Quantitative Economics 13 (2022), 761–786 1759-7331/20220761

Revealing a preference for mixtures: An experimental study of risk

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Using a revealed preference approach, we conduct an experiment where subjects make choices from linear convex budgets in the domain of risk. We find that many individuals prefer mixtures of lotteries in ways that systematically rule out expected utility behavior. We explore the extent to which an individual's preference to choose mixtures is related to a preference for randomization by comparing choices from a convex choice task to the decisions made in a repeated discrete choice task. We find that a preference to mix is positively correlated with behavior from repeated discrete choice tasks.

Keywords. Risk preferences, preference for randomization, stochastic choice, revealed preferences.

JEL classification. C91, D81, D91.

1. Introduction

Understanding whether individuals have a preference for choosing mixtures over lotteries is critical for understanding risk preferences and stochastic choice. We say that the risk preference of an individual has a preference for mixtures when an individual chooses a (nondegenerate) mixture of distinct lotteries from a convex budget.[1] A preference for mixtures is naturally related to stochastic choice as described in Machina (1985). For instance, if an individual prefers mixtures, then for *repeated* binary choice

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We are grateful to Marina Agranov, Jim Andreoni, David Blau, Zach Breig, Daniel Burghart, Mauricio Fernández-Duque, David Dillenberger, Marcelo Ariel Fernández, Raymond Fisman, P.J. Healy, Shachar Kariv, Ali Khan, Jay Lu, Mark Machina, Ben Meiselman, Bin Miao, Kirby Nielsen, Pietro Ortoleva, Koji Shirai, Charlie Sprenger, Joel Sobel, John Quah, and Songfa Zhong for their thoughtful comments. We thank three referees for helpful comments. Excellent research assistance was provided by Jacqueline Vallon. Funding for the project was provided by the National Science Foundation's Dissertation Improvement Grant 1824121.

[1] In the paper, we examine convex budgets generated from p^1 and p^2 that give the simple lottery defined by $p = \lambda p^1 + (1-\lambda)p^2$. For example, an individual chooses a mixture of lotteries p^1 and p^2 from a convex budget when they choose a lottery $\lambda p^1 + (1-\lambda)p^2$ with $\lambda \in (0, 1)$.

Available at http://qeconomics.org. https://doi.org/10.3982/QE1694

tasks between lotteries p^1 and p^2 they may choose both p^1 and p^2 in a way that replicates their most preferred mixture of the lotteries.[2] We refer to the distributional behavior that results when an individual makes repeated discrete choice problems as their *preference for randomization*. In this paper, we begin to experimentally explore the link between risk preferences and a preference for randomization.

To explore the link between risk preferences and a preference for randomization, we perform an incentivized within-subject experiment where subjects answer questions from convex choice tasks and repeated discrete choice tasks. We first examine whether individuals prefer mixtures in the convex choice tasks. Next, we check whether the decisions an individual makes in a convex choice task are related to the choices from a repeated discrete choice task. Our two main hypotheses are stated below.

Hypothesis 1. *Many individuals will choose mixtures, and behavior will be inconsistent with expected utility predictions.*

Hypothesis 2. *The decisions from the convex choice tasks and the repeated discrete choice tasks will be positively correlated with one another.*

We find support for our two main hypotheses. In particular, we find 136/144 (94.4%) subjects have some preference for mixing and only 1/144 (0.7%) subjects can be described by expected utility. The reason few individuals can be described by expected utility is because individuals either choose a mixture from budgets with two different slopes or choices alternate between the lotteries at the "corners" of the budget sets. For the questions we examine, we find evidence that choices from the convex choice tasks and repeated discrete choice tasks are positively correlated with one another. This evidence suggests a preference for mixing and preference for randomization are linked as suggested by Machina (1985). This suggests that applied researchers may better describe choices by using models that allow mixing such as Chew et al. (1991), Cerreia-Vioglio, Dillenberger, and Ortoleva (2015), or Fudenberg, Iijima, and Strzalecki (2015) rather than using an expected utility model.[3]

Support for Hypothesis 1 is further experimental evidence that one should consider nonexpected utility theories (Machina (1982, 1989)) that allow mixing when studying risk preferences. While violations of expected utility have been pointed out in numerous experiments, experiments often use binary choice tasks to look for violations. If an individual has a preference for mixing and this is linked to a preference to randomize,

[2]Thus, an individual would choose p^1 and p^2 in the same proportion from repeated discrete choice problems as their preference for mixtures dictates.

[3]If a researcher studies expected utility with additive errors, then this distinction is more subtle. For example, a model of expected utility with additive errors is observationally equivalent to a perturbed utility model of risk preferences following results from Fudenberg, Iijima, and Strzalecki (2015) and Allen and Rehbeck (2019).

then responses to binary choice tasks will necessarily be random.[4] Thus, this experiment provides evidence of mixing in a straightforward manner.

The support of Hypothesis 2 is broadly consistent with models of deliberate stochastic choice as considered in Machina (1985). Deliberate stochastic choice supposes that an individual has an ideal distribution of choices in mind and can use a randomization device to simulate this distribution when facing a decision problem. Recently, models of deliberate stochastic choice have been studied theoretically in Fudenberg, Iijima, and Strzalecki (2015), Cerreia-Vioglio et al. (2019), and Allen and Rehbeck (2019). In particular, Allen and Rehbeck (2019) show a class of deliberate stochastic choice models nests the behavior of additive random utility models. The results in this paper suggest that individual repeated discrete choices might be generated by an individual's most preferred distribution chosen from the convex budget.

To elicit choices from a convex budget, we build on the interface pioneered by Sopher and Mattison Narramore (2000). This interface gives individuals a slide-rule to choose a mixture between two lotteries and displays the mixture as a simple lottery in a pie chart. We do not display the pie chart to individuals until they interact with the slide-rule to mitigate demand effects. We also include a verification step to the interface from Sopher and Mattison Narramore (2000) to encourage individuals to think about their choice to reduce noise.

While the features mentioned above should reduce experimenter demand effects and noise, it is difficult to evaluate whether these design choices are successful. However, taking a revealed preference approach, we compare the individual behavior to benchmarks related to experimenter demand effects and noisy behavior. This approach allows us to evaluate whether the interface used in this experiment could be useful for eliciting preferences in different domains (e.g., mixed strategy elicitation). In the experiment, we find that individual choices are "closer" to a well-defined risk preference than benchmark behavior that accounts for experimenter demand effects or noise. Thus, we believe the subjects' responses to the convex choice tasks reveal preference information.

We now describe some other features of the experiment. We use a large number of convex choice tasks (seventy-nine) to elicit risk preferences. We direct power at the convex choice tasks because a priori we were concerned about individuals being drawn to the extremes of the budget as predicted by expected utility (von Neumann and Morgenstern (1944)) and betweenness preferences (Dekel (1986)).[5] However, we found that mixing behavior was present in aggregate behavior for all convex choice tasks. Given the large number of questions used to address this concern, we only compare the behavior of convex choice tasks and repeated discrete choice tasks for two unique decision problems. Thus, while the results are suggestive, they are far from the final word.

[4]Examples of experiments that show violations of expected utility include Allais (1953), Ellsberg (1961), Loomes, Starmer, and Sugden (1992), Birnbaum and Chavez (1997), Charness, Karni, and Levin (2007), and Burghart (2019) among many others. Other experiments that examine nonexpected utility with binary choice tasks include Mosteller and Nogee (1951), Camerer and Ho (1994), Harless and Camerer (1994a), Hey and Orme (1994) among many others. The majority of this work has binary choice tasks.

[5]The choice of questions also helps us differentiate whether the choices made by experimental subjects differs from benchmark demand effects and benchmark noisy behavior.

