Supplement to "Specification and estimation of network formation and network interaction models with the exponential probability distribution"

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This Supplementary Appendix provides (1) additional discussion on the specification of global network effects in the utility function; (2) the likelihood function for the full model with both uncensored and censored activity variables and the corresponding Bayesian posterior analysis; (3) an extensive simulation study to show the issue of model misspecification and the associated estimation biases; (4) additional empirical tables and figures.

Appendix B: Specification of global network structure effects

The empirical specification of the global network effects in equation (10) of the paper are based on which network structural statistics may represent potential benefits and costs of network connections. When treating the local network effects in equation (3) as the first- order interaction terms among network links, the set of global effects in equation (10) covers the reciprocality effect, the congestion effect, and the popularity effect as second-order interaction terms and the transitive triads effect and the three-cycle effect as third-order interaction terms.

As a result, the deterministic part of the network value, $V(W)$ consists of the sum of first-, second-, and third-order interaction terms. The sum of first-order interaction terms $\sum_{i=1}^m \sum_{j=1}^m w_{ij}\psi_{ij}$ can be written as $\mathrm{tr}(\Psi W')$, where Ψ is a $m \times m$ matrix with the (i,j) entry equal to ψ_{ij}. This term reflects the local network effect in the explicit preference over the network structure. Second, the sums of second-order interaction terms $\sum_{i=1}^m \sum_{j=1}^m w_{ij}w_{ji}$, $\sum_{i=1}^m \sum_{j=1}^m \sum_{k\neq j}^m w_{ij}w_{ik}$ and $\sum_{i=1}^m \sum_{j=1}^m \sum_{k\neq i}^m w_{ij}w_{kj}$, respectively, are equivalent to $\mathrm{tr}(W^2)$, $l_m' W' W l_m - l_m' W l_m$ and $l_m' W W' l_m - l_m' W l_m$. These terms reflect, respectively, the reciprocity, and the level term of the congestion and the

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popularity effects in the in the explicit preference over the network structure. The square term of the congestion effect will be reflected by $\sum_{i=1}^{m}\sum_{j=1}^{m} w_{ij}(\sum_{k\neq j}^{m} w_{ik})^2$, which can be written as $l_m' W' \text{Diag}(W l_m) W l_m - 2l' W' W l_m + l_m' W l_m$, where $\text{Diag}(A)$ is a $n \times n$ diagonal matrix of a $n \times 1$ vector A. Third, the sum of third-order interaction terms $\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m} w_{ij}w_{ik}w_{kj}$, or equivalent to $\text{tr}(W^2 W')$, reflects the transitive triads effects. The term $\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m} w_{ij}w_{jk}w_{ki}$, equivalent to $\text{tr}(W^3)$, reflects the three cycles effect.

The exponential random graph model (Frank and Strauss (1986)), or more generally, the p^* model (Wasserman and Pattison (1996)) in the statistics literature also takes a log linear form with some specified network statistics. The most common statistics include the number of reciprocal ties $\sum_{i=1}^{m}\sum_{j=1}^{m} w_{ij}w_{ji}$, the number of two stars $\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m} w_{ij}w_{ik}$, or the number of triangles $\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m} w_{ij}w_{jk}w_{ki}$, etc. The coefficients in the p^* model represent how the corresponding network statistics contribute to the occurrence of the network but do not provide any causal interpretation. Compared to the p^* model, our network model is originated from the consideration of network structural effects in the link associated utility, and thus captures economic reasoning in the formulation.

Mele (2017) provided some alternative network effects that might also be specified in the model. He proposed a network model that considers the effects from direct, mutual, and indirect friends. Using the terminology in our paper, they correspond to the exogenous effect, reciprocality effect, congestion effect, and popularity effect. The exogenous and the reciprocality effects are considered in both Mele's and our models. However, Mele (2017) looked at the congestion effect from the receiver's perspective and the popularity effect from the sender's perspective, whereas we look at sender's congestion effect and receiver's popularity effect. We believe that our specifications are intuitively appealing for our named (direct) friends network. Although different, both sets of effects all belong to the second- order interaction terms that can be specified in the link associated utility. They coincide for a network having mutual friendships where W becomes a symmetric matrix, that is, an undirected network. Note that two effects considered by Mele have the same statistics $l_m' W^2 l_m - \text{tr}(W^2)$ in the transferable utility function, and hence, they have the same effect (due to the model coherency requirement). On the contrary, our two interaction terms can have different effects for the named friendship (directed) network. The third-order interaction terms—transitive triads and three cycles effects are only considered in our model but not in Mele's.

Appendix C: Likelihood function for the full model with a censored activity intensity

For the censored activity intensity $y_{ig} \in \mathbb{R}_+$, the equilibrium outcome vector based on equation (5) can be expressed as

$$\begin{aligned} Y_g &= \max(0, \ddot{Y}_g), \\ \ddot{Y}_g &= \lambda W_g Y_g + X_g \beta_1 + W_g X_g \beta_2 + Z_g \rho_1 + W_g Z_g \rho_2 + l_g \alpha_g + \xi_g. \end{aligned} \tag{C.1}$$

We can divide the m_g individuals in group g into two blocks, such that the first m_{g1} individuals have activity variables equal to zero, and the remaining individuals who are arranged from $m_{g1}+1$ to m_g have positive values. $\ddot{Y}_g$ of equation (C.1) and the network W_g can be conformably decomposed into

$$\begin{aligned}\begin{pmatrix}\ddot{Y}_{g1}\\ Y_{g2}\end{pmatrix} &= \lambda\begin{pmatrix}W_{11,g} & W_{12,g}\\ W_{21,g} & W_{22,g}\end{pmatrix}\begin{pmatrix}Y_{g1}\\ Y_{g2}\end{pmatrix}+\begin{pmatrix}X_{g1}\\ X_{g2}\end{pmatrix}\beta_1\\ &+\begin{pmatrix}W_{11,g} & W_{12,g}\\ W_{21,g} & W_{22,g}\end{pmatrix}\begin{pmatrix}X_{g1}\\ X_{g2}\end{pmatrix}\beta_2\\ &+\begin{pmatrix}Z_{g1}\\ Z_{g2}\end{pmatrix}\rho_1+\begin{pmatrix}W_{11,g} & W_{12,g}\\ W_{21,g} & W_{22,g}\end{pmatrix}\begin{pmatrix}Z_{g1}\\ Z_{g2}\end{pmatrix}\rho_2+\begin{pmatrix}l_{g1}\\ l_{g2}\end{pmatrix}\alpha_g+\begin{pmatrix}\xi_{g1}\\ \xi_{g2}\end{pmatrix},\end{aligned} \tag{C.2}$$

where $Y_{g2}>0$ and $Y_{g1}=0$, with the corresponding latent $\ddot{Y}_{g1}\leq 0$. Based on equation (C.2), the joint probability function of Y_g and W_g can be written as

$$\begin{aligned}&P(Y_g, W_g|\theta_g,\alpha_g,Z_g)\\ &=P(Y_{g1}=0, Y_{g2}, W_g|\theta_g,\alpha_g,Z_g)\\ &=\int I(Y_{g1}=0,\ddot{Y}_{g1})\cdot P(\ddot{Y}_{g1}, Y_{g2}, W_g|\theta_g,\alpha_g,Z_g)\cdot d\ddot{Y}_{tg1}\\ &=\int_{-\infty}^{-(\lambda W_{12,g}Y_{g2}+X_{g1}\beta_1+(W_{11,g}X_{g1}+W_{12,g}X_{g2})\beta_2+Z_{g1}\rho_1+(W_{11,g}Z_{g1}+W_{12,g}Z_{g2})\rho_2+l_{g1}\alpha_g)}\\ &\quad |I_{m_g-m_{g1}}-\lambda W_{22,g}|\cdot f(\xi_{g1},\xi_{g2}|\theta_g,\alpha_g,Z_g)\\ &\quad\times P(W_g|\xi_{g1},\xi_{g2},\theta_g,\alpha_g,Z_g)\cdot d\xi_{g1},\end{aligned} \tag{C.3}$$

where $I(Y_{g1}=0,\ddot{Y}_{g1})$ is a dichotomous indicator that is equal to 1 when $\ddot{Y}_{g1}$ is negative, and equal to 0, otherwise; $\xi_{g2}=(I_{m_g-m_{g1}}-\lambda W_{22,g})Y_{g2}-X_{g2}\beta_1-(W_{21,g}X_{g1}+W_{22,g}X_{g2})\beta_2-Z_{g2}\rho_1-(W_{21,g}Z_{g1}+W_{22,g}Z_{g2})\rho_2-l_{g2}\alpha_g$ and $\theta_g=(\gamma',\eta',\delta,\lambda,\beta',\rho',\sigma^2_{\xi_g})$.

Appendix D: Posterior analysis for the model with both uncensored and censored activity variables

In this Appendix, we provide the posterior analysis of the parameters in the full model with one uncensored activity variable ($y_{uc,ig}$) and one censored activity variable ($y_{c,ig}$), where the subscripts *uc* and *c* represent "uncensored" and "censored," respectively.

D.1 *Likelihood function*

With two activity intensity equations in the model, the disturbances $\xi_{uc,ig}$ and $\xi_{c,ig}$ of the two equations are assumed following a bivariate normal distribution:

$$(\xi_{uc,ig},\xi_{c,ig})\sim \text{i.i.d.}\mathcal{N}_2\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}\sigma^2_{\xi_{uc,g}} & \sigma_{\xi_{ucc,g}}\\ \sigma_{\xi_{ucc,g}} & \sigma^2_{\xi_{c,g}}\end{pmatrix}\right),\quad i=1,\dots,m_g. \tag{D.1}$$

From equation (D.1), conditional on $\xi_{uc,g}$, we have

$$\xi_{c,g} = \sigma_{\xi_{ucc,g}} \sigma^{-2}_{\xi_{uc,g}} \xi_{uc,g} + u_g, \quad u_g \sim \mathcal{N}_{m_g}(0, \sigma^2_{u_g} I_{m_g}), \tag{D.2}$$

where u_g is independent of $\xi_{uc,g}$ and $\sigma^2_{u_g} = (\sigma^2_{\xi_{c,g}} - \sigma_{\xi_{ucc,g}} \sigma^{-2}_{\xi_{uc,g}} \sigma_{\xi_{ucc,g}})$.

Performing the decomposition of $\ddot{Y}_{c,g}$, as in Supplementary Appendix C:

$$\begin{aligned}\begin{pmatrix} \ddot{Y}_{c,g1} \\ Y_{c,g2} \end{pmatrix} &= \lambda_c \begin{pmatrix} W_{11,g} & W_{12,g} \\ W_{21,g} & W_{22,g} \end{pmatrix} \begin{pmatrix} Y_{c,g1} \\ Y_{c,g2} \end{pmatrix} + \begin{pmatrix} X_{g1} \\ X_{g2} \end{pmatrix} \beta_{1,c} + \begin{pmatrix} W_{11,g} & W_{12,g} \\ W_{21,g} & W_{22,g} \end{pmatrix} \begin{pmatrix} X_{g1} \\ X_{g2} \end{pmatrix} \beta_{2,c} \\ &+ \begin{pmatrix} Z_{g1} \\ Z_{g2} \end{pmatrix} \rho_{1,c} + \begin{pmatrix} W_{11,g} & W_{12,g} \\ W_{21,g} & W_{22,g} \end{pmatrix} \begin{pmatrix} Z_{g1} \\ Z_{g2} \end{pmatrix} \rho_{2,c} + \begin{pmatrix} l_{g1} \\ l_{g2} \end{pmatrix} \alpha_{c,g} + \begin{pmatrix} \xi_{c,g1} \\ \xi_{c,g2} \end{pmatrix}, \end{aligned} \tag{D.3}$$

where $Y_{c,g2} > 0$ and $Y_{c,g1} = 0$ with the corresponding latent $\ddot{Y}_{c,g1} \le 0$. We let $\theta_g = (\gamma', \eta', \delta_{uc}, \delta_{uc}, \lambda_{uc}, \lambda_c, \beta'_{uc}, \beta'_c, \rho'_{uc}, \rho'_c, \sigma^2_{\xi_{uc,g}}, \sigma^2_{\xi_{c,g}}, \sigma_{\xi_{ucc,g}})$. The joint probability function of $Y_{uc,g}$, $Y_{c,g}$, and W_g is

$$\begin{aligned} &P(Y_{c,g}, Y_{uc,g}, W_g | \theta_g, \alpha_{uc,g}, \alpha_{c,g}, Z_g) \\ &= \int_{-\infty}^{-(\lambda_c W_{12,g} Y_{c,g2} + X_{g1}\beta_{1,c} + (W_{11,g} X_{g1} + W_{12,g} X_{g2})\beta_{2,c} + Z_{g1}\rho_{1,c} + (W_{11,g} Z_{g1} + W_{12,g} Z_{g2})\rho_{2,c} + l_{g1}\alpha_{c,g})} \\ &\quad |I_{m_g - m_{g1}} - \lambda_c W_{22,g}| \cdot f(\xi_{c,g} | \xi_{uc,g}, \theta_g, \alpha_{c,g}, \alpha_{uc,g}, Z_g) \\ &\quad \times \left|S_g(W_g)\right| \cdot f(\xi_{uc,g} | \theta_{cg}, \alpha_{uc,g}, Z_g) \\ &\quad \times P(W_g | \xi_{uc,g}, \xi_{c,g}, \theta_g, \alpha_{c,g}, \alpha_{uc,g}, Z_g) \cdot d\xi_{c,g1}, \end{aligned} \tag{D.4}$$

where $S_g(W_g) = I_{m_g} - \lambda_{uc} W_g$.

D.2 *Prior distributions*

By Bayes' theorem, the joint posterior distribution of the parameters and latent variables in the full model is

$$\begin{aligned} &P\big(\{\theta_g\}, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\}, \{\ddot{Y}_{c,g1}\} | \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}\big) \\ &\quad \propto \pi\big(\{\theta_g\}, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\}\big) \\ &\qquad \times \prod_{g=1}^{G} \Bigg\{ \Bigg(\prod_{i=1}^{m_{g1}} I(y_{c,ig} = 0) \cdot I(\ddot{y}_{c,ig} \le 0) \Bigg) \\ &\qquad \times P(Y_{c,g}, Y_{uc,g}, W_g, \ddot{Y}_{c,g1} | \theta_g, \alpha_{uc,g}, \alpha_{c,g}, Z_g) \Bigg\}, \end{aligned} \tag{D.5}$$

where $\pi(\cdot)$ represents the prior density function. We block the parameters and latent variables in the model and assign the prior distributions as follows:

(i) Unobserved individual latent variables in both the network formation model and activity intensity equation,

$$z_{i,g} \sim \mathcal{N}_{\bar{\ell}}(\mu_{z,g}, I_{\bar{\ell}}) \quad \text{and} \quad \mu_{z,g} \sim \mathcal{N}_{\bar{\ell}}(0, \varsigma I_{\bar{\ell}}) \quad i = 1, \ldots, m_g;\, g = 1, \ldots, G.$$

(ii) Coefficients of the network formation model,

$$\phi = \left(\gamma', \eta', \delta_{uc}, \delta_c\right) \sim \mathcal{TN}_{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2}(\phi_0, \Phi_0 I_{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2}, O),$$
$$O = \left\{\phi \in \mathbb{R}^{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2} \middle| |\gamma_{41}| \geq |\gamma_{42}| \geq \cdots \geq |\gamma_{4\bar{\ell}}|, \delta_{uc} \geq 0, \delta_c \geq 0\right\}.$$

(iii) Endogenous peer effect parameters in the activity intensity equation,

$$\lambda_{uc} \sim U[-1/\Delta_{uc}, 1/\Delta_{uc}] \quad \text{and} \quad \lambda_c \sim U[-1/\Delta_c, 1/\Delta_c].$$

(iv) Coefficients of own and contextual effects in the activity intensity equation,

$$\beta_{uc} = \left(\beta'_{1,uc}, \beta'_{2,uc}\right)' \sim \mathcal{N}_{2k}(\beta_0, B_0 I_{2k}) \quad \text{and} \quad \beta_c = \left(\beta'_{1,c}, \beta'_{2,c}\right)' \sim \mathcal{N}_{2k}(\beta_0, B_0 I_{2k}).$$

(v) Coefficients of own and contextual correlated effects in the activity intensity equation,

$$\rho_{uc} = \left(\rho'_{1,uc}, \rho'_{2,uc}\right)' \sim \mathcal{N}_{2\bar{\ell}}(\rho_0, R_0 I_{2\bar{\ell}}) \quad \text{and} \quad \rho_c = \left(\rho'_{1,c}, \rho'_{2,c}\right)' \sim \mathcal{N}_{2\bar{\ell}}(\rho_0, R_0 I_{2\bar{\ell}}).$$

(vi) Variances and covariance of disturbance in the activity intensity equation,

$$\sigma_g = \left(\sigma^2_{\xi_{uc,g}}, \sigma^2_{\xi_{c,g}}, \sigma_{\xi_{ucc,g}}\right) \sim \mathcal{N}_3(\sigma_0, \Sigma_0 I_3), \quad \sigma_g \in T_g,\, g = 1, \ldots, G.$$

(vii) Group fixed effects in the activity intensity equation,

$$\alpha_{uc,g} \sim \mathcal{N}(\alpha_0, A_0) \quad \text{and} \quad \alpha_{c,g} \sim \mathcal{N}(\alpha_0, A_0), \quad g = 1, \ldots, G.$$

Most of the above prior distributions are conjugate priors used commonly in the Bayesian literature. However, following Hsieh and Lee (2016) and Hsieh and Van Kippersluis (2018), we set up a hierarchical prior for $z_{i,g}$. That is, we assume $z_{i,g}$ is normally distributed having a unit variance and a prior mean equal to $\mu_{z,g}$, reflecting the identification restrictions on $z_{i,g}$, as presented in Section 3.2. Then we further assume that $\mu_{z,g}$ is normally distributed having a hyperprior mean of zero and a hyperprior variance of ς. As a result, latent variables $\{Z_g\}$ only add G "real" parameters ($\{\mu_{z,g}\}$) in addition to their coefficients γ_4 into the estimation procedure.

Note that the prior of ϕ is constrained to a parameter space O in which $|\gamma_{41}| \geq |\gamma_{42}| \geq \cdots \geq |\gamma_{4\bar{\ell}}|$, So the prior distribution of ϕ is truncated normal ($\mathcal{TN}$) on O. for the identification of latent variables Z. We also assume that δ is nonnegative to maintain the coherence with our microeconomic model. The prior on λ is also constrained on $[-1/\Delta, 1/\Delta]$, where $\Delta = \bar{n}$, to ensure that Proposition 1 holds and that the equilibrium is always unique and generates a well-defined data generating process (Kelejian and Prucha (2010)). The use of a uniform prior follows Smith and LeSage (2004).

We put $\sigma^2_{\xi_{uc,g}}$, $\sigma^2_{\xi_{c,g}}$, and $\sigma_{\xi_{ucc,g}}$ into a group σ_g and specify a multivariate normal distribution truncated to the area $T_g = \{\sigma_g \in \mathbb{R}^3 | \sigma^2_{\xi_{uc,g}} > 0, \sigma^2_{\xi_{c,g}} > 0, \sigma^2_{\xi_{uc,g}}\sigma^2_{\xi_{c,g}} - \sigma^2_{\xi_{ucc,g}} \geq 0\}$ so that $\sigma^2_{\xi_{uc,g}}$, $\sigma^2_{\xi_{c,g}}$, and $\sigma_{\xi_{ucc,g}}$ form a proper covariance matrix.

In the specification of (vii), we treat the group effects $\alpha_{uc,g}$ and $\alpha_{c,g}$ as fixed effects with the hyperparameters α_0 and A_0 fixed in their prior distributions. The distinction between fixed and random effects in a Bayesian approach lies on prior assignment at the second and third levels of hierarchy (Lancaster (2004), Rendon (2013)). For a fixed effect model, a Bayesian approach updates distributions of fixed effect parameters, whereas a random effect model updates distributions of hyperparameters in the prior distribution of random-effect parameters. If it is preferred to model the correlation between covariates and group effects explicitly, one may follow Mundlak (1978) to have a correlated random effect specification that allows the mean of the random effect (i.e., the mean hyperparameter in the prior distribution of the random effect) to be a linear function of covariates (e.g., Boucher and Goussé (2009)). To determine if there is any impact due to the specification of random group effects, we also examine estimation results of our model based on the correlated random effect specification for a robustness check.

In the case of correlated random group effects (Mundlak (1978)), we change the prior settings for $\alpha_{uc,g}$ and $\alpha_{c,g}$ in (vii). We specify $\alpha_{uc,g}$ and $\alpha_{c,g}$ as follows:

$$\alpha_{uc,g} = \overline{X}_g\beta_{3,uc} + \overline{Z}_g\rho_{3,uc} + \zeta_{uc,g},\ \zeta_{uc,g} \sim \mathcal{N}\left(0, \sigma^2_{\alpha,uc}\right),$$
$$\alpha_{c,g} = \overline{X}_g\beta_{3,c} + \overline{Z}_g\rho_{3,c} + \zeta_{c,g},\ \zeta_{c,g} \sim \mathcal{N}\left(0, \sigma^2_{\alpha,c}\right),$$

where $\overline{X}_g$ and $\overline{Z}_g$ are, respectively, the group averages of X_g and Z_g. $\beta_{3,uc}$, $\rho_{3,uc}$, $\beta_{3,c}$, $\rho_{3,c}$, $\sigma^2_{\alpha,uc}$, and $\sigma^2_{\alpha,c}$ are unknown parameters, and we specify the following prior distributions for them: $\beta_{3,uc} \sim \mathcal{N}_k(\beta_0, B_0 I_k)$, $\rho_{3,uc} \sim \mathcal{N}_{\bar{\ell}}(\rho_0, R_0 I_{\bar{\ell}})$, $\beta_{3,c} \sim \mathcal{N}_k(\beta_0, B_0 I_k)$, $\rho_{3,c} \sim \mathcal{N}_{\bar{\ell}}(\rho_0, R_0 I_{\bar{\ell}})$, $\sigma^2_{\alpha,uc} \sim \mathcal{IG}(\frac{\kappa_0}{2}, \frac{\nu_0}{2})$, and $\sigma^2_{\alpha,c} \sim \mathcal{IG}(\frac{\kappa_0}{2}, \frac{\nu_0}{2})$.

D.3 *Conditional posterior distributions*

Here, we list the set of conditional posterior distributions required by the Gibbs sampler:

(i) $P(\ddot{Y}_{c,g1} | \theta_g, \alpha_{uc,g}, \alpha_{c,g}, Z_g, Y_{uc,g}, Y_{c,g}, W_g)$, $g = 1, \ldots, G$.
By applying Bayes' theorem, we have

$$\begin{aligned} &P(\ddot{Y}_{c,g1} | \theta_g, \alpha_{uc,g}, \alpha_{c,g}, Z_g, Y_{uc,g}, Y_{c,g}, W_g) \\ &\propto \left(\prod_{i=1}^{m_{g1}} \mathrm{I}(y_{c,ig} = 0) I(\ddot{y}_{c,ig} \leq 0)\right) \\ &\quad \times P(\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g | \theta_g, \alpha_{uc,g}, \alpha_{c,g}, Z_g), \end{aligned} \tag{D.6}$$

for $g = 1, \ldots, G$.

(ii-1) $P(z_{i,g} | \ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g, \theta_g, \alpha_{uc,g}, \alpha_{c,g}, Z_{-i,g})$, $i = 1, \ldots, m_g$; $g = 1, \ldots, G$.

By applying Bayes' theorem, we have

$$\begin{aligned}
&P(z_{i,g}|\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g, \theta_g, \alpha_{uc,g}, \alpha_{c,g}, Z_{-i,g}) \\
&\quad \propto \mathcal{N}_{\bar{\ell}}(z_{i,g}; \mu_{z,g}, I_{\bar{\ell}}) \\
&\qquad \times P(\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g|\theta_g, \alpha_{uc,g}, \alpha_{c,g}, Z_g),
\end{aligned} \tag{D.7}$$

for $g = 1, \ldots, G$.

