Supplement to "Asymmetric information in secondary insurance markets: Evidence from the life settlements market"

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This appendix collects supplemental material for the main article. Appendix A provides relevant technical details, particularly an expected utility version of the model from Section 2.2 (Appendix A.1), a description of the estimation approach for the proportional hazards model used in Section 3 of the main text (Appendix A.2), a detailed discussion of the impact of the mixed nature of the dataset described in Section 3 of the main text (Appendix A.3), the derivation of the nonparametric estimates of *excess hazard* used in Sections 4 and 5 of the main text (Appendix A.4), and an extension of the model from Section 2.2 with price uncertainty that is relevant for robustness analyses with regards to omitted variables in Section 6.1 of the main text (Appendix A.5). Appendix B presents additional results for the analyses in Sections 3, 4, and 5 in the main text. Appendix C presents a Monte Carlo experiment on the settlement process when brokers or policyholders "cherry-pick" among LEs or offers as discussed in Section 5.2, part (iii) in the main text.

Appendix A: Technical appendix

A.1 *Expected utility version of the model from Section 2.2*

The policyholder's proclivity for settling, ψ, can be characterized in an expected utility framework. Consider the setting from Section 2.2, where the policyholder is endowed with a one-period life insurance policy with face value f and wealth w. The policyholder's utility function when alive is $u(\cdot)$, with $u'' < 0 < u'$. If the policyholder dies by the end of the period, she receives utility, $v(\cdot)$, from her dependents' consumption of bequests, with $v'' < 0 < v'$. Again we set the risk-free rate to zero.

The policyholder solves two distinct optimization problems to obtain her total expected utility. When not settling her policy, the policyholder chooses an optimal con-

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sumption level at time 0, c_n, to maximize her expected utility in

$$V_n = \max_{c_n} u(c_n) + \left[(1-q) \times u(w-c_n) + q \times v(w+f-c_n)\right].$$

Here, q is once again the probability for dying before time one. On the other hand, when accepting a settlement at price π for the policy, the policyholder's optimization problem becomes

$$V_s = \max_{c_s} u(c_s) + \left[(1-q) \times u(w+\pi-c_s) + q \times v(w+\pi-c_s)\right].$$

Equating V_n and V_s results in a *reservation offer price* π^* that makes the policyholder indifferent between keeping or settling her policy, and it immediately follows from equation (1) that $\psi = qf - \pi^*$, that is, the policyholder will gladly settle at any price that is higher than $qf - \psi = \pi^*$.

In what follows, we assume that $v(\cdot) = b \times u(\cdot)$, where $b \in [0, 1]$ measures the intensity of the bequest motive (see Fischer (1973), e.g.). We further adopt a logarithmic specification of the utility, $u(\cdot) = \ln(\cdot)$, as a special case of the constant relative risk aversion utility function (relative risk aversion is one). This allows us to obtain explicit expression of ψ, and analyze how settling is affected by the primitives of our model.

LEMMA A.1. *With $u(\cdot) = \ln(\cdot)$, the optimal consumption levels are*

$$c_n = \frac{3w+2f-qf-qw+bqw-\sqrt{(3w+2f-qf-qw+bqw)^2-4(2+bq-q)\left(w^2+wf\right)}}{2\times(2+bq-q)}$$

and

$$c_s = \frac{w+\pi}{2+bq-q}.$$

PROOF. The first-order condition of V_n w.r.t. c_n yields

$$\frac{1}{c_n} - \frac{1-q}{w-c_n} - \frac{bq}{w+f-c_n} = 0, \tag{A.1}$$

which can be expressed as

$$(2+bq-q)c_n^2 - (3w+2f-qf-qw+bqw)c_n + \left(w^2+wf\right) = 0.$$

Given $c_n < w$, the optimal consumption c_n is the left root of the above quadratic equation.

Similarly, the first-order condition of V_s w.r.t. c_s yields

$$\frac{1}{c_s} - \frac{1+bq-q}{w+\pi-c_s} = 0, \tag{A.2}$$

which immediately gives the optimal consumption $c_s = (w+\pi)/(2+bq-q)$. □

PROPOSITION A.1. *The policyholder's proclivity for settling, ψ, is*

$$\psi = w + qf - \frac{A_1^{\frac{1}{2+bq-q}} \times A_2^{\frac{1-q}{2+bq-q}} \times A_3^{\frac{bq}{2+bq-q}}}{(1+bq-q)^{\frac{1+bq-q}{2+bq-q}}},$$

where

$$A_1 \equiv \frac{3w + 2f - qf - qw + bqw - \sqrt{(3w + 2f - qf - qw + bqw)^2 - 4(2 + bq - q)\big(w^2 + wf\big)}}{2},$$

$$A_2 \equiv (2 + bq - q)w - A_1,$$

and

$$A_3 \equiv (2 + bq - q)(w + f) - A_1.$$

PROOF. Plugging the optimal consumption levels c_n and c_s from Lemma A.1 back into V_n and V_s, we obtain

$$\begin{aligned} V_n &= u(c_n) + (1-q)u(w - c_n) + bqu(w + f - c_n) \\ &= \ln\left(\frac{A_1}{2+bq-q}\right) + (1-q)\ln\left(\frac{A_2}{2+bq-q}\right) + bq\ln\left(\frac{A_3}{2+bq-q}\right) \\ &= \ln\big(A_1 \times A_2^{1-q} \times A_3^{bq}\big) - (2+bq-q)\ln(2+bq-q) \end{aligned}$$

and

$$\begin{aligned} V_s &= u(c_s) + (1+bq-q)u(w + \pi - c_s) \\ &= \ln\left(\frac{w+\pi}{2+bq-q}\right) + (1+bq-q)\ln\left(\frac{w+\pi}{2+bq-q} \times (1+bq-q)\right) \\ &= (2+bq-q)\ln(w+\pi) - (2+bq-q)\ln(2+bq-q) + (1+bq-q)\ln(1+bq-q). \end{aligned}$$

Equating V_n and V_s yields

$$\ln\big(A_1 \times A_2^{1-q} \times A_3^{bq}\big) = (2+bq-q)\ln\big(w + \pi^*\big) + (1+bq-q)\ln(1+bq-q),$$

which gives the reservation offer price π^* as

$$\pi^* = \frac{A_1^{\frac{1}{2+bq-q}} \times A_2^{\frac{1-q}{2+bq-q}} \times A_3^{\frac{bq}{2+bq-q}}}{(1+bq-q)^{\frac{1+bq-q}{2+bq-q}}} - w,$$

and hence

$$\psi = qf - \pi^* = w + qf - \frac{A_1^{\frac{1}{2+bq-q}} \times A_2^{\frac{1-q}{2+bq-q}} \times A_3^{\frac{bq}{2+bq-q}}}{(1+bq-q)^{\frac{1+bq-q}{2+bq-q}}}. \qquad \square$$

We now evaluate how the proclivity for settling (ψ) varies with wealth (w), face value (f), and bequest motive (b). The following lemma proves helpful in this pursuit.

LEMMA A.2. *When* $\pi = \pi^*$, $c_s \geq c_n$.