We also have additional results that may be of general interest. First, we construct the convex choice tasks in a way that allows us to look at how demand for a numeraire lottery changes with an implied relative price.[6] In particular, this variation allows us to look at aggregate demand curves. We find that demand curves are downward sloping and follow a linear-log demand system. However, while aggregate risk preferences follow simple demand systems, there is substantial heterogeneity in individual risk preferences. Thus, we find the hypothesis of Becker (1962) that individual heterogeneity can lead to downward-sloping aggregate demand curves holds in the domain of risk preferences.

We briefly mention some references here, but we include a more thorough literature review in Section 6. First, this paper builds on the work of Agranov and Ortoleva (2017) and Dwenger, Kübler, and Weizsäcker (2018) that recently look at whether individuals have a preference to randomize. In particular, we differ from the experiment of Agranov and Ortoleva (2017) since we do not look at how choice difficulty affects randomization and do not introduce an external randomization device. Instead, we focus on whether behavior from convex choice tasks and repeated discrete choice tasks are related for individuals. The results in this paper are also relevant for work that builds off the random expected utility models developed in Gul and Pesendorfer (2006). In particular, since we see mixtures chosen in many budgets, this suggests the assumption that lotteries chosen from convex budgets occur at extreme points is a poor descriptive axiom.

The paper proceeds as follows: Section 2 describes the basic theory on risk preferences. Section 3 describes the experimental procedures. Section 4 provides the main results. Section 5 contains the additional results on aggregate demand and individual heterogeneity. Section 6 elaborates how our paper fits within the broader literature on stochastic choice, risk preferences, and revealed preferences. Section 7 provides our final remarks.

2. Theoretical preliminaries

In this section, we describe the domain of lotteries in the experiment and discuss properties of risk preferences. We examine individual preferences over objective lotteries when there are three distinct monetary prizes: A low prize (x_L), a middle prize (x_M), and a high prize (x_H) where $x_L < x_M < x_H$. In the experiment, the low, middle, and high prizes are \$2, \$10, and \$30, respectively. We refer to the low and high prize as the *extreme prizes* since they are the highest and lowest amounts of money an individual can win in a lottery. We denote lotteries over the prizes using the vector $p = (p_L, p_M, p_H)$ where p_L is the probability of receiving the low prize, p_M is the probability of receiving the middle prize, and p_H is the probability of receiving the high prize. Since the vector p represents a lottery, the entries of p are nonnegative and sum to one.

Within the paper, we use the Marschak–Machina (MM) triangle (Marschak (1950), Machina (1982)) to parsimoniously describe the domain of lotteries. However, the experimental subjects did not interact with the MM triangle in the experiment. An exam-

[6] The numeraire lottery is formally defined in Section 2. A rough definition of the numeraire lottery is a lottery that is fixed and used to generate a set of convex budgets.

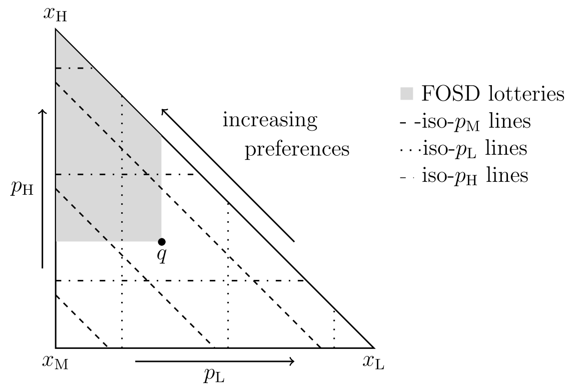


Figure 1. Marschak–Machina triangle.

ple of the MM triangle with three prizes is shown in Figure 1. In Figure 1, the probability of receiving the high prize is on the vertical axis, and the probability of receiving the low prize is on the horizontal axis. Therefore, the point $(0, 0)$ represents the middle prize with certainty, the point $(1, 0)$ represents the low prize with certainty, and the point $(0, 1)$ represents the high prize with certainty.

We now describe some features of the MM triangle from Figure 1. First, the dashed lines in Figure 1 all have the same probability of receiving the middle prize. We refer to the dashed lines as iso-p_M lines. As the dashed lines move northeast, the level of p_M decreases. Similarly, the dotted lines all have the same probability of receiving the low price, so we refer to these as iso-p_L lines. As the dotted lines move east, the probability of receiving the low prize increases. Finally, the dash-dotted lines have the same probability of p_H so we refer to these as iso-p_H lines. As these lines move north, the probability of receiving the high prize increases.

The one property we assume for risk preferences is first-order stochastic dominance (FOSD). The lottery $p' = (p'_L, p'_M, p'_H)$ first-order stochastic dominates the lottery $p = (p_L, p_M, p_H)$ when $p'_H \geq p_H$ and $p'_H + p'_M \geq p_H + p_M$ with one inequality strict. For example, the lotteries that FOSD the lottery q in Figure 1 are those that lie to the west (less p_L more p_M) or north (less p_M more p_H) of q. Since preferences respect first-order stochastic dominance, the lotteries to the northwest are more preferred, which is indicated with the arrow labeled "increasing preferences" in Figure 1.

For the experiment, we elicit individual choices from convex budget sets. Importantly, a convex budget allows individuals to choose lotteries that are a mixture of the "corners" of the budget set. This design feature is essential since many theories of non-expected utility allow an individual to prefer mixtures of lotteries from convex budgets.[7] In contrast, if an individual has expected utility preferences, then they will almost always choose lotteries on the boundary of the MM triangle and cannot choose mixtures from budgets with two different slopes. We argue it is important to elicit a preference for mixing using convex budget sets since an individual may "convexify" discrete choice

[7]One notable class that does not allow mixtures except for the case of indifference is the betweenness class of preferences Dekel (1986).

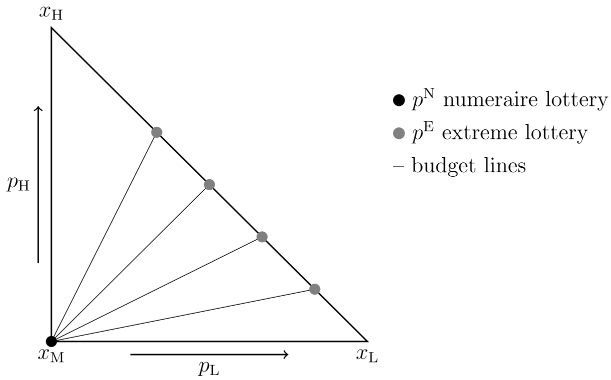


FIGURE 2. "Price" variation.

problems by randomizing among the lotteries to mimic their most preferred lottery from a convex budget set.

We introduce some terminology to describe how budget sets are constructed. Each budget in the experiment is generated by a line connecting two lotteries that lie on the boundary of the MM triangle. In particular, we choose an *extreme lottery* and a *numeraire lottery* to generate each budget line. The *extreme lottery* is denoted p^{E} and places probability exclusively on the extreme high and low prizes. The *numeraire lottery* is denoted p^{N}, places some probability on the middle prize and is on the boundary of the MM triangle, serves as a reference lottery that budget lines pivot around, and allows us to examine changes in the relative price for a given numeraire lottery. A series of example budgets that pivot around a numeraire lottery are shown in Figure 2.

The reason we examine a series of budgets that pivot around a numeraire lottery is to examine a "demand" curve for the numeraire lottery. To see this, note that for a fixed numeraire lottery, p^{N}, the slope of a budget line is essentially a relative price for the numeraire lottery. To better understand the analogy between the slope and a "price" for the numeraire lottery, note that as the probability of receiving the high prize of the extreme lottery increases, the extreme lottery becomes more attractive, and the slope of the budget line increases. Thus, we say the relative price (r) for the numeraire lottery is given by

$$r = \frac{p_{\mathrm{H}}^{\mathrm{E}} - p_{\mathrm{H}}^{\mathrm{N}}}{p_{\mathrm{L}}^{\mathrm{E}} - p_{\mathrm{L}}^{\mathrm{N}}}.$$

We later examine how the aggregate demands for the numeraire lotteries vary with the relative prices. We note that the interpretation of r as a relative price of the numeraire lottery is a feature of linear budget sets and does not depend on preferences.

