Supplement to "Differences in euro-area household finances and their relevance for monetary-policy transmission"

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APPENDIX C: RECURSIVE SOLUTION

This Appendix relies on the description of the model presented in Section 2 and explains its solution based on the recursive formulation. For ease of exposition, we refer to the discrete-choice options in a compact way here by using the numerical codes 0, 1, 2 (in that order) for the discrete-choice options *owning-and-not-adjusting, owning-and-adjusting, renting*.

C.1 *Normalizing the household problem*

First, we **normalize** the household problem such that the price level p_t does not enter as a separate state variable. We define *price-transformed variables* in the following way:

$$\bar{s}_j = p_t \hat{s}_j, \qquad h_{j+1} = p_t \hat{h}_{j+1}, \qquad f_j = p_t \hat{f}_j.$$

The normalization uses the assumption of a constant price-growth factor $\Pi = \frac{p_t}{p_{t-1}}$.

Normalizing the objective In terms of price-transformed units, $\bar{s}_j = p_t \hat{s}_j$, the utility function is expressed as

$$u(c_j, \hat{s}_j) = \theta \log c_j + (1-\theta) \log\left(\frac{1}{p_t} p_t \hat{s}_j\right) = \theta \log c_j + (1-\theta)\log(\bar{s}_j) - (1-\theta)\log p_t.$$

For characterizing consequences of the endogenous choices of c_j and $\bar{s}_j$, utility can equivalently be described by

$$U(c_j, \bar{s}_j) = \theta \log c_j + (1-\theta)\log(\bar{s}_j)$$

Resources relevant for bequests contain the term $p_{t+1}\hat{h}_{j+1}$, which can be expressed as Πh_{j+1}. Given the separability in discounted expected life-cycle utility, the normalization extends to the forward-looking objective of the household.

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In the following, we are going to show that, for any possible discrete choice d_j, also the constraint sets can equivalently be expressed in terms of price-transformed variables.

Normalizing the constraints for each discrete choice

Ownership choice, not adjusting *If the household chooses to* consume housing as an **owner, not adjusting** the housing stock, we code this as $d_j = 0$. We first make precise what non-adjustment means in terms of valued units. Non-adjustment of housing is naturally defined in terms of having the same *physical* (i.e., utility generating) quantity in two consecutive periods, meaning that

$$\hat{h}_{j+1} = \hat{h}_j.$$

Multiplying by p_t and using the definition of Π,

$$p_t \hat{h}_{j+1} = p_t \hat{h}_j = p_t \frac{1}{p_{t-1}} p_{t-1} \hat{h}_j = \Pi p_{t-1} \hat{h}_j.$$

In terms of price-transformed units, **physical non-adjustment** therefore implies that

$$h_{j+1} = \Pi h_j.$$

Ownership of housing implies that rented physical housing units $\hat{f}_j = 0$, and hence $p_t \hat{f}_j = 0$. Therefore,

$$f_j = 0.$$

For the physical service flow in the non-adjustment case, we have $\hat{s}_j = \phi \hat{h}_j$, implying $p_t \hat{s}_j = \phi p_t \hat{h}_j$ and, therefore,

$$\bar{s}_j = \phi \Pi h_j.$$

The budget constraint is

$$c_j + a_{j+1} = y_j(s_j) + (1 + r_{t-1}) a_j,$$

and the collateral constraint $(1 + r_t) a_{j+1} \geq -\mu p_t \hat{h}_j - g_{y,j+1}$ can be expressed as

$$(1 + r_t) a_{j+1} \geq -\mu \Pi h_j - g_{y,j+1}.$$

Ownership choice, adjusting *If the household chooses to* consume housing as an **owner, adjusting** the housing stock, coded as $d_j = 1$, $\hat{f}_j = 0$ implies

$$f_j = 0.$$

The physical service flow $\hat{s}_j = \phi \hat{h}_{j+1}$ implies $p_t \hat{s}_j = \phi p_t \hat{h}_{j+1}$ and, therefore,

$$\bar{s}_j = \phi h_{j+1}.$$

The adjustment cost function can be written as

$$\begin{aligned}\boldsymbol{\alpha}_p(\hat{h}_j, \hat{h}_{j+1}) &= \alpha_1 p_t \hat{h}_j + \alpha_2 p_t \hat{h}_{j+1} \\ &= \alpha_1 \frac{p_t}{p_{t-1}} h_j + \alpha_2 h_{j+1} \\ &= \alpha_1 \Pi h_j + \alpha_2 h_{j+1}.\end{aligned}$$

