Quantitative Economics 14 (2023), 419–470 1759-7331/20230419

Risk aversion in share auctions: Estimating import rents from TRQs in Switzerland

Samuel Häfner
School of Economics and Political Sciences, University of St. Gallen

This paper analyzes risk aversion in discriminatory share auctions. I generalize the k-step share auction model of Kastl (2011, 2012) and establish that marginal profits are set-identified for any given coefficient of constant absolute risk aversion. I also derive necessary conditions for best-response behavior, which allows determining risk preferences from bidding data. Further, I show how the bidders' optimality conditions allow computing bounds on the marginal profits that are tighter than those currently available. I use my results to estimate import rents from Swiss tariff-rate quotas on high-quality beef. Rents are overestimated when ignoring risk aversion, and rent extraction is underestimated. Small bidders (small, privately owned butcheries) are more risk averse than large bidders (general retailers). Best response violations are few and uniform across bidder sizes.

Keywords. Discriminatory share auctions, estimation, risk aversion, best-response violations, tariff-rate quota, import rent.

JEL classification. C57, D44, F14.

1. Introduction

Risk neutrality is often a convenient assumption when estimating auctions. Yet, in many settings, it is unclear whether it is also accurate. In this paper, I analyze this question in the context of share auctions. I generalize the current state-of-the-art model (Kastl (2011, 2012)) to risk aversion, develop a method to determine risk preferences from observed bids, and show with real-life data how properly accounting for risk aversion affects rent estimation.

Share auctions are widely used mechanisms to sell (almost) perfectly divisible goods such as large quantities of treasury bills or electricity. A bid typically consists of a finite number of price-quantity pairs, indicating a bidder's marginal willingness to pay. The central insight from the literature is that, under the assumption of risk neutrality, the

Samuel Häfner: samuel.haefner@unisg.ch
I am indebted to Yvan Lengwiler, Georg Nöldeke, and Marek Pycia for many helpful comments and suggestions. I also benefited from comments by Stefan Bühler, Allan Collard-Wexler, Maarten Janssen, Daniel Marszalec, James Roberts, Orly Sade, and Kyle Woodwards. I received several excellent referee reports but the effort by one particular referee was outstanding and greatly helped to improve the paper. Support from a *Forschungskredit* by the University of Zurich is gratefully acknowledged. I thank the Swiss Federal Office for Agriculture for providing me with the data. Calculations were performed at sciCORE (https://scicore.unibas.ch/) scientific computing core facility at the University of Basel. Replication files including a data set are available at https://github.com/SamuelHaefner/RiskAversionInShareAuctions. A Supplementary Appendix is available in the Online Supplementary Material (Häfner (2023)).

 https://doi.org/10.3982/QE1907

observed bids allow point estimating bidders' marginal values at the submitted quantity points (see Kastl (2017, 2020) for excellent overviews). Risk neutrality is a natural assumption in the context of treasury bill auctions or electricity markets (e.g., Nyborg, Rydqvist, and Sundaresan (2002)). It is potentially problematic in other contexts, though. In particular, bidders that are small relative to the goods on sale are often suspected—and found—to be risk averse (e.g., Li, Lu, and Zhao (2015), Kong (2020)).[1]

In this paper, I consider data from tariff-rate quota auctions run by the Swiss Federal Office for Agriculture. These auctions allocate rights to import high-quality beef at a (low) in-quota tariff. The number of bidders in the auctions is relatively high (72 on average), and most of the bidders only bid for a small fraction of the overall quota (cf. Section 2 for more details). These bidders act on behalf of small- or medium-sized butcheries that serve distinct customer bases in restricted geographical areas and make up a substantial part of the Swiss retail meat market. The bidders are often the owners of the butcheries, lack financial flexibility, and rely on the highly lucrative high-quality beef imports. We might thus worry that the bidders are risk averse, which would render the current estimation methods inapplicable.[2] Having an accurate picture of the import rents and how well the auctions capture these rents is of quite some policy relevance.

The analysis starts with the theoretical model in Section 3. There is a fixed quantity of a perfectly divisible good on auction. Bids correspond to (fixed-length) tuples of price-quantity pairs, and payment is discriminatory. Each bidder acts on behalf of a firm, which obtains decreasing marginal profit from the good. The firm's marginal profit function is private information of the respective bidder. The bidders evaluate the value of the received quantity in the auction, net of the resulting payment, with a commonly known, possibly nonlinear utility function. I allow for arbitrary increasing utility functions in the general model, for which I establish equilibrium existence (Proposition 1) and derive the equilibrium characterization (Proposition 2). I restrict attention to the class of CARA utility functions in all results relevant to the empirical analysis.

I present three main theoretical results in Section 4. First, I show that for any given CARA parameter, the optimality conditions for the quantity points together with the assumption of decreasing marginal profits allow us to set identify a firm's marginal profit function at the submitted quantity points in a straightforward way (Lemma 1). The central insight is that under CARA preferences, the optimal choice of a quantity point is independent of the marginal profits on inframarginal quantities. This gives a recursive formulation, starting with the highest quantity point, of upper and lower bounds on the marginal profits that can rationalize the observed bids (Proposition 3). Under risk neutrality, the bounds correspond to the bounds in Kastl (2012).

Second, I formulate necessary conditions for best-response behavior. I derive the conditions from the bidders' optimality conditions. The first set of conditions stems

[1]More recent papers on risk aversion in (procurement) auctions include Bolotnyy and Vasserman (2019), Luo and Takahashi (2019), and Aryal et al. (2022).

[2]Indeed, the butcheries consider fluctuating import rights a major issue. In a survey by the Federal Office for Agriculture from 2010, the Swiss association of meat producers complained that, especially for smaller firms, the allocation through auctions was just too uncertain: "For the individual firm, the allocation resembles a lottery [...], impeding planning security also towards the customers" (my translation). The report (in German) is available under http://www.news.admin.ch/NSBSubscriber/message/attachments/20686.pdf.

from the optimality of the quantity points, requiring that the submitted quantity points are consistent with the existence of a decreasing marginal profit function. The second set of conditions requires that, among the decreasing marginal profit functions that are consistent with the quantity points, there are functions that are also consistent with the optimality conditions of the price points. The conditions are inequalities and stated in Proposition 4. The inequalities depend on a bidder's risk parameter and are the basis for my empirical strategy to determine risk preferences.

Third, I address a caveat from Proposition 3: The marginal profits at the quantity points are merely set-identified under risk aversion. Hence, the corresponding upper and lower bounds on the possible marginal profit functions are unlikely to be very tight. Here, the optimality conditions for the price points again turn out to be helpful, allowing me to formulate tighter upper and lower bounds as the least fixed point of a mapping that we can estimate from the data (Proposition 5). In particular, we can obtain the least fixed point through a fixed-point iteration that uses the bounds derived from Proposition 3 as initial conditions.