3. EXPERIMENTAL DESIGN

In this section, we describe the details of our experimental design. Subjects completed 85 different choice tasks in the experiment. For each choice task, budgets consist of lotteries over the monetary prizes $x_{\mathrm{L}} = \$2$, $x_{\mathrm{M}} = \$10$, and $x_{\mathrm{H}} = \$30$.[8] In each task, a subject selects a preferred lottery from a given budget set. There are two types of choice tasks: convex choice tasks and repeated discrete choice tasks. The subjects first faced 79 convex choice tasks from linear budget sets that consist of convex combinations of a numeraire lottery (p^{N}) and an extreme lottery (p^{E}) in a random order. The remaining six tasks were discrete choice tasks where a subject could choose either the numeraire lottery (p^{N}) or the extreme lottery (p^{E}). In particular, these six tasks were three repeated discrete choices from two distinct choice tasks.

One hundred and forty-four undergraduates from UC San Diego participated in this study. Six sessions were conducted from January 22–23 in 2018. The tasks were conducted on internet-enabled laptops through a web browser and the design was coded in oTree (Chen, Schonger, and Chris (2016)). The experiment was separated into three sections: examples, convex choice tasks (79 tasks), and discrete choice tasks (6 tasks). Instructions were provided to each subject before the session and read aloud by the experimenter. The full set of instructions can be found in Appendix A (Feldman and Rehbeck (2022)). Subjects were paid according to one randomly chosen decision in the experiment to ensure truthful revelation of preferences. We use a physical randomization device to resolve all uncertainty. Further details on how the randomization was carried out and why we chose to pay one decision are in Appendix B. There was a \$10 show-up fee. On average subjects earned \$23.15 in total. Subjects spent an average of 15.8 seconds per convex choice task and 12.3 seconds per binary choice task.

3.1 *Convex choice tasks*

The first set of choice tasks have convex budgets generated from a numeraire lottery and an extreme lottery as in Figure 2. In more detail, an individual could choose any distribution from

$$p(\alpha) = \frac{\alpha}{100} p^{\mathrm{N}} + \frac{100 - \alpha}{100} p^{\mathrm{E}}, \tag{1}$$

where $\alpha \in \{0, \dots, 100\}$. Thus, a choice of the extreme lottery, p^{E}, corresponds to $\alpha = 0$, a choice of the numeraire lottery, p^{N}, corresponds to $\alpha = 100$, and a choice of a mixture corresponds to $\alpha \in \{1, \dots, 99\}$. The fineness of the convexification of the budget mimics that of other experiments (e.g., Choi et al. (2007)).

While convex budgets in the MM triangle are easy for economists to understand, we use a more familiar representation of probabilities to elicit choices from subjects. In particular, we build on the interface pioneered by Sopher and Mattison Narramore (2000) that uses a pie chart to display the simple lottery that corresponds to a given α mixture. An example of the pie chart and the choice interface is shown in Figure 3. For each choice task, the prizes $x_{\mathrm{L}} = \$2$, $x_{\mathrm{M}} = \$10$, and $x_{\mathrm{H}} = \$30$ are displayed at the top of the interface and color-coded to match the appropriate region on the pie chart. As the subject interacts with the slide-rule, the pie chart updates to reflect the lottery induced by mixing a

[8]These payments are the same order of magnitude as other studies of risk (see, e.g., Harless and Camerer (1994), Hey and Orme (1994), Sprenger (2015) among others). Our key design choice was to operate in the domain of only gains to not involve loss aversion.

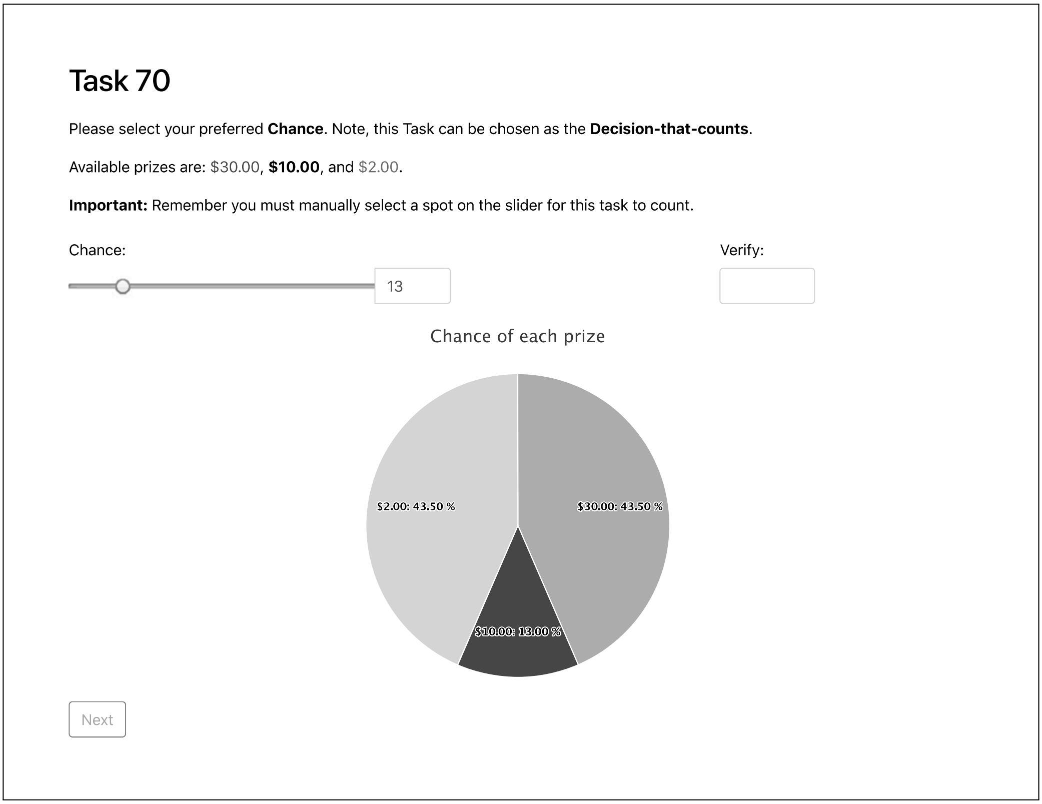


FIGURE 3. Example of a task.

numeraire and extreme lottery for a given α mixture. Our interface was chosen because it is intuitive for representing three outcome lotteries, makes the trade-off between the numeraire and extreme lottery salient, and has already been used in the experimental literature (Sopher and Mattison Narramore (2000), Karni, Salmon, and Sopher (2008)). Moreover, Simkin and Hastie (1987) find that pie charts are an efficient mechanism for emphasizing the likelihood of each outcome relative to the whole.

We now discuss some important design features of the interface. At the start of each task, the pie chart is not displayed until the subject interacts with the slide rule and the slide rule is positioned at $\alpha = 50$. Since the pie chart is not displayed, we believe framing effects from the slide rule at $\alpha = 50$ should be mitigated since a subject needs to interact with the slide rule to see the lottery. There are also advantages to keeping the slide rule initialized at a fixed position. First, we do not introduce idiosyncratic noise in the choices by using a random starting point on all tasks. We were also concerned that the random starting point would induce framing effects for the repeated discrete choice tasks since subjects could be drawn to the lottery that is closest to the random starting point. Finally, the choice of a fixed starting point allows us to examine experimenter demand effects since we can compare the behavior of subjects to behavior that simulates possible demand effects. In particular, we compare the choices of subjects in the experiment to noisy choices around the midpoint of $\alpha = 50$. One final difference of the design

TABLE 1. Reference lotteries.

	N1	N2	N3	N4	N5	N6	N7	N8
$p_{\$2}$	0.50	0	0	0	0	0	0	0
$p_{\$10}$	0.50	1	0.90	0.75	0.60	0.45	0.30	0.15
$p_{\$30}$	0	0	0.10	0.25	0.40	0.55	0.70	0.85

from Sopher and Mattison Narramore (2000) is that subjects cannot proceed to the next choice task until they interact with the slider and reaffirm their choice by typing it in the box below "Verify." This design feature helps to ensure that subjects are expressing a preference for whatever α mixture they select.