Denoting

$$\boldsymbol{\alpha}(h_j, h_{j+1}) = \alpha_1 \Pi h_j + \alpha_2 h_{j+1},$$

the budget constraint

$$c_j + a_{j+1} + p_t \hat{h}_{j+1} + \boldsymbol{\alpha}_p(\hat{h}_j, \hat{h}_{j+1}) = y_j(s_j) + (1 + r_{t-1})a_j + p_t \hat{h}_j$$

becomes

$$c_j + a_{j+1} + h_{j+1} + \boldsymbol{\alpha}(h_j, h_{j+1}) = y_j(s_j) + (1 + r_{t-1})a_j + p_t \frac{p_{t-1}}{p_{t-1}} \hat{h}_j,$$

which, using the price growth factor, can be written as

$$c_j + a_{j+1} + h_{j+1} + \boldsymbol{\alpha}(h_j, h_{j+1}) = y_j(s_j) + (1 + r_{t-1})a_j + \Pi h_j.$$

The collateral constraint $(1 + r_t)a_{j+1} \geq -\mu p_t \hat{h}_{j+1} - g_{y,j+1}$ can be expressed as

$$(1 + r_t)a_{j+1} \geq -\mu h_{j+1} - g_{y,j+1}.$$

Rental choice *If the household chooses to* consume housing as a **renter**, coded as $d_j = 2$, the choice of nonownership of housing $\hat{h}_{j+1} = 0$ implies $p_t \hat{h}_{j+1} = 0$ and, therefore,

$$h_{j+1} = 0.$$

The physical service flow $\hat{s}_j = \phi_R \hat{f}_j$ implies $p_t \hat{s}_j = \phi_R p_t \hat{f}_j$ and, therefore,

$$\bar{s}_j = \phi_R f_j.$$

The adjustment cost function can be expressed as

$$\begin{aligned}\boldsymbol{\alpha}_{pR}(\hat{h}_j) &= \alpha_1 p_t \hat{h}_j \\ &= \alpha_1 \frac{p_t}{p_{t-1}} h_j \\ &= \alpha_1 \Pi h_j.\end{aligned}$$

Denoting

$$\boldsymbol{\alpha}_R(h_j) = \alpha_1 \Pi h_j,$$

and using the rent-to-price ratio k_t to express the rental price $q_t = k_t p_t$, the budget constraint

$$c_j + a_{j+1} + q_t \hat{f}_j + \boldsymbol{\alpha}_{pR}(\hat{h}_j) = y_j(s_j) + (1 + r_{t-1})a_j + p_t \hat{h}_j$$

becomes

$$c_j + a_{j+1} + k_t p_t \hat{f}_j + \boldsymbol{\alpha}_R(h_j) = y_j(s_j) + (1 + r_{t-1})a_j + p_t \frac{p_{t-1}}{p_{t-1}} \hat{h}_j,$$

which, using $f_j = p_t \hat{f}_j$ and the price growth factor, can be written as

$$c_j + a_{j+1} + k_t f_j + \boldsymbol{\alpha}_R(h_j) = y_j(s_j) + (1 + r_{t-1})a_j + \Pi h_j.$$

The collateral constraint is

$$(1 + r_t)a_{j+1} \geq -g_{y,j+1}.$$

C.2 *Using liquidable wealth as a state variable*

We define an auxiliary state variable, which turns out to be convenient for the solution, and rewrite all constraints using that variable. The auxiliary state variable x_j, which may be interpreted as liquidable wealth, is defined as

$$x_j = (1 + r_{t-1})a_j + (1 - \alpha_1)\Pi h_j.$$

For the two cases (not adjusting and adjusting) of ownership choice, the budget constraint reads

$$c_j + a_{j+1} + h_{j+1} + \mathbf{1}_{d_j=1}\boldsymbol{\alpha}(h_j, h_{j+1}) = y_j(s_j) + (1 + r_{t-1})a_j + \Pi h_j,$$

where $\mathbf{1}_{d_j=1}$ denotes an indicator function, which takes the value of 1 if an adjustment is made and zero otherwise.

