}|s_t = \mathbf{P}, \underbrace{(p_t, x_t, r_t, 0),}_{\mathbf{y}_t \equiv (p_t, x_t, r_t, m_t) \text{ in the baseline history}} I_{t-1}\big) \\
&= \begin{cases}
\mathrm{E}\left(y_{t+k}|\boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, v_{rt} = r_t - r_t^e + \delta_r, \underbrace{s_{t-1} = \mathbf{P}, \mathbf{y}_{t-1}, \ldots}_{I_{t-1}}\right) \\
\quad - \mathrm{E}\left(y_{t+k}|\boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, v_{rt} = r_t - r_t^e, \underbrace{s_{t-1} = \mathbf{P}, \mathbf{y}_{t-1}, \ldots}_{I_{t-1}}\right) \\
\quad \text{if } s_{t-1} = \mathbf{P}, \\
\mathrm{E}\left(y_{t+k}|\boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, v_{rt} = r_t - r_t^e + \delta_r, v_{\overline{\pi}t} \le \pi_t - \overline{\pi}, \underbrace{s_{t-1} = \mathbf{Z}, \mathbf{y}_{t-1}, \ldots}_{I_{t-1}}\right) \\
\quad - \mathrm{E}\left(y_{t+k}|\boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, v_{rt} = r_t - r_t^e, v_{\overline{\pi}t} \le \pi_t - \overline{\pi}, \underbrace{s_{t-1} = \mathbf{Z}, \mathbf{y}_{t-1}, \ldots}_{I_{t-1}}\right) \\
\quad \text{if } s_{t-1} = \mathbf{Z}.
\end{cases}
\end{aligned} \tag{C.22}
$$

Therefore, the *only* difference in the configuration of the shocks between the baseline and the alternative conditional expectations is that the interest rate shock v_{rt} differs by δ_r in the alternative.

The QE effect (6.2). Using (C.15) and (C.17), we obtain the translation: for $m_t > 0$ and $m_t + \delta_m > 0$,

$$
\begin{aligned}
&\mathrm{E}\big(y_{t+k}|s_t = \mathbf{Z}, \underbrace{(p_t, x_t, \overline{r}_t, m_t + \delta_m),}_{\mathbf{y}_t \equiv (p_t, x_t, r_t, m_t) \text{ in the alternative history}} I_{t-1}\big) \\
&\quad - \mathrm{E}\big(y_{t+k}|s_t = \mathbf{Z}, \underbrace{(p_t, x_t, \overline{r}_t, m_t),}_{\mathbf{y}_t \equiv (p_t, x_t, r_t, m_t) \text{ in the baseline history}} I_{t-1}\big)
\end{aligned}
$$

$$= \begin{cases} \mathrm{E}\left(y_{t+k} | \boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, v_{rt} \leq \overline{r}_t - r_t^e, v_{st} = m_t - m_{st}^e + \delta_m, \underbrace{s_{t-1} = \mathbf{P}, \mathbf{y}_{t-1}, \dots}_{I_{t-1}} \right) \\ \quad - \mathrm{E}\left(y_{t+k} | \boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, v_{rt} \leq \overline{r}_t - r_t^e, v_{st} = m_t - m_{st}^e, \right. \\ \quad \left. \underbrace{s_{t-1} = \mathbf{P}, \mathbf{y}_{t-1}, \dots}_{I_{t-1}} \right) \quad \text{if } s_{t-1} = \mathbf{P}, \\ \mathrm{E}\left(y_{t+k} | \boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, \left(v_{rt} \leq \overline{r}_t - r_t^e \text{ or } v_{\overline{\pi}t} > \pi_t - \overline{\pi}\right), v_{st} = m_t - m_{st}^e + \delta_m, \right. \\ \quad \left. \underbrace{s_{t-1} = \mathbf{Z}, \mathbf{y}_{t-1}, \dots}_{I_{t-1}} \right) \\ \quad - \mathrm{E}\left(y_{t+k} | \boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, \left(v_{rt} \leq \overline{r}_t - r_t^e \text{ or } v_{\overline{\pi}t} > \pi_t - \overline{\pi}\right), \right. \\ \quad \left. v_{st} = m_t - m_{st}^e, \underbrace{s_{t-1} = \mathbf{Z}, \mathbf{y}_{t-1}, \dots}_{I_{t-1}} \right) \quad \text{if } s_{t-1} = \mathbf{Z}. \end{cases} \tag{C.23}$$

Again, the *only* difference in the configuration of the shocks is that the excess reserve shock v_{st} differs by δ_m in the alternative.