(ii-2) $P(\mu_{z,g}|\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g, \theta_g, \alpha_{uc,g}, \alpha_{c,g}, Z_g), g = 1, \ldots, G$.
By applying Bayes' theorem, we have

$$\begin{aligned}
&P(\mu_{z,g}|\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g, \theta_g, \alpha_{uc,g}, \alpha_{c,g}, Z_g) \\
&\quad \propto \mathcal{N}_{\bar{\ell}}\left(\frac{m_g \bar{Z}_g}{m_g + 1/\varsigma^2}, \frac{1}{m_g + 1/\varsigma^2} I_{\bar{\ell}}\right),
\end{aligned} \tag{D.8}$$

where $\bar{Z}_g = \frac{1}{m_g}\sum_{i=1}^{m_g} z_{i,g}$.

(iii) $P(\phi|\{\ddot{Y}_{c,g1}\}, \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}, \{\theta_g\}\backslash\phi, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\})$, where $\{\theta_g\}\backslash\phi$ stands for $\{\theta_g\}$ excluding ϕ.
By applying Bayes' theorem, we have

$$\begin{aligned}
&P\big(\phi|\{\ddot{Y}_{c,g1}\}, \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}, \{\theta_g\}\backslash\phi, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\}\big) \\
&\quad \propto \mathcal{TN}_{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2}(\phi; \phi_0, \Phi_0 I_{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2}, O) \\
&\qquad \times \prod_{g=1}^{G} P(\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g|\theta_g, \alpha_{c,g}, \alpha_{uc,g}, Z_g),
\end{aligned} \tag{D.9}$$

where $\mathcal{TN}_{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2}(\phi; \phi_0, \Phi_0 I_{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2}, O)$ is the prior density function of ϕ truncated at O.

(iv) $P(\lambda_{uc}|\{\ddot{Y}_{c,g1}\}, \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}, \{\theta_g\}\backslash\lambda_{uc}, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\})$.
By applying Bayes' theorem, we have

$$\begin{aligned}
&P\big(\lambda_{uc}|\{\ddot{Y}_{c,g1}\}, \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}, \{\theta_g\}\backslash\lambda_{uc}, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\}\big) \\
&\quad \propto \prod_{g=1}^{G} P(\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g|\theta_g, \alpha_{uc,g}, \alpha_{c,g}, Z_g) \\
&\qquad \times \mathrm{I}\big(\lambda_{uc} \in [-1/\Delta_{uc}, 1/\Delta_{uc}]\big).
\end{aligned} \tag{D.10}$$

(v) $P(\lambda_c|\{\ddot{Y}_{c,g1}\}, \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}, \{\theta_g\}\backslash\lambda_c, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\})$.
By applying Bayes' theorem, we have

$$P\big(\lambda_c|\{\ddot{Y}_{c,g1}\}, \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}, \{\theta_g\}\backslash\lambda_c, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\}\big)$$

$$\propto \prod_{g=1}^{G} P(\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g|\theta_g, \alpha_{c,g}, \alpha_{uc,g}, Z_g)$$
$$\times \mathrm{I}\big(\lambda_c \in [-1/\Delta_c, 1/\Delta_c]\big). \tag{D.11}$$

(vi) $P(\beta_{uc}|\{\ddot{Y}_{c,g1}\}, \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}, \{\theta_g\}\backslash\beta_{uc}, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\})$.
By applying Bayes' theorem, we have

$$P\big(\beta_{uc}|\{\ddot{Y}_{c,g1}\}, \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}, \{\theta_g\}\backslash\beta_{uc}, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\}\big)$$
$$\propto \mathcal{N}_{2k}(\beta_{uc}; \beta_0, B_0 I_{2k})$$
$$\times \prod_{g=1}^{G} P(\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g|\theta_g, \alpha_{c,g}, \alpha_{uc,g}, Z_g), \tag{D.12}$$

where $\mathcal{N}_{2k}(\beta_{uc}; \beta_0, B_0 I_{2k})$ is the prior normal density function of β_{uc}.

(vii) $P(\beta_c|\{\ddot{Y}_{c,g1}\}, \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}, \{\theta_g\}\backslash\beta_c, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\})$.
By applying Bayes' theorem, we have

$$P\big(\beta_c|\{\ddot{Y}_{c,g1}\}, \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}, \{\theta_g\}\backslash\beta_c, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\}\big)$$
$$\propto \mathcal{N}_{2k}(\beta_c; \beta_0, B_0 I_{2k})$$
$$\times \prod_{g=1}^{G} P(\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g|\theta_g, \alpha_{c,g}, \alpha_{uc,g}, Z_g), \tag{D.13}$$

where $\mathcal{N}_{2k}(\beta_c; \beta_0, B_0 I_{2k})$ is the prior normal density function of β_c.

(viii) $P(\rho_{uc}|\{\ddot{Y}_{c,g1}\}, \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}, \{\theta_g\}\backslash\rho_{uc}, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\})$.
By applying Bayes' theorem, we have

$$P\big(\rho_{uc}|\{\ddot{Y}_{c,g1}\}, \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}, \{\theta_g\}\backslash\rho_{uc}, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\}\big)$$
$$\propto \mathcal{N}_{2\bar{\ell}}(\rho_{uc}; \rho_0, R_0 I_{2\bar{\ell}})$$
$$\times \prod_{g=1}^{G} P(\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g|\theta_g, \alpha_{c,g}, \alpha_{uc,g}, Z_g), \tag{D.14}$$

where $\mathcal{N}_{2\bar{\ell}}(\rho_{uc}; \rho_0, R_0 I_{2\bar{\ell}})$ is the prior normal density function of ρ_{uc}.

(ix) $P(\rho_c|\{\ddot{Y}_{c,g1}\}, \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}, \{\theta_g\}\backslash\rho_c, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\})$.
By applying Bayes' theorem, we have

$$P\big(\rho_c|\{\ddot{Y}_{c,g1}\}, \{Y_{uc,g}\}, \{Y_{c,g}\}, \{W_g\}, \{\theta_g\}\backslash\rho_c, \{\alpha_{uc,g}\}, \{\alpha_{c,g}\}, \{Z_g\}\big)$$
$$\propto \mathcal{N}_{2\bar{\ell}}(\rho_c; \rho_0, R_0 I_{2\bar{\ell}})$$
$$\times \prod_{g=1}^{G} P(\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g|\theta_g, \alpha_{c,g}, \alpha_{uc,g}, Z_g), \tag{D.15}$$

where $\mathcal{N}_{2\bar{\ell}}(\rho_c; \rho_0, R_0 I_{2\bar{\ell}})$ is the prior normal density function of ρ_c.

(x) $P(\sigma_g|\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g, \theta_g \backslash \sigma_g, \alpha_{uc,g}, \alpha_{c,g}, Z_g)$, $g = 1, \dots, G$.
By applying Bayes' theorem, we have

$$\begin{aligned} &P(\sigma_g|\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g, \theta_g \backslash \sigma_g, \alpha_{uc,g}, \alpha_{c,g}, Z_g) \\ &\quad \propto \mathcal{N}_3(\sigma_g; \sigma_0, \Sigma_0) \cdot \mathrm{I}(\sigma_g \in T_g) \\ &\qquad \times P(\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g|\theta_g, \alpha_{c,g}, \alpha_{uc,g}, Z_g), \end{aligned} \tag{D.16}$$

for $g = 1, \dots, G$.

(xi) $P(\alpha_{uc,g}|\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g, \theta_g, \alpha_{c,g}, Z_g)$, $g = 1, \dots, G$.
By applying Bayes' theorem, we have

$$\begin{aligned} &P(\alpha_{uc,g}|\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g, \theta_g, \alpha_{c,g}, Z_g) \\ &\quad \propto \mathcal{N}(\alpha_{uc,g}; \alpha_0, A_0) \\ &\qquad \times P(\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g|\theta_g, \alpha_{c,g}, \alpha_{uc,g}, Z_g), \\ &\quad g = 1, \dots, G, \end{aligned} \tag{D.17}$$

where $\mathcal{N}(\alpha_{uc,g}; \alpha_0, A_0)$ is the prior normal density function of $\alpha_{uc,g}$.

(xii) $P(\alpha_{c,g}|\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g, \theta_g, \alpha_{uc,g}, Z_g)$, $g = 1, \dots, G$.
By applying Bayes' theorem, we have

$$\begin{aligned} &P(\alpha_{c,g}|\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g, \theta_g, \alpha_{uc,g}, Z_g) \\ &\quad \propto \mathcal{N}(\alpha_{c,g}; \alpha_0, A_0) \cdot P(\ddot{Y}_{c,g1}, Y_{uc,g}, Y_{c,g}, W_g|\theta_g, \alpha_{c,g}, \alpha_{uc,g}, Z_g), \\ &\quad g = 1, \dots, G, \end{aligned} \tag{D.18}$$

where $\mathcal{N}(\alpha_{c,g}; \alpha_0, A_0)$ is the prior normal density function of $\alpha_{c,g}$.

D.4 *Detailed MCMC sampling steps*

At the tth run of the iteration, we perform the following steps sequentially. For each step, the pseudo MCMC algorithm outlined in the main text is implemented.

Step I. For $g = 1, \dots, G$, simulate $\ddot{Y}^{(t)}_{c,g1}$ from $P(\ddot{Y}_{c,g1}|Y_{c,g}, Y_{uc,g}, W_g, Z^{(t-1)}_g, \theta^{(t-1)}_g, \alpha^{(t-1)}_{c,g}, \alpha^{(t-1)}_{uc,g})$ by the double M–H algorithm.

(a) Propose $\widetilde{\ddot{Y}}_{c,g1}$ from a random walk proposal density $q(\ddot{Y}_{c,g1}|\ddot{Y}^{(t-1)}_{c,g1})$.

(b) Calculate the implied residuals from activity intensity equations, $\widetilde{\xi}_{c,g1} = \widetilde{\ddot{Y}}_{c,g1} - \lambda^{(t-1)}_c W_{12,g} Y_{c,g2} - X_{g1}\beta^{(t-1)}_{1,c} - (W_{11,g}X_{g1} + W_{12,g}X_{g2})\beta^{(t-1)}_{2,c} - Z^{(t-1)}_{g1}\rho^{(t-1)}_{1,c} - (W_{11,g}Z^{(t-1)}_{g1} + W_{12,g}Z^{(t-1)}_{g2})\rho^{(t-1)}_{2,c} - l_{g1}\alpha^{(t-1)}_{c,g}$ and $\xi^{(t-1)}_{c,g2} = (I_{m_g - m_{g1}} - \lambda^{(t-1)}_c W_{22,g}) \times Y_{c,g2} - X_{g2}\beta^{(t-1)}_{1,c} - (W_{21,g}X_{g1} + W_{22,g}X_{g2})\beta^{(t-1)}_{2,c} - Z^{(t-1)}_{g2}\rho^{(t-1)}_{1,c} - (W_{21,g}Z^{(t-1)}_{g1} + W_{22,g}Z^{(t-1)}_{g2})\rho^{(t-1)}_{2,c} - l_{g2}\alpha^{(t-1)}_{c,g}$. Denote $\widetilde{\xi}_{c,g} = (\widetilde{\xi}'_{c,g1}, \xi^{(t-1)'}_{c,g2})'$. To make a distinction, denote $\xi^{(t-1)}_{c,g} = (\xi^{(t-1)'}_{c,g1}, \xi^{(t-1)'}_{c,g2})'$ with $\xi^{(t-1)}_{c,g1} = \ddot{Y}^{(t-1)}_{c,g1} - \lambda^{(t-1)}_c W_{12,g} Y_{c,g2} -$

$X_{g1}\beta_{1,c}^{(t-1)} - (W_{11,g}X_{g1} + W_{12,g}X_{g2})\beta_{2,c}^{(t-1)} - Z_{g1}^{(t-1)}\rho_{1,c}^{(t-1)} - (W_{11,g}Z_{g1}^{(t-1)} + W_{12,g}Z_{g2}^{(t-1)})\rho_{2,c}^{(t-1)} - l_{g1}\alpha_{c,g}^{(t-1)}$. Also calculate $\xi_{uc,g}^{(t-1)} = (I_{m_g} - \lambda_{uc}^{(t-1)}W_g)Y_{uc,g} - X_g\beta_{1,uc}^{(t-1)} - W_gX_g\beta_{2,uc}^{(t-1)} - Z_g^{(t-1)}\rho_{1,uc}^{(t-1)} - W_gZ_g^{(t-1)}\rho_{2,uc}^{(t-1)} - l_{m_g}\alpha_{uc,g}^{(t-1)}$.

(c) Given $\widetilde{\xi}_{c,g}$ and $\xi_{uc,g}^{(t-1)}$ from (b), simulate an auxiliary network $\widetilde{W}_g$ by the M–H algorithm based on

$$P(W_g|\widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}) = \frac{\exp(\mathrm{V}(W_g, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}))}{\sum_W \exp(\mathrm{V}(W, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}))}$$

starting from the observed W_g, that is, set the initial auxiliary network $\widetilde{W}_g^{(0)}$ equal to W_g. Update $\widetilde{W}_g^{(0)}$ by looping all entries of $\widetilde{W}_g^{(0)}$ with either a local or a global update:

(i) local update: for all ij, we propose $\widetilde{w}_{ij,g}^{(r)} = 1 - \widetilde{w}_{ij,g}^{(r-1)}$. With the probability

$$\alpha_{\mathrm{MH,local}}(\widetilde{w}_{ij,g}^{(r)}, \widetilde{w}_{ij,g}^{(r-1)}) = \min\left\{\frac{\exp(\mathrm{V}(\widetilde{w}_{ij,g}^{(r)}, \widetilde{W}_{-ij,g}^{(r-1)}, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}))}{\exp(\mathrm{V}(\widetilde{w}_{ij,g}^{(r-1)}, \widetilde{W}_{-ij,g}^{(r-1)}, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}))}, 1\right\},$$

accept $\widetilde{w}_{ij,g}^{(r)}$; otherwise, set $\widetilde{w}_{ij,g}^{(r)} = \widetilde{w}_{ij,g}^{(r-1)}$.

(ii) global update: despite implementation of the local step for most of the time, with a probability P_{inv}, we propose $\widetilde{W}_g^{(r)} = l_{m_g}l'_{m_g} - I_{m_g} - \widetilde{W}_g^{(r-1)}$. With the probability

$$\alpha_{\mathrm{MH,large}}(\widetilde{W}_g^{(r)}, \widetilde{W}_g^{(r-1)}) = \min\left\{\frac{\exp(\mathrm{V}(\widetilde{W}_g^{(r)}, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}))}{\exp(\mathrm{V}(\widetilde{W}_g^{(r-1)}, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}))}, 1\right\},$$

accept $\widetilde{W}_g^{(r)}$; otherwise, set $\widetilde{W}_g^{(r)} = \widetilde{W}_g^{(r-1)}$.

Notice that when proposing any change on network link, the auxiliary uncensored activity outcome vector $\widetilde{Y}_{uc,g}^*(\widetilde{W}_g, \widetilde{\xi}_{uc,g})$ and the censored activity outcome vector $\widetilde{Y}_{c,g}^*(\widetilde{W}_g, \widetilde{\xi}_{c,g})$ are implicitly calculated from the reduced form of equation (4) and the contraction mapping of equation (5) in the main text. These auxiliary outcome vectors will be used in evaluating the value of function V in the above acceptance probabilities, $\alpha_{\mathrm{MH,local}}$ and $\alpha_{\mathrm{MH,large}}$. Repeat the described updating procedure $R = 2$ times and obtain the realization of $\widetilde{W}_g^{(R)}$ as the simulation result. If the obtained auxiliary network does not belong to $\Omega_{\bar{n},g}$, reject it and rerun the simulation.

(d) With the acceptance probability equal to

$$\begin{aligned}
&\alpha_{\mathrm{MH}}(\tilde{\ddot{Y}}_{c,g1}, \ddot{Y}_{c,g1}^{(t-1)}) \\
&= \min\left\{ \frac{P(\tilde{\ddot{Y}}_{c,g1}, Y_{c,g}, Y_{uc,g}, W_g | \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, Z_g^{(t-1)})}{P(\ddot{Y}_{c,g1}^{(t-1)}, Y_{c,g}, Y_{uc,g}, W_g | \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, Z_g^{(t-1)})} \right. \\
&\quad \left. \times \frac{P(\tilde{W}_g | \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, Z_g^{(t-1)})}{P(\tilde{W}_g | \tilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, Z_g^{(t-1)})} \cdot \frac{I(\tilde{\ddot{Y}}_{c,g1} < 0)}{I(\ddot{Y}_{c,g1}^{(t-1)} < 0)}, 1 \right\} \\
&= \min\left\{ \frac{f(\tilde{\xi}_{c,g} - \sigma_{\xi_{ucc,g}}^{(t-1)} (\sigma_{\xi_{c,g}}^{2(t-1)})^{-1} \xi_{uc,g}^{(t-1)}) \exp(\mathrm{V}(W_g, \tilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, Z_g^{(t-1)}))}{f(\xi_{c,g}^{(t-1)} - \sigma_{\xi_{ucc,g}}^{(t-1)} (\sigma_{\xi_{c,g}}^{2(t-1)})^{-1} \xi_{uc,g}^{(t-1)}) \exp(\mathrm{V}(W_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, Z_g^{(t-1)}))} \right. \\
&\quad \left. \times \frac{\exp(\mathrm{V}(\tilde{W}_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, Z_g^{(t-1)}))}{\exp(\mathrm{V}(\tilde{W}_g, \tilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, Z_g^{(t-1)}))} \cdot \frac{I(\tilde{\ddot{Y}}_{c,g1} < 0)}{I(\ddot{Y}_{c,g1}^{(t-1)} < 0)}, 1 \right\}
\end{aligned}$$

set $\ddot{Y}_{c,g1}^{(t)} = \tilde{\ddot{Y}}_{c,g1}$. Otherwise, set $\ddot{Y}_{c,g1}^{(t)} = \ddot{Y}_{c,g1}^{(t-1)}$.

Step II-1. Simulate $z_{i,g}^{(t)}$ from $P(z_{i,g} | \ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g, Z_{-i,g}^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)},)$ by the double M–H algorithm, for $i = 1, \ldots, m_g$; $g = 1, \ldots, G$.

(a) Propose $\tilde{z}_{i,g}$ from a random walk proposal density $q(z_{i,g} | z_{i,g}^{(t-1)})$.
Denote $\tilde{Z}_g = (z_{1,g}^{(t)}, \ldots, z_{i-1,g}^{(t)}, \tilde{z}_{i,g}, z_{i+1,g}^{(t-1)} \cdots, z_{m_g,g}^{(t-1)})$ and $Z_g^{(t-1)} = (z_{1,g}^{(t)}, \ldots, z_{i-1,g}^{(t)}, z_{i,g}^{(t-1)}, z_{i+1,g}^{(t-1)}, \ldots, z_{m_g,g}^{(t-1)})$.

(b) Calculate the implied residuals from the activity intensity equations $\tilde{\xi}_{c,g1} = \ddot{Y}_{c,g1}^{(t)} - \lambda_c^{(t-1)} W_{12,g} Y_{c,g2} - X_{g1} \beta_{1,c}^{(t-1)} - (W_{11,g} X_{g1} + W_{12,g} X_{g2}) \beta_{2,c}^{(t-1)} - \tilde{Z}_{g1} \rho_{1,c}^{(t-1)} - (W_{11,g} \tilde{Z}_{g1} + W_{12,g} \tilde{Z}_{g2}) \rho_{2,c}^{(t-1)} - l_{g1} \alpha_{c,g}^{(t-1)}$ and $\tilde{\xi}_{c,g2} = (I_{m_g - m_{g1}} - \lambda_c^{(t-1)} W_{22,g}) Y_{c,g2} - X_{g2} \beta_{1,c}^{(t-1)} - (W_{21,g} X_{g1} + W_{22,g} X_{g2}) \beta_{2,c}^{(t-1)} - \tilde{Z}_{g2} \rho_{1,c}^{(t-1)} - (W_{21,g} \tilde{Z}_{g1} + W_{22,g} \tilde{Z}_{g2}) \rho_{2,c}^{(t-1)} - l_{g2} \alpha_{c,g}^{(t-1)}$. Denote $\tilde{\xi}_{c,g} = (\tilde{\xi}'_{c,g1}, \tilde{\xi}'_{c,g2})'$. To make a distinction, denote $\xi_{c,g}^{(t-1)} = (\xi_{c,g1}^{(t-1)'}, \xi_{c,g2}^{(t-1)'})'$ with $\xi_{c,g1}^{(t-1)} = \ddot{Y}_{c,g1}^{(t)} - \lambda_c^{(t-1)} W_{12,g} Y_{c,g2} - X_{g1} \beta_{1,c}^{(t-1)} - (W_{11,g} X_{g1} + W_{12,g} X_{g2}) \beta_{2,c}^{(t-1)} - Z_{g1}^{(t-1)} \rho_{1,c}^{(t-1)} - (W_{11,g} Z_{g1}^{(t-1)} + W_{12,g} Z_{g2}^{(t-1)}) \rho_{2,c}^{(t-1)} - l_{g1} \alpha_{c,g}^{(t-1)}$ and $\xi_{c,g2}^{(t-1)} = (I_{m_g - m_{g1}} - \lambda_c^{(t-1)} W_{22,g}) Y_{c,g2} - X_{g2} \beta_{1,c}^{(t-1)} - (W_{21,g} X_{g1} + W_{22,g} X_{g2}) \beta_{2,c}^{(t-1)} - Z_{g2}^{(t-1)} \rho_{1,c}^{(t-1)} - (W_{21,g} Z_{g1}^{(t-1)} + W_{22,g} Z_{g2}^{(t-1)}) \rho_{2,c}^{(t-1)} - l_{g2} \alpha_{c,g}^{(t-1)}$.
Also calculate $\tilde{\xi}_{uc,g} = (I_{m_g} - \lambda_{uc}^{(t-1)} W_g) Y_{uc,g} - X_g \beta_{1,uc}^{(t-1)} - W_g X_g \beta_{2,uc}^{(t-1)} - \tilde{Z}_g \rho_{1,uc}^{(t-1)} - W_g \tilde{Z}_g \rho_{2,uc}^{(t-1)} - l_{m_g} \alpha_{uc,g}^{(t-1)}$ and $\xi_{uc,g}^{(t-1)} = (I_{m_g} - \lambda_{uc}^{(t-1)} W_g) Y_{uc,g} - X_g \beta_{1,uc}^{(t-1)} - W_g X_g \beta_{2,uc}^{(t-1)} - Z_g^{(t-1)} \rho_{1,uc}^{(t-1)} - W_g Z_g^{(t-1)} \rho_{2,uc}^{(t-1)} - l_{m_g} \alpha_{uc,g}^{(t-1)}$.

(c) Given $\tilde{\xi}_{c,g}$ and $\tilde{\xi}_{uc,g}$ from (b), simulate an auxiliary network $\tilde{W}_g$ by R runs of the M–H algorithm based on

$$P(W_g | \tilde{\xi}_{c,g}, \tilde{\xi}_{uc,g}, \tilde{Z}_g, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)})$$

$$= \frac{\exp\big(\mathrm{V}\big(W_g, \widetilde{\xi}_{c,g}, \widetilde{\xi}_{uc,g}, \widetilde{Z}_g, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}\big)\big)}{\sum_W \exp\big(\mathrm{V}\big(W, \widetilde{\xi}_{c,g}, \widetilde{\xi}_{uc,g}, \widetilde{Z}_g, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}\big)\big)}$$

starting from the observed W_g. See details in Step I, part (c). If the obtained auxiliary network $\widetilde{W}_g$ does not belong to $\Omega_{\bar{n},g}$, reject it and rerun the simulation.