In the case of **non-adjustment** of housing, where the discrete choice variable is $d_j = 0$, and $h_{j+1} = \Pi h_j$, the budget constraint can be expressed as

$$c_j = y_j(s_j) + x_j - (1 - \alpha_1)\Pi h_j - a_{j+1}.$$

In the case of **adjustment** of housing, where the discrete choice variable is $d_j = 1$, we have

$$c_j = y_j(s_j) + x_j - a_{j+1} - h_{j+1} - \alpha_2 h_{j+1}.$$

In both cases, adjustment and non-adjustment, the next-period asset positions need to satisfy the collateral constraint

$$(1 + r_t)a_{j+1} \geq -\mu h_{j+1} - g_{y,j+1}$$

which, in terms of our auxiliary variable can be expressed as derived in the following. For the next age, the definition of the auxiliary state variable can be solved for the financial asset

$$(1 + r_t)a_{j+1} = x_{j+1} - (1 - \alpha_1)\Pi h_{j+1}.$$

Substituting for $(1+r_t)a_{j+1}$ in the collateral constraint, we obtain

$$x_{j+1} \geq \big[(1-\alpha_1)\Pi - \mu\big]h_{j+1} - g_{y,j+1}.$$

For the case of **rental choice**, where the discrete choice is $d_j = 2$, and $h_{j+1} = 0$, the budget constraint

$$c_j + a_{j+1} + k_t f_j + \boldsymbol{\alpha}_R(h_j) = y_j(s_j) + (1+r_{t-1})a_j + \Pi h_j$$

is expressed in terms of the auxiliary variable as follows:

$$c_j + k_t f_j = y_j(s_j) + x_j - a_{j+1},$$

and the collateral constraint is

$$x_{j+1} \geq -g_{y,j+1}.$$

C.3 *Solving the recursive problem*

In the recursive problem, restated here for convenience, we denote

$$\begin{aligned} &W_j(x_j, h_j, s_j) \\ &\quad = \max_{d_j, c_j, f_j, a_{j+1}, h_{j+1}} \big\{U(c_j, \bar{s}_j) + (1-\iota_j)\beta \mathop{E}_{s_{j+1}|s_j} W_{j+1}(x_{j+1}, h_{j+1}, s_{j+1}) + \iota_j \Psi(x_{j+1})\big\}, \end{aligned}$$

where the expectation operator $\mathop{E}_{s'|s} f(\cdot, s') = \sum_{s' \in S} \pi_{s,s'} f(\cdot, s')$. The probability of death in period j is denoted by ι_j. We consider a warm-glow bequest motive, represented by utility from bequeathing, as captured by the function $\Psi(x_{j+1})$, whose argument is therefore to be interpreted as liquidable wealth after death. The bequest utility function is parameterized as follows:

$$\Psi(x_{j+1}) = \psi_0 \log(\psi_1 + \psi_2 x_{j+1}).$$

Given that $\psi_1 > g_{y,j+1}$ for all j in our calibration, the bequest utility function is well-defined for borrowers in the feasible borrowing set of our model.

Henceforth, we denote by β_j the product of the survival probability in age j and the discount factor β, that is,

$$\beta_j \equiv (1-\iota_j)\beta.$$

By the same token, we define

$$\Psi_j(x_{j+1}) \equiv \iota_j \Psi(x_{j+1}).$$

Conditional on the discrete choice,

$$w_j(x_j, h_j, s_j | d_j) = \max_{c_j, f_j, a_{j+1}, h_{j+1}} \big\{U(c_j, \bar{s}_j) + \beta_j \mathop{E}_{s_{j+1}|s_j} W_{j+1}(x_{j+1}, h_{j+1}, s_{j+1}) + \Psi_j(x_{j+1})\big\}.$$

So far, there is uncertainty about death, earnings, and future interest rates in the model. We handle the discrete-choice options in the recursive problem according to the approach suggested by Iskhakov, Jørgensen, Rust, and Schjerning (2017). More specifically, we consider the addition of a random component to the valuation of discrete-choice options, and assume that this component is distributed according to an extreme-value (type I) distribution so that, keeping for simplicity the same notation for functions $W_j(\cdot)$ and $w_j(\cdot)$,

$$W_j(x_j, h_j, s_j, \boldsymbol{\eta}_j) = \max_{d_j \in D_j} \{ w_j(x_j, h_j, s_j | d_j) + \eta_{d_j} \},$$

where η_{d_j} denotes the realization of the random component specific to a discrete choice d_j, and the vector $\boldsymbol{\eta}_j$ contains the collection of all realizations at age j for the set of all available discrete choices D_j. This randomness is considered for the discrete-choice-specific value functions so that