Throughout the experiment, the location of the extreme lottery, p^{E}, is fixed at $\alpha = 0$ and the location of the numeraire lottery, p^{N}, is fixed at $\alpha = 100$. We made this design choice so that individuals would be able to quickly understand the structure of the experiment. Since it is easy to discern extreme lotteries from numeraire lotteries, we believe this makes it easier for individuals to make choices consistent with no mixing behavior. Lastly, we display the lottery from the mixture $\frac{\alpha}{100} p^{\mathrm{N}} + \frac{100-\alpha}{100} p^{\mathrm{E}}$ as a simple one-stage lottery. This design feature eliminates any errors that result when individuals fail to reduce compound lotteries.

Next, we describe the budget sets that individuals face in the experiment. We used eight different numeraire lotteries to generate convex budgets. We refer to these lotteries in the text as N1–N8, where the lotteries take the values in Table 1. The reference lotteries N1–N8 are ordered by first-order stochastic dominance so that when $k > j$, the lottery $p^{\mathrm{N}k}$ first-order stochastic dominates $p^{\mathrm{N}j}$.[9]

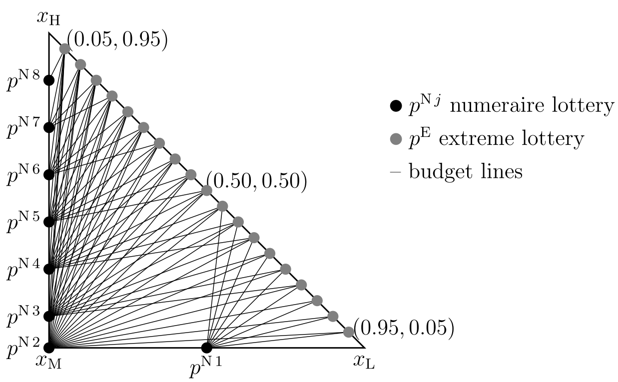


FIGURE 4. Budget lines in the Marshack–Machina triangle.

[9]We focus on numeraire lotteries on the high/middle axis to direct power where the expected value is high enough to have payoffs which are nontrivial.

We summarize all budgets for the convex choice tasks in the MM triangle in Figure 4.[10] Recall that the budget lines are designed to pivot around the numeraire lotteries to examine how mixing behavior varies with the relative price of a numeraire lottery. All subjects face the same set of 79 budgets, but the order of the budgets is randomized for each subject. The budget lines all have strictly positive slopes in the MM triangle so that no lotteries in the budget are ordered by first-order stochastic dominance.[11] This feature allows us to focus on violations resulting from mixing behavior without worrying about subjects satisfying the FOSD property within a budget. This feature is similar to other revealed preference experiments that restrict subjects from expressing satiation (Andreoni and Miller (2002), Choi et al. (2007), Andreoni and Sprenger (2011, 2012a,b)).

3.2 *Discrete choice tasks*

Here, we describe the six binary discrete choice tasks each subject faces. These tasks were completed after all seventy-nine convex choice tasks and use the same interface described above. The one difference is that α could only take the values of zero or one hundred when moved from the initial position. Thus, a subject could only choose either a numeraire lottery or an extreme lottery. In particular, we examine the binary choice tasks $D_1 = \{(0, 0.9, 0.1), (0.65, 0, 0.35)\}$ and $D_2 = \{(0.5, 0.5, 0), (0.95, 0, 0.05)\}$ where the first lottery is a numeraire lottery and the second is an extreme lottery. The numeraire lottery in D1 is N3, while the numeraire lottery in D2 is N1. Each subject first faced three repeated discrete choice tasks from D1, then faced three repeated discrete choice tasks from D2. For each budget, the subject knew before choosing that they would face each budget set three consecutive times with each repetition appearing on a different screen. This follows the experimental design in Agranov and Ortoleva (2017) to prevent the realization of several random shocks to elicit a preference for randomization.

Each discrete choice budget is linked with a convex choice task in the first part of the experiment. While the convex choice tasks are designed to elicit information about whether an individual prefers mixtures of lotteries, these choice tasks are designed to see whether mixing behavior is related to behavior from repeated discrete choices. In particular, if this behavior is related, then individuals should have repeated discrete choices that mimic the choice from the convex budget. The implied mixtures a subject could choose from three discrete choice tasks are shown in Figure 5. We later show that choices in the two domains are positively correlated. We take this as evidence that an individual's nonlinear risk preferences and preference for randomization are linked.

We note that the relative price, r, of the numeraire lottery in both environments favors the numeraire lottery since $r_{D_1} \approx 0.38$ and $r_{D_2} \approx 0.11$ both of which are less than one. If there is little mixing, then we should be better able to detect whether there is any correlation between behavior from the two elicitation methods since boundary choices

[10] We piloted sessions where individuals were restricted to choose mixtures on the interior of the simplex and found individuals made choices at the boundary. For this reason, we allowed the choices to go to the boundary. The results are qualitatively similar.

[11] There were more budgets for questions when the numeraire is of intermediate value to increase the potential violations of utility maximization in the revealed preference analysis described in Section 4.3.

are intuitively easier for individuals to make. Thus, one could interpret the relationship we find between behavior in the convex choice task and repeated discrete choice task as an upper bound on the correlation between tasks.

4. Main results

In this section, we present the main results of the experiment. We restate our two main hypotheses below.

Hypothesis 1. *Many individuals will choose mixtures, and behavior will be inconsistent with expected utility predictions.*

Hypothesis 2. *The decisions from the convex choice tasks and the repeated discrete choice tasks will be positively correlated with one another.*

We briefly summarize the main results and provide additional detail in the following subsections. We find support for Hypothesis 1 since 136/144 subjects mix at least once, and only 1/144 subject can be described by expected utility. We also find support for Hypothesis 2. In particular, we find that most of the mixture lotteries implied by the three repeated discrete choices are within one choice from the behavior in the convex choice task. We also find that behavior from the convex and repeated discrete choice tasks are positively correlated. These results suggest that an individual's preference for mixtures is related to behavior in the repeated discrete choice task.

4.1 *Mixing behavior from convex choice tasks*

For the convex choice tasks, we say a subject mixes in a convex choice task when $\alpha \in \{1, 2, \ldots, 99\}$.[12] Result 1 collects the main evidence that supports Hypothesis 1.

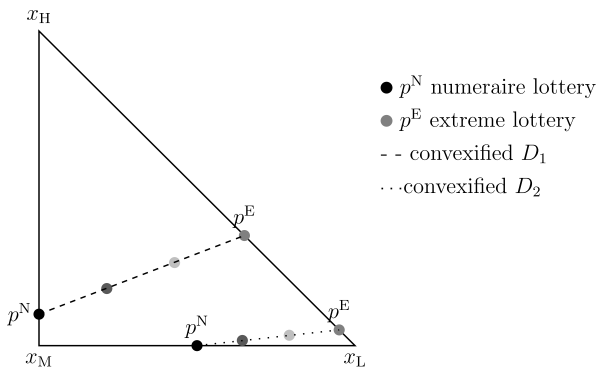


Figure 5. Discrete-choice tasks and implied mixtures.

[12]Results are robust to the threshold used to define "mixing." For example, when we say a subject mixes in a task for $\alpha \in \{0 + c, \ldots, 100 - c\}$ results are qualitatively similar for $c = 2$ or $c = 5$. See Appendix D for details.