PROOF. Plugging π^* back into c_s, we obtain

$$c_s = \frac{A_1^{\frac{1}{2+bq-q}} \times A_2^{\frac{1-q}{2+bq-q}} \times A_3^{\frac{bq}{2+bq-q}}}{(1+bq-q)^{\frac{1+bq-q}{2+bq-q}} \times (2+bq-q)},$$

whereas c_n can be expressed as

$$c_n = \frac{A_1}{2+bq-q}.$$

Therefore, $c_s \geq c_n$ is equivalent to

$$\frac{A_1^{\frac{1}{2+bq-q}} \times A_2^{\frac{1-q}{2+bq-q}} \times A_3^{\frac{bq}{2+bq-q}}}{(1+bq-q)^{\frac{1+bq-q}{2+bq-q}}} \geq A_1,$$

which simplifies to

$$A_2^{\frac{1-q}{1+bq-q}} \times A_3^{\frac{bq}{1+bq-q}} \overset{?}{\geq} A_1 \times (1+bq-q). \tag{A.3}$$

We note that the F.O.C. (A.1) can also be expressed as

$$\frac{1}{A_2} \times \frac{1-q}{(1+bq-q)} + \frac{1}{A_3} \times \frac{bq}{(1+bq-q)} = \frac{1}{A_1} \times \frac{1}{(1+bq-q)}.$$

Since $\frac{1-q}{1+bq-q} \in [0,1]$, $\frac{bq}{1+bq-q} \in [0,1]$, and $\frac{1-q}{1+bq-q} + \frac{bq}{1+bq-q} = 1$, it immediately follows from the *weighted AM-GM inequality* that

$$\begin{aligned}\frac{1}{A_1} \times \frac{1}{(1+bq-q)} &= \frac{1}{A_2} \times \frac{1-q}{(1+bq-q)} + \frac{1}{A_3} \times \frac{bq}{(1+bq-q)} \\ &\geq \left(\frac{1}{A_2}\right)^{\frac{1-q}{(1+bq-q)}} \times \left(\frac{1}{A_3}\right)^{\frac{bq}{(1+bq-q)}},\end{aligned}$$

which gives

$$A_1 \times (1+bq-q) \leq A_2^{\frac{1-q}{1+bq-q}} \times A_3^{\frac{bq}{1+bq-q}}.$$

Therefore, inequality (A.3) holds, and thus $c_s \geq c_n$. □

PROPOSITION A.2. *The policyholder's proclivity for settling* (ψ) *decreases in wealth and bequest motive, and increases in face value, ceteris paribus. That is,*

$$\frac{d\psi}{dw} \leq 0, \qquad \frac{d\psi}{db} \leq 0, \quad \textit{and} \quad \frac{d\psi}{df} \geq 0.$$

Moreover, the proclivity for settling per unit of face value, ψ/f, increases in face value in the right-hand limit. That is,

$$\frac{d(\psi/f)}{df} > 0, \quad \text{as } f \to \infty.$$

Proof. Define $V \equiv V_n - V_s = 0$. We use implicit differentiation by treating ψ as an implicit function of w, b, or f, when taking derivative of V to each of them, respectively.

(i) $d\psi/dw \leq 0$

Taking the derivative of V with respect to w, we have

$$\begin{aligned}
0 &= \frac{dV}{dw} \\
&= \frac{1}{c_n}\frac{dc_n}{dw} + \frac{1-q}{w-c_n}\left(1-\frac{dc_n}{dw}\right) + \frac{bq}{w+f-c_n}\left(1-\frac{dc_n}{dw}\right) \\
&\quad - \left\{\frac{1}{c_s}\left(\frac{dc_s}{dw} + \frac{dc_s}{d\psi}\times\frac{d\psi}{dw}\right) + \frac{1+bq-q}{w+qf-\psi-c_s}\left(1-\frac{d\psi}{dw}-\frac{dc_s}{dw}-\frac{dc_s}{d\psi}\frac{d\psi}{dw}\right)\right\}.
\end{aligned}$$

Using the F.O.C.s (A.1) and (A.2), we can simplify the above equation to

$$\frac{1-q}{w-c_n} + \frac{bq}{w+f-c_n} - \frac{1+bq-q}{w+qf-\psi-c_s}\left(1-\frac{d\psi}{dw}\right) = 0.$$

It is then straightforward that $d\psi/dw \leq 0$ is equivalent to

$$\frac{1-q}{w-c_n} + \frac{bq}{w+f-c_n} \geq \frac{1+bq-q}{w+qf-\psi-c_s}.$$

Once again, using equations (A.1) and (A.2), the inequality can be further modified to

$$\frac{1}{c_n} \geq \frac{1}{c_s},$$

which is shown in Lemma A.2.

(ii) $d\psi/db \leq 0$

Taking the derivative of V with respect to b, we have

$$\begin{aligned}
0 &= \frac{dV}{db} \\
&= \frac{1}{c_n}\frac{dc_n}{db} - \frac{1-q}{w-c_n}\frac{dc_n}{db} - \frac{bq}{w+f-c_n}\frac{dc_n}{db} + q\ln(w+f-c_n) \\
&\quad - \left\{\frac{1}{c_s}\left(\frac{dc_s}{db} + \frac{dc_s}{d\psi}\times\frac{d\psi}{db}\right) - \frac{1+bq-q}{w+qf-\psi-c_s}\left(\frac{d\psi}{db}+\frac{dc_s}{db}+\frac{dc_s}{d\psi}\frac{d\psi}{db}\right) + q\ln(w+qf-\psi-c_s)\right\}.
\end{aligned}$$

Using the F.O.C.s (A.1) and (A.2), this simplifies to

$$q\ln\left(\frac{w+f-c_n}{w+qf-\psi-c_s}\right) + \frac{1+bq-q}{w+qf-\psi-c_s}\frac{d\psi}{db} = 0.$$

Since $(1+bq-q)/(w+qf-\psi-c_s)>0$, $d\psi/db \le 0$ is equivalent to $w+f-c_n \ge w+qf-\psi-c_s \Leftrightarrow c_s \ge c_n+qf-\psi-f$, which is also immediately available from Lemma A.2 since $qf-\psi-f=\pi^*-f\le 0$.[1]

(iii) $d\psi/df \ge 0$

Taking the derivative of V with respect to f, we have

$$\begin{aligned}
0 &= \frac{dV}{df} \\
&= \frac{1}{c_n}\frac{dc_n}{df} + \frac{1-q}{w-c_n}\left(-\frac{dc_n}{df}\right) + \frac{bq}{w+f-c_n}\left(1-\frac{dc_n}{df}\right) \\
&\quad - \left\{\frac{1}{c_s}\times\frac{dc_s}{d\psi}\frac{d\psi}{df} + \frac{1+bq-q}{w+qf-\psi-c_s}\left(q-\frac{d\psi}{df}-\frac{dc_s}{d\psi}\frac{d\psi}{df}\right)\right\}.
\end{aligned}$$

Using the F.O.C.s (A.1) and (A.2), this simplifies to

$$\frac{bq}{w+f-c_n} + \frac{1+bq-q}{w+qf-\psi-c_s}\left(\frac{d\psi}{df}-q\right)=0.$$

Since both $bq/(w+f-c_n)$ and $(1+bq-q)/(w+qf-\psi-c_s)$ are positive, it is trivial to show that $0\le d\psi/df<q$ is equivalent to

$$\frac{1+bq-q}{w+qf-\psi-c_s}\times q \ge \frac{bq}{w+f-c_n},$$

which can be further modified to

$$(1+bq-q)(w+f-c_n)\ge b(w+qf-\psi-c_s).$$

This inequality holds since $1+bq-q-b=(1-b)(1-q)\ge 0$, and $w+f-c_n\ge w+qf-\psi-c_s$ as proven in part (ii) above.