$$\begin{aligned} & w_j(x_j, h_j, s_j | d_j) \\ & \quad = \max_{c_j, f_j, a_{j+1}, h_{j+1}} \Big\{ U(c_j, \bar{s}_j) + \beta_j \mathop{E}_{s_{j+1}|s_j} \Big[\mathop{E}_{\boldsymbol{\eta}_{j+1}} W_{j+1}(x_{j+1}, h_{j+1}, s_{j+1}, \boldsymbol{\eta}_{j+1}) \Big] + \Psi_j(x_{j+1}) \Big\} \\ & \quad = \max_{c_j, f_j, a_{j+1}, h_{j+1}} \Big\{ U(c_j, \bar{s}_j) + \beta_j \mathop{E}_{s_{j+1}|s_j} \lambda\big(\mathbf{w}_{j+1}(x_{j+1}, h_{j+1}, s_{j+1} | d_{j+1}), D_{j+1}; \sigma\big) + \Psi_j(x_{j+1}) \Big\}. \end{aligned}$$

Assuming that the random components for discrete-choice taste shocks are distributed according to an extreme-value (type I) distribution, the relevant expectations can then be expressed by using the well-known *log-sum* formula with a scale parameter σ for taste shocks[37]

$$\lambda\big((\mathbf{x}|d_{j+1}), D_{j+1}; \sigma\big) = \sigma \log\Big[\Sigma_{d_{j+1} \in D_{j+1}} \exp \frac{(x|d_{j+1})}{\sigma} \Big].$$

Ownership choice, not adjusting In the case of non-adjustment, where $h_{j+1} = \Pi h_j$, using the budget constraint for this case, we have

$$\begin{aligned} w_j(x_j, h_j, s_j | d_j = 0) = \max_{a_{j+1}} \Big\{ & U\big(y_j(s_j) + x_j - (1-\alpha_1)\Pi h_j - a_{j+1}, \phi \Pi h_j\big) \\ & + \beta_j \mathop{E}_{s_{j+1}|s_j} \lambda\big(\mathbf{w}_{j+1}(x_{j+1}, \Pi h_j, s_{j+1} | d_{j+1}), D_{j+1}; \sigma\big) + \Psi_j(x_{j+1}) \Big\}, \end{aligned}$$

subject to the collateral constraint

$$x_{j+1} \geq \big[(1-\alpha_1)\Pi - \mu\big]\Pi h_j - g_{y,j+1}.$$

Ownership choice, adjusting Inserting the budget constraint and the adjustment cost function, the recursive problem in the case of adjustment is

$$w_j(x_j, h_j, s_j | d_j = 1) = \max_{a_{j+1}, h_{j+1}} \big\{ U\big(y_j(s_j) + x_j - a_{j+1} - h_{j+1} - \alpha_2 h_{j+1}, \phi h_{j+1}\big)$$

[37]The notation with a boldface variable $\mathbf{x}$ in the expression $(\mathbf{x}|d_{j+1}), D_{j+1}$ is shorthand for denoting the corresponding collection of discrete-choice-specific variables by $\{(x|d_{j+1}) : d_{j+1} \in D_{j+1}\}$.

$$+ \beta_j \mathop{E}_{s_{j+1}|s_j} \lambda\big(\mathbf{w}_{j+1}(x_{j+1}, h_{j+1}, s_{j+1}|d_{j+1}), D_{j+1}; \sigma\big) + \Psi_j(x_{j+1})\big\}.$$

The next-period asset positions need to satisfy the collateral constraint

$$x_{j+1} \geq \big[(1-\alpha_1)\Pi - \mu\big]h_{j+1} - g_{y,j+1}.$$

Note that in this discrete-choice-specific problem any dependence on h_j is captured by its contribution to x_j. Apart from this contribution, the problem conditional on choosing to adjust is independent of h_j, which is convenient for the numerical solution.

Rental choice Using the budget constraint for the case of renting, considering the service flow obtained as $\bar{s}_j = \phi_R f_j$, and taking into account nonhomeownership for the next-period state, $h_{j+1} = 0$, we have

$$w_j(x_j, h_j, s_j|d_j = 2) = \max_{f_j, a_{j+1}} \big[U\big(y_j(s_j) + x_j - a_{j+1} - k_t f_j, \phi_R f_j\big)$$
$$+ \beta_j \mathop{E}_{s_{j+1}|s_j} \lambda\big(\mathbf{w}_{j+1}(x_{j+1}, 0, s_{j+1}|d_{j+1}), D_{j+1}; \sigma\big) + \Psi_j(x_{j+1})\big].$$

The collateral constraint in this case is

$$x_{j+1} \geq -g_{y,j+1}.$$

Note that also for this discrete-choice-specific problem any dependence on h_j is captured by its contribution to x_j. Separate from this contribution, the problem conditional on choosing to rent is independent of h_j, which conveniently simplifies the numerical solution.