(d) With the acceptance probability equals to

$$\begin{aligned}
&\alpha_{\mathrm{MH}}\big(\widetilde{z}_{i,g}, z_{i,g}^{(t-1)}\big) \\
&= \min\Bigg\{\frac{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g|\theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, \widetilde{Z}_g\big)}{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g|\theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, Z_g^{(t-1)}\big)} \\
&\quad \times \frac{P\big(\widetilde{W}_g|\xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, Z_g^{(t-1)}\big)}{P\big(\widetilde{W}_g|\widetilde{\xi}_{c,g}, \widetilde{\xi}_{uc,g}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, \widetilde{Z}_g\big)} \cdot \frac{\mathcal{N}_{\bar{\ell}}(\widetilde{z}_{i,g}; \mu_{z,g}, I_{\bar{\ell}})}{\mathcal{N}_{\bar{\ell}}\big(z_{i,g}^{(t-1)}; \mu_{z,g}, I_{\bar{\ell}}\big)}, 1\Bigg\} \\
&= \min\Bigg\{\frac{f\big(\widetilde{\xi}_{c,g} - \sigma_{\xi_{ucc,g}}^{(t-1)}\big(\sigma_{\xi_{c,g}}^{2(t-1)}\big)^{-1}\widetilde{\xi}_{uc,g}\big)\exp\big(\mathrm{V}\big(W_g, \widetilde{\xi}_{c,g}, \widetilde{\xi}_{uc,g}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, \widetilde{Z}_g\big)\big)}{f\big(\xi_{c,g}^{(t-1)} - \sigma_{\xi_{ucc,g}}^{(t-1)}\big(\sigma_{\xi_{c,g}}^{2(t-1)}\big)^{-1}\xi_{uc,g}^{(t-1)}\big)\exp\big(\mathrm{V}\big(W_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, Z_g^{(t-1)}\big)\big)} \\
&\quad \times \frac{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, Z_g^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \widetilde{\xi}_{c,g}, \widetilde{\xi}_{uc,g}, \theta_g^{(t-1)}, \alpha_{c,g}^{(t-1)}, \alpha_{uc,g}^{(t-1)}, \widetilde{Z}_g\big)\big)} \cdot \frac{\mathcal{N}_{\bar{\ell}}(\widetilde{z}_{i,g}; \mu_{z,g}, I_{\bar{\ell}})}{\mathcal{N}_{\bar{\ell}}\big(z_{i,g}^{(t-1)}; \mu_{z,g}, I_{\bar{\ell}}\big)}, 1\Bigg\}
\end{aligned}$$

set $z_{i,g}^{(t)} = \widetilde{z}_{i,g}$. Otherwise, set $z_{i,g}^{(t)} = z_{i,g}^{(t-1)}$.

Step II-2. Simulate $\mu_{z,g}^{(t)}$ by a standard Gibbs step, from $P(\mu_{z,g}|\ddot{Y}_{c,g1}^{(t)}, Y_{uc,g}, Y_{c,g}, W_g, \theta_g^{(t-1)}, \alpha_{uc,g}^{(t-1)}, \alpha_{c,g}^{(t-1)}, Z_g^{(t)}) \propto \mathcal{N}_{\bar{\ell}}(\frac{m_g \bar{Z}_g^{(t)}}{m_g+1/\varsigma^2}, \frac{1}{m_g+1/\varsigma^2} I_{\bar{\ell}})$, where $\bar{Z}_g^{(t)} = \frac{1}{m_g}\sum_{i=1}^{m_g} z_{i,g}^{(t)}$.

Step III. Simulate $\phi^{(t)}$ from $P(\phi|\{\ddot{Y}_{c,g1}^{(t)}\}, \{Y_{c,g}\}, \{Y_{uc,g}\}, \{W_g\}, \{Z_g^{(t)}\}, \Upsilon^{(t-1)})$ by the double M–H algorithm, where $\Upsilon^{(t-1)}$ denotes the remaining parameters evaluated at the $(t-1)$th iteration, including $\{\theta_g^{(t-1)}\}$, $\{\alpha_{uc,g}^{(t-1)}\}$, and $\{\alpha_{c,g}^{(t-1)}\}$.

(a) Propose $\widetilde{\phi}$ from a random walk proposal density $q(\phi|\phi^{(t-1)})$.

(b) For $g = 1, \ldots, G$, calculate the implied residuals from the activity intensity equations $\xi_{c,g1}^{(t-1)} = \ddot{Y}_{c,g1}^{(t)} - \lambda_c^{(t-1)} W_{12,g} Y_{c,g2} - X_{g1}\beta_{1,c}^{(t-1)} - (W_{11,g}X_{g1} + W_{12,g}X_{g2})\beta_{2,c}^{(t-1)} - Z_{g1}^{(t)}\rho_{1,c}^{(t-1)} - (W_{11,g}Z_{g1}^{(t)} + W_{12,g}Z_{g2}^{(t)})\rho_{2,c}^{(t-1)} - l_{g1}\alpha_{c,g}^{(t-1)}$ and $\xi_{c,g2}^{(t-1)} = (I_{m_g-m_{g1}} - \lambda_c^{(t-1)} W_{22,g}) Y_{c,g2} - X_{g2}\beta_{1,c}^{(t-1)} - (W_{21,g}X_{g1} + W_{22,g}X_{g2})\beta_{2,c}^{(t-1)} - Z_{g2}^{(t)}\rho_{1,c}^{(t-1)} - (W_{21,g}Z_{g1}^{(t)} + W_{22,g}Z_{g2}^{(t)})\rho_{2,c}^{(t-1)} - l_{g2}\alpha_{c,g}^{(t-1)}$. Denote $\xi_{c,g}^{(t-1)} = (\xi_{c,g1}^{(t-1)\prime}, \xi_{c,g2}^{(t-1)\prime})'$. Also calculate $\xi_{uc,g}^{(t-1)} = (I_{m_g} - \lambda_{uc}^{(t-1)} W_g) Y_{uc,g} - X_g\beta_{1,uc}^{(t-1)} - W_g X_g \beta_{2,uc}^{(t-1)} - Z_g^{(t)}\rho_{1,uc}^{(t-1)} - W_g Z_g^{(t)}\rho_{2,uc}^{(t-1)} - l_{m_g}\alpha_{uc,g}^{(t-1)}$.

(c) For $g = 1, \ldots, G$, given $\xi_{c,g}^{(t-1)}$ and $\xi_{uc,g}^{(t-1)}$ from (b), simulate an auxiliary network $\widetilde{W}_g$ by R runs of the M–H algorithm based on

$$P\big(W_g|\xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \widetilde{\phi}, Y^{(t-1)}\big) = \frac{\exp\big(\mathrm{V}\big(W_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \widetilde{\phi}, Y^{(t-1)}\big)\big)}{\sum_W \exp\big(\mathrm{V}\big(W, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \widetilde{\phi}, Y^{(t-1)}\big)\big)}$$

starting from W_g. See details in Step I, part (c). If the obtained auxiliary network $\widetilde{W}_g$ does not belong to $\Omega_{\bar{n},g}$, reject it and rerun the simulation.

(d) With the acceptance probability equal to

$$\begin{aligned}
&\alpha_{\mathrm{MH}}\big(\widetilde{\phi}, \phi^{(t-1)}\big) \\
&= \min\Bigg\{\prod_{g=1}^{G}\Bigg(\frac{P\big(W_g|\xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \widetilde{\phi}, Y^{(t-1)}\big)}{P\big(W_g|\xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t-1)}, Y^{(t-1)}\big)} \\
&\quad\times \frac{P\big(\widetilde{W}_g|\xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t-1)}, Y^{(t-1)}\big)}{P\big(\widetilde{W}_g|\xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \widetilde{\phi}, Y^{(t-1)}\big)}\Bigg) \\
&\quad\times \frac{\mathcal{N}_{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2}(\widetilde{\phi}|\phi_0, \Phi_0 I_{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2})}{\mathcal{N}_{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2}\big(\phi^{(t-1)}|\phi_0, \Phi_0 I_{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2}\big)} \cdot \frac{I(\widetilde{\phi} \in O)}{I\big(\phi^{(t-1)} \in O\big)}, 1\Bigg\} \\
&= \min\Bigg\{\prod_{g=1}^{G}\Bigg(\frac{\exp\big(\mathrm{V}\big(W_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \widetilde{\phi}, Y^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(W_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t-1)}, Y^{(t-1)}\big)\big)} \\
&\quad\times \frac{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t-1)}, Y^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \widetilde{\phi}, Y^{(t-1)}\big)\big)}\Bigg) \\
&\quad\times \frac{\mathcal{N}_{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2}(\widetilde{\phi}|\phi_0, \Phi_0 I_{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2})}{\mathcal{N}_{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2}\big(\phi^{(t-1)}|\phi_0, \Phi_0 I_{2\bar{s}+\bar{q}+\bar{\ell}+\bar{h}+2}\big)} \cdot \frac{I(\widetilde{\phi} \in O)}{I\big(\phi^{(t-1)} \in O\big)}, 1\Bigg\},
\end{aligned}$$

set $\phi^{(t)} = \widetilde{\phi}$. Otherwise, set $\phi^{(t)} = \phi^{(t-1)}$.

Step IV. Simulate $\lambda_{uc}^{(t)}$ from $P(\lambda_{uc}|\{\ddot{Y}_{c,g1}^{(t)}\}, \{Y_{c,g}\}, \{Y_{uc,g}\}, \{W_g\}, \{Z_g^{(t)}\}, \phi^{(t)}, Y^{(t-1)})$ by the double M–H algorithm.

(a) Propose $\widetilde{\lambda}_{uc}$ from a random walk proposal density $q(\lambda_{uc}|\lambda_{uc}^{(t-1)})$.

(b) For $g = 1, \ldots, G$, calculate the implied residuals from the activity intensity equations $\widetilde{\xi}_{uc,g} = (I_{m_g} - \widetilde{\lambda}_{uc} W_g) Y_{uc,g} - X_g \beta_{1,uc}^{(t-1)} - W_g X_g \beta_{2,uc}^{(t-1)} - Z_g^{(t)} \rho_{1,uc}^{(t-1)} - W_g Z_g^{(t)} \rho_{2,uc}^{(t-1)} - l_{m_g} \alpha_{uc,g}^{(t-1)}$ and $\xi_{uc,g}^{(t-1)} = (I_{m_g} - \lambda_{uc}^{(t-1)} W_g) Y_{uc,g} - X_g \beta_{1,uc}^{(t-1)} - W_g X_g \beta_{2,uc}^{(t-1)} - Z_g^{(t)} \rho_{1,uc}^{(t-1)} - W_g Z_g^{(t)} \rho_{2,uc}^{(t-1)} - l_{m_g} \alpha_{uc,g}^{(t-1)}$. Also calculate $\xi_{c,g1}^{(t-1)} = \ddot{Y}_{c,g1}^{(t)} - \lambda_c^{(t-1)} W_{12,g} Y_{c,g2} - X_{g1} \beta_{1,c}^{(t-1)} - (W_{11,g} X_{g1} + W_{12,g} X_{g2}) \beta_{2,c}^{(t-1)} - Z_{g1}^{(t)} \rho_{1,c}^{(t-1)} - (W_{11,g} Z_{g1}^{(t)} + W_{12,g} Z_{g2}^{(t)}) \rho_{2,c}^{(t-1)} - l_{g1} \alpha_{c,g}^{(t-1)}$ and $\xi_{c,g2}^{(t-1)} = (I_{m_g - m_{g1}} - \lambda_c^{(t-1)} W_{22,g}) Y_{c,g2} -$

$X_{g2}\beta_{1,c}^{(t-1)} - (W_{21,g}X_{g1} + W_{22,g}X_{g2})\beta_{2,c}^{(t-1)} - Z_{g2}^{(t)}\rho_{1,c}^{(t-1)} - (W_{21,g}Z_{g1}^{(t)} + W_{22,g}Z_{g2}^{(t)}) \times \rho_{2,c}^{(t-1)} - l_{g2}\alpha_{c,g}^{(t-1)}$. Denote $\xi_{c,g}^{(t-1)} = (\xi_{c,g1}^{(t-1)\prime}, \xi_{c,g2}^{(t-1)\prime})'$.

(c) For $g = 1, \ldots, G$, given $\widetilde{\xi}_{uc,g}$ and $\xi_{c,g}^{(t-1)}$ in (b), simulate an auxiliary network $\widetilde{W}_g$ by R runs of the M–H algorithm based on

$$P\big(W_g|\widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \widetilde{\lambda}_{uc}, Y^{(t-1)}\big) = \frac{\exp\big(\mathrm{V}\big(W_g, \widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \widetilde{\lambda}_{uc}, Y^{(t-1)}\big)\big)}{\sum_W \exp\big(\mathrm{V}\big(W, \widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \widetilde{\lambda}_{uc}, Y^{(t-1)}\big)\big)}$$

starting from W_g. See details in Step I, part (c). If the obtained auxiliary network $\widetilde{W}_g$ does not belong to $\Omega_{\bar{n},g}$, reject it and rerun the simulation.

(d) With the acceptance probability equal to

$$\begin{aligned}
&\alpha_{\mathrm{MH}}\big(\widetilde{\lambda}_{uc}, \lambda_{uc}^{(t-1)}\big) \\
&= \min\Bigg\{\prod_{g=1}^{G}\Bigg(\frac{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g|Z_g^{(t)}, \phi^{(t)}, \widetilde{\lambda}_{uc}, Y^{(t-1)}\big)}{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g|Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t-1)}, Y^{(t-1)}\big)} \\
&\quad \times \frac{P\big(\widetilde{W}_g|\xi_{uc,g}^{(t-1)}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t-1)}, Y^{(t-1)}\big)}{P\big(\widetilde{W}_g|\widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \widetilde{\lambda}_{uc}, Y^{(t-1)}\big)}\Bigg) \\
&\quad \times \frac{I\big(\widetilde{\lambda}_{uc} \in [-1/\Delta_{uc,G}, 1/\Delta_{uc,G}]\big)}{I\big(\lambda_{uc}^{(t-1)} \in [-1/\Delta_{uc,G}, 1/\Delta_{uc,G}]\big)}, 1\Bigg\} \\
&= \min\Bigg\{\prod_{g=1}^{G}\Bigg(\frac{f\big(\xi_{c,g}^{(t-1)} - \sigma_{\xi_{ucc,g}}^{(t-1)}\big(\sigma_{\xi_{c,g}}^{2(t-1)}\big)^{-1}\widetilde{\xi}_{uc,g}\big)}{f\big(\xi_{c,g}^{(t-1)} - \sigma_{\xi_{ucc,g}}^{(t-1)}\big(\sigma_{\xi_{c,g}}^{2(t-1)}\big)^{-1}\xi_{uc,g}^{(t-1)}\big)} \cdot \frac{|I_{m_g} - \widetilde{\lambda}_{uc}W_g|}{|I_{m_g} - \lambda_{uc}^{(t-1)}W_g|} \cdot \frac{f(\widetilde{\xi}_{uc,g})}{f\big(\xi_{uc,g}^{(t-1)}\big)} \\
&\quad \times \frac{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t-1)}, Y^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \xi_{c,g}^{(t-1)}, \widetilde{\xi}_{uc,g}, Z_g^{(t)}, \phi^{(t)}, \widetilde{\lambda}_{uc}, Y^{(t-1)}\big)\big)}\Bigg) \\
&\quad \times \frac{I\big(\widetilde{\lambda}_{uc} \in [-1/\Delta_{uc}, 1/\Delta_{uc}]\big)}{I\big(\lambda_{uc}^{(t-1)} \in [-1/\Delta_{uc}, 1/\Delta_{uc}]\big)}, 1\Bigg\},
\end{aligned}$$

set $\lambda_{uc}^{(t)} = \widetilde{\lambda}_{uc}$. Otherwise, set $\lambda_{uc}^{(t)} = \lambda_{uc}^{(t-1)}$.

Step V. Simulate $\lambda_c^{(t)}$ from $P(\lambda_c|\{\ddot{Y}_{c,g1}^{(t)}\}, \{Y_{c,g}\}, \{Y_{uc,g}\}, \{W_g\}, \{Z_g^{(t)}\}, \phi^{(t)}, \lambda_{uc}^{(t)}, Y^{(t-1)})$ by the double M–H algorithm.

(a) Propose $\widetilde{\lambda}_c$ from a random walk proposal density $q(\lambda_c|\lambda_c^{(t-1)})$.

(b) For $g = 1, \ldots, G$, calculate the implied residuals from activity intensity equations $\widetilde{\xi}_{c,g1} = \ddot{Y}_{c,g1}^{(t)} - \widetilde{\lambda}_c W_{12,g} Y_{c,g2} - X_{g1}\beta_{1,c}^{(t-1)} - (W_{11,g}X_{g1} + W_{12,g}X_{g2})\beta_{2,c}^{(t-1)} - Z_{g1}^{(t)}\rho_{1,c}^{(t-1)} - (W_{11,g}Z_{g1}^{(t)} + W_{12,g}Z_{g2}^{(t)})\rho_{2,c}^{(t-1)} - l_{g1}\alpha_{c,g}^{(t-1)}$ and $\widetilde{\xi}_{c,g2} = (I_{m_g - m_{g1}} - \widetilde{\lambda}_c W_{22,g})Y_{c,g2} -$

$X_{g2}\beta_{1,c}^{(t-1)} - (W_{21,g}X_{g1} + W_{22,g}X_{g2})\beta_{2,c}^{(t-1)} - Z_{g2}^{(t)}\rho_{1,c}^{(t-1)} - (W_{21,g}Z_{g1}^{(t)} + W_{22,g}Z_{g2}^{(t)}) \times \rho_{2,c}^{(t-1)} - l_{g2}\alpha_{c,g}^{(t-1)}$. Denote $\widetilde{\xi}_{c,g} = (\widetilde{\xi}'_{c,g1}, \widetilde{\xi}'_{c,g2})'$. To make a distinction, denote $\xi_{c,g}^{(t-1)} = (\xi_{c,g1}^{(t-1)'}, \xi_{c,g2}^{(t-1)'})'$ with $\xi_{c,g1}^{(t-1)'}$ and $\xi_{c,g2}^{(t-1)'}$ calculated based on $\lambda_c^{(t-1)}$. Also calculate $\xi_{uc,g}^{(t-1)} = (I_{m_g} - \lambda_{uc}^{(t)}W_g)Y_{uc,g} - X_g\beta_{1,uc}^{(t-1)} - W_gX_g\beta_{2,uc}^{(t-1)} - Z_g^{(t)}\rho_{1,uc}^{(t-1)} - W_gZ_g^{(t)}\rho_{2,uc}^{(t-1)} - l_{m_g}\alpha_{uc,g}^{(t-1)}$.

(c) For $g = 1, \dots, G$, given $\widetilde{\xi}_{c,g}$ and $\xi_{uc,g}^{(t-1)}$ in (b), simulate an auxiliary network $\widetilde{W}_g$ by R runs of the M–H algorithm based on

$$P\big(W_g|\widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \widetilde{\lambda}_c, Y^{(t-1)}\big) = \frac{\exp\big(\mathrm{V}\big(W_g, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \widetilde{\lambda}_c, Y^{(t-1)}\big)\big)}{\sum_W \exp\big(\mathrm{V}\big(W, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \widetilde{\lambda}_c, Y^{(t-1)}\big)\big)}$$

starting from W_g. See details in Step I, part (c). If the obtained auxiliary network $\widetilde{W}_g$ does not belong to $\Omega_{\bar{n},g}$, reject it and rerun the simulation.

(d) With the acceptance probability equal to

$$\begin{aligned}
&\alpha_{\mathrm{MH}}\big(\widetilde{\lambda}_c, \lambda_c^{(t-1)}\big) \\
&= \min\Bigg\{\prod_{g=1}^{G}\Bigg(\frac{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g|Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \widetilde{\lambda}_c, Y^{(t-1)}\big)}{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g|Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t-1)}, Y^{(t-1)}\big)} \\
&\quad \times \frac{P\big(\widetilde{W}_g|\xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t-1)}, Y^{(t-1)}\big)}{P\big(\widetilde{W}_g|\widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \widetilde{\lambda}_c, Y^{(t-1)}\big)}\Bigg) \\
&\quad \times \frac{I\big(\tilde{\lambda}_c \in [-1/\Delta_{c,G}, 1/\Delta_{c,G}]\big)}{I\big(\lambda_c^{(t-1)} \in [-1/\Delta_{c,G}, 1/\Delta_{c,G}]\big)}, 1\Bigg\} \\
&= \min\Bigg\{\prod_{g=1}^{G}\Bigg(\frac{|I_{m_g-m_{g1}} - \widetilde{\lambda}_c W_{22,g}|f\big(\widetilde{\xi}_{c,g} - \sigma_{\xi_{ucc,g}}^{(t-1)}\big(\sigma_{\xi_{c,g}}^{2(t-1)}\big)^{-1}\xi_{uc,g}^{(t-1)}\big)}{|I_{m_g-m_{g1}} - \lambda_c^{(t-1)}W_{22,g}|f\big(\xi_{c,g}^{(t-1)} - \sigma_{\xi_{ucc,g}}^{(t-1)}\big(\sigma_{\xi_{c,g}}^{2(t-1)}\big)^{-1}\xi_{uc,g}^{(t-1)}\big)} \\
&\quad \times \frac{\exp\big(\mathrm{V}\big(W_g, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \widetilde{\lambda}_c, Y^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(W_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t-1)}, Y^{(t-1)}\big)\big)} \\
&\quad \times \frac{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t-1)}, Y^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \widetilde{\lambda}_c, Y^{(t-1)}\big)\big)}\Bigg) \\
&\quad \times \frac{I\big(\widetilde{\lambda}_c \in [-1/\Delta_c, 1/\Delta_c]\big)}{I\big(\lambda_c^{(t-1)} \in [-1/\Delta_c, 1/\Delta_c]\big)}, 1\Bigg\},
\end{aligned}$$

set $\lambda_c^{(t)} = \widetilde{\lambda}_c$. Otherwise, set $\lambda_c^{(t)} = \lambda_c^{(t-1)}$.

Step VI. Simulate $\beta_{uc}^{(t)}$ from $P(\beta_{uc}|\{\ddot{Y}_{c,g1}^{(t)}\}, \{Y_{c,g}\}, \{Y_{uc,g}\}, \{W_g\}, \{Z_g^{(t)}\}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, Y^{(t-1)})$ by the double M–H algorithm.

(a) Propose $\widetilde{\beta}_{uc}$ from a random walk proposal density $q(\beta_{uc}|\beta_{uc}^{(t-1)})$.

(b) For $g = 1, \ldots, G$, calculate the implied residuals from activity intensity equations $\widetilde{\xi}_{uc,g} = (I_{m_g} - \lambda_{uc}^{(t)} W_g)Y_{uc,g} - X_g\widetilde{\beta}_{1,uc} - W_g X_g \widetilde{\beta}_{2,uc} - Z_g^{(t)}\rho_{1,uc}^{(t-1)} - W_g Z_g^{(t)}\rho_{2,uc}^{(t-1)} - l_{m_g}\alpha_{uc,g}^{(t-1)}$ and $\xi_{uc,g}^{(t-1)} = (I_{m_g} - \lambda_{uc}^{(t)} W_g)Y_{uc,g} - X_g\beta_{1,uc}^{(t-1)} - W_g X_g \beta_{2,uc}^{(t-1)} - Z_g^{(t)}\rho_{1,uc}^{(t-1)} - W_g Z_g^{(t)}\rho_{2,uc}^{(t-1)} - l_{m_g}\alpha_{uc,g}^{(t-1)}$.

Also calculate $\xi_{c,g1}^{(t-1)} = \ddot{Y}_{c,g1}^{(t)} - \lambda_c^{(t)} W_{12,g} Y_{c,g2} - X_{g1}\beta_{1,c}^{(t-1)} - (W_{11,g}X_{g1} + W_{12,g}X_{g2})\beta_{2,c}^{(t-1)} - Z_{g1}^{(t)}\rho_{1,c}^{(t-1)} - (W_{11,g}Z_{g1}^{(t)} + W_{12,g}Z_{g2}^{(t)})\rho_{2,c}^{(t-1)} - l_{g1}\alpha_{c,g}^{(t-1)}$ and $\xi_{c,g2}^{(t-1)} = (I_{m_g - m_{g1}} - \lambda_c^{(t)} W_{22,g})Y_{c,g2} - X_{g2}\beta_{1,c}^{(t-1)} - (W_{21,g}X_{g1} + W_{22,g}X_{g2})\beta_{2,c}^{(t-1)} - Z_{g2}^{(t)}\rho_{1,c}^{(t-1)} - (W_{21,g}Z_{g1}^{(t)} + W_{22,g}Z_{g2}^{(t)})\rho_{2,c}^{(t-1)} - l_{g2}\alpha_{c,g}^{(t-1)}$. Denote $\xi_{c,g}^{(t-1)} = (\xi_{c,g1}^{(t-1)\prime}, \xi_{c,g2}^{(t-1)\prime})'$.