(iv) $d(\psi/f)/df>0$, *as* $f\to\infty$

Recall that $\psi=qf-\pi^*$. Therefore, $d(\psi/f)/df>0$ is equivalent to $d(\pi^*/f)/df<0$. As $f\to\infty$, we obtain

$$V_n \propto bq\ln(f) \quad \text{and} \quad V_s\propto(2+bq-q)\ln(\pi).$$

Therefore, by equating V_n and V_s as $f\to\infty$, we can express π^* as

$$\pi^* = C\times f^{\frac{bq}{2+bq-q}},$$

where C is a positive constant. It immediately follows that $\pi^*/f = C\times f^{(q-2)/(2+bq-q)}$ with first-order derivative

$$\frac{d(\pi^*/f)}{df} = \underbrace{C}_{>0}\times\underbrace{\frac{q-2}{2+bq-q}}_{<0}\times\underbrace{f^{\frac{q-2}{2+bq-q}-1}}_{>0}<0, \quad f\to\infty$$

and, therefore, $d(\psi/f)/df>0$, as $f\to\infty$. □

[1] By comparing V_n and V_s, it is immediately clear that $\pi^*\le f$ because otherwise V_s will always be strictly higher than V_n.

A.2 *Estimation approach for the proportional hazards model*

Consider a proportional hazards model of the form:

$$\mu_t(Z) = \beta_0(t) \times \exp\{\beta' \cdot Z(t)\}, \tag{A.4}$$

in which β is the vector of regression parameters and $Z(t)$ are (possibly) time-varying covariates. The nonparametric term $\beta_0(t)$ is also referred to as the *nuisance* parameter when the emphasis is on the estimation of the regression coefficients.

For a set of n independent subjects, define the observation time $X_i = \min\{T_i, C_i\}$, $i \in \{1, \dots, n\}$, where T_i is the failure time (time until death) and C_i is the censoring time (time until end of observation period) for subject i. The indicator for observed death of subject i is defined as $\Delta_i = \boldsymbol{1}_{\{T_i < C_i\}}$. We further define the ith at-risk and observed-death counting processes as $Y_i(t) = \boldsymbol{1}_{\{X_i \geq t\}}$ and $N_i(t) = \boldsymbol{1}_{\{T_i \leq t, \Delta_i = 1\}}$, respectively.

The partial log-likelihood function of (A.4) was first derived in Cox (1975), and in our specification with time-varying covariates can be expressed as (see equation (1.2) in Andersen and Gill (1982)):

$$l(\beta) = \sum_{i=1}^{n} \Delta_i \left\{ \beta' \cdot Z_i(T_i) - \ln\left(\sum_{j=1}^{n} Y_j(T_i) \exp\{\beta' \cdot Z_j(T_i)\} \right) \right\}.$$

Differentiating yields the (partial-likelihood) score function:

$$\begin{aligned} U(\beta) = \frac{\partial l(\beta)}{\partial \beta} &= \sum_{i=1}^{n} \Delta_i \left\{ Z_i(T_i) - \underbrace{\frac{\sum_{j=1}^{n} Y_j(T_i) Z_j(T_i) \exp\{\beta' \cdot Z_j(T_i)\}}{\sum_{j=1}^{n} Y_j(T_i) \exp\{\beta' \cdot Z_j(T_i)\}}}_{\bar{Z}(T_i)} \right\} \\ &= \sum_{i=1}^{n} \int_0^{\infty} \left[Z_i(t) - \bar{Z}(t) \right] dN_i(t). \end{aligned}$$

The parameter vector is then estimated as the solution to the score equation $U(\beta) = 0$, without estimating the nuisance parameter $\beta_0(t)$.

In our specification (4), the estimated hazard $\hat{\mu}_t^{(i)}$ is the only time-varying covariate. Here, we assume that the estimated hazard only changes on a monthly basis, so that within each month all covariates remain constant. Hence, we can write the integral in the score function as

$$\begin{aligned} U(\beta) &= \sum_{i=1}^{n} \left\{ \int_0^{1/12} \left[Z_i(t) - \bar{Z}(t) \right] dN_i(t) + \int_{1/12}^{2/12} \left[Z_i(t) - \bar{Z}(t) \right] dN_i(t) + \cdots \right\} \\ &= \sum_{i=1}^{n} \{ \left[Z_i(0) - \bar{Z}(0) \right] \cdot \left[N_i(1/12) - N_i(0) \right] \\ &\quad + \left[Z_i(1/12) - \bar{Z}(1/12) \right] \cdot \left[N_i(2/12) - N_i(1/12) \right] + \cdots \}, \end{aligned}$$

and then numerically solve for the maximum (partial) likelihood estimate $\hat{\beta}$. We refer to Section 4.2 of Schnaubel and Wei (2007) for a similar discretization technique in a setting with time-varying covariates.

In the presence of unobserved heterogeneity (see Sections 3.3 and 5.2 in the main text), Lin, Wei, Yang, and Ying (2000) showed that the point estimates remain consistent for the *mean function* or the *cumulative rates*. However, Lin and Wei (1989) argued that the standard Fisher information matrix $\hat{I}(\hat{\beta})$ no longer provides an adequate estimator for the variance of $\hat{\beta}$. Thus, here we calculate so-called sandwich variance estimates (Lin and Wei (1989)) that account for model misspecification in the parametric setting. In particular, the asymptotic covariance matrix is estimated via

$$\hat{V}(\hat{\beta}) = \hat{I}^{-1}(\hat{\beta}) \times \underbrace{\left(\frac{1}{n}\sum_{i=1}^{n} U_i(\hat{\beta}) \times U_i'(\hat{\beta})\right)}_{\hat{\Sigma}(\hat{\beta})} \times \hat{I}^{-1}(\hat{\beta}),$$

where $U_i(\beta) = \int_0^\infty [Z_i(t) - \bar{Z}(t)]\, dN_i(t)$ is the contribution from the ith observation to the total score function $U(\beta)$.

A.3 *Impact of the mixed nature of the remaining subsample*

As discussed in Section 3 of the main text, the estimate for the regression coefficient γ of the *Settled-and-Observed* variable generally does not provide a consistent estimate for the difference between closed and nonclosed cases due to the mixed nature of the subsample of remaining policies. Put differently, since the remaining cases include both nonclosed and closed cases, γ will not constitute a suitable adjustment for closed cases relative to individuals that did not settle their policy but will only amount to a fraction of the “true” difference and, therefore, needs to be inflated. In what follows, we derive appropriate inflation formulae for the proportional hazards model and the additive hazards model.

Proportional hazards model To illustrate, consider the following simplified version of our proportional hazards model (4):

$$\mu_t^{(i)} = \beta_0(t) \times \exp\{\gamma \mathrm{SaO}_i\}. \tag{A.5}$$

Denote by Y_t all remaining observations at time t, by $Y_t^{(1)}$ all remaining settled/closed cases at time t (unobserved), $p_t = Y_t^{(1)}/Y_t$, and by $Y_t^{(2)}$ all remaining Settled-and-Observed cases at time t, $q_t = Y_t^{(2)}/Y_t$. Denote by $N_i(t)$ the death counting process for policyholder i akin to Section A.2. Furthermore, denote by γ^{act} the *unknown* actual regression coefficient for the model in which the econometrist observes all settlement decisions—effectively replacing the Settled-and-Observed variable (SaO_i) by a corresponding Settled variable (Set_i) in (A.5), and by γ^{our} our coefficient based on the Settled-and-Observed cases only. For simplicity, assume further that at any time t, the probability that a settlement decision is observed is constant. Therefore, based on Lin and Ying (1994, equation (2.6)), γ^{act} and γ^{our} will be solutions to the following *(partial) score functions*, respectively:

$$0 = \sum_{i=1}^{n} \int_{t=0}^{\infty} \left[\mathrm{Set}_i - \frac{p_t}{(1-p_t) \times \exp\{-\hat{\gamma}^{\mathrm{act}}\} + p_t} \right] dN_i(t) \quad \text{and}$$

$$0 = \sum_{i=1}^{n} \int_{t=0}^{\infty} \left[\mathrm{SaO}_i - \frac{q_t}{(1-q_t) \times \exp\{-\hat{\gamma}^{\mathrm{our}}\} + q_t} \right] dN_i(t).$$