The empirical part (Section 5) proceeds in three steps. First, I estimate the equilibrium distribution of opponent demand (Section 5.1). To do so, I follow the recent literature and use a resampling procedure that draws from observed bids (Hortaçsu and McAdams (2010), Kastl (2011)). To account for potential heterogeneity among the bidders, I follow Kastl (2011) and divide the bidders into three groups $g = 1, 2, 3$ based on average quantity bids, where I assume symmetry in the bid distribution among the members of a group.

In a second step, I determine risk preferences (Section 5.2). To do so, I plug the opponent demand distribution estimates into the necessary inequality conditions for best-response behavior. Assuming that all bidders are risk-neutral, I find that more than 50% of the estimated inequalities fail to hold. To account for risk aversion, I assume that all bidders in a bidder group $g = 1, 2, 3$ have an identical CARA parameter ρ_g. I find that the share of the best-response violations is U-shaped in ρ for all three groups. At the values that minimize the percentage of best-response violations in the respective group, a mere 21% of all estimated inequalities fail to hold. The average number of price-quantity pairs that a bidder submits is 4.4, so there is a best-response violation in less than one price-quantity pair per bidder.

Of the submitted bids, only a tiny fraction violates the necessary conditions from quantity point optimality. I take this as a justification for the assumption of decreasing marginal profits and conclude that most best-response violations are due to suboptimally chosen price points. Overall, the number of best response violations is comparable to what Chapman, McAdams, and Paarsch (2006) find for Canadian term deposit auctions under risk neutrality. Chapman, McAdams, and Paarsch (2006) consider a multiunit auction with a discrete bid space. Their setting yields optimality conditions that differ from the ones derived in this paper, which require both optimal price and quantity bids.

The values of the CARA parameter that minimizes the share of best-response violations in a given group, ρ_g^*, are inversely related to the average quantity bidders bid for in a group. In other words, small bidders are more risk averse than large bidders. Such a finding is intuitive given the structure of the Swiss meat market. The large bidders correspond to large general retail chains that have the necessary means to mitigate some of the auction-specific risks. The small bidders are smaller local butcheries that do not have these means.

In a third step, I estimate upper and lower bounds on the firms' average profits per kg of imported beef (Section 5.3). My results show that assuming risk aversion indeed makes the bounds from the optimality conditions for the quantity points less precise. Nevertheless, we can fully compensate for this loss by considering the tighter bounds that also use the optimality conditions for the price points. Moreover, appropriately accounting for risk aversion considerably affects the estimates. For example, the upper bound on average profits per kg after auction payments is estimated at CHF 7.93 under risk neutrality but only at CHF 3.83 under risk aversion. Also, the lower bound decreases from CHF 1.82 to CHF 0.80. This corresponds to drops of 52% and 55%, respectively.

The sign of the bias is intuitive: Because risk-averse bidders bid closer to their marginal profit than risk-neutral bidders do, not accounting for risk aversion yields upward-biased estimates. In particular, the results imply that import rents are much lower, and the auctions perform much better in extracting rents than they would appear when assuming risk neutrality. As we will see, appropriately accounting for risk preferences yields magnitudes of rent extraction that are comparable to what the literature has found for treasury bill auctions (cf. Kastl (2011, 2017)).

The empirical literature on share auctions was pioneered by Hortaçsu and McAdams (2010) and Kastl (2011, 2012) and has been fast-growing since then. Recent contributions include Hortaçsu, Kastl, and Zhang (2018) who assess the market power of primary dealers in the US treasury market, and Elsinger, Schmidt-Dengler, and Zulehner (2019) who analyze the role of competition in Austrian treasury auctions (cf. Kastl (2017, 2020) for overviews and further references). Hortaçsu and McAdams (2010) analyze a share auction without restricting bid schedules to step functions, akin to the original share auction model proposed by Wilson (1979). They apply their model to Turkish treasury auctions.[3] All these papers consider the risk-neutral case.

There is also related theoretical literature on share auctions comparing uniform and discriminatory pricing (Back and Zender (1993)), considering endogenous supply (Back and Zender (2001)), and analyzing risk aversion with mean-variance preferences (Wang and Zender (2002)). In contrast to the model I study, these papers assume common values and constant marginal profits. More recent contributions include Anderson, Holmberg, and Philpott (2013) and Pycia and Woodward (2020), both analyzing discriminatory payment in models with complete information about preferences and stochastic supply.

Further, my study is related to the seminal analysis of the Texas electricity spot market in Hortaçsu and Puller (2008).[4] Restricting attention to linear bid schedules, Hortaçsu and Puller (2008) find that large bidders comply with behavior predicted by Nash equilibrium while smaller firms submit demand schedules that are too steep. In a recent

[3] See Marszalec (2017) for a discussion and an application to Polish treasury auction.

[4] Other empirical studies of electricity auctions include Wolak (2000, 2003) and Fabra and Reguant (2014).

follow-up study, Hortaçsu et al. (2019) show that this can be rationalized by a cognitive hierarchy model in which small bidders are less sophisticated than large bidders. My analysis suggests that risk aversion gives rise to a similar pattern in the TRQ auctions: risk aversion goes along with less bid-shading, and smaller bidders are substantially more risk averse.

Last, my paper contributes to the literature on estimating risk preferences in the field (cf. Barseghyan et al. (2018) for a recent overview). Due to the complexities of the share auction, I restrict myself to a particular class of risk aversion. For first-price auctions, the identification of more general risk preferences is explored in the seminal papers of Guerre, Perrigne, and Vuong (2009) and Campo et al. (2011). The former uses fluctuating participation across auctions for identification, while the latter uses heterogeneity in objects across auctions.[5] In contrast, I frame the problem of determining risk preferences as a model selection problem, where the number of best response violations serves as a metric for model fit. While certainly an avenue for future research, a formal test for a given degree of risk aversion in the share auction is out of the scope of this paper.

2. The Swiss TRQ auctions for high-quality beef

Worldwide, many imports of agricultural products are managed by so-called tariff-rate quotas. A tariff-rate quota (TRQ) is a two-tiered tariff regime that allows imports up to a given quota at a low in-quota tariff and puts a high over-quota tariff on imports outside the quota. In 2016, the WTO counted a total of 1128 tariff-rate quotas in more than 40 countries (cf. WTO Committee on Agriculture (2018)).

Tariff-rate quotas naturally entail rents for the importers that can import at the in-quota tariff (e.g., Boughner, de Gorter, and Sheldon (2000)). Many countries have thus adopted some form of a sale mechanism for the in-quota import rights to distribute these import rents back to the general public. Given a specific tariff-rate quota and its allocation mechanism, two natural and related questions are the following. What is the magnitude of the import rents that the TRQ creates? And how well does the sale mechanism perform in capturing these rents? Accurately answering these questions is a prerequisite for any informed policy debate about the TRQ and its allocation mechanism. Yet, this requires profit estimates at a very disaggregated level, which are typically not readily available. If the sale mechanism is an auction—which is the case in about 5% of all TRQs worldwide (WTO Committee on Agriculture (2018))—we may obtain profit estimates from the bidders' behavior.