(c) For $g = 1, \ldots, G$, given $\widetilde{\xi}_{uc,g}$ and $\xi_{c,g}^{(t-1)}$ from (b), simulate an auxiliary network $\widetilde{W}_g$ by R runs of the M–H algorithm based on

$$P\big(W_g|\widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \widetilde{\beta}_c, Y^{(t-1)}\big) = \frac{\exp\big(\mathrm{V}\big(W_g, \widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \widetilde{\beta}_{uc}, Y^{(t-1)}\big)\big)}{\sum_W \exp\big(\mathrm{V}\big(W, \widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \widetilde{\beta}_{uc}, Y^{(t-1)}\big)\big)}$$

starting from W_g. See details in Step I, part (c). If the obtained auxiliary network $\widetilde{W}_g$ does not belong to $\Omega_{\bar{n},g}$, reject it and rerun the simulation.

(d) With the acceptance probability equal to

$$\begin{aligned}
&\alpha_{\mathrm{MH}}\big(\widetilde{\beta}_{uc}, \beta_{uc}^{(t-1)}\big) \\
&= \min\Bigg\{\prod_{g=1}^{G}\Bigg(\frac{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g|Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \widetilde{\beta}_{uc}, Y^{(t-1)}\big)}{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g|Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t-1)}, Y^{(t-1)}\big)} \\
&\quad \times \frac{P\big(\widetilde{W}_g|\xi_{uc,g}^{(t-1)}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t-1)}, Y^{(t-1)}\big)}{P\big(\widetilde{W}_g|\widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \widetilde{\beta}_{uc}, Y^{(t-1)}\big)}\Bigg) \\
&\quad \times \frac{\mathcal{N}_{2k}(\widetilde{\beta}_{uc}|\beta_0, B_0 I_{2k})}{\mathcal{N}_{2k}\big(\beta_{uc}^{(t-1)}|\beta_0, B_0 I_{2k}\big)}, 1\Bigg\} \\
&= \min\Bigg\{\prod_{g=1}^{G}\Bigg(\frac{f\big(\xi_{c,g}^{(t-1)} - \sigma_{\xi_{ucc,g}}^{(t-1)}\big(\sigma_{\xi_{c,g}}^{2(t-1)}\big)^{-1}\widetilde{\xi}_{uc,g}\big)}{f\big(\xi_{c,g}^{(t-1)} - \sigma_{\xi_{ucc,g}}^{(t-1)}\big(\sigma_{\xi_{c,g}}^{2(t-1)}\big)^{-1}\xi_{uc,g}^{(t-1)}\big)} \cdot \frac{f\big(\widetilde{\xi}_{uc,g}\big)}{f\big(\xi_{uc,g}^{(t-1)}\big)} \\
&\quad \times \frac{\exp\big(\mathrm{V}\big(W_g, \xi_{c,g}^{(t-1)}, \widetilde{\xi}_{uc,g}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \widetilde{\beta}_{uc}, Y^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(W_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t-1)}, Y^{(t-1)}\big)\big)}
\end{aligned}$$

$$\times \frac{\exp\big(\mathrm{V}(\widetilde{W}_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t-1)}, Y^{(t-1)})\big)}{\exp\big(\mathrm{V}(\widetilde{W}_g, \xi_{c,g}^{(t-1)}, \widetilde{\xi}_{uc,g}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \widetilde{\beta}_{uc}, Y^{(t-1)})\big)}\Bigg)$$
$$\times \frac{\mathcal{N}_{2k}(\widetilde{\beta}_{uc}|\beta_0, B_0 I_{2k})}{\mathcal{N}_{2k}\big(\beta_{uc}^{(t-1)}|\beta_0, B_0 I_{2k}\big)}, 1\Bigg\}$$

set $\beta_{uc}^{(t)} = \widetilde{\beta}_{uc}$. Otherwise, set $\beta_{uc}^{(t)} = \beta_{uc}^{(t-1)}$.

Step VII. Simulate $\beta_c^{(t)}$ from $P(\beta_c|\{\ddot{Y}_{c,g1}^{(t)}\}, \{Y_{c,g}\}, \{Y_{uc,g}\}, \{W_g\}, \{Z_g^{(t)}\}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, Y^{(t-1)})$ by the double M–H algorithm.

(a) Propose $\widetilde{\beta_c}$ from a random walk proposal density $q(\beta_c|\beta_c^{(t-1)})$.

(b) For $g = 1, \ldots, G$, calculate the implied residuals from activity intensity equations $\widetilde{\xi}_{c,g1} = \ddot{Y}_{c,g1}^{(t)} - \lambda_c^{(t)} W_{12,g} Y_{c,g2} - X_{g1}\widetilde{\beta}_{1,c} - (W_{11,g}X_{g1} + W_{12,g}X_{g2})\widetilde{\beta}_{2,c} - Z_{g1}^{(t)}\rho_{1,c}^{(t-1)} - (W_{11,g}Z_{g1}^{(t)} + W_{12,g}Z_{g2}^{(t)})\rho_{2,c}^{(t-1)} - l_{g1}\alpha_{c,g}^{(t-1)}$ and $\widetilde{\xi}_{c,g2} = (I_{m_g - m_{g1}} - \lambda_c^{(t)} W_{22,g})Y_{c,g2} - X_{g2}\widetilde{\beta}_{1,c} - (W_{21,g}X_{g1} + W_{22,g}X_{g2})\widetilde{\beta}_{2,c} - Z_{g2}^{(t)}\rho_{1,c}^{(t-1)} - (W_{21,g}Z_{g1}^{(t)} + W_{22,g}Z_{g2}^{(t)})\rho_{2,c}^{(t-1)} - l_{g2}\alpha_{c,g}^{(t-1)}$. Denote $\widetilde{\xi}_{c,g} = (\widetilde{\xi}'_{c,g1}, \widetilde{\xi}'_{c,g2})'$. To make a distinction, $\xi_{c,g}^{(t-1)} = (\xi_{c,g1}^{(t-1)'}, \xi_{c,g2}^{(t-1)'})'$ is calculated based on $\beta_c^{(t-1)}$. Also calculate $\xi_{uc,g}^{(t-1)} = (I_{m_g} - \lambda_c^{(t)} W_g)Y_{uc,g} - X_g\beta_{1,uc}^{(t)} - W_g X_g \beta_{2,uc}^{(t)} - Z_g^{(t)}\rho_{1,uc}^{(t-1)} - W_g Z_g^{(t)}\rho_{2,uc}^{(t-1)} - l_{m_g}\alpha_{uc,g}^{(t-1)}$.

(c) For $g = 1, \ldots, G$, given $\widetilde{\xi}_{c,g}$ and $\xi_{uc,g}^{(t-1)}$ from (b), simulate an auxiliary network $\widetilde{W}_g$ by R runs of the M–H algorithm based on

$$P\big(W_g|\widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \widetilde{\beta_c}, Y^{(t-1)}\big)$$
$$= \frac{\exp\big(\mathrm{V}(W_g, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \widetilde{\beta_c}, Y^{(t-1)})\big)}{\sum_W \exp\big(\mathrm{V}(W, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \widetilde{\beta_c}, Y^{(t-1)})\big)}$$

See details in Step I. part (c). If the obtained auxiliary network $\widetilde{W}_g$ does not belong to $\Omega_{\bar{n},g}$, reject it and rerun the simulation.

(d) With the acceptance probability equal to

$$\alpha\big(\widetilde{\beta}_t, \beta_c^{(t-1)}\big)$$
$$= \min\Bigg\{\prod_{g=1}^{G}\Bigg(\frac{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g|Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \widetilde{\beta}_t, Y^{(t-1)}\big)}{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g|Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t-1)}, Y^{(t-1)}\big)}$$
$$\times \frac{P\big(\widetilde{W}_g|\xi_{uc,g}^{(t-1)}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t-1)}, Y^{(t-1)}\big)}{P\big(\widetilde{W}_g|\widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \widetilde{\beta}_t, Y^{(t-1)}\big)}\Bigg)$$
$$\times \frac{\mathcal{N}_{2k}(\widetilde{\beta}_t|\beta_0, B_0 I_{2k})}{\mathcal{N}_{2k}\big(\beta_c^{(t-1)}|\beta_0, B_0 I_{2k}\big)}, 1\Bigg\}$$

$$= \min\Bigg\{\prod_{g=1}^{G}\Bigg(\frac{f\big(\widetilde{\xi}_{c,g} - \sigma^{(t-1)}_{\xi_{ucc,g}}\big(\sigma^{2(t-1)}_{\xi_{c,g}}\big)^{-1}\xi^{(t-1)}_{uc,g}\big)}{f\big(\xi^{(t-1)}_{c,g} - \sigma^{(t-1)}_{\xi_{ucc,g}}\big(\sigma^{2(t-1)}_{\xi_{c,g}}\big)^{-1}\xi^{(t-1)}_{uc,g}\big)}$$

$$\times \frac{\exp\big(\mathrm{V}\big(W_g, \widetilde{\xi}_{c,g}, \xi^{(t-1)}_{uc,g}, Z^{(t)}_g, \phi^{(t)}, \lambda^{(t)}_{uc}, \lambda^{(t)}_c, \beta^{(t)}_{uc}, \widetilde{\beta}_t, Y^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(W_g, \xi^{(t-1)}_{c,g}, \xi^{(t-1)}_{uc,g}, Z^{(t)}_g, \phi^{(t)}, \lambda^{(t)}_{uc}, \lambda^{(t)}_c, \beta^{(t)}_{uc}, \beta^{(t-1)}_c, Y^{(t-1)}\big)\big)}$$

$$\times \frac{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \xi^{(t-1)}_{c,g}, \xi^{(t-1)}_{uc,g}, Z^{(t)}_g, \phi^{(t)}, \lambda^{(t)}_{uc}, \lambda^{(t)}_c, \beta^{(t)}_{uc}, \beta^{(t-1)}_c, Y^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \widetilde{\xi}_{c,g}, \xi^{(t-1)}_{uc,g}, Z^{(t)}_g, \phi^{(t)}, \lambda^{(t)}_{uc}, \lambda^{(t)}_c, \beta^{(t)}_{uc}, \widetilde{\beta}_t, Y^{(t-1)}\big)\big)}\Bigg)$$

$$\times \frac{\mathcal{N}_{2k}(\widetilde{\beta}_t|\beta_0, B_0 I_{2k})}{\mathcal{N}_{2k}\big(\beta^{(t-1)}_c|\beta_0, B_0 I_{2k}\big)}, 1\Bigg\}$$

set $\beta^{(t)}_c = \widetilde{\beta}_t$. Otherwise, set $\beta^{(t)}_c = \beta^{(t-1)}_c$.

Step VIII. Simulate $\rho^{(t)}_{uc}$ from $P(\rho_{uc}|\{\ddot{Y}^{(t)}_{1g}\}, \{Y_{c,g}\}, \{Y_{uc,g}\}, \{W_g\}, \{Z^{(t)}_g\}, \phi^{(t)}, \lambda^{(t)}_{uc}, \lambda^{(t)}_c, \beta^{(t)}_{uc}, \beta^{(t)}_c, Y^{(t-1)})$ by the double M–H algorithm.

(a) Propose $\widetilde{\rho}_{uc}$ from a random walk proposal density $q(\rho_{uc}|\rho^{(t-1)}_{uc})$.

(b) For $g = 1, \ldots, G$, calculate the implied residuals from activity intensity equations, $\widetilde{\xi}_{uc,g} = (I_{m_g} - \lambda^{(t)}_{uc} W_g)Y_{uc,g} - X_g\beta^{(t)}_{1,uc} - W_g X_g \beta^{(t)}_{2,uc} - Z^{(t)}_g \widetilde{\rho}_{1,uc} - W_g Z^{(t)}_g \widetilde{\rho}_{2,uc} - l_{m_g}\alpha^{(t-1)}_{uc,g}$ and $\xi^{(t-1)}_{uc,g} = (I_{m_g} - \lambda^{(t)}_{uc} W_g)Y_{uc,g} - X_g\beta^{(t)}_{1,uc} - W_g X_g\beta^{(t)}_{2,uc} - Z^{(t)}_g \rho^{(t-1)}_{1,uc} - W_g Z^{(t)}_g \rho^{(t-1)}_{2,uc} - l_{m_g}\alpha^{(t-1)}_{uc,g}$. Also calculate $\xi^{(t-1)}_{c,g1} = \ddot{Y}^{(t)}_{c,g1} - \lambda^{(t)}_c W_{12,g} Y_{c,g2} - X_{g1}\beta^{(t)}_{1,c} - (W_{11,g}X_{g1} + W_{12,g}X_{g2})\beta^{(t)}_{2,c} - Z^{(t)}_{g1}\rho^{(t-1)}_{1,c} - (W_{11,g}Z^{(t)}_{g1} + W_{12,g}Z^{(t)}_{g2})\rho^{(t-1)}_{2,c} - l_{g1}\alpha^{(t-1)}_{c,g}$ and $\xi^{(t-1)}_{c,g2} = (I_{m_g - m_{g1}} - \lambda^{(t)}_c W_{22,g})Y_{c,g2} - X_{g2}\beta^{(t)}_{1,c} - (W_{21,g}X_{g1} + W_{22,g}X_{g2})\beta^{(t)}_{2,c} - Z^{(t)}_{g2}\rho^{(t-1)}_{1,c} - (W_{21,g}Z^{(t)}_{g1} + W_{22,g}Z^{(t)}_{g2})\rho^{(t-1)}_{2,c} - l_{g2}\alpha^{(t-1)}_{c,g}$. Denote $\xi^{(t-1)}_{c,g} = (\xi^{(t-1)'}_{c,g1}, \xi^{(t-1)'}_{c,g2})'$.

(c) For $g = 1, \ldots, G$, given $\widetilde{\xi}_{uc,g}$ and $\xi^{(t-1)}_{c,g}$ from (b), simulate an auxiliary network $\widetilde{W}_g$ by R runs of the M–H algorithm based on

$$P\big(W_g|\widetilde{\xi}_{uc,g}, \xi^{(t-1)}_{c,g}, Z^{(t)}_g, \phi^{(t)}, \lambda^{(t)}_{uc}, \lambda^{(t)}_c, \beta^{(t)}_{uc}, \beta^{(t)}_c, \widetilde{\rho}_{uc}, Y^{(t-1)}\big)$$
$$= \frac{\exp\big(\mathrm{V}\big(W_g, \widetilde{\xi}_{uc,g}, \xi^{(t-1)}_{c,g}, Z^{(t)}_g, \phi^{(t)}, \lambda^{(t)}_{uc}, \lambda^{(t)}_c, \beta^{(t)}_{uc}, \beta^{(t)}_c, \widetilde{\rho}_{uc}, Y^{(t-1)}\big)\big)}{\sum_W \exp\big(\mathrm{V}\big(W, \widetilde{\xi}_{uc,g}, \xi^{(t-1)}_{c,g}, Z^{(t)}_g, \phi^{(t)}, \lambda^{(t)}_{uc}, \lambda^{(t)}_c, \beta^{(t)}_{uc}, \beta^{(t)}_c, \widetilde{\rho}_{uc}, Y^{(t-1)}\big)\big)}$$

starting from W_g. See details in Step I, part (c). If the obtained auxiliary network $\widetilde{W}_g$ does not belong to $\Omega_{\bar{n},g}$, reject it and rerun the simulation.

(d) With the acceptance probability equal to

$$\begin{aligned}
&\alpha_{\text{MH}}\big(\widetilde{\rho}_{uc}, \rho_{uc}^{(t-1)}\big) \\
&= \min\Bigg\{\prod_{g=1}^{G}\Bigg(\frac{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g|Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \widetilde{\rho}_{uc}, Y^{(t-1)}\big)}{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g|Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t-1)}, Y^{(t-1)}\big)} \\
&\quad \times \frac{P\big(\widetilde{W}_g|\xi_{uc,g}^{(t-1)}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t-1)}, Y^{(t-1)}\big)}{P\big(\widetilde{W}_g|\widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \widetilde{\rho}_{uc}, Y^{(t-1)}\big)}\Bigg) \\
&\quad \times \frac{\mathcal{N}_{2\ell}(\widetilde{\rho}_{uc}|\rho_0, R_0 I_{2\ell})}{\mathcal{N}_{2\ell}\big(\rho_{uc}^{(t-1)}|\rho_0, R_0 I_{2\ell}\big)}, 1\Bigg\} \\
&= \min\Bigg\{\prod_{g=1}^{G}\Bigg(\frac{f\big(\xi_{c,g}^{(t-1)} - \sigma_{\xi_{ucc,g}}^{(t-1)}\big(\sigma_{\xi_{c,g}}^{2(t-1)}\big)^{-1}\widetilde{\xi}_{uc,g}\big)}{f\big(\xi_{c,g}^{(t-1)} - \sigma_{\xi_{ucc,g}}^{(t-1)}\big(\sigma_{\xi_{c,g}}^{2(t-1)}\big)^{-1}\xi_{uc,g}^{(t-1)}\big)} \cdot \frac{f(\widetilde{\xi}_{uc,g})}{f\big(\xi_{uc,g}^{(t-1)}\big)} \\
&\quad \times \frac{\exp\big(\mathrm{V}\big(W_g, \xi_{c,g}^{(t-1)}, \widetilde{\xi}_{uc,g}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \widetilde{\rho}_{uc}, Y^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(W_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t-1)}, Y^{(t-1)}\big)\big)} \\
&\quad \times \frac{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t-1)}, Y^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \xi_{c,g}^{(t-1)}, \widetilde{\xi}_{uc,g}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \widetilde{\rho}_{uc}, Y^{(t-1)}\big)\big)}\Bigg) \\
&\quad \times \frac{\mathcal{N}_{2\ell}(\widetilde{\rho}_{uc}|\rho_0, R_0 I_{2\ell})}{\mathcal{N}_{2\ell}\big(\rho_{uc}^{(t-1)}|\rho_0, R_0 I_{2\ell}\big)}, 1\Bigg\}
\end{aligned}$$

set $\rho_{uc}^{(t)} = \widetilde{\rho}_{uc}$. Otherwise, set $\rho_{uc}^{(t)} = \rho_{uc}^{(t-1)}$.

Step IX. Simulate $\rho_c^{(t)}$ from $P(\rho_c|\{\ddot{Y}_{1g}^{(t)}\}, \{Y_{c,g}\}, \{Y_{uc,g}\}, \{W_g\}, \{Z_g^{(t)}\}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, Y^{(t-1)})$ by the double M–H algorithm.

(a) Propose $\widetilde{\rho}_c$ from a random walk proposal density $q(\rho_c|\rho_c^{(t-1)})$.

(b) For $g = 1, \dots, G$, calculate the implied residuals from activity intensity equations, $\widetilde{\xi}_{c,g1} = \ddot{Y}_{c,g1}^{(t)} - \lambda_c^{(t)} W_{12,g} Y_{c,g2} - X_{g1}\beta_{1,c}^{(t)} - (W_{11,g}X_{g1} + W_{12,g}X_{g2})\beta_{2,c}^{(t)} - Z_{g1}^{(t)}\widetilde{\rho}_{1,c} - (W_{11,g}Z_{g1}^{(t)} + W_{12,g}Z_{g2}^{(t)})\widetilde{\rho}_{2,c} - l_{g1}\alpha_{c,g}^{(t-1)}$ and $\widetilde{\xi}_{c,g2} = (I_{m_g - m_{g1}} - \lambda_c^{(t)} W_{22,g})Y_{c,g2} - X_{g2}\beta_{1,c}^{(t)} - (W_{21,g}X_{g1} + W_{22,g}X_{g2})\beta_{2,c}^{(t)} - Z_{g2}^{(t)}\widetilde{\rho}_{1,c} - (W_{21,g}Z_{g1}^{(t)} + W_{22,g}Z_{g2}^{(t)})\widetilde{\rho}_{2,c} - l_{g2}\alpha_{c,g}^{(t-1)}$. Denote $\widetilde{\xi}_{c,g} = (\widetilde{\xi}'_{c,g1}, \widetilde{\xi}'_{c,g2})'$. To make a distinction, $\xi_{c,g}^{(t-1)} = (\xi_{c,g1}^{(t-1)'}, \xi_{c,g2}^{(t-1)'})'$ is calculated based on $\beta_c^{(t-1)}$. Also calculate $\xi_{uc,g}^{(t-1)} = (I_{m_g} - \lambda_{uc}^{(t)} W_g)Y_{uc,g} - X_g\beta_{1,uc}^{(t)} - W_g X_g \beta_{2,uc}^{(t)} - Z_g^{(t)}\rho_{1,uc}^{(t)} - W_g Z_g^{(t)}\rho_{2,uc}^{(t)} - l_{m_g}\alpha_{uc,g}^{(t-1)}$.

(c) For $g = 1, \dots, G$, given $\widetilde{\xi}_{c,g}$ and $\xi_{uc,g}^{(t-1)}$ from (b), simulate an auxiliary network $\widetilde{W}_g$ by R runs of the M–H algorithm based on

$$P\big(W_g|\widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \widetilde{\rho}_c, Y^{(t-1)}\big)$$

$$= \frac{\exp\big(\mathrm{V}\big(W_g, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \widetilde{\rho}_c, Y^{(t-1)}\big)\big)}{\sum_W \exp\big(\mathrm{V}\big(W, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \widetilde{\rho}_c, Y^{(t-1)}\big)\big)}$$

See details in Step I. part (c). If the obtained auxiliary network $\widetilde{W}_g$ does not belong to $\Omega_{\bar{n},g}$, reject it and rerun the simulation.

(d) With the acceptance probability equal to

$$\begin{aligned}
&\alpha_{\mathrm{MH}}\big(\widetilde{\rho}_c, \rho_c^{(t-1)}\big) \\
&= \min\Bigg\{\prod_{g=1}^{G}\Bigg(\frac{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g | Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \widetilde{\rho}_c, Y^{(t-1)}\big)}{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g | Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t-1)}, Y^{(t-1)}\big)} \\
&\quad \times \frac{P\big(\widetilde{W}_g | \xi_{uc,g}^{(t-1)}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t-1)}, Y^{(t-1)}\big)}{P\big(\widetilde{W}_g | \widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \widetilde{\rho}_c, Y^{(t-1)}\big)}\Bigg) \\
&\quad \times \frac{\mathcal{N}_{2\ell}(\widetilde{\rho}_c | \rho_0, R_0 I_{2\ell})}{\mathcal{N}_{2\ell}\big(\rho_c^{(t-1)} | \rho_0, R_0 I_{2\ell}\big)}, 1\Bigg\} \\
&= \min\Bigg\{\prod_{g=1}^{G}\Bigg(\frac{f\big(\widetilde{\xi}_{c,g} - \sigma_{\xi_{ucc,g}}^{(t-1)}\big(\sigma_{\xi_{c,g}}^{2(t-1)}\big)^{-1}\xi_{uc,g}^{(t-1)}\big)}{f\big(\xi_{c,g}^{(t-1)} - \sigma_{\xi_{ucc,g}}^{(t-1)}\big(\sigma_{\xi_{c,g}}^{2(t-1)}\big)^{-1}\xi_{uc,g}^{(t-1)}\big)} \\
&\quad \times \frac{\exp\big(\mathrm{V}\big(W_g, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \widetilde{\rho}_c, Y^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(W_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t-1)}, Y^{(t-1)}\big)\big)} \\
&\quad \times \frac{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \xi_{c,g}^{(t-1)}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t-1)}, Y^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \widetilde{\rho}_c, Y^{(t-1)}\big)\big)}\Bigg) \\
&\quad \times \frac{\mathcal{N}_{2\ell}(\widetilde{\rho}_c | \rho_0, R_0 I_{2\ell})}{\mathcal{N}_{2\ell}\big(\rho_c^{(t-1)} | \rho_0, R_0 I_{2\ell}\big)}, 1\Bigg\}
\end{aligned}$$

set $\rho_c^{(t)} = \widetilde{\rho}_c$. Otherwise, set $\rho_c^{(t)} = \rho_c^{(t-1)}$.