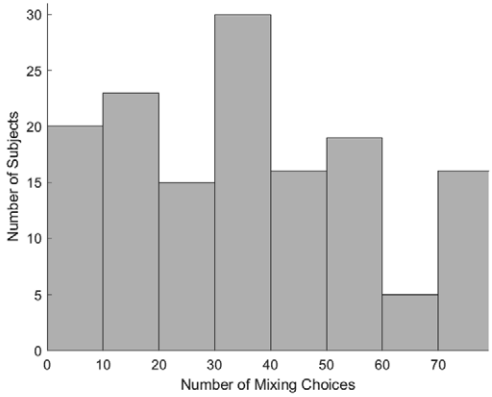


FIGURE 6. Histogram of mixing behavior.

RESULT 1. Mixing behavior is common in the convex choice tasks.

(a) 136/144 (94.4%) subjects mix at least once.

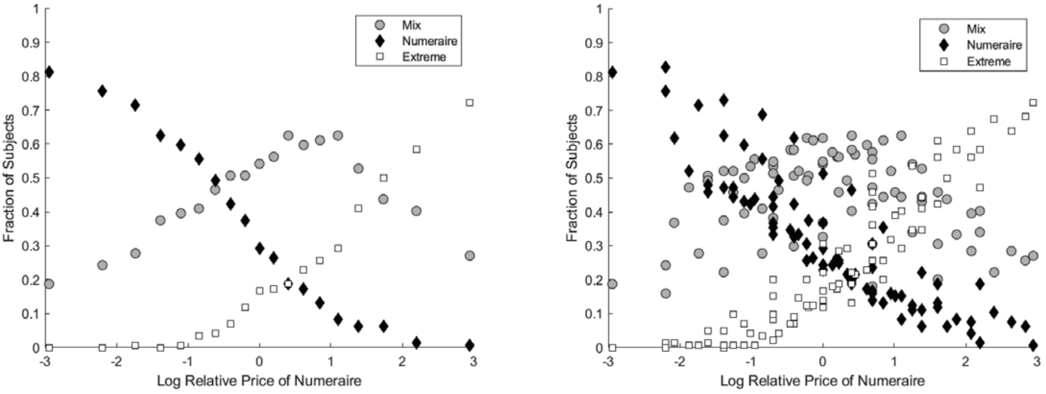
(b) 44.6% of all choices exhibit a preference for mixing.

(c) 1/144 subjects has choices that can be described by expected utility.

First, we show the histogram of how often subjects chose a mixture in Figure 6. We find that 56/144 (38.9%) subjects mix in over thirty-nine convex choice tasks (over half of all choice tasks). There are also 16/144 (11.1%) individuals choosing to mix in seventy or more convex choice tasks. We take these results as evidence against expected utility since this theory predicts (almost) no mixing. Moreover, we find 131/144 (91%) subjects chose mixtures from budgets with different slopes, which is a quick heuristic that refutes expected utility. In fact, expected utility is satisfied by only one subject.

Next, we show the percentage of choices for the numeraire lottery, extreme lottery, and mixtures for budgets generated from different numeraire lotteries in Table 2. We find that the most mixing occurs in convex choice tasks with numeraire lottery N3, which has a 90% chance of \$10 and a 10% chance of \$30 (or $(0, 0.90, 0.10)$). The least mixing occurs for the convex choice task for N8, which likely occurs from the single relative price used at these budgets. We do not have the same prices at each numeraire lottery since we were trying to maximize the ability to compare individual choices to certain benchmark behavior in a later revealed preference analysis. Thus, other than the fact that mixing occurs for all numeraire lotteries, we encourage caution in drawing conclusion from the comparative static with regards to mixing and first-order stochastic dominating numeraire lotteries.

Intuitively, the relative price of the numeraire lottery should affect the amount of mixing behavior that is observed. We examine this comparative static in Figure 7, which shows the percentage of choices for the numeraire lottery, extreme lottery, and mixtures as a function of the natural log of the relative price ($\log(r)$). Thus, when $r \approx 1$ it follows that $\log(r) \approx 0$.



(a) Numeraire lottery 2 (b) All convex tasks

FIGURE 7. Log prices versus mixing behavior.

In Figure 7(a), we focus on how choices change with log prices when focused on the numeraire lottery N2 ($10 for sure). We see that the extent of mixing behavior from N2 is similar to mixing from all convex tasks, as shown in Figure 7(b). Additional figures for each numeraire lottery are provided in Appendix E. We collect some stylized facts on how behavior responds to the relative price of the numeraire lottery.

STYLIZED FACT 1. Aggregate behavior is responsive to the relative price of the numeraire lottery. We find that:

(a) As the relative price of the numeraire lottery increases, the numeraire lottery is chosen weakly less often, and the extreme lottery is chosen weakly more often.

(b) As the relative price of the numeraire lottery increases, mixtures are chosen weakly more often until a price of approximately $r = 1.2$, after which mixtures are chosen less often.

(c) Mixing occurs even when the numeraire has high or low relative prices.

From the stylized fact, we conclude several things. First, mixing behavior is responsive to the implied price of the numeraire similar to standard consumer theory. Second, since mixtures are chosen more often at a price of $r = 1.2$ this may mean that mixing

TABLE 2. Percentage of choices at each numeraire lottery.

	N1	N2	N3	N4	N5	N6	N7	N8	Total
Mix	33%	45%	51%	50%	45%	42%	30%	18%	44%
Numeraire	51%	35%	28%	26%	26%	24%	30%	31%	31%
Extreme	16%	20%	21%	24%	29%	33%	39%	51%	24%
Obs	1296	2736	2304	1872	1440	1008	576	144	11,376

occurs most often when the trade-offs between the numeraire and extreme lottery are almost equal. Finally, we note that even at extreme prices mixing behavior occurs so that descriptive models of risk preference should allowing mixing even when the implied price difference of lotteries is large.

4.2 *Comparison of convex and discrete choice tasks*

In this section, we show that the choices from the convex choice tasks are related to those from the repeated discrete choice tasks. In particular, we find that the mixture implied by the repeated discrete choice tasks is "close" to the lottery chosen in the convex choice task. We also find that these choices are positively correlated with one another. Result 2 collects the evidence relevant for Hypothesis 2.

Result 2. Behavior from the convex and discrete choice tasks are related. In particular:

(a) For the repeated discrete choice task D_1, 93/144 (64.6%) subjects are one choice away from the mixture chosen in the associated convex choice task;

(b) For the repeated discrete choice task D_2, 122/144 (84.7%) subjects are one choice away from the mixture chosen in the associated convex choice task;

(c) The correlation between behavior in the convex choice tasks and repeated discrete choice tasks are 0.42 for D_1 and 0.33 for D_2.

We now describe the results above in more detail. First, we examine how "close" the mixture implied by the repeated discrete choice tasks is to the mixture chosen in the convex choice task. Recall in the repeated discrete choice task, the individual is only able to induce mixtures where $\alpha \in \{0, 33\frac{1}{3}, 66\frac{2}{3}, 100\}$ so there are inherent measurement discrepancies between the two tasks. To account for this discreteness, we measure closeness in "number of choices" from the convex task.

To better understand this measure of distance, let the mixture implied from the repeated discrete choice task be α_d and the mixture from the convex choice task be α_c. If an individual has the same mixture in each task, then $|\alpha_d - \alpha_c|$ equals zero. When $|\alpha_d - \alpha_c|$ is less than 16.6, this means the individual is as close as possible to the convex choice given the inherent coarseness of the repeated discrete choices. Finally, when $|\alpha_d - \alpha_c|$ is less than 33.3 (66.6), the mixture from the discrete choice is one (two) discrete choices away from the mixture chosen in the convex choice task. We present the results on the distance between behavior from the convex and repeated discrete choices in Table 3.

Table 3. Frequency of $|\alpha_d - \alpha_c|$ (total 144 subjects).

	0	≤16.6	≤33.3	≤66.6
D_1	58	93	126	139
D_2	113	122	130	139

We find a large number of subjects that *exactly* match the choice from the convex choice task with the repeated discrete choice task. This match occurs since many individuals who chose either the numeraire or extreme lottery in the convex choice task also repeatedly choose the same lottery in the repeated choice task. We also note that more subjects have behavior close to the convex choice task for the repeated discrete choice task with budget D_2. We suspect that this occurs since the relative price of the numeraire lottery is much lower for D_2 than D_1 ($r_{D_2} \approx 0.11$ vs $r_{D_1} \approx 0.38$). Thus, the numeraire lottery is more attractive in D_2, which leads to it being chosen more often deterministically in the convex task, and this behavior is matched in the repeated discrete choice tasks.