In this paper, I obtain rent estimates from TRQs on meat imports to Switzerland. The Federal Office for Agriculture (FOA) runs the TRQ auctions. Bidding is open to all residents of Switzerland, but prior registration is required. Depending on the meat category, the quota periods last between 1 to 3 months, and the quotas vary from period to period. The FOA sets and announces the quotas a couple of days before the respective periods. The FOA is required to set the quotas in a manner that accommodates seasonally fluctuating domestic demand without affecting domestic prices. After every auction, the FOA

[5] Lu and Perrigne (2008) use the fact that both ascending and sealed-bid auctions are used in timber sales to identify risk preferences.

publishes an online report containing the names of the firms in the auction, their allocated quantities, the market-clearing price, and the average price per kg that the successful bidders paid. Allocated import rights must be executed during the quota period and expire after that.[6]

The TRQs for high-quality beef My data set covers 39 auctions from 01/2008 to 12/2010 for the category of high-quality beef. High-quality beef subsumes beef cuts and carcasses of superior quality, such as tenderloin and sirloin steaks.[7] Imported beef—most of which originates from Germany—amounts to roughly 20% of total domestic beef consumption (Loi, Esposti, and Gentile (2016)). Switzerland is one of the most expensive countries for beef worldwide (cf. the 2017 Caterwings Meat-Price Index). The average retail price for sirloin steak in Switzerland between 2008 and 2010 was 60.67 CHF/kg (not discriminating between imported and domestically produced meat). The average US retail price for USDA Choice sirloin steak was 5.58 USD/lb. USD and CHF were roughly at parity during that period.[8]

The quota periods last roughly 30 days, and the quotas range from 67.5 tons to 630 tons, with a mean of 311.5 tons (cf. Table 1). The subcategories of high-quality beef are subject to different in-quota and over-quota tariffs, which remained unchanged over the period I consider. The highest spread between an in-quota tariff and the respective over-quota tariff is CHF 20.53 (CHF 1.59 vs. CHF 22.12).[9] The over-quota tariffs are

Table 1. Summary of the high-quality beef TRQ auction characteristics.

	Mean	Min	Pctl(25)	Pctl(75)	Max
Quotas, t	311.5	67.5	213.75	360	630
Bid-to-Cover ratio	2.96	1.80	2.41	3.37	5.43
Number of bidders	71.6	58	68	76	82
Market clearing price, CHF	8.22	3.21	6.125	9.52	14.41
Revenue, CHF mio.	2.76	0.67	1.71	3.46	7.98
Average total quantity bid per bidder, kg	8592	30	851	7785	143,072
Share of successful bidders	0.64	0.04	0.44	0.83	0.97
Success rate per bidder	0.60	0	0.47	0.84	1
Success rate per bidder*	0.71	0.28	0.58	0.85	1
Share of allocated quantity	0.0216	0.0001	0.0022	0.0222	0.4741

[6]Import rights are transferrable. In the meat category that I look at, high-quality beef, most imports are managed by one large firm (that does not bid itself). Unfortunately, little is known about after-auction interactions (Loi, Esposti, and Gentile (2016)).

[7]For further information, complare the memo "Definition of High-Quality-Beef," available at https://www.blw.admin.ch.

[8]The prices for Switzerland that I use are available from https://www.blw.admin.ch/blw/de/home/markt/marktbeobachtung/fleisch.html. The prices for the US are available from the Bureau of Labor Statistics, https://data.bls.gov/cgi-bin/srgate, using the series ID APU0000703613.

[9]Specifically, the beef subsumed under the category of high-quality beef is associated with four different tariff-numbers: 0201.2091/018 and 019 (fresh or chilled carcasses and half-carcasses with bone in), 0201.3091/018 and 019 (boneless), 0202.2091/018 and 019 (frozen carcasses and half-carcasses with bone in), 0202.3091/018 and 019 (boneless), where 018 stands for the in-quota tariff and 019 for the over-quota

prohibitive; that is, high-quality beef imports generally do not exceed the quota (Loi, Esposti, and Gentile (2016)).

The auctions The bidders can submit at most 5 pairs of price-quantity points. The submitted prices are in CHF, and the quantities are in kg. The average number of price-quantity pairs per bid is 4.42. There are a total number of 123 registered bidders. The number of active bidders in an auction varies between 58 and 82, with a mean of 72 bidders. The average bid-to-cover ratio is 2.96. With five exceptions, the bid-to-cover ratio is higher than 2; in one auction, it is more than 5.[10] The market-clearing prices range from 3.21 CHF/kg to 14.41 CHF/kg. Revenues per auction range from CHF 0.7 million to CHF 8 million, with a mean revenue per auction of CHF 2.8 million. The cumulated revenue over the 39 auctions amounts to CHF 107 million.

As mentioned in the Introduction, most bidders are relatively small compared to the total market size. This can be seen from Table 1, considering the average total quantity bid per bidder: for 75% of all bidders, the average quantity for which that bidder submits a positive price (conditional on being active) is below 7.8t, and thus corresponds to less than 3% of the average quota, which is 311.5t. These average quantity bids are heavily skewed to the right, with the mean of 8.5t being higher than the 75% quantile. A similar observation holds for the allocated share of the total quota, which ranges from a minimum of 0.0001 for a single bidder to a maximum of 0.47, with an average roughly equal to the 75% quantile. This heterogeneity, both in the bids and the outcomes, reflects the structure of the Swiss retail market for meat, consisting of a few large retailers and many smaller to medium-sized butcheries.

Despite this heterogeneity in the bids and the outcomes, it seems that the chances of obtaining a nonzero quantity are intact for everyone. The average share of successful bidders is 64% per auction. The minimum of 4% successful bidders is an outlier as the reported first quartile of 44% suggests. Moreover, the mean individual success rate for an active bidder (i.e., the ratio of auctions in which the bidder was successful to the number of auctions in which the bidder was active) amounts to 60%. For the 44 bidders who were active in at least 35 out of the 39 auctions, the mean success rate is even higher, at 71%; and the minimum success rate is 28% (marked with an asterisk * in Table 1).

3. Theoretical model

3.1 *Types, strategies, and equilibrium*

There are $n \geq 2$ bidders with corresponding firms $i = 1, \ldots, n$. Bidder i bids on behalf of firm i, which has decreasing and Lipschitz-continuous marginal profit $v_i(q)$ from importing a quantity $q \geq 0$ of meat. I assume that bidder i privately knows v_i and I call v_i the type of bidder i.

tariff. These tariffs are, respectively, 1.59 and 13.68 CHF/kg, 1.59 and 22.12 CHF/kg, 1.59 and 12.33 CHF/kg, and 1.09 and 20.57 CHF/kg (cf. https://xtares.admin.ch for the tariffs).

[10] I compute the bid-to-cover ratio by dividing the aggregate amount of quantities for which bidders submitted a positive price by the total quota on sale.