Step X. For $g = 1, \dots, G$, simulate $\sigma_g^{(t)}$ from $P(\sigma_g | \ddot{Y}_{1g}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, Y^{(t-1)})$ by the standard M–H algorithm.

(a) Propose $\widetilde{\sigma}_g$ from a random walk proposal density $q(\sigma_g | \sigma_g^{(t-1)})$.

(b) Calculate the implied residual from activity intensity equations, $\xi_{c,g1}^{(t-1)} = \ddot{Y}_{c,g1}^{(t)} - \lambda_c^{(t)} W_{12,g} Y_{c,g2} - X_{g1}\beta_{1,c}^{(t)} - (W_{11,g}X_{g1} + W_{12,g}X_{g2})\beta_{2,c}^{(t)} - Z_{g1}^{(t)}\rho_{1,c}^{(t)} - (W_{11,g}Z_{g1}^{(t)} + W_{12,g}Z_{g2}^{(t)})\rho_{2,c}^{(t)} - l_{g1}\alpha_{c,g}^{(t-1)}$ and $\xi_{c,g2}^{(t-1)} = (I_{m_g - m_{g1}} - \lambda_c^{(t)} W_{22,g}) Y_{c,g2} - X_{g2}\beta_{1,c}^{(t)} - (W_{21,g}X_{g1} + W_{22,g}X_{g2})\beta_{2,c}^{(t)} - Z_{g2}^{(t)}\rho_{1,c}^{(t)} - (W_{21,g}Z_{g1}^{(t)} + W_{22,g}Z_{g2}^{(t)})\rho_{2,c}^{(t)} - l_{g2}\alpha_{c,g}^{(t-1)}$. Denote $\xi_{c,g}^{(t-1)} = (\xi_{c,g1}^{(t-1)\prime}, \xi_{c,g2}^{(t-1)\prime})'$. Also calculate $\xi_{uc,g}^{(t-1)} = (I_{m_g} - \lambda_{uc}^{(t)} W_g) Y_{uc,g} - X_g\beta_{1,uc}^{(t)} - W_g X_g \beta_{2,uc}^{(t)} - Z_g^{(t)}\rho_{1,uc}^{(t)} - W_g Z_g^{(t)}\rho_{2,uc}^{(t)} - l_{m_g}\alpha_{uc,g}^{(t-1)}$.

(c) With the acceptance probability equal to

$$\begin{aligned}
&\alpha_{\mathrm{MH}}\big(\widetilde{\sigma}_g|\sigma_g^{(t-1)}\big) \\
&= \min\left\{\left(\frac{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g|Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \widetilde{\sigma}_g, Y^{(t-1)}\big)}{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g|Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t-1)}, Y^{(t-1)}\big)}\right)\right. \\
&\quad \left.\times \frac{\mathcal{N}_3(\widetilde{\sigma}_g|\sigma_0, \Sigma_0)}{\mathcal{N}_3\big(\sigma_g^{(t-1)}|\sigma_0, \Sigma_0\big)} \cdot \frac{I(\tilde{\sigma}_g \in T_g)}{I\big(\sigma_g^{(t-1)} \in T_g\big)}, 1\right\} \\
&= \min\left\{\left(\frac{f\big(\xi_{c,g}^{(t-1)} - \widetilde{\sigma}_{\xi_{ucc,g}}\big(\widetilde{\sigma}_{\xi_c}^2\big)^{-1}\xi_{uc,g}^{(t-1)}; \widetilde{\sigma}\big)}{f\big(\xi_{c,g}^{(t-1)} - \sigma_{\xi_{ucc,g}}^{(t-1)}\big(\sigma_{\xi_{c,g}}^{2(t-1)}\big)^{-1}\xi_{uc,g}^{(t-1)}; \sigma^{(t-1)}\big)} \cdot \frac{f\big(\xi_{uc,g}^{(t-1)}; \widetilde{\sigma}\big)}{f\big(\xi_{uc,g}^{(t-1)}; \sigma^{(t-1)}\big)}\right)\right. \\
&\quad \left.\times \frac{\mathcal{N}_3(\widetilde{\sigma}_g|\sigma_0, \Sigma_0)}{\mathcal{N}_3\big(\sigma_g^{(t-1)}|\sigma_0, \Sigma_0\big)} \cdot \frac{I(\tilde{\sigma} \in T_g)}{I\big(\sigma^{(t-1)} \in T_g\big)}, 1\right\},
\end{aligned}$$

set $\sigma_g^{(t)} = \widetilde{\sigma}_g$. Otherwise, set $\sigma_g^{(t)} = \sigma_g^{(t-1)}$.

Step XI. For $g = 1, \ldots, G$, simulate $\alpha_{uc,g}^{(t)}$ from $P(\alpha_{uc,g}|\ddot{Y}_{1g}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{c,g}^{(t-1)})$ by the double M–H algorithm.

(a) Propose $\widetilde{\alpha}_{uc,g}$ from a random walk proposal density $q(\alpha_{uc,g}|\alpha_{uc,g}^{(t-1)})$.

(b) Calculate the implied residuals from activity intensity equations, $\widetilde{\xi}_{uc,g} = (I_{m_g} - \lambda_{uc}^{(t)}W_g)Y_{uc,g} - X_g\beta_{1,uc}^{(t)} - W_gX_g\beta_{2,uc}^{(t)} - Z_g^{(t)}\rho_{1,uc}^{(t)} - W_gZ_g^{(t)}\rho_{2,uc}^{(t)} - l_{m_g}\widetilde{\alpha}_{uc,g}$ and $\xi_{uc,g}^{(t-1)} = (I_{m_g} - \lambda_{uc}^{(t)}W_g)Y_{uc,g} - X_g\beta_{1,uc}^{(t)} - W_gX_g\beta_{2,uc}^{(t)} - Z_g^{(t)}\rho_{1,uc}^{(t)} - W_gZ_g^{(t)}\rho_{2,uc}^{(t)} - l_{m_g}\alpha_{uc,g}^{(t-1)}$. Also calculate $\xi_{c,g1}^{(t-1)} = \ddot{Y}_{c,g1}^{(t)} - \lambda_c^{(t)}W_{12,g}Y_{c,g2} - X_{g1}\beta_{1,c}^{(t)} - (W_{11,g}X_{g1} + W_{12,g}X_{g2})\beta_{2,c}^{(t)} - Z_{g1}^{(t)}\rho_{1,c}^{(t)} - (W_{11,g}Z_{g1}^{(t)} + W_{12,g}Z_{g2}^{(t)})\rho_{2,c}^{(t)} - l_{g1}\alpha_{c,g}^{(t-1)}$ and $\xi_{c,g2}^{(t-1)} = (I_{m_g - m_{g1}} - \lambda_c^{(t)}W_{22,g})Y_{c,g2} - X_{g2}\beta_{1,c}^{(t)} - (W_{21,g}X_{g1} + W_{22,g}X_{g2})\beta_{2,c}^{(t)} - Z_{g2}^{(t)}\rho_{1,c}^{(t)} - (W_{21,g}Z_{g1}^{(t)} + W_{22,g}Z_{g2}^{(t)})\rho_{2,c}^{(t)} - l_{g2}\alpha_{c,g}^{(t-1)}$ Denote $\xi_{c,g}^{(t-1)} = (\xi_{c,g1}^{(t-1)'}, \xi_{c,g2}^{(t-1)'})'$.

(c) Given $\widetilde{\xi}_{uc,g}$ and $\xi_{c,g}^{(t-1)}$ from (b), simulate an auxiliary network $\widetilde{W}_g$ by R runs of the M–H algorithm based on

$$\begin{aligned}
&P\big(W_g|\widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \widetilde{\alpha}_{uc,g}, \alpha_{c,g}^{(t-1)}\big) \\
&\quad = \frac{\exp\big(\mathrm{V}\big(W_g, \widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \widetilde{\alpha}_{uc,g}, \alpha_{c,g}^{(t-1)}\big)\big)}{\sum_W \exp\big(\mathrm{V}\big(W, \widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \widetilde{\alpha}_{uc,g}, \alpha_{c,g}^{(t-1)}\big)\big)}
\end{aligned}$$

See details in Step I, part (c). If the obtained auxiliary network $\widetilde{W}_g$ does not belong to $\Omega_{\bar{n},g}$, reject it and rerun the simulation.

(d) With the acceptance probability equal to

$$\begin{aligned}
&\alpha_{\mathrm{MH}}\big(\widetilde{\alpha}_{uc,g}, \alpha_{uc,g}^{(t-1)}\big) \\
&= \min\Bigg\{ \frac{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{uc,g}, Y_{c,g}, W_g | Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \widetilde{\alpha}_{uc,g}, \alpha_{c,g}^{(t-1)}\big)}{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{uc,g}, Y_{c,g}, W_g | Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{uc,g}^{(t-1)}, \alpha_{c,g}^{(t-1)}\big)} \\
&\quad \times \frac{P\big(\widetilde{W}_g | Z_g^{(t)}, \xi_{uc,g}^{(t-1)}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{uc,g}^{(t-1)}, \alpha_{c,g}^{(t-1)}\big)}{P\big(\widetilde{W}_g | \widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \widetilde{\alpha}_{uc,g}, \alpha_{c,g}^{(t-1)}\big)} \\
&\quad \times \frac{\mathcal{N}(\widetilde{\alpha}_{uc,g} | \alpha_0, A_0)}{\mathcal{N}\big(\alpha_{uc,g}^{(t-1)} | \alpha_0, A_0\big)}, 1\Bigg\} \\
&= \min\Bigg\{ \Bigg(\frac{f\big(\xi_{c,g}^{(t-1)} - \sigma_{\xi_{tcg}}^{(t)} \big(\sigma_{\xi_{uc,g}}^{2(t)}\big)^{-1} \widetilde{\xi}_{uc,g}\big)}{f\big(\xi_{c,g}^{(t-1)} - \sigma_{\xi_{tcg}}^{(t)} \big(\sigma_{\xi_{uc,g}}^{2(t)}\big)^{-1} \xi_{uc,g}^{(t-1)}\big)} \cdot \frac{f(\widetilde{\xi}_{uc,g})}{f\big(\xi_{uc,g}^{(t-1)}\big)} \\
&\quad \times \frac{\exp\big(\mathrm{V}\big(W_g, \widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \widetilde{\alpha}_{uc,g}, \alpha_{c,g}^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(W_g, \xi_{uc,g}^{(t-1)}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{uc,g}^{(t-1)}, \alpha_{c,g}^{(t-1)}\big)\big)} \\
&\quad \times \frac{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \xi_{uc,g}^{(t-1)}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{uc,g}^{(t-1)}, \alpha_{c,g}^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \widetilde{\xi}_{uc,g}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \widetilde{\alpha}_{uc,g}, \alpha_{c,g}^{(t-1)}\big)\big)} \Bigg\} \\
&\quad \times \frac{\mathcal{N}(\widetilde{\alpha}_{uc,g} | \alpha_0, A_0)}{\mathcal{N}\big(\alpha_{uc,g}^{(t-1)} | \alpha_0, A_0\big)}, 1\}
\end{aligned}$$

set $\alpha_{uc,g}^{(t)} = \widetilde{\alpha}_{uc,g}$. Otherwise, set $\alpha_{uc,g}^{(t)} = \alpha_{uc,g}^{(t-1)}$.

Step XII. For $g = 1, \ldots, G$, simulate $\alpha_{c,g}^{(t)}$ from $P(\alpha_{c,g} | \ddot{Y}_{1g}^{(t)}, Y_{c,g}, Y_{uc,g}, W_g, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \sigma_{\xi_{uc,g}}^{2(t)}, \sigma_{\xi_{c,g}}^{2(t)}, \sigma_{\xi_{tcg}}^{(t)}, \alpha_{uc,g}^{(t)})$ by the double M–H algorithm.

(a) Propose $\widetilde{\alpha}_{c,g}$ from a random walk proposal density $q(\alpha_{c,g} | \alpha_{c,g}^{(t-1)})$.

(b) For $g = 1, \ldots, G$, calculate the implied residuals from activity intensity equations, $\widetilde{\xi}_{c,g1} = \ddot{Y}_{c,g1}^{(t)} - \lambda_c^{(t)} W_{12,g} Y_{c,g2} - X_{g1} \beta_{1,c}^{(t)} - (W_{11,g} X_{g1} + W_{12,g} X_{g2}) \beta_{2,c}^{(t)} - Z_{g1}^{(t)} \rho_{1,c}^{(t)} - (W_{11,g} Z_{g1}^{(t)} + W_{12,g} Z_{g2}^{(t)}) \rho_{2,c}^{(t)} - l_{g1} \widetilde{\alpha}_{c,g}$ and $\widetilde{\xi}_{c,g2} = (I_{m_g - m_{g1}} - \lambda_c^{(t)} W_{22,g}) Y_{c,g2} - X_{g2} \beta_{1,c}^{(t)} - (W_{21,g} X_{g1} + W_{22,g} X_{g2}) \beta_{2,c}^{(t)} - Z_{g2}^{(t)} \rho_{1,c}^{(t)} - (W_{21,g} Z_{g1}^{(t)} + W_{22,g} Z_{g2}^{(t)}) \rho_{2,c}^{(t)} - l_{g2} \widetilde{\alpha}_{c,g}$ Denote $\widetilde{\xi}_{c,g} = (\widetilde{\xi}'_{c,g1}, \widetilde{\xi}'_{c,g2})'$. To make a distinction, $\xi_{c,g}^{(t-1)} = (\xi_{c,g1}^{(t-1)'}, \xi_{c,g2}^{(t-1)'})'$ is calculated based on $\alpha_{c,g}^{(t-1)}$. Also calculate $\xi_{uc,g}^{(t)} = (I_{m_g} - \lambda_{uc}^{(t)} W_g) Y_{uc,g} - X_g \beta_{1,uc}^{(t)} - W_g X_g \beta_{2,uc}^{(t)} - Z_g^{(t)} \rho_{1,uc}^{(t)} - W_g Z_g^{(t)} \rho_{2,uc}^{(t)} - l_{m_g} \alpha_{uc,g}^{(t)}$.

(c) Given $\widetilde{\xi}_{c,g}$ and $\xi_{uc,g}^{(t)}$ from (b), simulate an auxiliary network $\widetilde{W}_g$ by R runs of the M–H algorithm based on

$$\begin{aligned}
&P\big(W_g | \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{uc,g}^{(t)}, \widetilde{\alpha}_{c,g}\big) \\
&\quad = \frac{\exp\big(\mathrm{V}\big(W_g, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{uc,g}^{(t)}, \widetilde{\alpha}_{c,g}\big)\big)}{\sum_W \exp\big(\mathrm{V}\big(W, \widetilde{\xi}_{c,g}, \xi_{uc,g}^{(t)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{uc,g}^{(t)}, \widetilde{\alpha}_{c,g}\big)\big)}
\end{aligned}$$

See details in Step I, part (c). If the obtained auxiliary network $\widetilde{W}_g$ does not belong to $\Omega_{\bar{n},g}$, reject it and rerun the simulation.

(d) With the acceptance probability equal to

$$
\begin{aligned}
&\alpha_{\mathrm{MH}}\big(\widetilde{\alpha}_{c,g}, \alpha_{c,g}^{(t-1)}\big) \\
&= \min\left\{ \frac{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{uc,g}, Y_{c,g}, W_g | Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{uc,g}^{(t)}, \widetilde{\alpha}_{c,g}\big)}{P\big(\ddot{Y}_{c,g1}^{(t)}, Y_{uc,g}, Y_{c,g}, W_g | Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{uc,g}^{(t)}, \alpha_{c,g}^{(t-1)}\big)} \right. \\
&\quad \times \frac{P\big(\widetilde{W}_g | \xi_{uc,g}^{(t)}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{uc,g}^{(t)}, \alpha_{c,g}^{(t-1)}\big)}{P\big(\widetilde{W}_g | \xi_{uc,g}^{(t)}, \widetilde{\xi}_{c,g}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{uc,g}^{(t)}, \widetilde{\alpha}_{c,g}\big)} \\
&\quad \left. \times \frac{\mathcal{N}(\widetilde{\alpha}_{c,g} | \alpha_0, A_0)}{\mathcal{N}\big(\alpha_{c,g}^{(t-1)} | \alpha_0, A_0\big)}, 1 \right\} \\
&= \min\left\{ \Big(\frac{f\big(\widetilde{\xi}_{c,g}^{(t-1)} - \sigma_{\xi_{tcg}}^{(t)} \big(\sigma_{\xi_{uc,g}}^{2(t)}\big)^{-1} \xi_{uc,g}^{(t)}\big)}{f\big(\xi_{c,g}^{(t-1)} - \sigma_{\xi_{tcg}}^{(t)} \big(\sigma_{\xi_{uc,g}}^{2(t)}\big)^{-1} \xi_{uc,g}^{(t)}\big)} \right. \\
&\quad \times \frac{\exp\big(\mathrm{V}\big(W_g, \xi_{uc,g}^{(t)}, \widetilde{\xi}_{c,g}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{c,g}^{(t)}, \widetilde{\alpha}_{c,g}\big)\big)}{\exp\big(\mathrm{V}\big(W_g, \xi_{uc,g}^{(t)}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{uc,g}^{(t)}, \alpha_{c,g}^{(t-1)}\big)\big)} \\
&\quad \left. \times \frac{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \xi_{uc,g}^{(t)}, \xi_{c,g}^{(t-1)}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{uc,g}^{(t)}, \alpha_{c,g}^{(t-1)}\big)\big)}{\exp\big(\mathrm{V}\big(\widetilde{W}_g, \xi_{uc,g}^{(t)}, \widetilde{\xi}_{c,g}, Z_g^{(t)}, \phi^{(t)}, \lambda_{uc}^{(t)}, \lambda_c^{(t)}, \beta_{uc}^{(t)}, \beta_c^{(t)}, \rho_{uc}^{(t)}, \rho_c^{(t)}, \sigma_g^{(t)}, \alpha_{uc,g}^{(t)}, \widetilde{\alpha}_{c,g}\big)\big)} \right\} \\
&\quad \times \frac{\mathcal{N}(\widetilde{\alpha}_{c,g} | \alpha_0, A_0)}{\mathcal{N}\big(\alpha_{c,g}^{(t-1)} | \alpha_0, A_0\big)}, 1\}
\end{aligned}
$$

set $\alpha_{c,g}^{(t)}$ with $\widetilde{\alpha}_{c,g}$. Otherwise, set $\alpha_{c,g}^{(t)} = \alpha_{c,g}^{(t-1)}$.

Appendix E: Simulation study

In this Appendix, we conduct a simulation study to examine the finite sample performance of the Bayesian MCMC sampler proposed in Section 3. The simulation study is designed to accommodate four different purposes.

First, we carry out a Monte Carlo experiment (with 100 repetitions) to demonstrate that the MCMC sampler can successfully recover the true parameters from the artificially generated network data. *Second,* the same Monte Carlo experiment can be used to show the issue of model misspecification and consequent estimation biases. *Third,* using samples from one simulation repetition, we begin the MCMC sampler with different initial values to see if the Markov chains converge toward the true values within a reasonable amount of draws, that is, examining the issues of nonconvergence and the slow mixing of the Markov chain. *Fourth,* we also report the computation time required by network samples of different sizes to offer users additional information on the feasibility of our approach.

We design the data generating process (DGP) throughout the simulation based on the exponential distribution of equation (6). Continuous (uncensored) activity variables are generated by the activity intensity equation of equation (7), and censored activity variables are generated by equation (5). We set the network size for the uncensored case at 30 and the censored case at 40—to compensate the loss of information due to censoring—and fix the number of networks at 30, that is, there are 900 and 1200 individual observations for the uncensored and censored cases, respectively, at each simulation repetition.

In the activity intensity equation (9), we generate the exogenous variable X from a normal distribution $\mathcal{N}(0,4)$. The group fixed effects are generated from $\mathcal{N}(3,1)$ for the uncensored case and $\mathcal{N}(-1.5,1)$ for the censored case. The disturbance term ξ is generated from $\mathcal{N}(0,0.5)$. The latent variable Z is specified as one-dimensional and generated from $\mathcal{N}(0,1)$. For the other effects on network formation, the local network effect is specified based on equation (8). We include a constant term and a dyad-specific exogenous variable c_{ij} that is generated as follows: first, we draw two uniform random variables from $U(0,1)$, denoted as U_1 and U_2. If U_1 and U_2 are both larger than 0.7 or less than 0.3, then we set c_{ij} to one. Otherwise, we set c_{ij} to zero. We also include the distance of latent variables $|z_i - z_j|$ as part of the local network effect. The global network effects are specified according to equation (10). All true parameter values of the DGP are reported in the second column of Tables F.1 and F.2.

Each artificial network W is simulated by the M–H algorithm from an empty network based on the exponential distribution of equation (6). The following steps are implemented iteratively corresponding to the local and global updates in Step (c) of the pseudo MCMC algorithm in Section 3.4. Activity intensity variables are simulated along with networks. The M–H algorithm runs through the entire network for a total of 10,000 iterations, and realizations of the network and activity intensity variables from the last iteration are used as the data. The networks generated out of the design have an average out-degree of 3.6978 for the uncensored case and 2.6458 for the censored case; the average density is 0.1275 (uncensored) and 0.0678 (censored), and the average clustering coefficient is 0.0493 (uncensored) and 0.0263 (censored).[1] These network statistics are comparable to those of the empirical samples in Section 5. The generated uncensored activity variable has a mean of 4.0767, and the censored variable has a mean of 1.1923. A total of 21.67% of the observations are censored.

To estimate the model, a total of 50,000 draws were simulated using the double M–H algorithm discussed in Section 3.3. The values of hyperparameters in the prior distributions described in Supplementary Appendix C are set as $\phi_0 = 0$; $\Phi_0 = 10$; $\beta_0 = 0$; $B_0 = 10$; $\rho_0 = 0$; $R_0 = 10$; $\sigma_0 = 0$; $\Sigma_0 = 10$; $\alpha_0 = 0$; $A_0 = 400$. These specified values of hyperparameters are chosen to form very flat prior densities over the range of parameter spaces so that estimation results are less influenced by choice of priors. We discard the first 10,000

[1] The out-degree for individual i in group g is calculated by $\sum_j w_{ij,g}$. The average out-degree is $\sum_i \sum_{j\neq i} w_{ij,g}/m_g$. The network density is obtained by further dividing the average degree by $(m_g - 1)$. The clustering coefficient is calculated as the total fraction of transitive triples in the network, that is,

$$C(W_g) = \frac{\sum_{i;j\neq i;k\neq i,j} w_{ij,g} w_{jk,g} w_{ik,g}}{\sum_{i;j\neq i;k\neq i,j} w_{ij,g} w_{jk,g}}.$$

draws and use the remaining 40,000 draws to compute the posterior mean as a point estimate. We summarize our findings from the simulation study below.

First, we report respectively the Monte Carlo simulation results for uncensored and censored activities in Tables F.1 and F.2. For both activities, we find that the proposed Bayesian estimation can successfully recover the true parameter values when the correct model—"full" model—is used. We also estimate four nested models under the full model to examine consequences of model misspecification: the "no latent" model that ignores the latent variable from network formation and activity intensity; the "no global" model that ignores the global network effects from network formation; the "latent only" model that only includes the latent variable; and the "activity only" model, that is, equation (7), that regards a network as exogenously given and only estimates the activity intensity equation. The results reveal different levels of estimation biases in these misspecified models.

When ignoring the latent variable, the "no latent" results show significant upward biases of the endogenous peer effect (λ) in equation (9) and some of the global network effects (e.g., η_1, η_2, and η_5) in equation (10), and there is a significant downward bias on the incentive effect (δ) in equation (1). These results demonstrate that omitting latent variables that affect both activity intensity and network formation not only causes an upward bias on the estimate of the endogenous peer effect (Hsieh and Lee (2016)), but also biases the estimates of other network effects.

When ignoring the global network effects, the "no global" results show that the estimated incentive effect (δ), which confounds with the uncontrolled global network effects in equation (1), becomes upward biased. Meanwhile, since the incentive effect and the endogenous peer effect (λ) are highly interdependent through the activity intensity, the upward bias on the estimated incentive effect also leads to the downward bias on the estimated endogenous peer effect. These results reveal the necessity of controlling the global network effects in our network formation model.