Integrating and using the assumption that we obtain the number of Settled-and-Observed deaths from multiplying the number of Settled deaths by the corresponding proportion (q_t/p_t), we obtain that

$$(1-p) \times \exp\{-\hat{\gamma}^{\mathrm{act}}\} + p \approx (1-q) \times \exp\{-\hat{\gamma}^{\mathrm{our}}\} + q,$$

where p is the (unknown) overall proportion of *settled cases* and q is the (known) overall proportion of *Settled-and-Observed cases* in the portfolio, which for simplicity we assume are constant. Thus, under the assumptions above, a suitable estimator for the actual difference between closed and non-closed cases under the proportion hazards assumption is

$$\hat{\gamma}^{\mathrm{act}} \approx -\ln\left(\frac{q-p}{1-p} + \exp\{-\hat{\gamma}^{\mathrm{our}}\} \times \frac{1-q}{1-p} \right), \tag{A.6}$$

where of course $\hat{\gamma}^{\mathrm{our}}$ corresponds to the estimate from specification (4). In particular, for $\hat{\gamma}^{\mathrm{our}} = 0$ we obtain $\hat{\gamma}^{\mathrm{act}} = 0$, and in case $\hat{\gamma}^{\mathrm{our}} < 0$ the actual coefficient $\hat{\gamma}^{\mathrm{act}}$ needs to be inflated ($\hat{\gamma}^{\mathrm{act}} < \hat{\gamma}^{\mathrm{our}} < 0$).

Additive hazards model Similar to above, we consider the following simplified version of our additive hazards model (see equation (B.1) in Section B.1):

$$\mu_t^{(i)} = \beta_0(t) + \gamma \mathrm{SaO}_i. \tag{A.7}$$

Using the same assumptions and notation, based on the estimates in Lin and Ying (1994, equation (2.8)) we have

$$\frac{\hat{\gamma}^{\mathrm{act}}}{\hat{\gamma}^{\mathrm{our}}} = \frac{\displaystyle\int_0^{\infty} Y_t^{(2)} [1-q_t]\, dt}{\displaystyle\int_0^{\infty} Y_t^{(2)} [1-p_t]\, dt},$$

and again using the assumption of constant proportions we obtain

$$\frac{\hat{\gamma}^{\mathrm{act}}}{\hat{\gamma}^{\mathrm{our}}} \approx \frac{(1-q)}{(1-p)}.$$

Thus, a suitable estimator for the actual difference between closed and nonclosed cases under the additive hazards assumption is

$$\hat{\gamma}^{\mathrm{act}} \approx \hat{\gamma}^{\mathrm{our}} \times \frac{(1-q)}{(1-p)}, \tag{A.8}$$

where $\hat{\gamma}^{\text{our}}$ corresponds to the estimate from specification (B.1). In particular, since the ratio $(1-q)/(1-p)$ is always greater than one, the inflated coefficient will again be greater (in its absolute value) than the one estimated from the mixed sample.

A.4 *Development of the nonparametric estimators*

Following the description in Section 4 of the main text, we derive nonparametric estimates for the *excess hazard* for policyholders that settled their policy as a function of time. To formalize our notion of *excess hazard*, assume we are given two individuals S and R with hazard rates $\{\mu_t^S\}_{t\geq 0}$ and $\{\mu_t^R\}_{t\geq 0}$, respectively, that differ only in the information regarding their settlement decision but are identical with respect to all observables. More precisely, assume that we know S settled her policy whereas the settlement decision for R is unknown. Then we can define the *multiplicative excess hazard* $\{\alpha(t)\}_{t\geq 0}$ and the *additive excess hazard* $\{\beta(t)\}_{t\geq 0}$ via the following relationships (see equation (7) in the main text):

$$\mu_t^S = \alpha(t) \times \mu_t^R \quad \text{and} \quad \mu_t^S = \beta(t) + \mu_t^R.$$

Andersen and Vaeth (1989) provide nonparametric estimators for the multiplicative and additive excess hazard by relying on the Nelson–Aalen (N–A) estimator for $\int_0^t \alpha(s)\,ds$ and the Kaplan–Meier (K–M) estimator for $\int_0^t \beta(s)\,ds$, respectively. However, their approach relies on the assumption that the baseline mortality (μ_t^R in our specification) is known, whereas we only have available estimates $\{\hat{\mu}_t^{(i)}\}_{t\geq 0}$, $i = 1, \ldots, n$, given by the LE provider. Therefore, for the estimation of the multiplicative excess hazard, we instead use the following three-step procedure that relies on a repeated application of the Andersen and Vaeth (1989) estimator:

1. We start with the specification,

$$\mu_t^{(i)} = A(t) \times \hat{\mu}_t^{(i)}, \quad i = 1, \ldots, n, \tag{A.9}$$

and use the Andersen and Vaeth (1989) excess hazard estimator to obtain an estimate for A, say $\hat{A}$, based on the full dataset. Hence, $\hat{A}$ corrects systematic deviations of the given estimates based on the observed times of death (in sample). We set

$$\bar{\mu}_t^{(i)} = \hat{A}(t) \times \hat{\mu}_t^{(i)}, \quad i = 1, \ldots, n$$

for the corrected individual baseline hazard rate.

2. We then use the specification:

$$\mu_t^{(i)} = \alpha(t) \times \bar{\mu}_t^{(i)} \tag{A.10}$$

for individual i in the closed subsample. Note that if we used the full dataset to estimate α, we would obtain $\alpha(t) \equiv 1$ and $\int_0^t \alpha(s)\,ds$ would be a straight line with slope one. However, when applying (A.10) to the subsample of closed policies, the resulting estimate for α—or rather $\int_0^t \alpha(s)\,ds$—picks up the residual hazard information due to the settlement decision.

3. Finally, we derive an estimate for α from the cumulative estimate $(\int_0^t \alpha(s)\,ds)$ using a suitable kernel function as in Muller and Wang (1994).

For the additive excess hazard, we proceed analogously replacing equations (A.9) and (A.10) by

$$\mu_t^{(i)} = B(t) + \hat{\mu}_t^{(i)} \quad \text{and} \quad \mu_t^{(i)} = \beta(t) + \underbrace{\big[\hat{B}(t) + \hat{\mu}_t^{(i)}\big]}_{=\bar{\mu}_t^{(i)}},$$

respectively.

In the context of Figures 2, 3, and A.2, for the derivation of the derivatives in Step 3, we use the Epanechnikov kernel with a fixed bandwidth of one.

A.5 *A version of the model from Section 2 with price uncertainty*

Assume the LS company has access to an additional estimate for the insured's probability of death q that is not known to the econometrist, say θ. Here, we assume that the underlying probability measure $\mathbb{P}$ reflects all available information and, to simplify the presentation, we ignore uncertainty in ψ and let $f = 1$.[2] Since we interpret θ as a signal for q, we assume (i) that a higher θ will result in a higher offer price, that is, $\pi(\theta)$ is increasing, and (ii) that q is stochastically increasing in θ. Then it is easy to see that

$$\mathbb{E}\big[q | q < \pi(\theta) + \psi, \theta\big] \text{ is increasing as a function of } \theta. \tag{A.11}$$

Indeed, it is sufficient to assume the weaker condition (A.11) holds, which solely indicates that the estimate for q conditional on a policyholder settling her policy is increasing in θ.