Remark 1 (Marginal profits). Although I take marginal profits v_i as the primitives of the model, they can be derived from a stylized partial equilibrium model of the market for imported high-quality beef as follows. There are $n \geq 2$ firms that compete in quantities. Each firm i has a private, weakly convex cost $C_i(q)$ of acquiring, importing, and processing a quantity $q \geq 0$ of beef. The market demand for imported meat is given by some inverse demand function $P(q)$. Imports are regulated with a TRQ. Motivated by the discussion in the previous section, we may assume that the over-quota tariff is prohibitively high so that the total quantity in the market is effectively restricted to the quota, $Q > 0$. If the in-quota-tariff is $\tau > 0$, then the (decreasing) marginal profit of a firm i when being allowed to import $q \leq Q$ is

$$v_i(q) = \max\{P(Q) - \tau - C_i'(q), 0\},$$

where the max-operator reflects that the firm will not import any additional quantity when the marginal gain is negative.

I assume that all types v_i are bounded above by some bidder-independent $\overline{v} > 0$ and let $\mathcal{V}$ be the space of all non-increasing, Lipschitz-continuous marginal profit functions $v_i : [0, Q] \to [0, \overline{v}]$ with a uniform (finite) Lipschitz constant. Throughout, I assume that $\mathcal{V}$ is equipped with the metric d_v induced by the supremum norm. Such a type space includes constant marginal profit functions on all quantities but excludes discontinuous marginal profits like, for example, step functions.[11]

The commonly known distribution of profit profiles $v = (v_1, \dots, v_n) \in \mathcal{V}^n$ is described by a probability measure η on the Borel subsets of $\mathcal{V}^n$ with a marginal distribution η_i on $\mathcal{V}$ for each bidder i.

(A1) The marginal profits $v_i \in \mathcal{V}$ are independently distributed.

(A2) For all bidders $i = 1, \dots, n$, it holds that, if $X \subset \mathcal{V}$ satisfies $\eta_i(X) > 0$, then there are $X', X'' \subset X$ with $\eta_i(X'), \eta_i(X'') > 0$ where $\forall f \in X'$ and $\forall g \in X''$ it holds that $f(q) > g(q), \forall q \in [0, Q]$.

Technically, Assumption (A1) can be weakened to type distributions η that are absolutely continuous with respect to the product of their marginals. Independence will be crucial for estimation later on, though. Assumption (A2) says that any set of profits with positive measure contains two sets also of nonzero measure, where all elements of one set dominate all elements of the other set in the pointwise partial order. This will be key for establishing equilibrium existence. Note that Assumption (A2) entails the assumption that η is atomless. I write $v_{-i} = (v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_n)$ for the elements of the profit profile v other than bidder i's type, and denote the distribution of opponent types v_{-i} by $\eta_{-i}(v_{-i})$.

[11]The theoretical results in this and the next section hold for any finite Lipschitz constant. The set $\mathcal{V}$ may include functions that are very close to step functions in the pointwise order if the Lipschitz constant is sufficiently high. While monotonicity alone is sufficient for equilibrium existence, continuity is required to characterize the optimal bids. A Lipschitz constant must be imposed so that the type space is compact (cf. the proof to Proposition 1 in the Supplementary Appendix). The upper bound on the marginal valuations, $\overline{v}$, will be important for the application (cf. also footnote 23).

I assume that all bidders evaluate their firms' monetary gains from the auction with a commonly known utility function $\phi : \mathbb{R} \to \mathbb{R}$. That is, if the quantity that bidder i's firm receives after the auction is q and the total payment is P, then his utility is

$$\phi\left(\int_0^q v_i(x)\,\mathrm{d}x - P\right).$$

When $\phi(x) = x$, the bidders are risk-neutral, and the utility function corresponds to that in Kastl (2012). An extension to heterogeneous (commonly known) utility functions ϕ_i is straightforward but omitted for a clean notation. All results go through when replacing ϕ with a bidder-dependent ϕ_i.[12]

Whereas the application will assume that ϕ is in the class of CARA utility functions, the general model only requires that the bidders' utility from the net profit in the auction is strictly monotone. Specifically, we have the following:

(A3) The function ϕ is strictly increasing, twice continuously differentiable, and satisfies $\phi(0) = 0$.

Bidders simultaneously submit their bids. Each bidder i submits $k \geq 1$ price-quantity pairs $(p_i^j, q_i^j) \in [0, \overline{p}] \times [0, Q]$, $j = 1, \ldots, k$, where $\overline{p}$ is finite and denotes the maximum price point that can be submitted. A feasible action of bidder i is a k-tuple b_i of price-quantity pairs,

$$b_i = \left\{\left(p_i^1, q_i^1\right), \left(p_i^2, q_i^2\right), \ldots, \left(p_i^k, q_i^k\right)\right\},$$

where the price points are decreasing, and the quantity points are increasing. This gives us the set $\mathcal{B}$ of feasible actions,

$$\mathcal{B} = \left\{ b_i \in \left[[0, \overline{p}] \times [0, Q]\right]^k : \begin{array}{l} \overline{p} \geq p_i^j \geq p_i^{j+1} \geq 0, \\ 0 \leq q_i^j \leq q_i^{j+1} \leq Q, \end{array} \quad \forall j \in \{1, \ldots, k-1\} \right\}.$$

The bids of all bidders are taken together in the vector $b \equiv (b_1, \ldots, b_n) \in \mathcal{B}^n$, which I call a bid profile.

It will be convenient to use the price-quantity pairs in $b_i \in \mathcal{B}$ to define left-continuous step functions $\beta_{b_i} : [0, Q] \to [0, \overline{p}]$,

$$\beta_{b_i}(q) = p_i^1 + \sum_{j=2}^{k+1} \left(p_i^j - p_i^1\right) \cdot \mathbf{1}_{q \in (q_i^{j-1}, q_i^j]},$$

where $\mathbf{1}_x$ denotes the indicator function, I let $(p_i^{k+1}, q_i^{k+1}) \equiv (0, Q)$, and I assume that $(q, q] = \emptyset$ for any $q \in [0, Q]$.[13] Further, I write $\beta_{b_i}^{-1}(p)$ for the inverse of β_{b_i}, returning the quantity demanded by player i at price p.

[12] As mentioned in the Introduction, the application in Section 5 will assume that we can group the bidders so that risk preferences differ across groups but are identical within groups.

[13] That is, when some submitted quantity points are equal, that is, $q_i^j = q_i^{j+1} = \cdots = q_i^{j+m}$ for some $j \in \{1, \ldots, k\}$ and $m \in \{1, \ldots, k - j\}$, then $\beta_{b_i}(q_i^j) = p_i^j$. This amounts to assuming that the auctioneer always considers the highest price bid on any quantity, and ignores the other price points.

Remark 2 (Bidders Submitting less than k price-quantity pairs). In the application, we will have $k = 5$; that is, bidders can submit at most five price-quantity pairs. Yet, bidders do not always submit five price-quantity pairs (the average number of price-quantity pairs per bidder is 4.4). Such behavior can be explained with a binding monotonicity constraint on at least one of the price points or quantity points, leading to a step function β_{b_i} whose graph has less than k steps. I will use the term distinct price-quantity pairs to refer to the smallest set of price-quantity pairs that are required to characterize the graph of such a step function (see Definition 2 in Section 3.2). As we will see, knowing the distinct price-quantity pairs of a bid is sufficient to apply the theoretical results to the data. Hence, we can treat the submitted price-quantity pairs as the distinct pairs of a bid (whose other price-quantity pairs need not be known).