When ignoring the global network effects and the incentive effect, the estimate of endogenous peer effect (λ) is also upward biased in the "latent only" results. This highlights the importance of the new avenue explored with our network formation model, that is, the incentive effect, in which the issue of network endogeneity on social interactions can be formulated, as well as extending the existing literature of jointly modeling network formation and interactions with latent variables (Goldsmith-Pinkham and Imbens (2013), Hsieh and Lee (2016), Johnsson and Moon (forthcoming)). Finally, the results of the "activity only" model display the most severe upward bias on the endogenous peer effect (λ) among the four misspecified models due to uncontrolled network endogeneity.

Next, we inspect the convergence of the Markov chain in our Bayesian MCMC estimation under different initial values to determine if there are issues of nonconvergence and slow mixing. To implement this inspection, we take the network and activity samples from the first simulation repetition of the above Monte Carlo experiment and estimate the model with the proposed MCMC procedure in Section 3.3. We focus on two important parameters of the model—the endogenous peer effect (λ) and the incentive effect (δ)—with different initial values to begin the MCMC sampling. To keep the exercise tractable, we let the sampling of other parameters begin with the true values.

For illustration purposes, we consider the case of uncensored activity and present the results of inspection by the trace plots of MCMC draws in Figure F.1 and Figure F.2. To interpret these trace plots, we take the endogenous peer effect λ in Figure F.1 as an example. The true value of λ in the simulation is set at 0.05 (see Table F.1). Accordingly, we assign five evenly spaced values between 0 and 0.1, namely 0, 0.025, 0.05, 0.075, and 0.1, to start the MCMC sampling. With these five different initial values, we run five independent MCMC samplings (for all parameters) and plot the first 5000 draws of λ in each chain. The plots in Figure F.1 show that regardless of the different initial values, the draws of λ converge and stabilize swiftly near the true value. We also find a similar pattern in the other figures for λ in the censored outcome and for δ in both the censored and uncensored outcomes; this occurs despite the convergence of δ requiring slightly more draws than λ.[2] Similar findings are found in the plots for the censored activity case in Figures F.3 and F.4.

Finally, we illustrate the computational cost of our estimation algorithm. We focus on the case of uncensored activity and generate artificial data with different network sizes of 20, 40, 60, 80, and 100. We fix the number of network groups at 30 given that the number of groups is less of a concern for computation because we can easily digest the cost of many groups by applying the parallel computation at the group level.

In Figure F.5, we show the computation cost, measured by the average CPU time (in seconds) of one MCMC iteration for the five studied models (in Table F.1); these models include the full model, three nested network formation models, and the activity intensity equation alone. The timing task is done using a desktop PC having an Intel i7-6700 CPU (4.00 GHz). We see that the computational cost increases exponentially with network size whenever estimation of a corresponding model requires the double M–H algorithm. Taking the median network size (60) in this simulation—which is also close to the average network size in the empirical study of Section 4—as a reference, completing the estimation of the full model with 100,000 MCMC iterations will require roughly 155 hours, a manageable amount of time.

[2] The draws of λ and δ have posterior means close to but not exactly equal to the true parameter values. This occurs as a result of one simulation repetition where sampling errors exist.

Appendix F: Additional tables and figures

Table F.1. Results of Monte Carlo experiments for the uncensored activity variable.

		Full		No Latent		No Global		Latent Only		Activity Alone	
Parameter	True	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
λ	0.0500	0.0500	0.0073	0.0679	0.0091	0.0167	0.0052	0.0595	0.0091	0.0846	0.0073
β_1	0.3000	0.2817	0.0280	0.2986	0.0283	0.2992	0.0299	0.3038	0.0271	0.2965	0.0281
β_2	0.1000	0.0927	0.0099	0.1036	0.0196	0.1084	0.0116	0.1317	0.0185	0.1243	0.0214
ρ_1	0.3000	0.2838	0.0605	–	–	0.2359	0.0689	0.2713	0.0923	–	–
ρ_2	0.1000	0.0946	0.0248	–	–	0.0696	0.0316	0.0864	0.0685	–	–
γ_0	−0.5000	−0.5262	0.0655	−1.9885	0.1387	−1.4266	0.0617	−1.0418	0.0362	–	–
γ_3	0.3000	0.3106	0.0505	0.2733	0.0538	0.3519	0.0723	0.3620	0.0611	–	–
γ_4	−1.0000	−1.1367	0.0715	–	–	−1.1137	0.0784	−1.0580	0.0514	–	–
η_1	0.3000	0.2959	0.0405	0.4401	0.0499	–	–	–	–	–	–
η_2	0.2000	0.2172	0.0475	0.3101	0.0602	–	–	–	–	–	–
η_3	−0.1000	−0.1003	0.0084	−0.1098	0.0100	–	–	–	–	–	–
η_4	0.0400	0.0418	0.0036	0.0325	0.0042	–	–	–	–	–	–
η_5	0.3000	0.3023	0.0241	0.4015	0.0285	–	–	–	–	–	–
η_6	−0.2000	−0.1976	0.0216	−0.1777	0.0273	–	–	–	–	–	–
δ	0.3000	0.3000	0.0259	0.2094	0.0569	1.0049	0.1001	–	–	–	–
σ_ξ^2	0.5000	0.5381	0.1759	0.6852	0.1843	0.6647	0.2403	0.5259	0.1764	0.6740	0.1784

Note: This Monte Carlo study consists of 100 repetitions. The values reported in this table are the mean and the standard deviation of parameter estimates across repetitions. The parameters λ, β_1, and β_2 capture, respectively, endogenous, own, and contextual peer effects in equation (9). The parameters γ_0, γ_3, and γ_4 capture local network effects specified in equation (8). The parameters η_1 to η_6 capture global network effects specified in equation (10). The parameter δ captures the incentive effect specified in equation (6). The parameter σ_ξ^2 is the variance of error term ξ in equation (9). In each repetition, we estimate each of the corresponding models with 50,000 MCMC draws. We drop the first 10,000 draws due to burn-in and calculate the (posterior) mean of the remaining draws as parameter estimates.

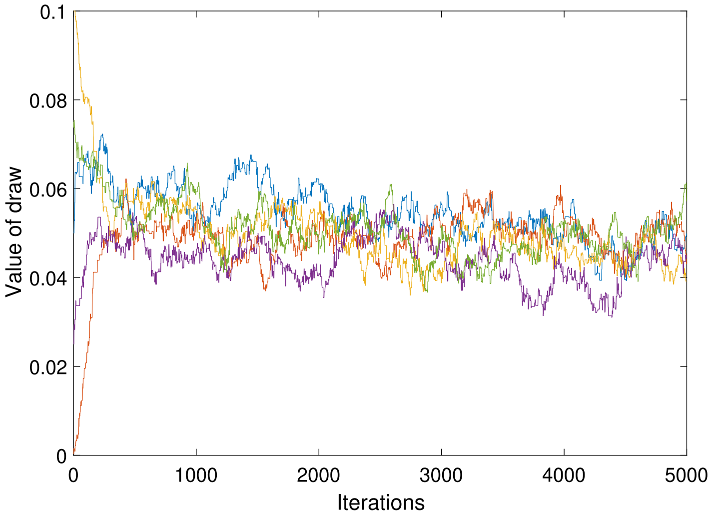

Figure F.1. Trace plot of MCMC draws for the parameter λ at different initial values (for the uncensored outcome). The true value of λ in the simulation is 0.05.

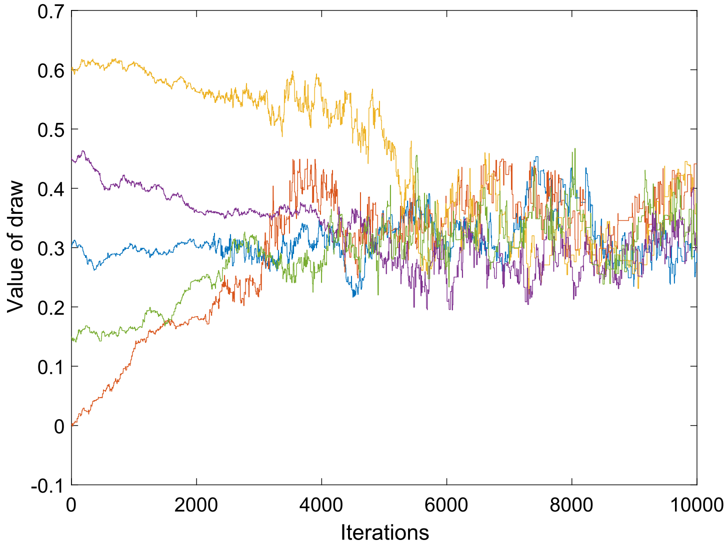

FIGURE F.2. Trace plot of MCMC draws for the parameter δ at different initial values (for the uncensored outcome). The true value of δ in the simulation is 0.3.

TABLE F.2. Results of Monte Carlo experiments for the censored activity variable.

		Full		No Latent		No Global		Latent Only		Activity Alone	
Parameter	True	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
λ	0.0500	0.0492	0.0131	0.0890	0.0157	0.0242	0.0121	0.0568	0.0131	0.1187	0.0147
β_1	0.3000	0.2695	0.0169	0.2798	0.0130	0.2726	0.0153	0.2779	0.0122	0.2761	0.0123
β_2	0.1000	0.0820	0.0091	0.0880	0.0060	0.0685	0.0085	0.1094	0.0058	0.0935	0.0065
ρ_1	0.3000	0.2464	0.0320	–	–	0.2567	0.0615	0.2496	0.0478	–	–
ρ_2	0.1000	0.0972	0.0228	–	–	0.1145	0.0249	0.1073	0.0401	–	–
γ_0	−1.7000	−1.6505	0.1155	−2.7295	0.1327	−1.9584	0.0895	−1.7348	0.0502	–	–
γ_3	0.3000	0.3022	0.0426	0.2673	0.0474	0.3211	0.0556	0.3265	0.0530	–	–
γ_4	−1.0000	−1.0715	0.0848	–	–	−1.0638	0.1144	−1.1122	0.0692	–	–
η_1	0.3000	0.2873	0.0392	0.4504	0.0520	–	–	–	–	–	–
η_2	0.2000	0.2085	0.0487	0.2107	0.0684	–	–	–	–	–	–
η_3	−0.1000	−0.0998	0.0099	−0.0975	0.0129	–	–	–	–	–	–
η_4	0.0400	0.0391	0.0041	0.0352	0.0044	–	–	–	–	–	–
η_5	0.3000	0.2802	0.0325	0.4073	0.0370	–	–	–	–	–	–
η_6	−0.2000	−0.2062	0.0252	−0.1609	0.0312	–	–	–	–	–	–
δ	0.3000	0.3166	0.0117	0.1885	0.0521	0.4211	0.1242	–	–	–	–
σ_ξ^2	0.5000	0.5163	0.1406	0.6132	0.1477	0.5348	0.1611	0.4739	0.1289	0.6013	0.1425

Note: This Monte Carlo study consists of 100 repetitions. The values reported in this table are the mean and the standard deviation of parameter estimates across repetitions. The parameters λ, β_1, and β_2 capture, respectively, endogenous, own, and contextual peer effects in equation (9). The parameters γ_0, γ_3, and γ_4 capture local network effects specified in equation (8). The parameters η_1 to η_6 capture global network effects specified in equation (10). The parameter δ captures the incentive effect specified in equation (6). The parameter σ_ξ^2 is the variance of error term ξ in equation (9). In each repetition, we estimate each of the corresponding models with 50,000 MCMC draws. We drop the first 10,000 draws due to burn-in and calculate the (posterior) mean of the remaining draws as parameter estimates.

TABLE F.3. Estimation results based on GPA.

	Full	No Latent	No Global	Latent Only	Activity Alone
Local & global & incentive effects					
Constant (γ_0)	−4.9278	−4.9817	−3.3236	1.0592	–
	(0.0989)	(0.1811)	(0.1918)	(0.1265)	–
Experience in school (sender) (γ_1)	−0.0202	−0.0242	0.0826	0.0540	–
	(0.0208)	(0.0230)	(0.0193)	(0.0141)	–
Experience in school (receiver) (γ_2)	0.0320	0.0228	0.1442	0.1310	–
	(0.0222)	(0.0251)	(0.0221)	(0.0151)	–
Same age (γ_{31})	0.3957	0.3662	0.9478	0.4688	–
	(0.0525)	(0.0964)	(0.0618)	(0.0500)	–
Same sex (γ_{32})	0.3272	0.3572	0.3273	0.4472	–
	(0.0464)	(0.0824)	(0.0747)	(0.0462)	–
Same race (γ_{33})	0.1616	0.1510	0.3281	0.3493	–
	(0.0590)	(0.0902)	(0.0700)	(0.0672)	–
Latent distance (γ_{41})	−0.0729	–	−0.4315	−2.8823	–
	(0.0283)	–	(0.0554)	(0.0533)	–
Latent distance (γ_{42})	−0.0467	–	−0.3452	−2.7970	–
	(0.0237)	–	(0.0534)	(0.0515)	–
Latent distance (γ_{43})	0.0160	–	−0.2251	−2.5115	–
	(0.0126)	–	(0.0813)	(0.0979)	–
Reciprocity (η_1)	1.3611	1.3593	-	–	–
	(0.0520)	(0.0645)	-	–	–
Congestion (η_2)	0.3065	0.3173	-	–	–
	(0.0204)	(0.0317)	-	–	–
Congestion (η_3)	−0.0266	−0.0273	-	–	–
	(0.0017)	(0.0024)	-	–	–
Popularity (η_4)	−0.0058	−0.0067	-	–	–
	(0.0060)	(0.0071)	-	–	–
Trans. triads (η_5)	0.4813	0.4873	-	–	–
	(0.0203)	(0.0203)	-	–	–
Three cycles (η_6)	−0.2145	−0.2197	-	–	–
	(0.0172)	(0.0175)	-	–	–

(*Continues*)

TABLE F.3. *Continued.*

	Full		No Latent		No Global		Latent Only		Activity Alone	
Incentive from GPA (δ)	0.2056		0.1966		0.3777		–		–	
	(0.0841)		(0.0854)		(0.1087)		–		–	
Activity outcome—GPA										
Endogenous (λ)	0.0179		0.0279		0.0139		0.0284		0.0383	
	(0.0065)		(0.0088)		(0.0079)		(0.0080)		(0.0086)	
	Own	Contex.	Own	Contex.	Own	Contex.	Own	Contex.	Own	Contex.
Age	−0.0398	0.0007	−0.0303	0.0000	−0.0351	−0.0016	−0.0320	−0.0023	−0.0235	−0.0027
	(0.0181)	(0.0021)	(0.0101)	(0.0026)	(0.0109)	(0.0020)	(0.0157)	(0.0026)	(0.0139)	(0.0025)
Male	−0.2426	−0.0247	−0.1871	−0.0338	−0.1822	−0.0278	−0.1865	−0.0414	−0.1986	−0.0387
	(0.0377)	(0.0199)	(0.0335)	(0.0221)	(0.0277)	(0.0170)	(0.0368)	(0.0227)	(0.0269)	(0.0163)
Black	0.0470	−0.0688	−0.0477	−0.0737	0.0838	−0.0463	−0.0469	−0.0662	0.0074	−0.0758
	(0.0410)	(0.0216)	(0.0589)	(0.0194)	(0.0336)	(0.0185)	(0.0658)	(0.0184)	(0.0501)	(0.0161)
Asian	0.1642	0.0587	0.0774	−0.1502	0.1692	0.1088	−0.0050	−0.0418	0.0420	0.0284
	(0.0264)	(0.0374)	(0.0938)	(0.0604)	(0.0336)	(0.0393)	(0.1132)	(0.0607)	(0.0452)	(0.0658)
Hispanic	−0.1332	0.0139	−0.1627	−0.0291	−0.0136	0.0398	−0.2696	−0.0146	−0.1968	−0.0210
	(0.0508)	(0.0368)	(0.0666)	(0.0462)	(0.0450)	(0.0245)	(0.0664)	(0.0435)	(0.0287)	(0.0425)
Other race	−0.1156	−0.0290	−0.2559	−0.0349	−0.0317	−0.0695	−0.2385	−0.0327	−0.1787	−0.0623
	(0.0274)	(0.0378)	(0.0756)	(0.0370)	(0.0320)	(0.0310)	(0.0622)	(0.0417)	(0.0572)	(0.0357)
Both parents	0.0358	−0.0455	0.0690	−0.0636	0.0491	−0.0222	0.1038	−0.0351	0.0475	−0.0477
	(0.0421)	(0.0218)	(0.0439)	(0.0249)	(0.0267)	(0.0180)	(0.0421)	(0.0238)	(0.0304)	(0.0226)
Less HS	−0.1967	−0.0979	−0.3728	−0.1249	−0.2118	−0.1060	−0.3255	−0.1002	−0.2655	−0.1296
	(0.0436)	(0.0274)	(0.0693)	(0.0401)	(0.0310)	(0.0227)	(0.0666)	(0.0458)	(0.0449)	(0.0333)
More HS	0.2010	0.0388	0.1411	0.0484	0.2186	0.0369	0.1474	0.0437	0.1722	0.0402
	(0.0283)	(0.0221)	(0.0430)	(0.0199)	(0.0315)	(0.0192)	(0.0405)	(0.0223)	(0.0384)	(0.0193)
Edu missing	−0.1320	−0.0420	−0.1642	0.0159	−0.0866	−0.0434	−0.1847	0.0041	−0.1487	−0.0057
	(0.0372)	(0.0223)	(0.0675)	(0.0317)	(0.0257)	(0.0243)	(0.0622)	(0.0386)	(0.0460)	(0.0338)
Welfare	−0.0519	0.0109	−0.1931	−0.2907	0.0157	0.0976	−0.0087	−0.0616	−0.0726	0.0195
	(0.0657)	(0.0341)	(0.1009)	(0.1370)	(0.0448)	(0.0429)	(0.1721)	(0.0931)	(0.0315)	(0.0399)
Job missing	−0.1337	−0.0278	−0.0686	−0.0023	−0.1459	−0.0013	−0.1046	−0.0132	−0.0626	−0.0072
	(0.0387)	(0.0243)	(0.0556)	(0.0420)	(0.0545)	(0.0217)	(0.0672)	(0.0370)	(0.0484)	(0.0331)

(*Continues*)

TABLE F.3. *Continued.*

	Full		No Latent		No Global		Latent Only		Activity Alone	
Professional	−0.0659	−0.0080	−0.0685	−0.0247	−0.0775	0.0044	−0.0986	−0.0083	−0.0866	−0.0197
	(0.0239)	(0.0300)	(0.0481)	(0.0277)	(0.0235)	(0.0220)	(0.0410)	(0.0262)	(0.0371)	(0.0224)
Other jobs	−0.0306	0.0360	−0.0303	0.0330	−0.0569	0.0603	−0.0604	0.0431	−0.0599	0.0316
	(0.0296)	(0.0180)	(0.0398)	(0.0216)	(0.0337)	(0.0197)	(0.0418)	(0.0231)	(0.0316)	(0.0180)
Num. of students at home	0.0153	0.0072	0.0136	0.0115	0.0089	0.0162	0.0125	0.0136	0.0031	0.0111
	(0.0184)	(0.0106)	(0.0245)	(0.0123)	(0.0230)	(0.0092)	(0.0223)	(0.0109)	(0.0186)	(0.0114)
Latent (ρ_1)	−0.1722	−0.0090	–	–	−0.1674	−0.0496	−0.0106	0.0136	–	–
	(0.0286)	(0.0210)	–	–	(0.0527)	(0.0144)	(0.0441)	(0.0108)	–	–
Latent (ρ_2)	−0.1650	−0.0607	–	–	−0.1111	−0.0248	−0.0521	−0.0111	–	–
	(0.0323)	(0.0161)	–	–	(0.0421)	(0.0220)	(0.0387)	(0.0106)	–	–
Latent (ρ_3)	−0.0861	−0.0416	–	–	0.0137	0.0168	−0.0089	−0.0216	–	–
	(0.0414)	(0.0303)	–	–	(0.0306)	(0.0209)	(0.0394)	(0.0120)	–	–
Group fixed effect	Yes		Yes		Yes		Yes		Yes	
$\sigma_\xi^{2(*)}$	0.3697		0.4596		0.4037		0.4438		0.4520	
	(0.1477)		(0.1510)		(0.1489)		(0.1456)		(0.1458)	

Note: The full model contains the activity outcome equation with interactions and the network formation model, where the network formation model involves the latent characteristic variables and the incentive effect. In the second column, we remove the latent variables from the network formation model. In the third column, we remove the incentive effect from the network formation model. In the fourth column, we estimate only the activity outcome equation. The MCMC runs for 100,000 iterations, and the first 50,000 runs are dropped due to burn-in. The values in parentheses are the standard deviation of draws from MCMC. The parameter $\sigma_\xi^{2(*)}$ denotes the average of the estimated variances of the error term in the activity intensity equation of GPA from different groups. The value in the parentheses is the average standard deviation.

Table F.4. Estimation results based on smoking.