Now if the econometrist finds a negative correlation between settling and dying, in the context of this extended model this means

$$\mathbb{E}\big[q | q < \pi(\theta) + \psi\big] < \mathbb{E}[q], \tag{A.12}$$

where the conditional expectation on the left-hand side incorporates all the information available to the econometrist (reflected in $\mathbb{P}$) and the observation that the policyholder settled. However, the question from the point of view of the LS company is whether there exists asymmetric information, indicated by a negative correlation, when incorporating all pricing-relevant information, particularly θ:

$$\mathbb{E}\big[q | q < \pi(\theta) + \psi, \theta\big] \overset{?}{<} \mathbb{E}[q | \theta]$$

for at least some choices of θ. When aggregating over all policyholders,

$$\mathbb{E}\big[\mathbb{E}\big[q | q < \pi(\theta) + \psi, \theta\big]\big] \overset{?}{<} \mathbb{E}\big[\mathbb{E}[q | \theta]\big] = \mathbb{E}[q]. \tag{A.13}$$

[2] The assumption of $f = 1$ is equivalent to defining π and ψ as settlement price and proclivity *per dollar face value*.

Therefore, the question of whether the observed relationship (A.12) provides definite evidence for the relevant relationship (A.13) depends on the relationship between the expectations on the left-hand sides of (A.12) and (A.13). In particular, the implication will hold if

$$\mathbb{E}\big[\mathbb{E}\big[q|q<\pi(\theta)+\psi,\theta\big]\big] \le \mathbb{E}\big[q|q<\pi(\theta)+\psi\big]. \tag{A.14}$$

We need the following lemma.

Lemma A.3. *Let X be a real random variable, g be an increasing function such that $\mathbb{E}[g(X)]=0$, and h be an increasing and positive function. Then $\mathbb{E}[g(X)h(X)]\ge 0$.*

Proof. Let $K=\operatorname{argmax}_x\{g(x)\le 0\}$. Then

$$0=\mathbb{E}\big[g(X)\big]=\mathbb{E}\big[g(X)|X\le K\big]\mathbb{P}(X\le K)+\mathbb{E}\big[g(X)|X>K\big]\mathbb{P}(X>K).$$

Now clearly $g(X)h(K)\le g(X)h(X)$ on $\{X\le K\}$, so that

$$\mathbb{E}\big[g(X)h(K)|X\le K\big]\le \mathbb{E}\big[g(X)h(X)|X\le K\big].$$

Similarly,

$$\mathbb{E}\big[g(X)h(K)|X> K\big]\le \mathbb{E}\big[g(X)h(X)|X> K\big].$$

Thus,

$$\begin{aligned} 0&=\mathbb{E}\big[g(X)h(K)|X\le K\big]\mathbb{P}(X\le K)+\mathbb{E}\big[g(X)h(K)|X>K\big]\mathbb{P}(X>K)\\ &\le \mathbb{E}\big[g(X)h(X)|X\le K\big]\mathbb{P}(X\le K)+\mathbb{E}\big[g(X)h(X)|X>K\big]\mathbb{P}(X>K)\\ &=\mathbb{E}\big[g(X)h(X)\big]. \end{aligned}$$

□

Now, by the tower property of conditional expectations, (A.14) is equivalent to

$$\begin{aligned} &\frac{\mathbb{E}\big[\mathbb{E}[q\mathbf{1}_{\{q<\pi(\theta)+\psi\}}|\theta]\big]}{\mathbb{P}\big(q<\pi(\theta)+\psi\big)}-\mathbb{E}\big[\mathbb{E}\big[q|q<\pi(\theta)+\psi,\theta\big]\big]\ge 0\\ \Leftrightarrow\quad &\mathbb{E}\Bigg[\mathbb{E}\big[q|q<\pi(\theta)+\psi,\theta\big]\underbrace{\bigg(\frac{\mathbb{P}\big(q<\pi(\theta)+\psi|\theta\big)}{\mathbb{P}\big(q<\pi(\theta)+\psi\big)}-1\bigg)}_{=g(\theta)}\Bigg]\ge 0. \end{aligned}$$

Since $\mathbb{E}[q|q<\pi(\theta)+\psi,\theta]$ is increasing as a function of θ by our assumption and since $\mathbb{E}[g(\theta)]=0$, with the lemma, relationship (A.13) will hold if g is increasing. Note that g is an affine transformation of the proportion of policyholders deciding to settle given the estimate θ, so that the pivotal relationship is the increasingness of this proportion in θ. Conversely, the implication will go in the other direction, so that that the econometrist's analysis will potentially overstate the effect, if the proportion of policyholders settling their policy is decreasing in the estimate.

Appendix B: Supplemental results

B.1 *Additive model specification*

In addition to the proportional hazards assumption in Section 3, we alternatively consider an additive hazards regression model (e.g., Aalen, Borgan, and Gjessing (2008)):

$$\begin{aligned}\mu_t^{(i)} &= \beta_0(t) + \beta_1\hat{\mu}_t^{(i)} + \beta_2\mathrm{DOU}_i + \beta_3\mathrm{AU}_i \\ &\quad + \beta_4\mathrm{SE}_i + \sum_{j=1}^{15}\beta_{5,j}\mathrm{PI}_{i,j} + \sum_{j=1}^{2}\beta_{6,j}\mathrm{SM}_{i,j} + \gamma\mathrm{SaO}_i,\end{aligned} \tag{B.1}$$

where the variables are defined as in equation (4). While less popular, the additive specification directly resembles the standard regression test for the coverage-risk correlation as described in Cohen and Siegelman (2010). We rely on the generalized least-squares (GLS) approach from Lin and Ying (1994) to estimate the coefficient vector and on their formula for the model likelihood. Column [A] in Table A.1 presents the results for the basic model (B.1).

Similar to the proportional model, the coefficients for underwriting age, sex, and the variables relating to smoking status are significant—although underwriting date is not significant here. The coefficient for the estimated hazard $\hat{\mu}_t^{(i)}$, while highly significant, with roughly 0.2 is now far away from one as would be the case for "perfect" estimates by the LE provider. This suggests that the proportional model may be better suited to capture residual effects. Important for our focus, the coefficient for the Settled-and-Observed covariate again is negative and highly significant. Thus, we again find a strong negative relationship between settlement and hazard, indicating the existence of asymmetric information in the life settlements market.

When augmenting the basic specification (B.1) by a linear time trend interacted with the Settled-and-Observed covariate, $\mathrm{SaO}_i \times t$, we obtain analogous effects as in the proportional model: The intercept more than doubles, and the coefficient for the time trend is positive and significant. The model likelihood also increases, and the coefficients for the nonsettlement related variables are very similar to the basic model. Column [B] in Table A.1 presents the corresponding estimates (see also Table A.2 for alternative trend specifications with lower likelihood values). Hence, here our result that the influence of the informational friction is most pronounced right after settlement but wears off over time also appears robust.

Similar to Section 6 of the main text, we also run robustness analyses that incorporate policy face value as a covariate, exclude deaths within 6 months of underwriting in the remaining subsample, and with latest observation date under the additive specification. The results are presented in columns [C] through [E] of Table A.1, where once again we find highly significant and consistent settlement coefficients.

B.2 *Additional regression results*

Table A.2 presents supplemental survival regression results based on proportional and additive hazards specifications. Columns [A] and [B] in the table show results for the earliest observation date with alternative time trend specification under the proportional

TABLE A.1. Additive hazards survival regression results. Column [A]: Basic regression (equation (B.1)), earliest observation date; [B]: With time trend, earliest observation date; [C]: Only considering cases with known face value (in the entire dataset) and with time trend, face value as covariate, earliest observation date; [D]: Excluding cases with times of death within 6 months of underwriting (in the remaining sample) and with time trend, earliest observation date; [E]: With time trend, latest observation date.