We now examine standard statistical measures of the relation between behavior in the convex and repeated discrete choice tasks. We find the Pearson linear correlation coefficient between the mixture in the convex and discrete choice task is 0.42 for D_1 and 0.33 for D_2 (both significant for the test not equal to zero for p-values < 0.0001). Thus, for both budgets, there is a positive correlation between behavior in the convex choice task and repeated discrete choice tasks.[13] We take this as evidence that supports a relationship between risk preferences and a preference for randomization. We also graph the convex mixtures against the implied discrete mixtures in the heat maps of Figure 8. Here, the 45°-line denotes an exact match between choices. This shows that there are regions where individuals mix and the convex mixture is close to the implied mixture from the repeated discrete choices. Given there is a large amount of mass at the boundaries, these results are suggestive but not conclusive evidence of a link between the preference for mixing and preference for randomization.

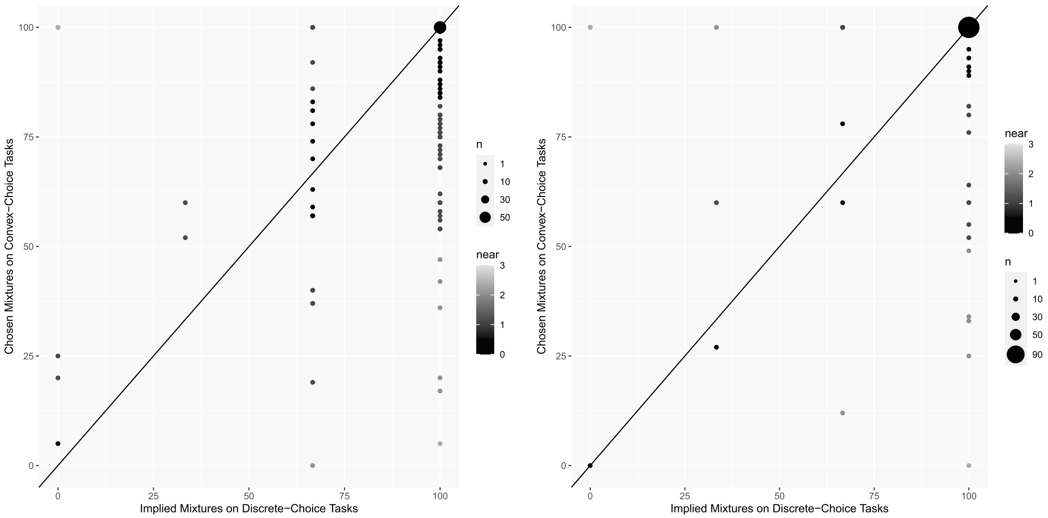
To check the sensitivity to the mass of points at $(1, 1)$, we also perform the correlation tests leaving out these observations. The remaining sample is 86 observations associated with D_1 and 32 observations associated with D_2. We find the Pearson linear correlation coefficient between the mixture in the convex and discrete choice task is 0.29 for D_1 (significant for the test not equal to zero for p-values < 0.01) and 0.56 for D_2 (significant for the test not equal to zero for p-values < 0.001). Thus, we still find that the correlation is positive and significant when leaving out the mass at $(1, 1)$.[14]

4.3 *Revealed preference results*

The previous sections show the prevalence of mixing behavior and show that behavior from convex choice tasks is closely related to the behavior in repeated discrete choice tasks. However, one concern is that the interface used to elicit preference either induces demand effects or is too noisy to reveal information about preferences. In this section, we use revealed preference methods to show that individual behavior is "closer" to a transitive preference relation that satisfies first-order stochastic dominance than benchmark behavior of demand effects or noisy

[13]The Spearman rho for the convex and discrete choice task is 0.38 for D_1 (significant for the test not equal to zero for p-values < 0.0001) and 0.2618 for D_2 (significant for the test not equal to zero for p-values < 0.005).

[14]The Spearman rho for the convex and discrete choice task is 0.21 for D_1 (significant for the test not equal to zero for p-values < 0.1) and 0.48 for D_2 (significant for the test not equal to zero for p-values < 0.1).



(a) Task D_1 (b) Task D_2

FIGURE 8. Convex mixtures (α_c) against implied discrete mixtures (α_d).

choice. Here, we assume benchmark behavior of a demand effect where an individual chooses by randomly perturbing the slide rule around $\alpha = 50$. In particular, we compare to simulated demand effects that are uniformly distributed with $\alpha \in [45, 55]$. We also examine benchmark noisy behavior drawn from a uniform distribution with $\alpha \in [0, 100]$.

Now, we describe revealed preference methods. The revealed preference approach places conditions on data sets of choices that are equivalent to the existence of a preference ordering. For example, we say the chosen lottery $p(\alpha) = \frac{\alpha}{100} p^{\mathrm{N}} + \frac{100-\alpha}{100} p^{\mathrm{E}}$ is *directly revealed preferred* to all lotteries on the budget line and "below" the budget line since it was chosen when the other lotteries were available.[15] Moreover, a chosen lottery is *strictly directly revealed preferred* to all lotteries strictly below the budget line. We consider a preference relation that respects first-order stochastic dominance. Formal details on the revealed preference relation and results are collected in Appendix C.

The revealed preference conditions prevent cycles of choices in the data set that lead to an observation being strictly revealed preferred to itself. One example of data that is ruled out using the revealed preference conditions for a preference that satisfies FOSD is given in Figure 9. Notice that if a lottery lies to the southeast of the budget set, then (assuming first-order stochastic dominance) it is less preferred to the chosen mixture.[16] Thus, the choices from Figure 9 generate a strict cycle where lottery p^1 is strictly

[15] Here, we interpret "below" the budget line to mean in the lotteries in the direction of x_L.

[16] This follows since any points to the north or west of a point to the southeast of the budget line are strictly preferred by first-order stochastic dominance. Making enough movements north or west will eventually lead to a point on the budget line. Finally, the point on the budget line is weakly less preferred to the chosen point by transitivity.

preferred to p^2 and lottery p^2 is strictly preferred to p^1, which is impossible for a well-defined preference relation.

Using the revealed preference approach, we find 14/144 (9.7%) subjects have behavior that is described by a well-defined preference relation that satisfies FOSD. We find that most behavior cannot be exactly described by a preference relation. To account for this issue, we use the Houtman–Maks index (HMI) developed in Houtman and Maks (1985) to find the largest number of choices that can be described by a preference relation. Here, a higher HMI indicates that behavior is closer to a well-defined preference relation conditional on the budget sets being the same. To better understand the HMI, note that the data in Figure 9 has an HMI of one since either observation can be removed, and the resulting data set can be described by a preference relation.

We display a relative probability distribution of the HMI for all subjects in Figure 10 relative to some benchmark behavior following ideas from Bronars (1987). Recall, we model behavior for benchmark demand effects as simulated individuals choosing α according to the uniform distribution on $[45, 55]$. Similarly, we model benchmark noisy behavior as simulated individuals choosing α according to the uniform distribution on $[0, 100]$. For each case, we generate 5000 simulated individuals. We plot the distribution of simulated HMI along with the distribution of subject HMI in Figure 10 for each benchmark.

First, we compare subject behavior to benchmark demand effects in Figure 10(a). We see that subjects have many more choices that are consistent with a preference relation that respects FOSD. In particular, we find that 144/144 (100%) subjects are "closer" to a well-defined preference than 95% of simulated benchmark demand effect behavior according to the HMI. Moreover, we reject the null that the distribution of subject HMI and benchmark demand effects are the same according to the Wilcoxon rank sum test ($p < 0.0001$) and according to the Kolmogorov–Smirnov test ($p < 0.0001$). We take this as evidence that demand effects are not substantive.