	Full	No Latent	No Global	Latent Only	Activity Alone
Local & global & incentive effects					
Constant (γ_0)	−5.2823	−5.4537	−3.8355	1.1758	–
	(0.1966)	(0.2355)	(0.1885)	(0.1310)	–
Experience in school (sender) (γ_1)	−0.0131	−0.0178	0.0969	0.0473	–
	(0.0265)	(0.0264)	(0.0226)	(0.0144)	–
Experience in school (receiver) (γ_2)	0.0254	0.0310	0.1559	0.1077	–
	(0.0242)	(0.0261)	(0.0211)	(0.0146)	–
Same age (γ_{31})	0.4026	0.4090	1.1439	0.5039	–
	(0.0893)	(0.0934)	(0.1069)	(0.0513)	–
Same sex (γ_{32})	0.3661	0.3732	0.3702	0.4670	–
	(0.0850)	(0.0951)	(0.0924)	(0.0455)	–
Same race (γ_{33})	0.3398	0.3225	0.3951	0.3544	–
	(0.1002)	(0.1058)	(0.1154)	(0.0663)	–
Latent distance (γ_{41})	−0.0971	–	−0.4542	−3.2074	–
	(0.0390)	–	(0.0950)	(0.1310)	–
Latent distance (γ_{42})	−0.0532	–	−0.2885	−2.8133	–
	(0.0258)	–	(0.0653)	(0.1020)	–
Latent distance (γ_{43})	−0.0249	–	−0.1344	−2.5520	–
	(0.0184)	–	(0.0704)	(0.1041)	–
Reciprocity (η_1)	1.4196	1.4041	–	–	–
	(0.0662)	(0.0667)	–	–	–
Congestion (η_2)	0.3570	0.3545	–	–	–
	(0.0329)	(0.0371)	–	–	–
Congestion (η_3)	−0.0290	−0.0290	–	–	–
	(0.0025)	(0.0026)	–	–	–
Popularity (η_4)	−0.0022	−0.0008	–	–	–
	(0.0068)	(0.0072)	–	–	–
Trans. triads (η_5)	0.4746	0.4720	–	–	–
	(0.0216)	(0.0248)	–	–	–

(Continues)

TABLE F.4. *Continued.*

	Full		No Latent		No Global		Latent Only		Activity Alone	
Three cycles (η_6)	−0.2121		−0.2106		–		–		–	
	(0.0188)		(0.0203)		–		–		–	
incentive from smoking (δ)	0.0260		0.0186		0.0282		–		–	
	(0.0174)		(0.0168)		(0.0158)		–		–	
Activity outcome—smoking										
Endogenous (λ)	0.0810		0.0930		0.0762		0.0786		0.0922	
	(0.0176)		(0.0175)		(0.0177)		(0.0165)		(0.0161)	
	Own	Contex.	Own	Contex.	Own	Contex.	Own	Contex.	Own	Contex.
Age	0.3224	−0.0066	0.1606	−0.0086	0.1956	−0.0082	0.1653	−0.0093	0.1615	−0.0087
	(0.0162)	(0.0041)	(0.0357)	(0.0040)	(0.0106)	(0.0024)	(0.0293)	(0.0044)	(0.0305)	(0.0036)
Male	0.1435	0.0223	0.2164	−0.0282	0.2123	−0.0808	0.1493	0.0100	0.0917	0.0186
	(0.0424)	(0.0432)	(0.0628)	(0.0428)	(0.0159)	(0.0282)	(0.0775)	(0.0445)	(0.0732)	(0.0396)
Black	−0.3546	0.0295	−0.5973	0.1402	−0.2062	0.0528	−0.1733	0.1129	−0.2240	0.0824
	(0.0481)	(0.0256)	(0.0680)	(0.0360)	(0.0583)	(0.0157)	(0.0414)	(0.0333)	(0.0824)	(0.0299)
Asian	−0.3802	−0.5119	−0.0521	−0.6845	−0.2020	0.0265	−0.0788	0.0994	−0.1767	0.1333
	(0.0479)	(0.0664)	(0.0771)	(0.1098)	(0.0219)	(0.1615)	(0.1752)	(0.1221)	(0.1347)	(0.0771)
Hispanic	0.4558	0.0399	0.3144	0.1299	0.2182	0.1288	0.4463	0.0715	0.2931	0.0521
	(0.0422)	(0.0374)	(0.0840)	(0.0723)	(0.0290)	(0.0432)	(0.1045)	(0.0624)	(0.0783)	(0.0646)
Other race	0.3661	0.2328	0.2555	0.1080	0.5930	0.3199	0.3661	0.1837	0.3149	0.1032
	(0.0373)	(0.0447)	(0.1006)	(0.0667)	(0.0697)	(0.0431)	(0.1344)	(0.0720)	(0.0957)	(0.0600)
Both parents	−0.0513	−0.1494	−0.1057	0.0065	−0.0613	−0.0504	0.0055	−0.0053	−0.0784	−0.0518
	(0.0316)	(0.0379)	(0.0613)	(0.0462)	(0.0295)	(0.0182)	(0.0811)	(0.0492)	(0.0519)	(0.0372)
Less HS	0.3995	0.0983	0.5019	0.0750	0.2180	0.0958	0.2737	0.1032	0.2805	0.0866
	(0.0481)	(0.0511)	(0.0786)	(0.0704)	(0.1201)	(0.0272)	(0.1140)	(0.0738)	(0.0893)	(0.0643)
More HS	−0.2639	0.0570	−0.1522	−0.0326	−0.1176	0.0561	−0.1297	−0.0446	−0.1811	−0.0260
	(0.0259)	(0.0338)	(0.0668)	(0.0441)	(0.0394)	(0.0352)	(0.0712)	(0.0409)	(0.0704)	(0.0367)
Edu missing	−0.1828	0.2687	−0.1529	−0.0534	−0.2366	0.0965	−0.1563	−0.0482	−0.2454	0.0192
	(0.0336)	(0.0398)	(0.0302)	(0.0655)	(0.0355)	(0.0526)	(0.0962)	(0.0572)	(0.0848)	(0.0496)
Welfare	1.4274	−0.1324	1.1840	−0.5384	2.2607	−0.2426	0.9059	−0.3657	1.0948	−0.0230
	(0.0894)	(0.0535)	(0.0955)	(0.0923)	(0.0979)	(0.1038)	(0.2358)	(0.2077)	(0.1156)	(0.1370)

(*Continues*)

TABLE F.4. *Continued.*

	Full		No Latent		No Global		Latent Only		Activity Alone	
Job missing	0.3730	−0.0081	0.3199	0.0293	0.2902	−0.0003	0.0476	0.0605	0.1015	0.0933
	(0.0413)	(0.0535)	(0.0971)	(0.0634)	(0.0490)	(0.0334)	(0.1011)	(0.0696)	(0.0681)	(0.0618)
Professional	0.3102	0.0566	0.2793	0.0039	0.3343	0.0197	0.1835	0.0367	0.2850	0.0452
	(0.0500)	(0.0345)	(0.0648)	(0.0487)	(0.0492)	(0.0304)	(0.0779)	(0.0536)	(0.0915)	(0.0429)
Other Jobs	0.1797	0.0515	0.1193	0.0621	0.2679	0.0975	0.0080	0.0650	0.0828	0.0823
	(0.0621)	(0.0258)	(0.0549)	(0.0428)	(0.0504)	(0.0228)	(0.0673)	(0.0435)	(0.0721)	(0.0337)
Num. of students at home	−0.1410	0.0325	−0.1193	0.0387	−0.0730	0.0081	−0.1183	0.0228	−0.1342	0.0254
	(0.0353)	(0.0227)	(0.0404)	(0.0223)	(0.0212)	(0.0173)	(0.0393)	(0.0201)	(0.0371)	(0.0206)
Latent (ρ_1)	0.0870	−0.0964	–	–	−0.0039	−0.1326	−0.0198	0.0037	–	–
	(0.0469)	(0.0712)	–	–	(0.0518)	(0.0227)	(0.0622)	(0.0186)	–	–
Latent (ρ_2)	−0.1053	−0.0675	–	–	−0.0076	−0.0480	0.0541	0.0146	–	–
	(0.0330)	(0.0731)	–	–	(0.0363)	(0.0373)	(0.0518)	(0.0234)	–	–
Latent (ρ_3)	−0.4018	−0.0394	–	–	0.0455	−0.0440	0.0751	0.0422	–	–
	(0.0568)	(0.0382)	–	–	(0.0874)	(0.0598)	(0.0272)	(0.0221)	–	–
Group fixed effect	Yes		Yes		Yes		Yes		Yes	
$\sigma_{\xi}^{2(*)}$	2.7869		3.1078		2.9894		3.0731		3.0733	
	(2.6676)		(2.8205)		(2.7645)		(2.8614)		(2.8507)	

Note: The full model contains the activity outcome equation with interactions and the network formation model, where the network formation model involves the latent characteristic variables and the incentive effect. In the second column, we remove the latent variables from the network formation model. In the third column, we remove the global effect from the network formation model. In the fourth column, we remove the global effect and the latent variables from the network formation model. In the fifth column, we estimate only the activity outcome equation. The MCMC runs for 100,000 iterations, and the first 50,000 runs are dropped due to burn-in. The values in parentheses are the standard deviation of draws from MCMC. The parameter $\sigma_{\xi}^{2(*)}$ denotes the average of the estimated variance of the error terms in the activity intensity equation of smoking from different groups. The values in the parentheses are the average standard deviation.

TABLE F.5. Estimation results for choosing latent dimensions in the network formation model.

	D1		D2		D3		D4	
Local effects								
Constant (γ_0)	−2.1872		−0.3380		1.1101		2.3569	
	(0.0846)		(0.1024)		(0.0958)		(0.1472)	
Experience in school (sender) (γ_1)	0.0602		0.0461		0.0528		0.0576	
	(0.0109)		(0.0132)		(0.0148)		(0.0180)	
Experience in school (receiver) (γ_2)	0.1145		0.1139		0.1301		0.1379	
	(0.0119)		(0.0121)		(0.0147)		(0.0177)	
Same age (γ_{31})	0.4598		0.4154		0.4450		0.4761	
	(0.0490)		(0.0529)		(0.0586)		(0.0513)	
Same sex (γ_{32})	0.4494		0.5289		0.5477		0.5595	
	(0.0385)		(0.0450)		(0.0517)		(0.0546)	
Same race (γ_{33})	0.3989		0.3500		0.3961		0.4446	
	(0.0538)		(0.0654)		(0.0595)		(0.0853)	
Latent distance (γ_{41})	−5.2440		−3.7725		−3.9387		−3.8920	
	(0.1786)		(0.1136)		(0.1214)		(0.1472)	
Latent distance (γ_{42})	-		−3.4090		−2.5592		−2.6235	
	-		(0.1430)		(0.1000)		(0.1498)	
Latent distance (γ_{43})	-		–		−2.3793		−2.0275	
	–		–		(0.0887)		(0.1181)	
Latent distance (γ_{44})	-		–		–		−1.6992	
	-		–		–		(0.1244)	
Activity outcome—GPA								
Endogenous (λ)	0.0296		0.0291		0.0239		0.0240	
	(0.0104)		(0.0107)		(0.0091)		(0.0081)	
	Own	Contex.	Own	Contex.	Own	Contex.	Own	Contex.
Age	−0.0437	−0.0003	−0.0406	0.0000	−0.0388	0.0009	−0.0437	0.0020
	(0.0186)	(0.0029)	(0.0167)	(0.0028)	(0.0202)	(0.0025)	(0.0163)	(0.0025)
Male	−0.1831	−0.0223	−0.1856	−0.0170	−0.1863	−0.0166	−0.1986	−0.0258
	(0.0405)	(0.0241)	(0.0398)	(0.0228)	(0.0381)	(0.0232)	(0.0428)	(0.0248)

(*Continues*)

Table F.5. *Continued.*

	D1		D2		D3		D4	
Black	0.0024	−0.0572	−0.0228	−0.0651	−0.0109	−0.0489	0.0180	−0.0748
	(0.0822)	(0.0246)	(0.0793)	(0.0234)	(0.0716)	(0.0219)	(0.0553)	(0.0245)
Asian	−0.0419	−0.1789	−0.0814	−0.2066	−0.0384	−0.1824	−0.0026	−0.1976
	(0.1105)	(0.0789)	(0.1049)	(0.0840)	(0.0902)	(0.0567)	(0.0847)	(0.0724)
Hispanic	−0.1961	−0.0528	−0.1902	−0.0319	−0.2433	−0.0502	−0.1702	−0.0506
	(0.0747)	(0.0609)	(0.0757)	(0.0450)	(0.0737)	(0.0500)	(0.0617)	(0.0559)
Other race	−0.0643	−0.0883	−0.0805	−0.0889	−0.0555	−0.0838	−0.0856	−0.0769
	(0.0718)	(0.0501)	(0.0813)	(0.0497)	(0.0575)	(0.0473)	(0.0511)	(0.0503)
Both parents	0.1120	−0.0575	0.1071	−0.0699	0.1145	−0.0602	0.1017	−0.0739
	(0.0429)	(0.0304)	(0.0484)	(0.0293)	(0.0427)	(0.0258)	(0.0456)	(0.0290)
Less HS	−0.3230	−0.0966	−0.2794	−0.0788	−0.2972	−0.0924	−0.2670	−0.0729
	(0.0825)	(0.0446)	(0.0811)	(0.0400)	(0.0623)	(0.0435)	(0.0616)	(0.0359)
More HS	0.1513	0.0246	0.1674	0.0326	0.1536	0.0261	0.1569	0.0333
	(0.0458)	(0.0254)	(0.0416)	(0.0245)	(0.0435)	(0.0248)	(0.0388)	(0.0225)
Edu missing	−0.1868	0.0441	−0.1607	0.0371	−0.1624	0.0472	−0.1756	0.0549
	(0.0660)	(0.0379)	(0.0576)	(0.0405)	(0.0634)	(0.0366)	(0.0417)	(0.0397)
Welfare	−0.1777	−0.2884	−0.1886	−0.3757	−0.1001	−0.2892	−0.1167	−0.3970
	(0.1725)	(0.1402)	(0.1462)	(0.1266)	(0.1589)	(0.1173)	(0.0952)	(0.0764)
Job missing	−0.1418	−0.0602	−0.1192	−0.0373	−0.1625	−0.0522	−0.1580	−0.0548
	(0.0648)	(0.0464)	(0.0706)	(0.0433)	(0.0503)	(0.0381)	(0.0536)	(0.0391)
Professional	−0.1268	0.0057	−0.1194	0.0079	−0.1347	0.0003	−0.1010	0.0079
	(0.0417)	(0.0312)	(0.0479)	(0.0272)	(0.0377)	(0.0264)	(0.0371)	(0.0266)
Other jobs	−0.0464	0.0256	−0.0417	0.0260	−0.0679	0.0214	−0.0273	0.0220
	(0.0408)	(0.0251)	(0.0455)	(0.0233)	(0.0377)	(0.0227)	(0.0368)	(0.0217)
Num. of other students at home	0.0180	0.0067	0.0203	0.0074	0.0216	0.0096	0.0217	0.0055
	(0.0243)	(0.0134)	(0.0260)	(0.0128)	(0.0232)	(0.0140)	(0.0251)	(0.0129)
Latent (ρ_{11})	0.0544*	−0.0145	−0.0376	0.0143	0.0272	−0.0170	0.1168	−0.0314
	(0.0316)	(0.0117)	(0.0368)	(0.0122)	(0.0369)	(0.0112)	(0.0402)	(0.0124)
Latent (ρ_{12})	–	–	0.0698	−0.0293	0.0721	−0.0235	−0.0274	0.0375
	–	–	(0.0427)	(0.0131)	(0.0472)	(0.0197)	(0.0450)	(0.0136)

(*Continues*)

Table F.5. *Continued.*

	D1		D2		D3		D4	
Latent (ρ_{13})	–	–	–	–	0.0724	−0.0045	0.0615	−0.0242
	–	–	–	–	(0.0466)	(0.0188)	(0.0473)	(0.0163)
Latent (ρ_{14})	–	–	–	–	–	–	−0.0228	0.0044
	–	–	–	–	–	–	(0.0471)	(0.0168)
Activity outcome—smoking								
Endogenous (λ)	0.1145		0.1126		0.1056		0.1051	
	(0.0191)		(0.0195)		(0.0197)		(0.0208)	
	Own	Contex.	Own	Contex.	Own	Contex.	Own	Contex.
Age	0.1550	−0.0026	0.1802	−0.0001	0.2051	−0.0120	0.1213	−0.0055
	(0.0351)	(0.0043)	(0.0414)	(0.0045)	(0.0455)	(0.0053)	(0.0465)	(0.0046)
Male	−0.0083	0.0335	0.0000	−0.0092	−0.0494	0.0414	0.1285	0.0336
	(0.0663)	(0.0443)	(0.0938)	(0.0493)	(0.0915)	(0.0436)	(0.0694)	(0.0403)
Black	−0.2077	0.0347	−0.1720	0.0402	−0.4153	0.0810	−0.3838	0.0468
	(0.2373)	(0.0329)	(0.2701)	(0.0328)	(0.2075)	(0.0377)	(0.1018)	(0.0340)
Asian	−0.7107	0.0976	−0.9664	0.1356	−1.2797	0.3093	−1.0802	0.3742
	(0.2862)	(0.1022)	(0.2530)	(0.0808)	(0.3075)	(0.1120)	(0.2477)	(0.0830)
Hispanic	−0.0116	0.0477	−0.0860	−0.0526	−0.0591	0.1509	−0.1509	0.1633
	(0.1285)	(0.0774)	(0.0743)	(0.0770)	(0.2321)	(0.0849)	(0.1112)	(0.0825)
Other race	0.2867	0.2504	0.0296	0.3143	0.1891	0.5043	−0.0123	0.2626
	(0.1204)	(0.1079)	(0.0724)	(0.1162)	(0.1459)	(0.0714)	(0.0853)	(0.0828)
Both parents	−0.0623	−0.0925	−0.1047	−0.0990	−0.0588	−0.0351	−0.0656	−0.1559
	(0.0810)	(0.0549)	(0.0930)	(0.0522)	(0.0878)	(0.0579)	(0.0904)	(0.0475)
Less HS	0.4121	0.0056	0.0507	−0.0555	0.2837	0.0762	0.5316	0.1003
	(0.0948)	(0.0780)	(0.1320)	(0.0812)	(0.0972)	(0.0667)	(0.1249)	(0.0665)
More HS	−0.0765	−0.0189	−0.1891	−0.0110	−0.1502	0.0131	−0.1992	0.0374
	(0.0740)	(0.0436)	(0.0646)	(0.0514)	(0.0906)	(0.0472)	(0.0782)	(0.0537)
Edu missing	0.0772	−0.0031	−0.2873	0.0266	−0.0775	−0.0273	−0.0605	0.0771
	(0.1111)	(0.0717)	(0.1334)	(0.0566)	(0.0874)	(0.0806)	(0.0617)	(0.0643)
Welfare	1.8921	0.1051	1.4732	0.5841	1.6348	0.0728	1.5210	0.5886
	(0.0926)	(0.3167)	(0.1305)	(0.1121)	(0.2791)	(0.4136)	(0.2065)	(0.1109)

(*Continues*)

Table F.5. *Continued.*

	D1		D2		D3		D4	
Job missing	0.1653	0.1481	0.5186	0.1601	0.5669	0.1639	0.5206	0.2061
	(0.2069)	(0.0732)	(0.0918)	(0.0481)	(0.1226)	(0.0907)	(0.1332)	(0.0760)
Professional	0.1461	0.0254	0.3500	−0.0609	0.4512	0.0344	0.5027	0.0370
	(0.0987)	(0.0552)	(0.0811)	(0.0371)	(0.1044)	(0.0559)	(0.0715)	(0.0556)
Other jobs	−0.0252	0.0234	0.0779	0.0072	0.1472	0.0682	0.0762	0.0728
	(0.1073)	(0.0381)	(0.0983)	(0.0362)	(0.0882)	(0.0450)	(0.0806)	(0.0477)
Num. of other students at home	−0.0932	0.0068	−0.0981	0.0142	−0.0636	0.0173	−0.0938	−0.0089
	(0.0448)	(0.0246)	(0.0450)	(0.0217)	(0.0472)	(0.0291)	(0.0481)	(0.0252)
Latent (ρ_{21})	0.0089	−0.0270	0.0149	0.0320	−0.0119	−0.0027	−0.0990	0.0141
	(0.0453)	(0.0216)	(0.0352)	(0.0201)	(0.0510)	(0.0201)	(0.0544)	(0.0209)
Latent (ρ_{22})	–	–	0.1039	−0.0555	−0.0332	0.0168	−0.1924	0.0262
	–	–	(0.0530)	(0.0199)	(0.0574)	(0.0290)	(0.0481)	(0.0223)
Latent (ρ_{23})	–	–	–	–	0.0758	−0.0579	0.0677	0.0418
	–	–	–	–	(0.0562)	(0.0216)	(0.0682)	(0.0237)
Latent (ρ_{24})	–	–	–	–	–	–	−0.0279	0.0427
	–	–	–	–	–	–	(0.0558)	(0.0300)
Group fixed effect	Yes		Yes		Yes		Yes	
$\sigma^{2(*)}_{\xi_{uc,g}}$ (GPA)	0.4781		0.4749		0.4668		0.4598	
	(0.1544)		(0.1555)		(0.1548)		(0.1599)	
$\sigma^{2(*)}_{\xi_{c,g}}$ (smoking)	3.5460		3.5775		3.6085		3.5416	
	(3.1150)		(3.1508)		(3.1239)		(3.1436)	
$\sigma^{(*)}_{\xi_{ucc,g}}$	−0.2502		−0.2538		−0.2542		−0.2426	
	(0.2741)		(0.2773)		(0.2755)		(0.2726)	
AICM	26,542		24,115		23,608		24,795	
se(AICM)	(51.89)		(63.80)		(78.42)		(115.31)	

Note: Models D1 to D4 stand for network formation models with only latent variables of dimensions one to four. The MCMC runs for 100,000 iterations, and the first 50,000 runs are dropped due to burn-in. The values in parentheses are the standard deviation of draws from MCMC. The parameters $\sigma^{2(*)}_{\xi_{uc,g}}$, $\sigma^{2(*)}_{\xi_{c,g}}$, and $\sigma^{(*)}_{\xi_{ucc,g}}$ denote the average of the estimated variance for error terms in the activity intensity equations of GPA, smoking, and their covariances from different groups. The values in the parentheses are the average standard deviation.

TABLE F.6. Estimation results based on both GPA and smoking with correlated random effects.

	Full	No Latent	No Global	Latent Only	Activity Alone
Local & global & incentive effects					
Constant (γ_0)	−4.9139	−5.6705	−3.9901	1.0454	–
	(0.0950)	(0.0960)	(0.2027)	(0.1067)	–
In school (sender) (γ_1)	−0.0331	−0.0080	0.0905	0.0542	–
	(0.0259)	(0.0191)	(0.0211)	(0.0149)	–
In school (receiver) (γ_2)	0.0299	0.0444	0.1613	0.1303	–
	(0.0250)	(0.0204)	(0.0221)	(0.0139)	–
Same age (γ_{31})	0.4263	0.4412	1.0434	0.4706	–
	(0.0746)	(0.0654)	(0.0777)	(0.0593)	–
Same sex (γ_{32})	0.3616	0.4689	0.3473	0.5394	–
	(0.0603)	(0.0493)	(0.0807)	(0.0565)	–
Same race (γ_{33})	0.1557	0.1845	0.4173	0.4537	–
	(0.0680)	(0.0685)	(0.0947)	(0.0773)	–
Latent distance (γ_{41})	−0.1455	–	−0.3712	−3.7634	–
	(0.0356)	–	(0.0764)	(0.2039)	–
Latent distance (γ_{42})	−0.0589	–	−0.2471	−2.6021	–
	(0.0428)	–	(0.0717)	(0.0955)	–
Latent distance (γ_{43})	−0.0523	–	−0.1084	−2.4456	–
	(0.0388)	–	(0.0679)	(0.1061)	–
Reciprocity (η_1)	1.4205	1.4431	–	–	–
	(0.0696)	(0.0454)	–	–	–
Congestion (η_2)	0.3145	0.3545	–	–	–
	(0.0354)	(0.0241)	–	–	–
Congestion (η_3)	−0.0280	−0.0306	–	–	–
	(0.0029)	(0.0021)	–	–	–
Popularity (η_4)	−0.0013	0.0028	–	–	–
	(0.0072)	(0.0053)	–	–	–
Trans. triads (η_5)	0.4956	0.4772	–	–	–
	(0.0255)	(0.0180)	–	–	–
Three cycles (η_6)	−0.2159	−0.2124	–	–	–
	(0.0217)	(0.0161)	–	–	–

(*Continues*)

TABLE F.6. *Continued.*

	Full		No Latent		No Global		Latent Only		Activity Alone	
Incentive from GPA (δ_1)	0.1885		0.2153		0.3221		–		–	
	(0.0858)		(0.0429)		(0.1019)		–		–	
Incentive from smoking (δ_2)	0.0130		0.0204		0.0217		–		–	
	(0.0127)		(0.0126)		(0.0127)		–		–	
Activity outcome—GPA										
Endogenous (λ)	0.0179		0.0206		0.0171		0.0241		0.0304	
	(0.0094)		(0.0101)		(0.0085)		(0.0085)		(0.0097)	
	Own	Contex.	Own	Contex.	Own	Contex.	Own	Contex.	Own	Contex.
Age	–0.0431	0.0015	–0.0410	0.0013	–0.0391	–0.0001	–0.0508	0.0015	–0.0422	–0.0001
	(0.0137)	(0.0027)	(0.0159)	(0.0028)	(0.0163)	(0.0024)	(0.0190)	(0.0024)	(0.0172)	(0.0025)
Male	–0.1826	–0.0163	–0.1788	–0.0189	–0.1841	–0.0101	–0.1943	–0.0088	–0.1913	–0.0200
	(0.0418)	(0.0255)	(0.0393)	(0.0245)	(0.0378)	(0.0221)	(0.0393)	(0.0271)	(0.0388)	(0.0259)
Black	0.0009	–0.0642	–0.0683	–0.0517	–0.0454	–0.0611	–0.0052	–0.0622	–0.0445	–0.0615
	(0.0658)	(0.0210)	(0.0872)	(0.0252)	(0.0578)	(0.0218)	(0.0630)	(0.0246)	(0.0695)	(0.0236)
Asian	–0.0163	–0.2411	–0.1246	–0.2570	–0.0516	–0.1793	0.0271	–0.2564	–0.0307	–0.2396
	(0.0838)	(0.0721)	(0.1031)	(0.0995)	(0.0917)	(0.0682)	(0.0886)	(0.0820)	(0.1297)	(0.0763)
Hispanic	–0.2441	–0.0393	–0.2413	–0.0376	–0.2502	–0.0439	–0.1860	–0.0863	–0.2132	–0.0454
	(0.0670)	(0.0565)	(0.0852)	(0.0613)	(0.0651)	(0.0486)	(0.0700)	(0.0500)	(0.0876)	(0.0588)
Other race	–0.0982	–0.1051	–0.1236	–0.0954	–0.0825	–0.1052	–0.0854	–0.0973	–0.0934	–0.1021
	(0.0645)	(0.0525)	(0.0836)	(0.0475)	(0.0592)	(0.0458)	(0.0819)	(0.0423)	(0.0658)	(0.0530)
Both parents	0.1071	–0.0687	0.1062	–0.0624	0.1145	–0.0546	0.1144	–0.0748	0.1118	–0.0671
	(0.0456)	(0.0256)	(0.0529)	(0.0291)	(0.0418)	(0.0250)	(0.0356)	(0.0295)	(0.0458)	(0.0322)
Less HS	–0.3320	–0.0984	–0.3193	–0.0915	–0.2783	–0.0991	–0.2561	–0.0673	–0.3158	–0.0844
	(0.0607)	(0.0363)	(0.0782)	(0.0448)	(0.0583)	(0.0394)	(0.0707)	(0.0460)	(0.0774)	(0.0415)
More HS	0.1483	0.0284	0.1626	0.0312	0.1516	0.0385	0.1613	0.0394	0.1569	0.0285
	(0.0396)	(0.0260)	(0.0448)	(0.0267)	(0.0346)	(0.0238)	(0.0417)	(0.0261)	(0.0403)	(0.0254)
Edu missing	–0.1862	0.0372	–0.1438	0.0320	–0.1673	0.0338	–0.1908	0.0410	–0.1921	0.0355
	(0.0509)	(0.0400)	(0.0576)	(0.0408)	(0.0479)	(0.0331)	(0.0625)	(0.0400)	(0.0589)	(0.0401)
Welfare	–0.1263	–0.2803	–0.2647	–0.3197	–0.1310	–0.2596	–0.1076	–0.3489	–0.2086	–0.2487
	(0.1124)	(0.1004)	(0.1868)	(0.1505)	(0.1269)	(0.0912)	(0.1370)	(0.1016)	(0.1609)	(0.1529)