	[A]	[B]	[C]	[D]	[E]
$1/14 \times \int_0^{14} \beta_0(t)\,dt$	−0.0090	−0.0090	−0.0010	−0.0094	−0.0108
Estimated hazard, $\hat{\mu}_t^{(i)}$	0.2032	0.2031	0.2739	0.1959	0.2264
	(0.0067)	(0.0067)	(0.0182)	(0.0066)	(0.0073)
Underwriting date, DOU_i	0.0001	0.0001	0.0024	3.2×10^{-5}	−0.0008
	(0.0002)	(0.0002)	(0.0007)	(0.0002)	(0.0002)
Age at underwriting, AU_i	0.0016	0.0016	0.0008	0.0017	0.0019
	(0.0001)	(0.0001)	(0.0002)	(0.0001)	(0.0001)
Sex, SE_i	0.0043	0.0042	0.0001	0.0043	0.0054
	(0.0007)	(0.0007)	(0.0016)	(0.0007)	(0.0008)
Smoker, $\mathrm{SM}_{i,1}$	0.0297	0.0297	0.0392	0.0292	0.0313
	(0.0026)	(0.0026)	(0.0095)	(0.0026)	(0.0029)
"Aggregate" smoking status, $\mathrm{SM}_{i,2}$	0.0115	0.0116	0.0117	0.0115	0.0135
	(0.0025)	(0.0025)	(0.0078)	(0.0025)	(0.0029)
Face Value, $\ln(1+\mathrm{FV})$			−0.0054		
			(0.0007)		
Settled-and-Observed, SaO_i	−0.0049	−0.0107	−0.0100	−0.0087	−0.0046
	(0.0007)	(0.0012)	(0.0016)	(0.0011)	(0.0015)
Settled-and-Observed × trend, $\mathrm{SaO}_i \times t$		0.0014	0.0033	0.0011	0.0011
		(0.0003)	(0.0004)	(0.0003)	(0.0004)
Log-likelihood value	−74,521.29	−74,304.23	−6,365.01	−75,599.54	−73,678.67

hazards specification (adding a quadratic trend, $\mathrm{SaO}_i \times \ln^2(1+t)$, in column [A] and adding a linear trend, $\mathrm{SaO}_i \times t$, in column [B]). Columns [D] and [E] show comparable results under the additive hazards specification (adding a quadratic trend, $\mathrm{SaO}_i \times t^2$, in column [D] and adding a logarithmic trend, $\mathrm{SaO}_i \times \ln(1+t)$, in column [E]). As is evident from the table, the quadratic trend components fail to be significant in both specifications, whereas the alternative time trend specification in either case provides qualitatively similar conclusions on informational frictions, yet with lower likelihood values when compared with the default trend choices in the main text.

Columns [C] and [F] of Table A.2 present results when using dummy variables for underwriting date and age at underwriting in the proportional and additive hazards specification, respectively. Comparing the estimates to the baseline results from Tables 2 and A.1, it is clear that using the log-linear/linear trend did not have a significant impact on the results. In particular, there is little change in the settlement-related variables that are in the focus of our analysis. This is also illustrated by Figure A.1 that plots coefficients of the corresponding dummy variables, from which we note that a basic increasing trend assumption can capture the relevant shape.

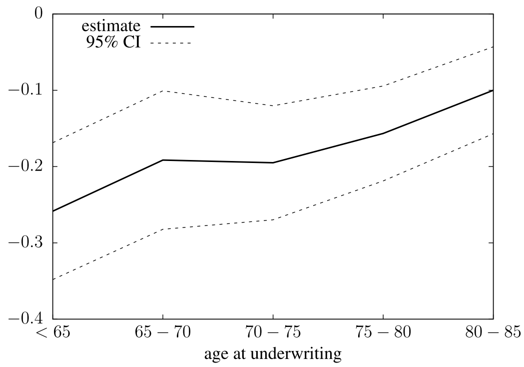
TABLE A.2. Survival regression supplemental results. Column [A]: Proportional hazards assumption with additional quadratic trend, earliest observation date; [B]: Proportional hazards assumption with linear time trend, earliest observation date; [C]: Proportional hazards assumption with logarithmic time trend and dummy variables for underwriting date and age at underwriting, earliest observation date; [D]: Additive hazards assumption with additional quadratic trend, earliest observation date; [E]: Additive hazards assumption with logarithmic time trend, earliest observation date; [F]: Additive hazards assumption with linear time trend and dummy variables for underwriting date and age at underwriting, earliest observation date.

	[A]	[B]	[C]	[D]	[E]	[F]
$1/14 \times \int_0^{14} \beta_0(t)\,dt$	0.0182	0.0189	0.4645	−0.0090	−0.0090	0.0061
Estimated hazard, $\hat{\mu}_t^{(i)}$	0.8966	0.8974	0.8804	0.2031	0.2031	0.1822
	(0.0101)	(0.0101)	(0.0125)	(0.0067)	(0.0067)	(0.0069)
Underwriting date, $\ln(1+\mathrm{DOU}_i)$	0.3048	0.3039	–	0.0001	0.0001	–
	(0.0277)	(0.0277)	–	(0.0002)	(0.0002)	–
Age at underwriting, $\ln(1+\mathrm{AU}_i)$	0.5880	0.5798	–	0.0016	0.0016	–
	(0.0828)	(0.0828)	–	(0.0001)	(0.0001)	–
Sex, SE_i	−0.0984	−0.0990	−0.0866	0.0042	0.0043	0.0049
	(0.0196)	(0.0196)	(0.0202)	(0.0007)	(0.0007)	(0.0007)
Smoker, $\mathrm{SM}_{i,1}$	0.3739	0.3733	0.3604	0.0298	0.0297	0.0268
	(0.0429)	(0.0429)	(0.0432)	(0.0026)	(0.0026)	(0.0026)
“Aggregate” smoking status, $\mathrm{SM}_{i,2}$	0.2117	0.2123	0.2213	0.0116	0.0116	0.0105
	(0.0551)	(0.0551)	(0.0552)	(0.0025)	(0.0025)	(0.0025)
Settled-and-Observed, SaO_i	−0.5994	−0.3300	−0.4650	−0.0113	−0.0143	−0.0101
	(0.1189)	(0.0419)	(0.0643)	(0.0016)	(0.0016)	(0.0012)
Settled-and-Observed × trend, $\mathrm{SaO}_i \times \ln(1+t)$	0.3993		0.2127		0.0063	
	(0.1691)		(0.0361)		(0.0011)	
Settled-and-Observed × quadratic trend, $\mathrm{SaO}_i \times \ln^2(1+t)$	−0.0608					
	(0.0567)					
Settled-and-Observed × trend, $\mathrm{SaO}_i \times t$		0.0411		0.0018		0.0014
		(0.0071)		(0.0009)		(0.0003)
Settled-and-Observed × quadratic trend, $\mathrm{SaO}_i \times t^2$				-4.1×10^{-5}		
				(0.0001)		
Log-likelihood value	−134,011.87	−134,015.18	−133,987.62	−74,322.44	−74,357.20	–

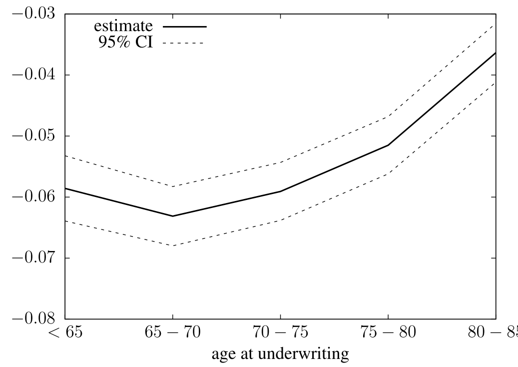
Tables A.3 and A.4 present additional results on the economic impact from Section 5.1 of the main text. In particular, we show results for representative US male and female policyholders under various model specification, using the approximate inflation formulae as derived in equations (A.6) and (A.8) in Appendix A.3. Among all cases, we estimate LE increases from roughly 1.8% to 13.9% for policyholders who choose to settle their policy. The difference in value of the insurance policy is roughly between −5.2% to −38.2%. We note that the differences are overall more pronounced under the time-constant trend assumption.