For a realized bid profile $b \in \mathcal{B}^n$ with at least one bidder submitting at least one strictly positive price point, the auctioneer determines the price $p^c > 0$ of the lowest served bid. The price p^c either corresponds to the market-clearing price or, if there is no such price, to the lowest strictly positive price point submitted; that is,

$$p^c = \max\left\{\sup\left\{p \in [0, \overline{p}] : \sum_{i=1}^{n} \beta_{b_i}^{-1}(p) \geq Q\right\},\right.$$
$$\left.\min\left\{p \in \left\{p_i^j\right\}_{i\in\{1,\dots,n\}, j\in\{1,\dots,k\}} : p > 0\right\}\right\},$$

where the supremum of the empty set is taken to be 0. The auctioneer retains the goods if no strictly positive price point is submitted.

If total demand at p^c is weakly smaller than Q, then all demand at p^c is served. If, on the other hand, total demand at p^c is strictly greater than Q, then at least one bidder will be rationed according to some prespecified rationing rule. To capture rationing formally, I say that, for any bid profile $b \in \mathcal{B}^n$ and any bidder i, the rationing rule induces a cumulative distribution $H_i^b : [0, Q] \to [0, 1]$ of the allocated quantity $q \in [0, Q]$, so that the payoff u_i that bidder i of type v_i receives when bids (b_i, b_{-i}) are submitted can be written as

$$u_i(b_i, b_{-i}, v_i) = \int_0^Q \phi\left(\int_0^q \left[v_i(\hat{q}) - \beta_{b_i}(\hat{q})\right] \mathrm{d}\hat{q}\right) \mathrm{d}H_i^b(q). \tag{1}$$

Specifically, the rationing rule used is pro-rata-on-the-margin, which is the standard rationing rule in share auctions (cf. Kastl (2011)). Suppose there are $m \geq 1$ bidders, collected in the set M, submitting a price point equal to p^c. For every bidder $i \in M$, let

$$Q_i \equiv \beta_{b_i}^{-1}(p^c) - \lim_{p \downarrow p^c} \beta_{b_i}^{-1}(p)$$

be the marginal demand at p^c. Then, for any bidder $i \in M$, the allocated quantity is

$$q_i^c \equiv \lim_{p \downarrow p^c} \beta_{b_i}^{-1}(p) + \frac{Q_i}{\sum_{j\in M} Q_j} \cdot \left[Q - \sum_{j\in N} \lim_{p \downarrow p^c} \beta_{b_j}^{-1}(p)\right],$$

where $N = \{1, \dots, n\}$ is the set of all bidders. For all the other bidders $i \in \{1, \dots, n\} \setminus M$, the allocated quantity is $\beta_{b_i}^{-1}(p_c)$. This gives the following.

(A4) The cumulative distribution function $H_i^b(q)$ of the allocated quantity for bidder i under a bid profile b is given by

$$H_i^b(q) = \begin{cases} 0, & \text{if } q \in [0, q_i^c) \text{ and there is } j \in \{1, \dots, k\} \text{ such that } p_i^j = p_c, \\ 0, & \text{if } q \in [0, \beta_{b_i}^{-1}(p_c)) \text{ and there is no } j \in \{1, \dots, k\} \text{ such that } p_i^j = p_c, \\ 1, & \text{else.} \end{cases}$$

Following Kastl (2012), I consider distributional strategies (Milgrom and Weber (1985)): A feasible strategy for bidder i consists in a probability measure μ_i over the product of bidder i's action space and type space, where the marginal distribution of the type space is equal to the type distribution. In other words, for any $X \subset \mathcal{V}$ we have $\mu_i(\mathcal{B} \times X) = \eta_i(X)$. The set of all such probability measures is denoted by $\mathcal{M}$, and the individual strategies are collected in the strategy profile $\mu = (\mu_1, \dots, \mu_n) \in \mathcal{M}^n$. I write $\mu_i(.|v_i)$ for bidder i's distribution over $\mathcal{B}$ conditional on being of type v_i.

Definition 1 (Equilibrium). An equilibrium is a strategy profile $\mu^* \in \mathcal{M}^n$ satisfying

$$\mu_i^* \in \underset{\mu_i \in \mathcal{M}}{\arg\max} \int_{\mathcal{B}^n \times \mathcal{V}^n} u_i(b_i, b_{-i}, v_i)\, \mathrm{d}\mu_1^*(b_1|v_1) \dots \mathrm{d}\mu_{i-1}^*(b_{i-1}|v_{i-1})$$
$$\times\, \mathrm{d}\mu_i(b_i|v_i)\, \mathrm{d}\mu_{i+1}^*(b_{i+1}|v_{i+1}) \dots \mathrm{d}\mu_n^*(b_n|v_n)\, \mathrm{d}\eta(v)$$

for all bidders $i = 1, \dots, n$.

3.2 *Equilibrium existence and best-response characterization*

I first establish existence of an equilibrium. A tie is when two or more bidders submit a price point equal to p^c, and total demand at p^c strictly exceeds the quota (i.e., rationing occurs). I can state the following result.[14]

Proposition 1. *An equilibrium exists. In any equilibrium, ties happen with probability zero.*

For risk-neutral bidders and continuous type distributions, the absence of ties in equilibrium is well known (Kastl (2011)). The strict monotonicity of ϕ posited in (A4) together with Assumption (A2) ensures that this continues to hold for alternative risk

[14]The proof, found in the Supplementary Appendix, first considers an auction with a discrete action space for which I establish existence by using the results of Milgrom and Weber (1985). Letting the discrete action space become dense in the continuous action space then allows me to construct a sequence of equilibria for which I show the limit to be an equilibrium of the auction with the continuous action space (using results developed in Reny (1999, 2011)). This approach is an alternative to that employed in Kastl (2012) who uses results from Reny and Zamir (2004). Note, however, that Kastl (2012) does not allow for risk aversion. A more direct proof of Proposition 1 could be obtained by extending the results of Olszewski and Siegel (2019) to the present setting, along analogous lines they suggest for multiunit auctions.

preferences. Roughly speaking, whenever ties were to happen, there would be a set of tying bidders with positive measure who would strictly prefer to avoid the tie.