Next, we compare subject behavior to benchmark noise in Figure 10(b). For this case, we find that 141/144 (97.9%) subjects are "closer" to a well-defined preference than 95%

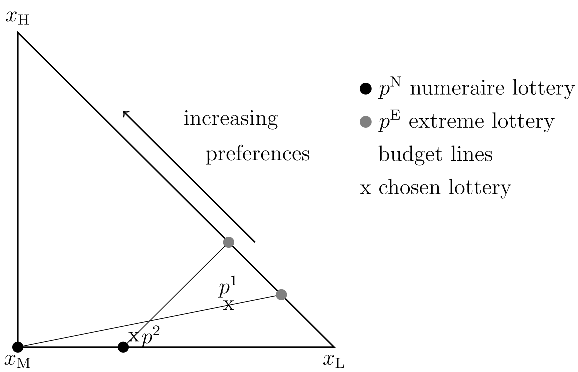


Figure 9. Example of choices that cannot be described by preferences.

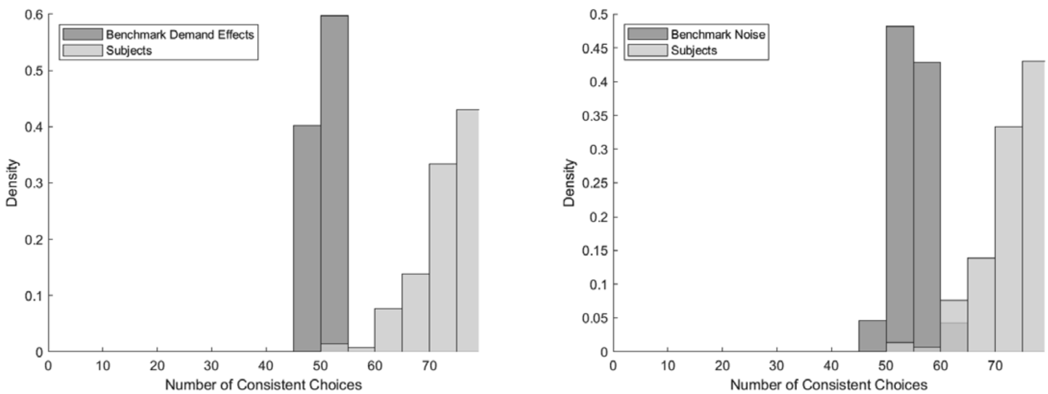
of simulated benchmark noisy behavior according to the HMI. Moreover, we reject the null that the distribution of subject HMI and benchmark noisy behavior are the same according to the Wilcoxon rank sum test ($p < 0.0001$) and according to the Kolmogorov–Smirnov test ($p < 0.0001$). We take this as evidence that the individuals are revealing information about their preferences.[17] Thus, this interface may be useful for eliciting behavior in other settings.

5. Additional results

Now that we have presented the main results, we explore two other themes: aggregate demand for numeraire lotteries and individual preference heterogeneity. The first section shows that when there is a numeraire lottery whose price varies, aggregate behavior can be well fit by a linear-log demand function. In particular, a regression of demand (α) on log relative prices ($\log(r)$) provides a good fit of aggregate behavior. The second section briefly describes some "types" of individual behavior that are present in the data. In particular, we find rich heterogeneity of individual behavior even in this small choice domain with three monetary prizes.

5.1 *Demand analysis*

As expressed in Machina (1982, 1985), the behavioral implications of nonexpected utility theory share many similarities with standard consumer theory. We examine this claim for aggregate demand of the numeraire lottery against the relative price. Recall that the demand for the numeraire lottery can be represented by the amount of α chosen on the slide rule in the choice interface. In Figure 11, we find that the average demand for the numeraire lottery approximately follows a downward sloping linear-log function when



(a) Demand effect benchmark (b) Noisy benchmark

Figure 10. HMI probability density of subjects and benchmark behavior.

[17]In the spirit of Andreoni, Feldman, and Sprenger (2017), we also compare subject behavior to bootstrapped samples and find similar qualitative results. See Appendix C.1 for details.

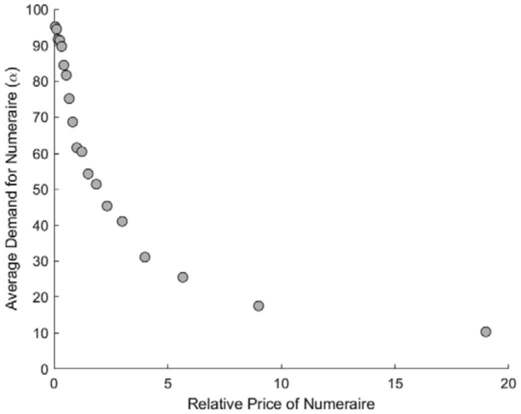


FIGURE 11. Demand behavior for numeraire against relative prices for N2.

analyzing choices for reference lottery N2 ($10 for certain).[18] This result is robust for N1–N7 as shown in Appendix F with regression results and additional figures. Thus, we conclude that aggregate behavior of risk can be described by relatively simple demand systems that satisfy the familiar property of downward sloping demand.

5.2 *Behavior patterns and individual heterogeneity*

While aggregate behavior is described by simple demand systems, there is heterogeneity in individual preferences. In Figure 12, we show the choices of six different individuals that represent different behavior in the convex choice tasks from numeraire lotteries N1, N2, and N5. The behavior of all individuals for all budgets in this graphical form is presented in Appendix G. The types of behavior we show in Figure 12 are representative of most behavior in the data. While we believe performing a formal preference classification exercise is interesting, it would shift the focus of the paper. For example, one could use methods that have been previously used in the work of Fudenberg and Liang (2019) and Peysakhovich and Naecker (2017).

We briefly describe the different types of behavior from Figure 12. First, Figure 12(a) represents the only individual who can be represented by the expected utility model. As expected, all choices lie on the extreme points of the MM triangle. The behavior in Figure 12(b) and (c) can be seen as thresholding behavior around an iso-p_{M} curve and local thresholding around iso-p_{L} curves, respectively. We call this thresholding behavior because behavior is "as if" an individual chooses some threshold amount of p_{M} or p_{L} and chooses the mixture that gives the threshold amount. Figure 12(d) represents a combination of multiple types of behavior. Figure 12(e) is labeled price responsive since the choices gradually respond to prices. Finally, Figure 12(f) has no discernible pattern.

Combining the large amount of individual preference heterogeneity with the downward sloping aggregate demand, we find support for the hypothesis of Becker (1962) that

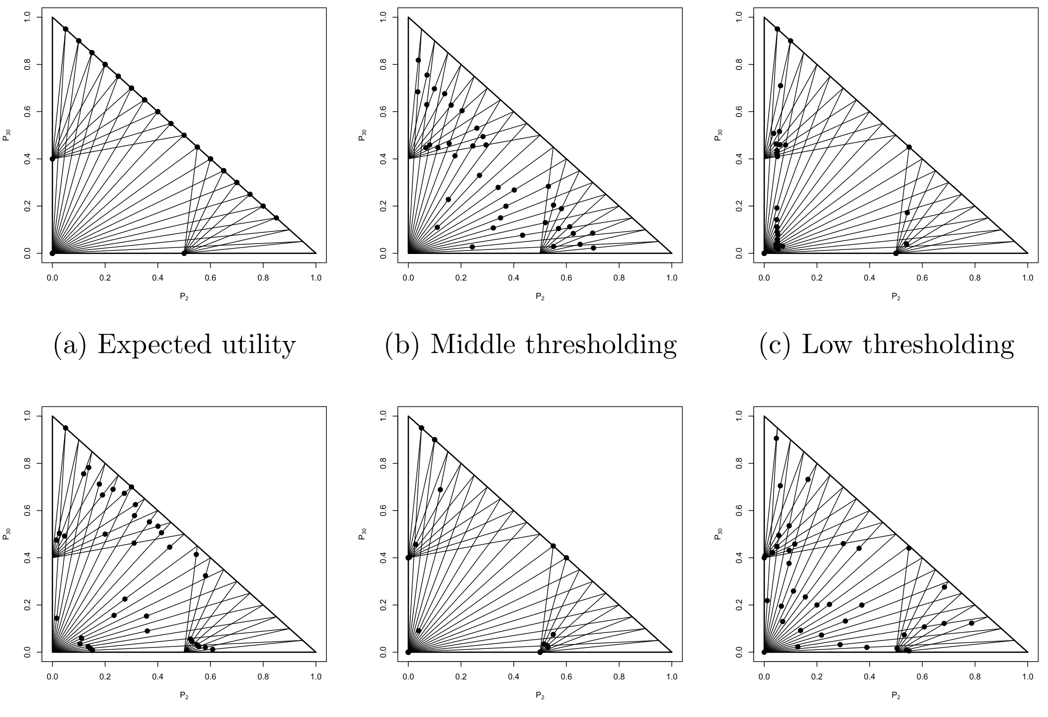
[18]This differs from the numeraire lottery being chosen less often in the presence of "income effects" since here a numeraire lottery is fixed.