The transitional effect (the first component in (6.4)). Using (C.11) and (C.19), and (C.13) and (C.21), we obtain the translation:

$$\lim_{r_t \downarrow \overline{r}_t} \mathrm{E}\big(y_{t+k} | s_t = \mathbf{P}, (p_t, x_t, r_t, 0), I_{t-1}\big) - \mathrm{E}\big(y_{t+k} | s_t = \mathbf{Z}, (p_t, x_t, \overline{r}_t, 0), I_{t-1}\big)$$

$$= \begin{cases} \lim_{r_t \downarrow \overline{r}_t} \mathrm{E}\left(y_{t+k} | \boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, v_{rt} = r_t - r_t^e, \underbrace{s_{t-1} = \mathbf{P}, \mathbf{y}_{t-1}, \dots}_{I_{t-1}} \right) \\ \quad - \mathrm{E}\left(y_{t+k} | \boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, v_{rt} \leq \overline{r}_t - r_t^e, v_{st} \leq -m_{st}^e, \right. \\ \quad \left. \underbrace{s_{t-1} = \mathbf{P}, \mathbf{y}_{t-1}, \dots}_{I_{t-1}} \right) \quad \text{if } s_{t-1} = \mathbf{P}, \\ \lim_{r_t \downarrow \overline{r}_t} \mathrm{E}\left(y_{t+k} | \boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, v_{rt} = r_t - r_t^e, v_{\overline{\pi}t} \leq \pi_t - \overline{\pi}, \right. \\ \quad \left. \underbrace{s_{t-1} = \mathbf{Z}, \mathbf{y}_{t-1}, \dots}_{I_{t-1}} \right) \\ \quad - \mathrm{E}\left(y_{t+k} | \boldsymbol{\varepsilon}_t = \begin{bmatrix} p_t - \widehat{p}_t \\ x_t - \widehat{x}_t \end{bmatrix}, \left(v_{rt} \leq \overline{r}_t - r_t^e \text{ or } v_{\overline{\pi}t} > \pi_t - \overline{\pi}\right), \right. \\ \quad \left. v_{st} \leq -m_{st}^e, \underbrace{s_{t-1} = \mathbf{Z}, \mathbf{y}_{t-1}, \dots}_{I_{t-1}} \right) \quad \text{if } s_{t-1} = \mathbf{Z}. \end{cases} \tag{C.24}$$

Appendix D: Two structural models with recursive SVAR representation

This Appendix provides two examples. In both examples, there are two variables (inflation and the policy rate) with two equations (which are the Fisher equation and the Taylor rule) with predetermined inflation. The agents of the model are forward-looking in the first example and backward-looking in the second. The model in each example admits the recursive SVAR representation in which the shock in the inflation equation is uncorrelated with the shock in the Taylor rule.

D.1 *A forward-looking example*

The first example is the following two-equation system:

$$\text{The Model:} \quad \begin{cases} \text{(Fisher equation)} & \mathrm{E}_{t-1}(r_t - \pi_{t+1}) = \rho + \varepsilon_{t-1}, \\ \text{(Active Taylor rule)} & r_t = \rho + \phi \pi_t + v_t,\ \phi > 1, \end{cases} \tag{D.1}$$

where E_{t-1} is the expectations operator conditional on information available in date $t-1$, r_t is the nominal interest rate in date t, π_{t+1} is the inflation from date t to $t+1$, and (ε_t, v_t) are serially independent and mutually independent. The Fisher equation states that the ex ante real interest rate be equal to some constant ρ plus the real rate shock ε. There is a 1-period information lag in that the ex ante real rate from date t to $t+1$ is formed in date $t-1$. We can reduce the system to one equation by eliminating r_t from the system:

$$\phi \mathrm{E}_{t-1}(\pi_t) - \mathrm{E}_{t-1}(\pi_{t+1}) = \varepsilon_{t-1}. \tag{D.2}$$

Define the expected inflation rate ξ_t as

$$\xi_t \equiv \mathrm{E}_t(\pi_{t+1}). \tag{D.3}$$

By the law of iterated expectations that $\mathrm{E}_{t-1}(\pi_{t+1}) = \mathrm{E}_{t-1}[\mathrm{E}_t(\pi_{t+1})]$, we can rewrite the above equation as: $\phi \xi_{t-1} - \mathrm{E}_{t-1}(\xi_t) = \varepsilon_{t-1}$. Shifting time forward by one period and rearranging, we obtain

$$\xi_t = \frac{1}{\phi} \mathrm{E}_t(\xi_{t+1}) + \frac{1}{\phi} \varepsilon_t. \tag{D.4}$$

This is the equation studied in, for example, Section II of Lubik and Schorfheide (2004, equation (7)), except that the variable ξ here has the interpretation of the expected, rather than actual, inflation rate (the difference in interpretation comes from our assumption of one-period information lag). The only stable solution is

$$\xi_t = \frac{1}{\phi} \varepsilon_t. \tag{D.5}$$

Now require that the inflation rate is *predetermined*. Then the inflation forecast error $\pi_{t+1} - \mathrm{E}_t(\pi_{t+1})$ is zero, so the actual inflation rate is determinate as in

$$\pi_{t+1} = \mathrm{E}_t(\pi_{t+1}) \equiv \xi_t = \frac{1}{\phi} \varepsilon_t. \tag{D.6}$$

By shifting time back by one period and denoting $\tilde{\varepsilon}_t \equiv \frac{1}{\phi}\varepsilon_{t-1}$, and supplementing the equation by the Taylor rule, we obtain a two-equation system

$$\text{The SVAR Representation:} \quad \begin{cases} \pi_t = \tilde{\varepsilon}_t, \\ r_t = \rho + \phi\pi_t + v_t. \end{cases} \tag{D.7}$$

This is a recursive VAR, with the serially uncorrelated reduced-form inflation shock $\tilde{\varepsilon}_t$ that is uncorrelated with the monetary policy shock v_t.