Next, I turn to the characterization of the equilibrium bids. To this end, let

$$\mathcal{B}_{p,q} = \left\{ b_{-i} \in \mathcal{B}^{n-1} : Q - \sum_{j \in \{1,\dots,n\} \setminus i} \beta_{b_j}^{-1}(p) \geq q \right\}$$

be the set of opponent bid profiles b_{-i} such that the residual supply faced by bidder i at price $p \in (0, \bar{p}]$ is weakly greater than $q \in (-\infty, Q]$. This allows to define

$$\begin{aligned} &W_i^*(p, q) \\ &\quad = \int_{\mathcal{V}^{n-1}} \int_{\mathcal{B}_{p,q}} \mathrm{d}\mu_1^*(b_1|v_1) \dots \mathrm{d}\mu_{i-1}^*(b_{i-1}|v_{i-1}) \, \mathrm{d}\mu_{i+1}^*(b_{i+1}|v_{i+1}) \dots \mathrm{d}\mu_n^*(b_n|v_n) \, \mathrm{d}\eta_{-i}(v_{-i}), \end{aligned}$$

which corresponds to the probability that the residual supply faced by bidder i at a price $p > 0$ when the other bidders play according to their equilibrium strategies in the strategy profile $\mu^* \in \mathcal{M}^n$ is greater than q. The absence of ties in equilibrium gives that $W_i^*(p, q)$ corresponds to the probability of winning at least a quantity of q when submitting a price p for that quantity. Let $w_i^*(p, q)$ denote the derivative of W^* with respect to p; that is, $w_i^*(p, q) \equiv \partial W_i^*(p, q)/\partial p$.

As observed in Remark 2 above, the restriction of the action space to decreasing p_i^j and increasing q_i^j might bind for some bidders. The optimal bid schedule b_i of a bidder for whom at least one of the restrictions binds either has $p_i^j = p_i^{j+1}$, or $q_i^j = q_i^{j-1}$, or both, for at least one $j \in \{1, \dots, k\}$. Such a bid b_i yields a step function β_{b_i} whose graph has less than k steps. The characterization of this graph only requires knowledge of a subset of the price-quantity pairs in b_i. In the following, I call the members of this subset the distinct price-quantity pairs. Formally, we have the following.

Definition 2 (Distinct price-quantity pairs). Let $b_i, b_i' \in \mathcal{B}$ satisfy $\beta_{b_i}(q) = \beta_{b_i'}(q)$ for all $q \in [0, Q]$, where b_i is of the following form: there is $\ell_i \leq k$ such that the price-quantity pairs in b_i satisfy $p_i^j > p_i^{j+1}$ and $q_i^{j+1} > q_i^j$ for all $j = 1, \dots, \ell_i - 1$, and if $\ell_i < k$, $(p_i^j, q_i^j) = (0, Q)$ for all $j = \ell_i + 1, \dots, k$. Then the price-quantity pairs $\{p_i^j, q_i^j\}_{j=1}^{\ell_i}$ are called the distinct price-quantity pairs of b_i'.

The following characterization result and those in Section 4 only require knowledge of the distinct price-quantity pairs. As mentioned in Remark 2, the application will take the submitted price-quantity pairs of a bidder to be the distinct price-quantity pairs of her bids, that is, ℓ_i will correspond to the number of submitted price-quantity pairs by bidder i.

Writing $V_i(q) = \int_0^q v_i(x) \, \mathrm{d}x$ for the gross value and $B_i(q) = \int_0^q \beta_{b_i}(x) \, \mathrm{d}x$ for the gross bid, we obtain our main characterization result.

Proposition 2. *Consider a bidder i that has submitted an equilibrium bid b_i with distinct price-quantity pairs $\{p_i^j, q_i^j\}_{j=1}^{\ell_i}$. Any price-quantity pair $(p_i^j, q_i^j) \in (0, \bar{p}) \times (0, Q)$ satisfies*

$$\begin{aligned}&\big[v_i(q_i^j) - p_i^j\big]W_i^*(p_i^j, q_i^j) - \big[v_i(q_i^j) - p_i^{j+1}\big]W_i^*(p_i^{j+1}, q_i^j)\\&\quad - \big[p_i^j - p_i^{j+1}\big]\sum_{m=j}^{\ell_i}\int_{q_i^m}^{q_i^{m+1}} \frac{\phi''\big(V_i(q) - B_i(q)\big)}{\phi'\big(V_i(q_i^j) - B_i(q_i^j)\big)}\big[v_i(q) - p_i^{m+1}\big]W_i^*\big(p_i^{m+1}, q\big)\,\mathrm{d}q\\&= 0.\end{aligned} \tag{2}$$

Furthermore, if $w_i^*(p, q)$ exists for the quantities $q \in [q_i^{j-1}, q_i^j]$ and is continuous in p, then

$$\begin{aligned}&\int_{q_i^{j-1}}^{q_i^j}\big[\phi'\big(V_i(q) - B_i(q)\big)\big[\big[v_i(q) - p_i^j\big]w_i^*\big(p_i^j, q\big) - W_i^*\big(p_i^j, q\big)\big]\\&\quad - \phi''\big(V_i(q) - B_i(q)\big)\big[q - q_i^{j-1}\big]\big[v_i(q) - p_i^j\big]W_i^*\big(p_i^j, q\big)\big]\,\mathrm{d}q\\&\quad - \big[q_i^j - q_i^{j-1}\big]\sum_{m=j}^{\ell_i}\int_{q_i^m}^{q_i^{m+1}} \phi''\big(V_i(q) - B_i(q)\big)\big[v_i(q) - p_i^{m+1}\big]W_i^*\big(p_i^{m+1}, q\big)\,\mathrm{d}q\\&= 0.\end{aligned} \tag{3}$$

Equation (2) follows from the bidder's optimality conditions for the quantity points, and equation (3) follows from the optimality conditions for the price points. When the utility function ϕ is an affine function, then $\phi''(q) = 0$ and (2) corresponds to the optimality condition identified by Kastl (2012).

3.3 *Discussion*

How does risk aversion affect incentives and equilibrium behavior? The proof to Proposition 2 shows that when all other bidders follow their strategies in an equilibrium profile μ^*, then the interim utility for bidder i submitting b_i and being of type v_i is

$$\Pi_i\big(b_i, v_i, \mu_{-i}^*\big) = \sum_{j=1}^{k+1}\int_{q_i^{j-1}}^{q_i^j} \phi'\big(V_i(q) - B_i(q)\big)\big[v_i(q) - \beta_{b_i}(q)\big]W_i^*\big(p_i^j, q\big)\,\mathrm{d}q. \tag{4}$$

In the standard risk-neutral case, we have $\phi'(x) = 1$, and hence, the relevant weight on the net profit from winning a certain amount q, $v_i(q) - \beta_{b_i}(q)$, under the integral in above expression is equal to the probability of winning at least q when bidding p_i^j for it, $W_i^*(p_i^j, q)$. In the case of risk aversion, this probability is multiplied by the factor $\phi'(V_i(q) - B_i(q))$, which decreases in q. Other than under risk neutrality, higher quantities receive relatively less weight than lower quantities.

From (4), we further see that under risk neutrality the price point submitted for a given quantity q, $p_i^j = \beta_{b_i}(q)$, just affects the winning probability, and thus the weight, for the quantities at this price, $q \in (q_i^{j-1}, q_i^j]$. Absent risk neutrality this is not true, because in that case the weights on all higher quantities are affected, too. This changes the marginal considerations, as can be seen from the additional terms in the optimality conditions (2) and (3) when $\phi'(x) \neq 1$ and $\phi''(x) \neq 0$.