(*Continues*)

TABLE F.6. *Continued.*

	Full		No Latent		No Global		Latent Only		Activity Alone	
Job missing	−0.1737	−0.0289	−0.1333	−0.0525	−0.1706	−0.0253	−0.1337	−0.0456	−0.1433	−0.0367
	(0.0522)	(0.0404)	(0.0733)	(0.0494)	(0.0614)	(0.0404)	(0.0838)	(0.0454)	(0.0730)	(0.0491)
Professional	−0.1429	0.0100	−0.1191	−0.0010	−0.1422	0.0175	−0.1482	−0.0035	−0.1089	0.0045
	(0.0423)	(0.0277)	(0.0465)	(0.0292)	(0.0454)	(0.0243)	(0.0507)	(0.0256)	(0.0609)	(0.0287)
Other jobs	−0.0726	0.0209	−0.0432	0.0229	−0.0740	0.0369	−0.0421	0.0142	−0.0348	0.0260
	(0.0451)	(0.0237)	(0.0468)	(0.0239)	(0.0399)	(0.0217)	(0.0444)	(0.0218)	(0.0501)	(0.0227)
Num. of students at home	0.0189	0.0091	0.0248	0.0091	0.0169	0.0142	0.0109	0.0056	0.0203	0.0061
	(0.0261)	(0.0122)	(0.0263)	(0.0130)	(0.0247)	(0.0118)	(0.0236)	(0.0127)	(0.0237)	(0.0123)
Latent (ρ_{11})	0.0756	0.0102	–	–	0.0646	0.0053	−0.0922	0.0054	–	–
	(0.0177)	(0.0263)	–	–	(0.0171)	(0.0239)	(0.0374)	(0.0114)	–	–
Latent (ρ_{12})	0.0020	−0.0065	–	–	0.0375	0.0052	−0.0292	−0.0072	–	–
	(0.0720)	(0.0278)	–	–	(0.0608)	(0.0240)	(0.0481)	(0.0175)	–	–
Latent (ρ_{13})	0.0591	0.0149	–	–	0.0130	−0.0030	0.0218	−0.0021	–	–
	(0.0551)	(0.0265)	–	–	(0.0538)	(0.0263)	(0.0358)	(0.0165)	–	–
Activity outcome—smoking										
Endogenous (λ)	0.1056		0.1116		0.1070		0.1065		0.1157	
	(0.0198)		(0.0211)		(0.0202)		(0.0206)		(0.0184)	
	Own	Contex.	Own	Contex.	Own	Contex.	Own	Contex.	Own	Contex.
Age	0.1729	−0.0074	0.1898	−0.0086	0.1541	−0.0050	0.1614	−0.0050	0.1689	−0.0078
	(0.0419)	(0.0040)	(0.0435)	(0.0042)	(0.0393)	(0.0043)	(0.0552)	(0.0042)	(0.0513)	(0.0040)
Male	0.0217	0.0116	−0.0600	0.0209	−0.0766	0.0461	0.0896	0.0046	0.0510	0.0034
	(0.0575)	(0.0400)	(0.0711)	(0.0415)	(0.0637)	(0.0352)	(0.0652)	(0.0469)	(0.0551)	(0.0379)
Black	−0.2923	0.0644	−0.0732	0.0793	−0.2327	0.0505	−0.3879	0.0379	0.0007	0.0789
	(0.1076)	(0.0344)	(0.1044)	(0.0414)	(0.0974)	(0.0319)	(0.1932)	(0.0365)	(0.1345)	(0.0356)
Asian	−1.1496	0.3273	−0.6471	0.1070	−1.0710	0.3737	−1.1972	0.1127	−0.4079	0.3377
	(0.1319)	(0.0771)	(0.1621)	(0.1192)	(0.1412)	(0.0935)	(0.2202)	(0.1172)	(0.2425)	(0.1387)
Hispanic	0.0895	0.0931	0.1771	−0.0020	0.0240	0.1370	−0.1233	0.0977	0.1711	0.0549
	(0.0798)	(0.0880)	(0.1198)	(0.0938)	(0.1236)	(0.0822)	(0.1852)	(0.0843)	(0.0733)	(0.0918)
Other race	0.1163	0.4460	0.3488	0.4000	0.1608	0.4234	0.1391	0.4786	0.0324	0.3383
	(0.0751)	(0.0795)	(0.1189)	(0.0785)	(0.0888)	(0.0687)	(0.1405)	(0.0948)	(0.1392)	(0.0913)

(*Continues*)

TABLE F.6. *Continued.*

	Full		No Latent		No Global		Latent Only		Activity Alone	
Both parents	−0.1407	−0.0553	−0.0744	−0.0268	−0.0291	−0.0878	−0.1246	−0.1005	−0.0520	−0.0335
	(0.0533)	(0.0437)	(0.0755)	(0.0460)	(0.0631)	(0.0424)	(0.0797)	(0.0591)	(0.0909)	(0.0452)
Less HS	0.2771	0.0044	0.2593	−0.0055	0.1805	0.0300	0.1745	0.0731	0.2152	0.0181
	(0.0966)	(0.0494)	(0.0898)	(0.0766)	(0.1025)	(0.0690)	(0.1078)	(0.0759)	(0.0799)	(0.0656)
More HS	−0.0977	−0.0438	−0.1478	−0.0515	−0.2080	0.0045	−0.2413	0.0512	−0.1184	−0.0164
	(0.0613)	(0.0496)	(0.0804)	(0.0442)	(0.0693)	(0.0402)	(0.0636)	(0.0450)	(0.0684)	(0.0418)
Edu missing	−0.1098	−0.0738	−0.1401	0.0198	−0.0695	0.0238	−0.0847	0.1071	−0.0339	0.0277
	(0.0609)	(0.0687)	(0.1049)	(0.0681)	(0.0814)	(0.0618)	(0.0720)	(0.0603)	(0.1015)	(0.0723)
Welfare	1.5716	0.0639	1.3942	−0.3763	1.9361	0.0343	2.3290	0.6267	1.1864	−0.0820
	(0.0864)	(0.0691)	(0.1838)	(0.1322)	(0.2247)	(0.1812)	(0.5425)	(0.0992)	(0.2050)	(0.2015)
Job missing	0.5180	0.2334	0.2237	0.2174	0.3828	0.1033	0.5810	0.1274	0.2110	0.1376
	(0.0699)	(0.0699)	(0.1305)	(0.0840)	(0.1044)	(0.0706)	(0.0867)	(0.0647)	(0.1617)	(0.0827)
Professional	0.3470	0.0869	0.2601	0.0598	0.3999	−0.0113	0.5637	−0.0386	0.2376	0.0385
	(0.0767)	(0.0403)	(0.1122)	(0.0483)	(0.0823)	(0.0463)	(0.0738)	(0.0548)	(0.1104)	(0.0487)
Other jobs	0.0910	0.0803	−0.0520	0.0700	0.1116	0.0312	0.1784	0.0089	−0.0415	0.0498
	(0.0640)	(0.0319)	(0.1014)	(0.0362)	(0.0925)	(0.0352)	(0.0925)	(0.0379)	(0.0931)	(0.0395)
Num. of students at home	−0.0778	−0.0012	−0.0912	0.0037	−0.0456	−0.0069	−0.0209	0.0191	−0.0726	0.0057
	(0.0432)	(0.0233)	(0.0472)	(0.0244)	(0.0417)	(0.0238)	(0.0469)	(0.0249)	(0.0413)	(0.0245)
Latent (ρ_{21})	0.0305	0.0051	–	–	−0.0434	−0.0362	0.0240	−0.0204	–	–
	(0.0509)	(0.0322)	–	–	(0.0481)	(0.0377)	(0.0603)	(0.0226)	–	–
Latent (ρ_{22})	0.0096	−0.0092	–	–	0.0485	−0.0040	0.0105	0.0256	–	–
	(0.0818)	(0.0480)	–	–	(0.0543)	(0.0457)	(0.0521)	(0.0230)	–	–
Latent (ρ_{23})	0.0783	−0.0039	–	–	0.0543	0.0246	0.0750	−0.0592	–	–
	(0.0561)	(0.0477)	–	–	(0.0547)	(0.0355)	(0.0788)	(0.0298)	–	–
Group correlated random effects										
	GPA	smoking	GPA	smoking	GPA	smoking	GPA	smoking	GPA	smoking
Age (group mean)	0.1574	−0.0229	0.1689	−0.0436	0.1657	−0.0195	0.1650	−0.0233	0.1725	−0.0218
	(0.9469)	(0.7536)	(0.7473)	(0.5689)	(0.9305)	(0.6984)	(0.9506)	(0.7350)	(0.7441)	(0.5463)
Male (group mean)	0.0869	−0.0153	−0.0102	0.2246	0.0528	−0.1029	0.0039	−0.1877	0.0636	0.0715
	(20.7801)	(18.1171)	(18.8131)	(16.1931)	(20.5633)	(17.5241)	(20.8496)	(17.8721)	(18.8444)	(15.7691)

(*Continues*)

TABLE F.6. *Continued.*

	Full		No Latent		No Global		Latent Only		Activity Alone	
Black (group mean)	0.3923	−0.2900	0.4153	−0.4260	0.4903	−0.2482	0.4987	−0.1541	0.4075	−0.5261
	(6.7101)	(5.4477)	(5.4776)	(4.2710)	(6.6682)	(5.1642)	(6.9952)	(5.5628)	(5.4720)	(4.0552)
Asian (group mean)	1.2234	−1.8222	1.7459	−2.3822	1.3625	−1.9639	1.1787	−1.9263	1.7860	−3.1009
	(29.9844)	(29.2814)	(29.4657)	(28.6543)	(29.8898)	(29.0662)	(29.9882)	(29.1971)	(29.5681)	(28.3376)
Hispanic (group mean)	2.0278	−0.7824	2.2743	−0.9126	2.1110	−1.0348	1.9569	−0.5607	2.2357	−0.7660
	(22.2424)	(19.3251)	(19.9353)	(16.5481)	(22.2359)	(18.7233)	(22.4462)	(18.9384)	(19.9441)	(15.9357)
Other race (group mean)	1.1491	−2.7396	1.4670	−3.3436	1.0165	−2.8801	0.8634	−2.6564	1.4475	−3.1639
	(22.7938)	(19.6939)	(20.3389)	(16.8477)	(22.5044)	(18.9372)	(22.7333)	(19.3327)	(20.2808)	(16.1445)
Both parents (group mean)	0.4395	−0.5457	0.1444	−0.2263	0.3969	−0.4131	0.6268	−0.3149	0.0612	−0.2458
	(17.7547)	(15.0297)	(15.2262)	(12.2757)	(17.6387)	(14.4283)	(17.7311)	(14.7450)	(15.2483)	(11.7482)
Less HS (group mean)	1.4726	−2.0600	2.1331	−2.6002	1.4712	−1.9634	1.3628	−1.9035	2.0071	−2.8791
	(24.3784)	(21.7740)	(21.8755)	(18.7165)	(24.1146)	(21.0380)	(24.3910)	(21.6937)	(21.8784)	(18.2459)
More HS (group mean)	1.2522	−1.7580	1.3978	−1.8341	1.1228	−1.9066	1.0302	−2.1076	1.3721	−1.8651
	(15.1748)	(13.2763)	(13.6783)	(11.4721)	(15.0841)	(12.7378)	(15.3148)	(13.0784)	(13.6439)	(11.0100)
Edu missing (group mean)	1.4077	−0.9712	1.5062	−0.7977	1.2593	−1.3228	1.2083	−0.9619	1.4995	−0.6485
	(25.9592)	(23.7459)	(24.0180)	(21.2605)	(25.9616)	(23.1844)	(25.9444)	(23.4406)	(24.0527)	(20.5102)
Welfare (group mean)	0.0012	−0.2173	0.2043	−0.0912	−0.0724	−0.2033	0.0719	−0.0938	0.0389	−0.1054
	(31.4629)	(31.3242)	(31.6158)	(31.0995)	(31.4503)	(31.4132)	(31.5226)	(31.3497)	(31.4217)	(31.2017)
Job missing (group mean)	0.3762	−0.8441	0.6378	−0.3175	0.5921	−0.6397	0.6112	−0.6572	0.5645	−0.1834
	(25.1399)	(22.7252)	(23.2301)	(19.9317)	(25.1515)	(22.0046)	(25.3033)	(22.4565)	(23.2752)	(19.3424)
Professional (group mean)	−0.0044	−0.8266	−0.1684	−0.8985	−0.0199	−0.4817	0.1076	−0.3670	−0.1584	−1.0599
	(20.4935)	(18.2802)	(18.0844)	(15.3369)	(20.3611)	(17.6343)	(20.2421)	(17.3508)	(18.0484)	(14.7901)
Other jobs (group mean)	−0.3286	0.6599	−0.4752	0.5195	−0.4610	0.9612	−0.4185	0.8056	−0.4446	0.6448
	(15.3348)	(12.7803)	(12.1830)	(9.7284)	(15.0281)	(12.0511)	(14.8921)	(12.1447)	(12.1616)	(9.4069)
Num. of students at home (group mean)	0.0669	−0.1695	0.0495	−0.3294	0.0296	−0.1615	0.0343	−0.3718	0.0616	−0.3609
	(6.5683)	(5.3758)	(5.2195)	(4.1161)	(6.4195)	(5.0290)	(6.4392)	(5.1635)	(5.2363)	(3.9479)
Latent 1 (group mean)	0.0741	−0.0954	–	–	−0.2080	0.2109	0.0241	0.1002	–	–
	(13.0829)	(10.8969)	–	–	(12.7508)	(10.3105)	(13.4819)	(11.1470)	–	–
Latent 2 (group mean)	−0.0568	0.0670	–	–	0.0232	−0.0403	0.0252	−0.2772	–	–
	(12.9157)	(10.8458)	–	–	(12.7316)	(10.2495)	(13.7154)	(11.3008)	–	–

(*Continues*)

TABLE F.6. *Continued.*

	Full		No Latent		No Global		Latent Only		Activity Alone	
Latent 3 (group mean)	0.0293	−0.0789	–	–	−0.0103	−0.2540	−0.0806	0.4535	–	–
	(13.0715)	(10.8881)	–	–	(12.7377)	(10.2508)	(13.1329)	(10.7256)	–	–
$\sigma^{2(*)}_{\xi_{uc,g}}$ (GPA)	0.4545		0.4818		0.4602		0.4730		0.4815	
	(0.1530)		(0.1533)		(0.1508)		(0.1546)		(0.1578)	
$\sigma^{2(*)}_{\xi_{c,g}}$ (smoking)	3.5382		3.5671		3.5606		3.6237		3.5804	
	(3.1026)		(3.1227)		(3.1299)		(3.1319)		(3.1490)	
$\sigma^{(*)}_{\xi_{ucc,g}}$	−0.2466		−0.2420		−0.2441		−0.2577		−0.2473	
	(0.2626)		(0.2684)		(0.2663)		(0.2839)		(0.2722)	
σ^2_a (GPA)	12.8010		12.4743		12.4091		13.3855		12.5212	
	(5.2290)		(5.2042)		(5.1912)		(5.6193)		(5.2314)	
σ^2_a (smoking)	5.6279		6.4484		4.5865		5.6291		5.8314	
	(3.5993)		(4.2203)		(3.1207)		(4.6745)		(4.1346)	

Note: The full model contains the activity outcome equations for GPA, smoking, and the network formation model, where the network formation model involves the latent characteristic variables, the global effect, and the incentive effect. In the second column, we remove the latent variables from the network formation model. In the third column, we remove the global effect from the network formation model. In the fourth column, we remove the global effect and the latent variables from the network formation model. In the fifth column, we estimate only the activity outcome equations. The MCMC runs for 100,000 iterations, and the first 50,000 runs are dropped due to burn-in. Values in parentheses are the standard deviations of draws from MCMC. The parameters $\sigma^{2(*)}_{\xi_{uc,g}}$, $\sigma^{2(*)}_{\xi_{c,g}}$, and $\sigma^{(*)}_{\xi_{ucc,g}}$ denote the average of the estimated variance for error terms in the activity intensity equations of GPA, smoking, and their covariances from different groups. The values in the parentheses are the average standard deviation.

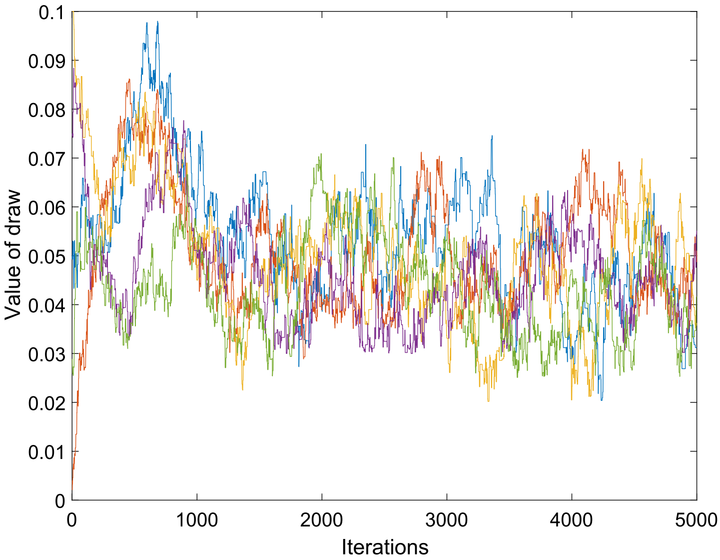

FIGURE F.3. Traceplot of MCMC draws for the parameter λ at different initial values (for censored outcome). The true value of λ in the simulation is 0.05.

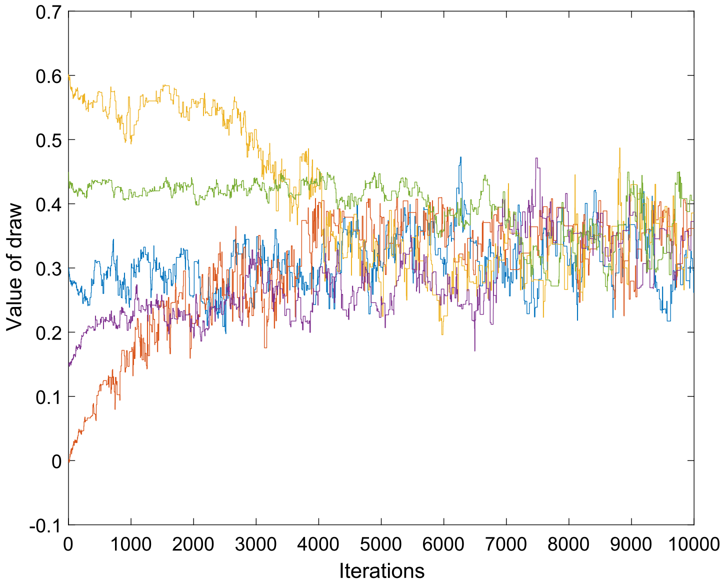

FIGURE F.4. Traceplot of MCMC draws for the parameter δ at different initial values (for censored outcome). The true value of δ in the simulation is 0.3.

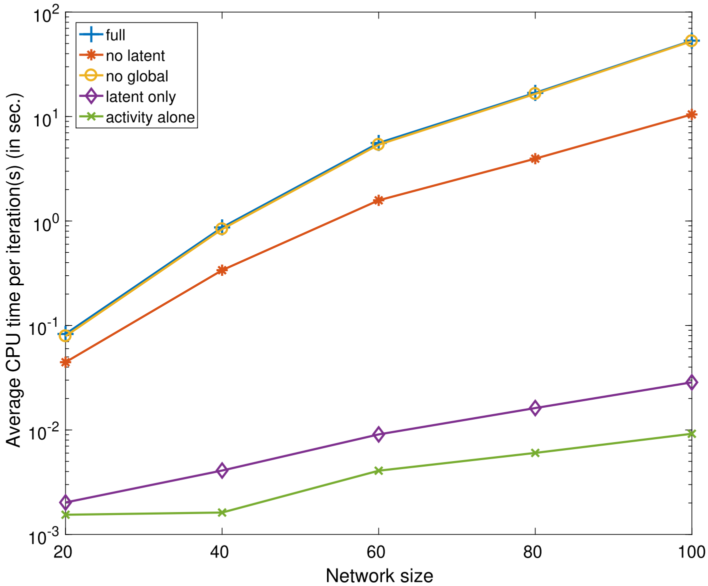

Figure F.5. The average computation time (in seconds) for a single MCMC iteration. The scale of the vertical axis is logarithmic.

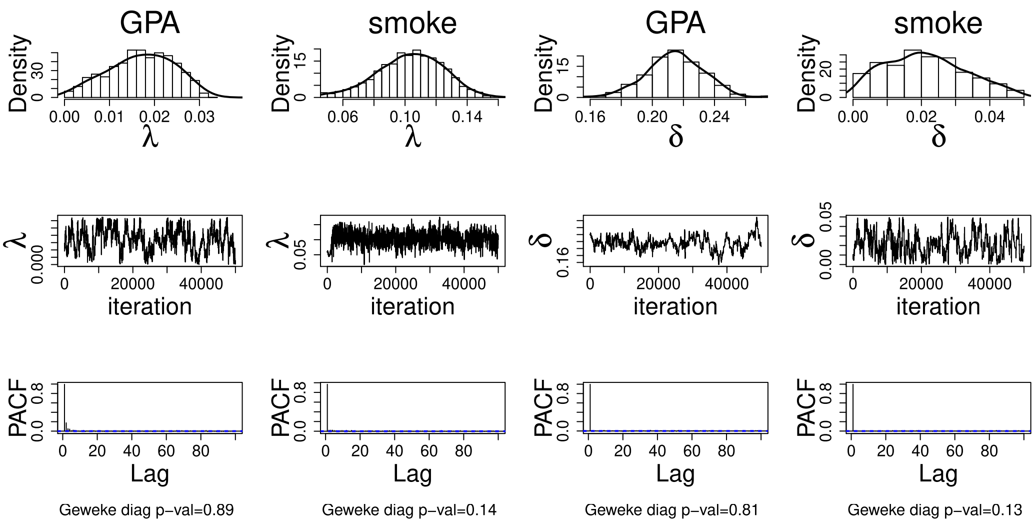

Figure F.6. MCMC trace plots and Geweke convergence diagnostics. Geweke convergence diagnostic tests for an equal mean of the first 10% versus the last 50% of the draws. We also try different proportions (e.g., 30% versus 70%), and the results are similar.

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