We implement the solution of the maximization operations present in the recursive formulation by exploiting the implied first-order and envelope conditions. This lets us take advantage of the method for solving portfolio choice problems suggested by Hintermaier and Koeniger (2010), identifying candidates for optimal portfolio choice combinations in a first step, and then using them to determine optimal policy functions for all continuous decision variables.

Appendix D: Bequest parameters

Consider a bequest $\flat$ that generates a perpetuity with annual payment flow $r\flat$, where r is the real interest rate. The bequeather considers the utility consequences of the bequest taking into account the average earnings of the offspring $\bar{y}$. The disposable income consists of the annual payment flow $r\flat$ and the offspring's earnings $\bar{y}$, and the per-period utility is $u(\bar{y} + r\flat)$.

If the bequeather considers the discounted sum of these future period utilities for the offspring, the utility generated by the bequest equals

$$u(\bar{y} + r\flat) + \beta u(\bar{y} + r\flat) + \beta^2 u(\bar{y} + r\flat) + \beta^3 u(\bar{y} + r\flat) \cdots = \frac{1}{1-\beta} u(\bar{y} + r\flat),$$

where the bequeather and the offspring discount with the same factor β. For a logarithmic per-period utility function, the bequest motive is then captured by the function

$$\Psi(\flat) = \frac{1}{1-\beta}\log(\bar{y} + r\flat). \tag{1}$$

Note that bequests are a luxury simply because the bequeather considers the utility consequences of the bequest taking into account the average earnings of the offspring.

We now refine the function for the bequest motive by allowing for aggregate income growth after death, which increases earnings of the offspring relative to the bequeather. Suppose that the bequests shall generate annual coupon payments z, which grow at the same rate g as average earnings. The size of z, which can be financed with a bequeathed amount $\flat$, is

$$z = \flat(r-g),$$

because

$$\begin{aligned}
(1+r)\flat &= z + \frac{1+g}{1+r}z + \left(\frac{1+g}{1+r}\right)^2 z + \left(\frac{1+g}{1+r}\right)^3 z + \cdots, \\
\iff \quad (1+r)\flat &= \frac{1}{1-\dfrac{1+g}{1+r}}z = \frac{1+r}{r-g}z, \\
\iff \quad \flat &= \frac{1}{r-g}z.
\end{aligned}$$

Note that without growth ($g=0$), $z = rb$ as in (1).

Now consider the sum of discounted utilities, with both the coupon z and average earnings $\bar{y}$ growing at rate $g < r$. Then

$$\begin{aligned}
\Psi(\flat) &= \log(\bar{y}+z) + \beta\log\big((1+g)(\bar{y}+z)\big) + \beta^2\log\big((1+g)^2(\bar{y}+z)\big) + \cdots \\
&= \log(\bar{y}+z) + \beta\log(\bar{y}+z) + \beta\log(1+g) + \beta^2\log(\bar{y}+z) + \beta^2 2\log(1+g) + \cdots \\
&= \frac{1}{1-\beta}\log(\bar{y}+z) + \log(1+g)\sum_{\tau=1}^{\infty}\beta^{\tau}\tau \\
&= \frac{1}{1-\beta}\log(\bar{y}+z) + \bar{R}_c,
\end{aligned}$$

with

$$\bar{R}_c \equiv \log(1+g)\frac{\beta}{(1-\beta)^2}.$$

The bequest motive of a bequeather who anticipates future growth is thus

$$\Psi(\flat) = \frac{1}{1-\beta}\log\big(\bar{y} + (r-g)\flat\big) + \bar{R}_c. \tag{2}$$

Note that the constant $\bar{R}_c$ is independent of choices and state variables in the decision problem of the bequeather so that we can abstract from it.

The marginal utility of bequests is

$$\Psi'(\flat) = \frac{1}{1-\beta}\frac{r-g}{\bar{y}+(r-g)\flat}.$$

In the quantitative application of the model, we set $\bar{y}$ to average earnings at the beginning of the life cycle. The bequest function could be further refined by choosing lower values to capture the extent to which the bequeather cares about the income risk, which the offspring faces.