In particular, under the assumption of risk aversion, the additional, subtracted terms in the optimality conditions (2) and (3) are all negative. Because W_i^* increases in its first argument, (2) gives that, of two bidders submitting the same price-quantity pairs, the profit function of the risk-averse bidder is closer to the price points (at the respective quantity points) than that of the risk-neutral bidder. So, loosely speaking and in line with what we know about equilibrium behavior in single-good auctions, we would expect risk-averse bidders to bid "closer" to their marginal profit functions than risk-neutral bidders do. The empirical results in Section 5 align with this intuition, showing that the average per-kg shading factor is lower when assuming risk aversion than when assuming risk neutrality.[15]

4. Marginal profits and risk preferences

For the results in this section, I assume that bidders have CARA utility,

$$\phi(x) = \begin{cases} \dfrac{1 - e^{-\rho x}}{\rho}, & \text{for } \rho > 0, \\ x, & \text{for } \rho = 0, \end{cases} \tag{5}$$

where ρ is the (commonly known) risk preference parameter of the bidders. For a distinct price-quantity pair $j \in \{1, \dots, \ell_i\}$ in a bid schedule b_i, let

$$\begin{aligned} \overline{\Pi}_i^j(b_i, v_i, \rho) = \sum_{m=j}^{\ell_i} \int_{q_i^m}^{q_i^{m+1}} & \exp\left(-\rho \int_{q_i^j}^{q} \left[v_i(x) - \beta_{b_i}(x)\right] \mathrm{d}x\right) \\ & \times \left[v_i(q) - p_i^{m+1}\right] W_i^*\left(p_i^{m+1}, q\right) \mathrm{d}q \end{aligned} \tag{6}$$

be the (normalized) equilibrium interim utility of bidder i from quantities above q_i^j when having type v_i (cf. the expression for interim utility in (4) above). Using (5) and the optimality conditions from Proposition 2, we get the following corollary to Proposition 2.

Corollary 1. *Consider a bidder i that has submitted an equilibrium bid b_i with distinct price-quantity pairs $\{p_i^j, q_i^j\}_{j=1}^{\ell_i}$. Any price-quantity pair $(p_i^j, q_i^j) \in (0, \bar{p}) \times (0, Q)$ satisfies*

$$v_i(q_i^j) = p_i^j + \left[p_i^j - p_i^{j+1}\right] \frac{W_i^*(p_i^{j+1}, q_i^j) - \rho \overline{\Pi}_i^j(b_i, v_i, \rho)}{W_i^*(p_i^j, q_i^j) - W_i^*(p_i^{j+1}, q_i^j)} \geq p_i^j. \tag{7}$$

[15] Risk aversion likely also affects the number of distinct price-quantity pairs a bidder would want to submit for a given profit function v_i. However, to analyze this question, we would require to know more about how risk aversion affects the distribution of residual supply (and consequently W_i^*). In particular, this would necessitate obtaining an analytical solution of the bidders' bid functions, which is currently unavailable even for the risk-neutral case.

Furthermore, if $w_i^*(p, q)$ exists for the quantities $q \in [q_i^{j-1}, q_i^j]$ and is continuous in p, then

$$\begin{aligned}\int_{q_i^{j-1}}^{q_i^j} \exp\left(\rho \int_q^{q_i^j} \left[v_i(x) - \beta_{b_i}(x)\right] \mathrm{d}x\right)\left[\left[v_i(q) - p_i^j\right]\right. \\ \times \left[w_i^*(p_i^j, q) + \rho\left[q - q_i^{j-1}\right] W_i^*(p_i^j, q)\right] - W_i^*(p_i^j, q)\left.\right] \mathrm{d}q \\ + \rho\left[q_i^j - q_i^{j-1}\right] \overline{\Pi}_i^j(b_i, v_i, \rho) = 0. \end{aligned} \tag{8}$$

From the equality in (7), we immediately see that, for two bidders submitting the same bid, bid shading at the submitted quantity points is lower under risk aversion ($\rho > 0$) than it is under risk neutrality ($\rho = 0$). Nevertheless, the price bid is always below the marginal profit at the corresponding quantity point for any risk preference $\rho > 0$, as the inequality in (7) asserts.

When $\rho = 0$, then the right-hand side of the equality in (7) is independent of v_i, and hence, provides a mapping from the submitted bid, b_i, and the distribution of the residual supply function, W_i^*, to the marginal profit function v_i at the submitted quantity points q_i^j. This corresponds to the core observation of Kastl (2012): the marginal profit function v_i is point identified at the submitted quantities. Yet, when $\rho > 0$ then point identification of $v_i(q_i^j)$, $j = 1, \dots, \ell_i - 1$, fails because the right-hand side of the inequality in (7) is not independent of v_i. Set identification still holds, though. And as we will see next, it has a particularly straightforward characterization in the case of CARA utility.

4.1 *Constructing bounds on the profits*

A crucial observation for the following is that, for every $j \in \{1, \dots, \ell_i\}$, the right side of the equality in (7) only depends on the segment of v_i that is on $[q_i^j, Q]$; that is, on the quantities greater than q_i^j.[16] Further, as the proof to Lemma 1 below shows, $\overline{\Pi}_i^j$ is monotone in v_i under the pointwise partial order on $\mathcal{V}$.

Definition 3 (Order on $\mathcal{V}$). Let $v_i, \tilde{v}_i \in \mathcal{V}$. We have $v_i \geq \tilde{v}_i$ iff $v_i(q) \geq \tilde{v}_i(q)$, $\forall q \in [0, Q]$.

With the assumption that marginal profits are decreasing, I recursively obtain bounds on the marginal profit function at the submitted quantity points as follows.

Lemma 1. *Consider a bidder i that has submitted an equilibrium bid b_i with distinct price-quantity pairs $\{p_i^j, q_i^j\}_{j=1}^{\ell_i}$. The tuples $\{\underline{v}_i^j, \overline{v}_i^j\}$, $j = 1, \dots, \ell_i$, recursively satisfying*

$$\underline{v}_i^{\ell_i} = \overline{v}_i^{\ell_i} = p_i^{\ell_i}, \tag{9}$$

[16] To get an intuition for this, observe that for any quantity point q_i^j, the inframarginal expected profits are unaffected by the particular choice of q_i^j, and hence, can be interpreted as a bidders' wealth. Because wealth effects are absent under CARA, these inframarginal profits are irrelevant for the optimal choice and, therefore, do not appear in (7). For other forms of risk aversion (e.g., CRRA), this does not hold, and hence, the constructions in the following are not possible.

as well as, for $j \in \{1, \dots, \ell_i - 1\}$,

$$\overline{v}_i^j = p_i^j + \left[p_i^j - p_i^{j+1}\right] \frac{W_i^*\left(p_i^{j+1}, q_i^j\right) - \rho \overline{\Pi}_i^j(b_i, v_l, \rho)}{W_i^*\left(p_i^j, q_i^j\right) - W_i^*\left(p_i^{j+1}, q_i^j\right)},$$

$$\textit{where } v_l(q) = \sum_{m=j}^{\ell_i - 1} \mathbf{1}_{q \in (q_i^m, q_i^{m+1}]} \cdot \underline{v}_i^{m+1} \textit{ for } q \in \left[q_i^j, Q\right], \tag{10}$$

and

$$\underline{v}_i^j = p_i^j + \left[p_i^j - p_i^{j+1}\right] \frac{W_i^*\left(p_i^{j+1}, q_i^j\right) - \rho \overline{\Pi}_i^j(b_i, v_u, \rho)}{W_i^*\left(p_i^j, q_i^j\right) - W_i^*\left(p_i^{j+1}, q_i^j\right)},$$

$$\textit{where } v_u(q) = \sum_{m=j}^{\ell_i} \mathbf{1}_{q \in (q_i^m, q_i^{m+1}]} \cdot \overline{v}_i^m \textit{ for } q \in \left[q_i^j, Q\right] \tag{11}$$

satisfy $\underline{v}_i^j \leq v_i(q_i^j) \leq \overline{v}_i^j$ *for all* $j = 1, \dots, \ell_i$.