aggregate demand is downward sloping even in the presence of preference heterogeneity in the domain of risk.

6. Related literature

This section discusses the relation of this paper to the literature on stochastic choice, risk preferences, and revealed preference. First, there are several papers that have examined stochastic choice from an experimental perspective. The work of Mosteller and Nogee (1951) was the first to find that individuals randomize when facing repeated discrete choice problems. Sopher and Mattison Narramore (2000) examine whether individuals choose mixtures of lotteries using a similar interface to this paper. Moreover, Sopher and Mattison Narramore (2000) examined repeated convex choice tasks with the same budget and found similar choices in the repetitions. They took this as evidence in support of a preference for randomization. We complement these papers by comparing the convex choice tasks to the repeated discrete choice tasks. We also have many more convex choice tasks and check revealed preference conditions to show that the interface from Sopher and Mattison Narramore (2000) is eliciting preference information.

Modern work on stochastic choice experiments includes Burghart (2019), Agranov and Ortoleva (2017), Dwenger, Kübler, and Weizsäcker (2018), Agranov, Healy, and



(a) Expected utility (b) Middle thresholding (c) Low thresholding

(d) Combination (e) Price responsive (f) Random

Figure 12. Example individual choice behavior from convex tasks N1, N2, and N5.

Nielsen (2020), and Agranov and Ortoleva (2020). The work of Burghart (2019) focuses on examining choices from convex discrete budgets and examines violations of the independence condition. The work of Agranov and Ortoleva (2017) and Dwenger, Kübler, and Weizsäcker (2018) both find a preference for purchasing external randomization devices. This evidence supports the idea that individuals have a preference for randomization. In particular, Agranov and Ortoleva (2017) find that a preference for the randomization device positively correlates with mixing behavior in repeated binary-choice tasks. Agranov, Healy, and Nielsen (2020) show a positive correlation between mixing in strategic and nonstrategic environments.[19] Finally, Agranov and Ortoleva (2020) show that individuals prefer to randomize for a range of monetary values of various lotteries. In all these papers, as well as this paper, there is evidence that individuals prefer mixtures either in convex choice tasks or repeated discrete choice tasks. This paper is unique in that it links behavior across the two domains through a within-individual design.

Models of a preference for randomization have also garnered theoretical interest. The interpretation of stochastic choice generated by nonexpected utility preferences was popularized by Machina (1985). Some recent papers that examine conditions to characterize different models that come from a preference for randomization include Fudenberg, Iijima, and Strzalecki (2015), Cerreia-Vioglio et al. (2019), and Allen and Rehbeck (2019).[20] The evidence here provides some support that a nonlinear risk preference may drive a preference for randomization in repeated discrete choices tasks. Moreover, there are modern demand systems developed in Fosgerau, Monardo, and De Palma that may be able to match behavior of nonlinear risk preferences when there are more than three prizes.

This experiment is also related to risk preferences more generally, but we do not further explore this relationship in depth here. For example, we find evidence that mixing behavior is common in convex choice tasks, which supports nonexpected utility theory (Machina (1982)) that allows mixing. More specifically, mixing behavior supports the hypothesis that there are regions of risk preferences that are quasi-concave. We also note that individual behavior suggests that risk preferences may only be locally well behaved. For example, it appears that the "low thresholding" individual in Figure 12(c) may have different thresholds that depend on the location of the budget set. This suggests additional study is needed to understand whether global risk preferences or local risk preferences are more appropriate for applications.

The evidence we find in this paper against expected utility also illuminates why there might be differences in estimates of risk preferences using different elicitation methods. The common method to measure risk preferences following Holt and Laury (2002a) uses choice lists with discrete choice. However, both Friedman et al. (2018) and Andreoni and

[19]This paper also examines mixing between lotteries when first-order stochastic dominance can be violated and finds correlation across strategic and nonstrategic choices. We do not allow subjects to choose first-order stochastic dominated lotteries.

[20]This contrasts from other interpretations of stochastic choices from random utility models in McFadden (1974), McFadden and Richter (1990), McFadden (2005), and Gul and Pesendorfer (2006).

Kuhn (2018) find that the different methods to elicit risk preferences yield different parameter estimates. Our research suggests part of the discrepancy may be that individuals prefer mixtures, which may induce randomization. This agrees with research by Chew et al. (2019) that shows switching behavior in repeated discrete choice is correlated with multiple switching behavior in choice lists.

Moreover, since we find risk preferences may only be locally well behaved, these discrepancies may also be caused by using a global model of risk preferences. While there are methods that use convex budgets to elicit risk preferences following Gneezy and Potters (1997) and Choi et al. (2007), these are currently less popular since the methods are slightly more complicated. The research in this paper suggests that this complication may be worthwhile due to nonlinearities in risk preferences. A promising approach that is close to existing practice is to use convex budgets with the choice list elicitation of Holt and Laury (2002b). Choices from these convex budgets also correlate with discrete choice behavior and are used in Cettolin and Riedl (2019) and Agranov and Ortoleva (2020).

Finally, this paper is related to experiments that take a revealed preference approach. General methods and theory of revealed preference are given a textbook treatment in Chambers and Echenique (2016). There are now numerous papers that use revealed preference methods, including some mentioned earlier. Other experimental papers that take a revealed preference approach include, Kagel et al. (1995) who examine the rationality of animals, Andreoni and Miller (2002) who examine altruistic behavior, Andreoni, Castillo, and Petrie (2003) who examine bargaining behavior, Choi et al. (2007) who examine Arrow securities, and Andreoni and Sprenger (2012a) who examine time preferences using convex budgets. Relative to this literature, our experimental methods are closest to Andreoni and Miller (2002), Andreoni, Castillo, and Petrie (2003), and Andreoni and Sprenger (2012a) since we use a fixed set of budgets for all subjects.

7. Concluding remarks

We designed an experiment to examine the prevalence of mixing behavior and examine whether behavior from convex choice tasks is related to behavior from repeated discrete choice tasks. Our exploratory study shows that mixing behavior is common and that behavior from repeated discrete choice tasks mimics behavior from convex choice tasks. We also show that the interface provided in this paper can elicit information on nonlinear risk preferences. While these results are suggestive that risk preferences may drive a preference for randomization in repeated discrete choice tasks, we believe more work is needed to see the extent of this relation. For example, interesting follow-up research could examine additional comparisons between convex budgets and repeated discrete choice tasks at different prices or examine how increasing the number of repetitions affects the closeness to the convex choice task. We also hope this motivates additional theoretical research following the work of Lu and Saito (2019) that looks at how repeated discrete choices are related to risk preferences.

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Co-editor Christopher Taber handled this manuscript.

Manuscript received 30 July, 2020; final version accepted 3 September, 2021; available online 2 December, 2021.