Appendix E: Decomposition of the effects of country-specific model inputs

We decompose how differences in model inputs across countries influence key model predictions reported in Table 3 of the main text. Figure 8 displays the decomposition

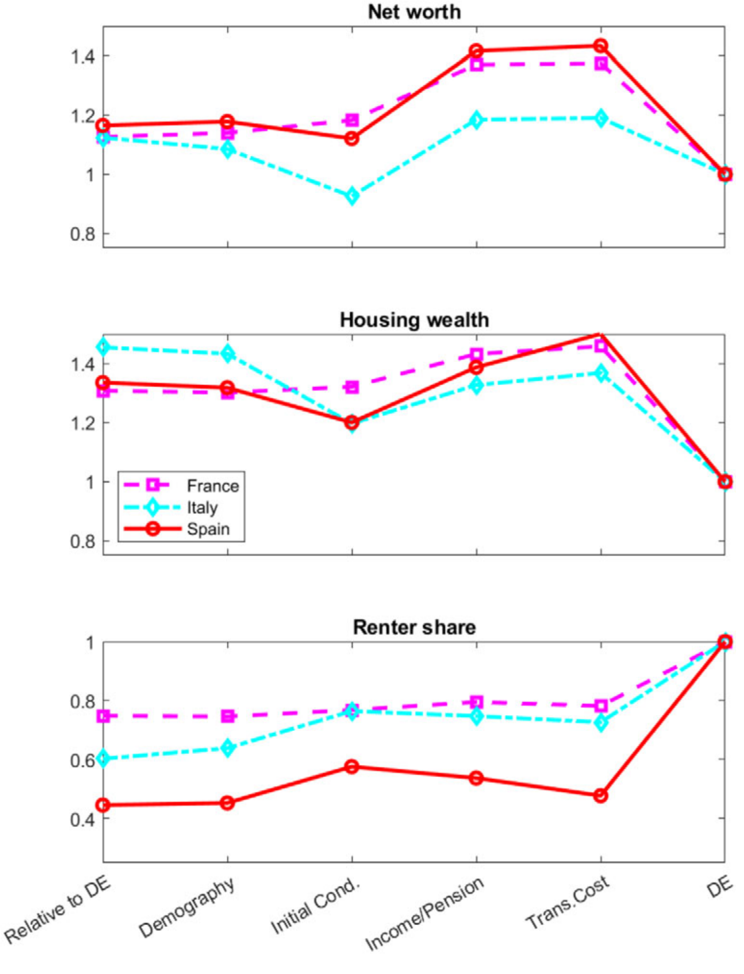

Figure 8. Decomposition of cross-country differences in net worth, housing wealth, and the renter share. Notes: Numbers reported relative to Germany. A value of 1 in the figure corresponds to the value of the respective statistic for Germany.

for France, Italy, and Spain relative to Germany, for the three key model predictions on housing wealth, net worth, and the housing renter share. Given the relative way of reporting, a value of 1 in the figure corresponds to the value of the respective statistic for Germany.

The changes are implemented incrementally and, as is well known, the sequence matters for the precise quantitative contribution that is attributed to each change of model input for the respective predicted statistic. The main point of the figure is thus to provide an indication for the order of magnitude with which a certain country-specific model input affects the model predictions. We comment on the results at the end of Section 3 of the main text.

Appendix F: The role of the interest spread in the life-cycle model with housing

It is instructive to comment on the role of the interest spread in our model. It is well known, at least since Kaplan and Violante (2014), that agents can have high marginal propensities to consume because of an interest spread, even if the borrowing constraint is not binding. If shocks are not large enough to make adjustment of the illiquid asset optimal, nonadjusting agents with illiquid wealth, and an intertemporal marginal rate of substitution between the interest factor implied by the borrowing and lending rate behave as (wealthy) hand-to-mouth consumers. Analogously, agents without illiquid wealth behave as hand-to-mouth consumers if the spread makes it optimal for them not to change their position of zero liquid wealth after a shock.