APPENDIX C: MONTE CARLO EXPERIMENT ON SETTLEMENT PROCESS

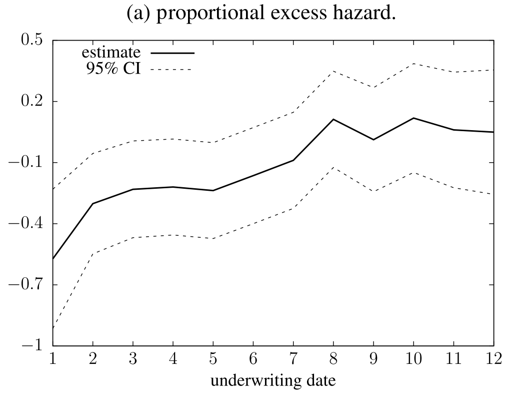
To implement a Monte Carlo version of the thought experiment on the winner's curse in Section 5.2, part (iii) of the main text, we first run a least-square regression of the



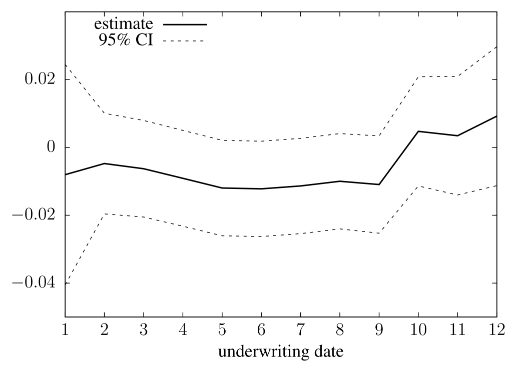
(a) proportional excess hazard.



(b) additive excess hazard.



(c) proportional excess hazard.



(d) additive excess hazard.

FIGURE A.1. Survival regression parameter estimations (left panels: Proportional; right panels: Additive) of dummy variables for age at underwriting (top panels) and underwriting date (bottom panels), with point-wise 95% confidence intervals (dashed curves); earliest observation date.

logarithm of the mortality multipliers on the observable characteristics (excluding the multipliers themselves and the settlement-related variables). We also include significant interactions of the terms so that we have 41 covariates in total. Based on the regression model, we then derive projected life expectancies as well as the standard deviation of the error term. The projected life expectancies will then be used as the benchmark of the assessment in the Monte Carlo experiment, that is, we assume these present the *true* life expectancies.

Now, following the logic from Section 5.2, assume that a fraction of all cases enter into life settlement transactions and the brokers commissioned with the sale "cherry-pick" among the available LEs (multiplier estimates). More precisely, assume that for these transactions, several LEs from various LE providers will be obtained but only the shortest LE (highest multiplier) is submitted. Alternatively, we may assume that there are several offers from various LS companies that base their pricing on different mortality multipliers, and the one with the highest bidding price (corresponding to the highest multiplier estimate) will make the trade. Importantly, while in the context of this experiment we assume the policyholder does not have private information on her survival prospects, note that there still exists an informational asymmetry—the broker and/or

TABLE A.3. Comparisons of average life expectancies as well as net policy values for a standard whole life insurance purchased 10 years ago, between population-level and settled US male policyholders; various model specifications.

	Proportion of closed policies (p)					
	24.5%	30%	40%	50%	60%	70%
Age 70 (nonadjusted LE 13.93, *value* 0.2092*)*						
Additive hazards; time-weakening effect						
Difference in LE (%)	3.16	3.42	4.00	4.82	6.06	8.16
Difference in value (%)	−10.37	−11.20	−13.11	−15.80	−19.87	−26.76
Additive hazards; time-constant effect						
Difference in LE (%)	4.79	5.18	6.08	7.36	9.32	12.70
Difference in value (%)	−13.95	−15.08	−17.69	−21.39	−27.05	−36.78
Age 75 (nonadjusted LE 10.48, *value* 0.2520*)*						
Additive hazards; time-weakening effect						
Difference in LE (%)	3.03	3.27	3.83	4.61	5.80	7.81
Difference in value (%)	−7.64	−8.25	−9.65	−11.62	−14.62	−19.69
Additive hazards; time-constant effect						
Difference in LE (%)	3.80	4.11	4.82	5.82	7.35	9.97
Difference in value (%)	−8.77	−9.48	−11.11	−13.42	−16.93	−22.93
Age 80 (nonadjusted LE 7.50, *value* 0.3024*)*						
Additive hazards; time-weakening effect						
Difference in LE (%)	2.83	3.06	3.58	4.31	5.43	7.31
Difference in value (%)	−5.38	−5.81	−6.79	−8.18	−10.29	−13.85
Additive hazards; time-constant effect						
Difference in LE (%)	2.91	3.15	3.68	4.44	5.60	7.56
Difference in value (%)	−5.20	−5.62	−6.58	−7.94	−9.99	−13.49

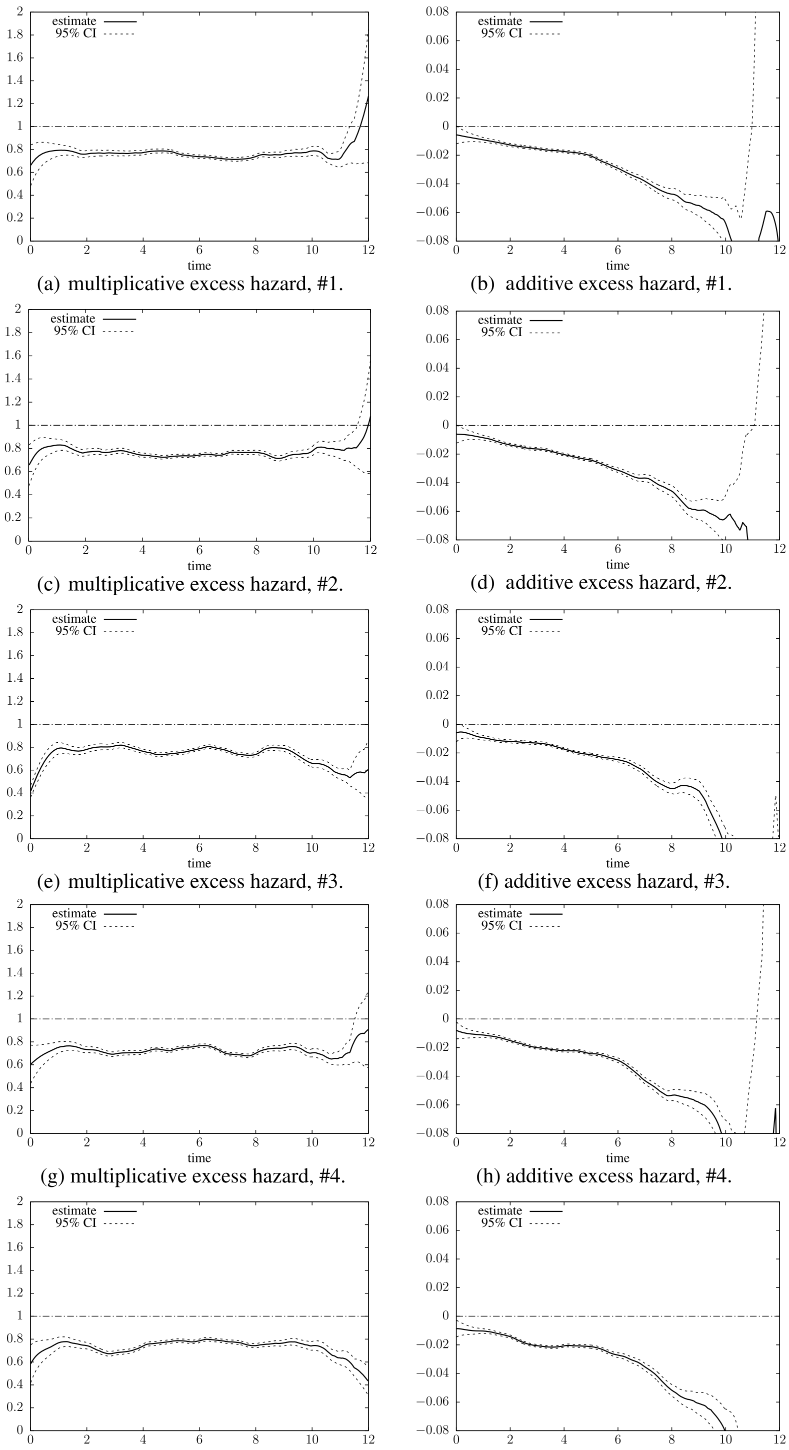
policyholder will have more information than the *winning* LS company—but this asymmetry emerges in the transaction process.