D.2 *A backward-looking example*

Drop the one-period information lag but continue to assume that inflation is *predetermined* (so $\pi_{t+1} = \mathrm{E}_t(\pi_{t+1})$). Assume passive monetary policy. The model is

$$\text{The model:} \quad \begin{cases} \text{(Fisher equation)} & r_t - \pi_{t+1} = \rho + \varepsilon_t, \\ \text{(passive Taylor rule)} & r_t = \rho + \phi\pi_t + v_t, 0 < \phi < 1. \end{cases} \tag{D.8}$$

Eliminating r_t from the system gives: $\pi_{t+1} = \phi\pi_t + (v_t - \varepsilon_t)$. With $0 < \phi < 1$, the only stable solution is the "backward" solution:

$$\pi_t = (v_{t-1} - \varepsilon_{t-1}) + \phi(v_{t-2} - \varepsilon_{t-2}) + \phi^2(v_{t-3} - \varepsilon_{t-3}) + \cdots . \tag{D.9}$$

So

$$\begin{aligned} r_{t-1} &= \rho + \phi\pi_{t-1} + v_{t-1} = \rho + v_{t-1} + \phi(v_{t-2} - \varepsilon_{t-2}) + \phi^2(v_{t-3} - \varepsilon_{t-3}) \\ &\quad + \phi^3(v_{t-4} - \varepsilon_{t-4}) + \cdots . \end{aligned} \tag{D.10}$$

Now take the Fisher equation, shift time back by one period, solve for π_t, and combine the resulting equation with the Taylor rule to obtain:

$$\text{The SVAR representation:} \quad \begin{cases} \pi_t = -\rho + r_{t-1} - \varepsilon_{t-1}, \\ r_t = \rho + \phi\pi_t + v_t. \end{cases} \tag{D.11}$$

This representation embodies the SVAR identification: (i) the first equation *is* a reduced form (the error term ε_t is uncorrelated with the RHS variable r_{t-1} (see (D.10)) and (ii) the reduced-form shock ε_{t-1} is uncorrelated with the monetary policy shock v_t.

Appendix E: Construction of error bands

The log likelihood function is additively separable in the partition $(\boldsymbol{\theta}_A, \boldsymbol{\theta}_B, \boldsymbol{\theta}_C)$. Consequently, if $\widehat{\boldsymbol{\theta}}_B$ is the ML estimator of $\boldsymbol{\theta}_B$, for example, and if $\mathrm{Avar}(\widehat{\boldsymbol{\theta}}_B)$ is its asymptotic variance, a consistent estimator, $\widehat{\mathrm{Avar}(\widehat{\boldsymbol{\theta}}_B)}$, of the asymptotic variance is the inverse of $1/T$ times the Hessian of the likelihood function where T is the sample size. For $\boldsymbol{\theta}_B$, we draw the parameter vector by generating a random vector from $\mathcal{N}(\widehat{\boldsymbol{\theta}}_B, \frac{1}{T}\widehat{\mathrm{Avar}(\widehat{\boldsymbol{\theta}}_B)})$. We do the same for and $\boldsymbol{\theta}_C$. For $\boldsymbol{\theta}_A$, we draw the parameter vector according to the RATS manual. That is, let $\widehat{\boldsymbol{\Sigma}}$ here be the ML estimator of the 2×2 variance-covariance matrix $\boldsymbol{\Sigma}$ of the bivariate error vector in the reduced form. It is simply the sample moment of the bivariate residual vector from the reduced form. We draw $\boldsymbol{\Sigma}$ from the inverse Wishart distribution with $T\widehat{\boldsymbol{\Sigma}}$ and $T - K$ as the parameters, where K is the number of regressors. Let $\tilde{\boldsymbol{\Sigma}}$ be the draw. We then draw reduced-form coefficient vector from $\mathcal{N}(\mathbf{b}, \tilde{\boldsymbol{\Sigma}} \otimes (T\mathbf{S}_{XX})^{-1})$, where $\mathbf{b}$ here is the estimated reduced-form coefficients and $\mathbf{S}_{XX}$ is the sample moment of the reduced-form regressors. The number of the parameter draws is 400 and the number of simulations for the Monte Carlo integration for each draw is 1000.

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