It is important to emphasize that the consolidation of balance sheets, required for consistency of the data and our model, implies a different interpretation of the spread and the corresponding incidence of households with zero *other wealth*, which is liquid in our model. Kaplan, Moll, and Violante (2018) consolidate housing wealth and mortgage liabilities to home equity as part of their illiquid wealth position so that negative liquid wealth has the interpretation of unsecured debt. The standard portfolio choice model, in which the house (not home equity) is a consumption good, an asset and serves as collateral, requires a different consolidation in a two-asset portfolio choice setting.[38] Other wealth besides housing wealth then is the consolidated position of all other assets and liabilities. A negative value of this position then typically implies secured mortgage debt for homeowners.

Not only does the different consolidation call for a calibration of a smaller spread, as noted in footnote 15 in the main text, also the interpretation and the economic implications of the incidence of zero other wealth for homeowners are different. Such homeowners have either amortized their mortgage or hold gross positions of other assets of the same value as the mortgage so that their net worth equals their housing wealth. Homeowners with such portfolio positions tend to be older, that is, at later stages in

[38]Extending the dimensionality of the portfolio choice problem by allowing for a third continuous endogenous state variable has proven prohibitively costly computationally so far. Such an extension would allow to distinguish features, such as liquidity, of assets and liabilities consolidated in the *other wealth* position.

their life cycle, and richer, and thus can more easily afford to pay adjustment costs to avoid the illiquidity and higher volatility of the marginal utility of consumption implied by the interest spread.

Let us elaborate on the interaction between adjustment costs and the interest spread in our setting compared with the literature. A key difference relative to models with an asset consolidation as in Kaplan, Moll, and Violante (2018) is that, in our model, the *housing asset* is illiquid but *home equity* is not. Households in our model can adjust their debt position and thus home equity without cost by changing their position in other wealth to finance consumption. Because of the spread, an adjustment of the position in other wealth may not be optimal if other wealth is zero. Net worth then contains no liquid resources so that households in these circumstances bear more consumption risk because adjusting the housing asset is costly. Home equity in our model thus becomes illiquid as in Kaplan, Moll, and Violante (2018) only when the interest spread implies that it is optimal to hold zero other wealth because the intertemporal marginal rate of substitution of consumption is between the interest factor implied by the borrowing and lending rate.

Agents can avoid this situation ex ante by adjusting the housing asset for obtaining liquidity, in order to prevent other wealth from being zero. The costs in terms of marginal utility for paying the adjustment costs can be smoothed intertemporally. This behavior of homeowners is analogous to standard quantitative models with occasionally binding constraints in which agents take precautions to avoid that these constraints bind, and thus distort their intertemporal consumption profile.

Appendix G: Using quadratic programming for the optimal mixing of heterogeneous household types

Let $m_{\mathfrak{s}}$ denote some statistic, where the index $\mathfrak{s} = 1, \dots, \mathfrak{S}$, meaning that a collection of $\mathfrak{S}$ such statistics is considered relevant for some objective, for example, for the calibration or estimation of a model.

The objective (loss) function considered is assumed to be quadratic and parameterized by

$$L = \frac{1}{2} \sum_{\mathfrak{s}=1}^{\mathfrak{S}} a_{\mathfrak{s}} (m_{\mathfrak{s}} - d_{\mathfrak{s}})^2, \tag{3}$$

where the coefficients $a_{\mathfrak{s}}$ may vary across the $\mathfrak{S}$ statistics considered, and where $d_{\mathfrak{s}}$ may be thought of as a data counterpart for a specific statistic.

Moreover, it is assumed that the statistics $m_{\mathfrak{s}}$ contained in the objective (3) can be obtained as a linear combination of statistics for a collection of types, indexed by $\tau = 1, \dots, \mathcal{T}$, considering the relative weight ω_τ, of the types involved:

$$m_{\mathfrak{s}} = \sum_{\tau=1}^{\mathcal{T}} \omega_\tau m_{\mathfrak{s},\tau}, \quad \mathfrak{s} = 1, \dots, \mathfrak{S}, \tag{4}$$

where $m_{\mathfrak{s},\tau}$ refers to the value of statistic $\mathfrak{s}$ for type τ.