Assume that each LE provider's estimate is based on the same projected mortality multiplier plus a varying error term (with mean of zero), according to our regression estimates. For simplicity, we assume that the submitted (highest) multiplier corresponds to the 75th percentile. Based on this logic, closed cases are systematically assessed with shorter LEs, whereas the remaining cases are not. We use the resulting multipliers to generate a hypothetical set of forecasts $\hat{\mu}_t^{(i)}$, $i = 1, \ldots, n$, where we use the skewed (75th percentile) multiplier for the (randomly sampled) closed cases and the projected (median) multiplier for the remaining cases. Based on the simulated sample, we derive nonparametric estimators similarly as in Section 4.

Figure A.2 presents the results for five different simulated data sets, where as for our actual dataset we assume 13,221 out of the 53,947 policyholders are Settled-and-Observed (Panels (a) and (b) are also shown as Panels (o) and (p) of Figure 3 in the main text). While the shapes and magnitudes differ between the simulated data sets, we observe that the multiplicative excess hazard roughly evolves according to a straight line below one, whereas the additive excess hazard diverges. This is consistent with the as-

Table A.4. Comparisons of average life expectancies as well as net policy values for a standard whole life insurance purchased 10 years ago, between population-level and settled US female policyholders; various model specifications.

	Proportion of closed policies (p)					
	24.5%	30%	40%	50%	60%	70%
Age 70 (nonadjusted LE 15.97*, value* 0.2181*)*						
Proportional hazards; time-weakening effect						
Difference in LE (%)	1.76	1.85	2.06	2.32	2.66	3.12
Difference in value (%)	−5.50	−5.80	−6.44	−7.26	−8.32	−9.77
Proportional hazards; time-constant effect						
Difference in LE (%)	4.52	4.85	5.58	6.59	8.03	10.27
Difference in value (%)	−11.38	−12.20	−14.04	−16.54	−20.11	−25.65
Additive hazards; time-weakening effect						
Difference in LE (%)	3.21	3.47	4.06	4.90	6.16	8.30
Difference in value (%)	−10.10	−10.91	−12.77	−15.39	−19.35	−26.07
Additive hazards; time-constant effect						
Difference in LE (%)	5.21	5.64	6.62	8.02	10.16	13.87
Difference in value (%)	−14.44	−15.62	−18.33	−22.18	−28.07	−38.21
Age 75 (nonadjusted LE 12.25*, value* 0.2645*)*						
Proportional hazards; time-weakening effect						
Difference in LE (%)	2.71	2.86	3.18	3.59	4.12	4.84
Difference in value (%)	−6.47	−6.83	−7.59	−8.56	−9.82	−11.54
Proportional hazards; time-constant effect						
Difference in LE (%)	5.06	5.43	6.25	7.38	9.00	11.52
Difference in value (%)	−10.32	−11.06	−12.74	−15.01	−18.27	−23.34
Additive hazards; time-weakening effect						
Difference in LE (%)	3.10	3.35	3.92	4.72	5.94	8.00
Difference in value (%)	−7.43	−8.03	−9.39	−11.32	−14.23	−19.17
Additive hazards; time-constant effect						
Difference in LE (%)	4.17	4.50	5.28	6.39	8.07	10.96
Difference in value (%)	−9.10	−9.83	−11.52	−13.92	−17.58	−23.84
Age 80 (nonadjusted LE 8.90*, value* 0.3205*)*						
Proportional hazards; time-weakening effect						
Difference in LE (%)	4.29	4.53	5.04	5.69	6.54	7.71
Difference in value (%)	−7.61	−8.03	−8.93	−10.08	−11.57	−13.63
Proportional hazards; time-constant effect						
Difference in LE (%)	5.62	6.03	6.96	8.21	10.01	12.83
Difference in value (%)	−9.00	−9.66	−11.12	−13.12	−15.98	−20.43
Additive hazards; time-weakening effect						
Difference in LE (%)	2.93	3.16	3.70	4.46	5.61	7.55
Difference in value (%)	−5.22	−5.63	−6.59	−7.94	−9.98	−13.44
Additive hazards; time-constant effect						
Difference in LE (%)	3.20	3.45	4.04	4.88	6.15	8.32
Difference in value (%)	−5.34	−5.77	−6.76	−8.15	−10.27	−13.88



(a) multiplicative excess hazard, #1.
(b) additive excess hazard, #1.
(c) multiplicative excess hazard, #2.
(d) additive excess hazard, #2.
(e) multiplicative excess hazard, #3.
(f) additive excess hazard, #3.
(g) multiplicative excess hazard, #4.
(h) additive excess hazard, #4.
(i) multiplicative excess hazard, #5.
(j) additive excess hazard, #5.

FIGURE A.2. Non-parametric estimates of excess hazard in Monte-Carlo experiment of settlement process.

TABLE A.5. Survival regression results for the five Monte-Carlo simulated datasets.

	Simulation				
	#1	#2	#3	#4	#5
Proportional hazards specification					
Settled-and-Observed, SaO_i	−0.3218	−0.2568	−0.2890	−0.4372	−0.3896
	(0.0596)	(0.0587)	(0.0589)	(0.0599)	(0.0598)
Settled-and-Observed × trend, $\text{SaO}_i \times \ln(1+t)$	−0.0133	−0.0499	−0.0358	0.0204	0.0181
	(0.0347)	(0.0342)	(0.0343)	(0.0347)	(0.0347)
Additive hazards specification					
Settled-and-Observed, SaO_i	−0.0009	0.0001	0.0001	−0.0029	−0.0023
	(0.0012)	(0.0012)	(0.0012)	(0.0012)	(0.0012)
Settled-and-Observed × trend, $\text{SaO}_i \times t$	−0.0009	−0.0010	−0.0012	−0.0008	−0.0007
	(0.0003)	(0.0003)	(0.0003)	(0.0003)	(0.0003)

sertions in Section 5.2. While it is possible that there are systematic differences in the underwriting process between the LE providers, it is difficult to construct a situation that yields the observed receding patterns from Section 4 based on this selection process.

Table A.5 confirms the nonparametric findings by presenting corresponding survival regression results based on the same five simulated datasets, using both proportional and additive specifications with time trend. For simplicity, we only show regressed coefficients for settlement-related covariates. We observe from the table consistent results across all simulation trials. In particular, for the proportional specification, the intercept of the trend starts at significantly negative values, however, the slope of the trend is insignificant and very close to zero, suggesting a persistent impact of settling on survival prospects. For the additive specification, the slope of the trend is no longer positive but significantly negative, which is again necessary to sustain the constant proportional trend.

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