Relative weights of types are assumed to be between 0 and 1 and sum up to 1:

$$0 \le \omega_\tau \le 1, \quad \tau = 1, \dots, \mathcal{T},$$
$$\sum_{\tau=1}^{\mathcal{T}} \omega_\tau = 1.$$

Using the type-weighted statistics obtained by the mixing formula (4) to substitute for the statistics $m_\mathfrak{s}$, $\mathfrak{s} = 1, \dots, \mathfrak{S}$, in (3), the objective function becomes

$$L = \frac{1}{2} \sum_{\mathfrak{s}=1}^{\mathfrak{S}} a_\mathfrak{s} \left(\sum_{\tau=1}^{\mathcal{T}} \omega_\tau m_{\mathfrak{s},\tau} - d_\mathfrak{s} \right)^2 .$$

Letting M denote the matrix containing the statistics $m_{\mathfrak{s},\tau}$, in rows $\mathfrak{s} = 1, \dots, \mathfrak{S}$ and columns $\tau = 1, \dots, \mathcal{T}$, and collecting the type weights in a vector $\omega = (\omega_1, \dots, \omega_\mathcal{T})^\top$, the objective can be expressed as

$$L = \frac{1}{2} \sum_{\mathfrak{s}=1}^{\mathfrak{S}} a_\mathfrak{s} \big(\omega^\top m_{\mathfrak{s},\cdot}^\top m_{\mathfrak{s},\cdot} \omega - 2 d_\mathfrak{s} m_{\mathfrak{s},\cdot} \omega + d_\mathfrak{s}^2 \big), \tag{5}$$

where $m_{\mathfrak{s},\cdot}$ denotes the $\mathfrak{s}th$ row of the matrix M.

For further revealing the structure of the objective, equation (5) can be written as

$$L = \frac{1}{2} \omega^\top \left(\sum_{\mathfrak{s}=1}^{\mathfrak{S}} a_\mathfrak{s} m_{\mathfrak{s},\cdot}^\top m_{\mathfrak{s},\cdot} \right) \omega - \left(\sum_{\mathfrak{s}=1}^{\mathfrak{S}} a_\mathfrak{s} d_\mathfrak{s} m_{\mathfrak{s},\cdot} \right) \omega + \frac{1}{2} \sum_{\mathfrak{s}=1}^{\mathfrak{S}} a_\mathfrak{s} d_\mathfrak{s}^2. \tag{6}$$

The last term on the right-hand side of equation (6) is independent of type weights. This constant term may be disregarded in a modified objective $\tilde{L}$, used for finding the optimal type weights:

$$\tilde{L} = \frac{1}{2} \omega^\top \left(\sum_{\mathfrak{s}=1}^{\mathfrak{S}} a_\mathfrak{s} m_{\mathfrak{s},\cdot}^\top m_{\mathfrak{s},\cdot} \right) \omega - \left(\sum_{\mathfrak{s}=1}^{\mathfrak{S}} a_\mathfrak{s} d_\mathfrak{s} m_{\mathfrak{s},\cdot} \right) \omega. \tag{7}$$

Using a formulation in terms of matrices and vectors, the modified objective in equation (7) can be written as

$$\tilde{L} = \frac{1}{2} \omega^\top \big(M^\top \operatorname{diag}(a) M \big) \omega - \big(d^\top \operatorname{diag}(a) M \big) \omega, \tag{8}$$

where $d = (d_1, \dots, d_\mathfrak{S})^\top$, $a = (a_1, \dots, a_\mathfrak{S})^\top$, and $\operatorname{diag}(a)$ is the diagonal matrix with the components of a on its main diagonal.

Finally, note that a common problem specification of numerical software, which performs quadratic programming, is the following:

$$\min_x \frac{1}{2} x^\top H x + f^\top x \quad \text{subject to constraints linear in } x.$$

Therefore, for capturing the relevant modified objective (8) such numerical optimization functions require the following specification of inputs:

$$H = M^\top \operatorname{diag}(a) M,$$
$$f = -M^\top \operatorname{diag}(a) d.$$

References

Hintermaier, Thomas and Winfried Koeniger (2010), "The method of endogenous gridpoints with occasionally binding constraints among endogenous variables." *Journal of Economic Dynamics and Control*, 34 (10), 2074–2088. [0007]

Iskhakov, Fedor, Thomas H. Jørgensen, John Rust, and Bertel Schjerning (2017), "The endogenous grid method for discrete-continuous dynamic choice models with (or without) taste shocks." *Quantitative Economics*, 8 (2), 317–365. [0006]

Kaplan, Greg, Benjamin Moll, and Gianluca Violante (2018), "Monetary policy according to HANK." *American Economic Review*, 108 (3), 697–743. [0010, 0011]

Kaplan, Greg and Gianluca Violante (2014), "A model of the consumption response to fiscal stimulus payments." *Econometrica*, 82 (4), 1199–1239. [0010]

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