If $\rho = 0$, then $\underline{v}_i^j = v_i(q_i^j) = \overline{v}_i^j$ for all $j = 1, \dots, \ell_i$, and the formulation boils down to the mapping from the data to the marginal profit identified in Kastl (2012). For $\rho > 0$, Proposition 5 establishes that the marginal profit function of any bidder is set identified at the submitted quantity points.

The proof makes use of the fact that the marginal profit function is point-identified at the last quantity point $q_i^{\ell_i}$ because the normalized interim utility beyond $q_i^{\ell_i}$ is zero, $\overline{\Pi}^{\ell_i}(b_i, v_i, \rho) = 0$. Then, because the normalized interim utility $\overline{\Pi}^j(b_i, v_i, \rho)$ is increasing in v_i and $\underline{v}_i^j$ provides a lower bound for the (decreasing) marginal profit v_i on the segment $(q_i^{\ell_i - 1}, q_i^{\ell_i}]$, we obtain an upper bound $\overline{v}_i^{\ell_i - 1}$ on the marginal profit at the second-to-last quantity point. Now, $\overline{v}_i^{\ell_i - 1}$ is an upper bound for the marginal profit v_i on $(q_i^{\ell_i - 1}, q_i^{\ell_i}]$, which in turn yields a lower bound on the marginal profit at the second-to-last quantity point, $q_i^{\ell_i - 2}$, and so on.

From Lemma 1 together with the upper bound $\overline{v}$ on the marginal profit functions in $\mathcal{V}$, we obtain the following upper and lower bounds on the marginal profit function of a bidder.

PROPOSITION 3. *Consider bidder* i *with type* $v_i \in \mathcal{V}$ *having submitted an equilibrium bid with distinct price-quantity pairs* $\{p_i^j, q_i^j\}_{j=1}^{\ell_i}$. *Then the marginal profit function* v_i *satisfies* $\underline{v}_i(q) \leq v_i(q) \leq \overline{v}_i(q)$ *for all* $q \in [0, Q]$, *where*

$$\overline{v}_i(q) \equiv \overline{v} + \sum_{j=1}^{\ell_i} \left(\min_{m \leq j}\{\overline{v}_i^m\} - \overline{v}\right) \cdot \mathbf{1}_{q \in (q_i^j, q_i^{j+1}]} \tag{12}$$

and

$$\underline{v}_i(q) \equiv \underline{v}_i^1 + \sum_{j=1}^{\ell_i} \left(\max_{m \geq j+1}\{\underline{v}_i^m\} - \underline{v}_i^1\right) \cdot \mathbf{1}_{q \in (q_i^j, q_i^{j+1}]} \tag{13}$$

with $q^{\ell_i + 1} = Q$ *and* $\underline{v}_i^{\ell_i + 1} = 0$.

The functions $\overline{v}_i(q)$ and $\underline{v}_i(q)$ are least upper and highest lower bounds on the (decreasing) marginal profit functions that are consistent with the optimality conditions (7). Again, if $\rho = 0$ then the bounds (12)–(13) correspond to the bounds used in Kastl (2011). The top panel of Figure 1 depicts a risk-neutral case. When $\rho > 0$, the bounds are less tight because point identification at the quantity points does not hold. The middle panel of Figure 1 depicts a case of risk aversion. The bottom panel of Figure 1 shows what happens when the upper and lower bounds from Lemma 1 are nonmonotone—which is not ruled out by the constructions in (10)–(11). In that case, the definitions in (12)–(13) give functions $\overline{v}_i$ and $\underline{v}_i$ that are sometimes tighter than the bounds from Lemma 1.

4.2 *Necessary conditions for best-response behavior*

The results of the last two sections allow us to formulate two sets of necessary conditions for best-response behavior for a given risk preference. The conditions are inequalities and formulated in Proposition 4 below.

The first set of conditions, (15), follows from the optimality conditions for the quantity points. The conditions require that there exists a decreasing marginal profit function between the bounds $\underline{v}_i$ and $\overline{v}_i$ defined in Proposition 3. In other words, (15) ensures that $\overline{v}_i(q) \geq \underline{v}_i(q)$ holds for all $q \in [0, Q]$. The second set of condition, (16), follows from the optimality conditions for the prices. Here, I use the left side of the optimality condition (8) to define

$$F_i^j(b_i, v_i, \rho) \equiv \int_{q_i^{j-1}}^{q_i^j} \exp\left(\rho \int_q^{q_i^j} \left[v_i(x) - \beta_{b_i}(x)\right] \mathrm{d}x\right) \left[\left[v_i(q) - p_i^j\right]\right.$$
$$\left. \times \left[w_i^*(p_i^j, q) + \rho\left[q - q_i^{j-1}\right] W_i^*(p_i^j, q)\right] - W_i^*(p_i^j, q)\right] \mathrm{d}q$$
$$+ \rho\left[q_i^j - q_i^{j-1}\right] \overline{\Pi}_i^j(b_i, v_i, \rho). \tag{14}$$

Then (16) must hold because $\overline{v}_i$ and $\underline{v}_i$ lie (in the pointwise partial order) below and above the (true) profit v_i that solves the optimality condition (8); that is, satisfies $F_i^j(b_i, v_i, \rho) = 0$.

Proposition 4. *Consider a bidder i that has submitted an equilibrium bid b_i with distinct price-quantity pairs $\{p_i^j, q_i^j\}_{j=1}^{\ell_i}$. Then it must hold for any $j \in \{1, \ldots, \ell_i - 1\}$ that*

$$\min_{m \leq j} \{\overline{v}_i^m\} \geq \max_{m \geq j+1} \{\underline{v}_i^m\}. \tag{15}$$

Further, assume that the function $F_i^j(b_i, v_i, \rho)$ increases in v_i for $\underline{v}_i \leq v_i \leq \overline{v}_i$ for some $j \in \{1, \ldots, \ell_i\}$. Then it must hold

$$F_i^j(b_i, \overline{v}_i, \rho) \geq 0 \geq F_i^j(b_i, \underline{v}_i, \rho). \tag{16}$$

In the application, the inequalities in (15) and (16) will be key for determining best-response violations. The interpretation of these conditions is as follows: If either of the

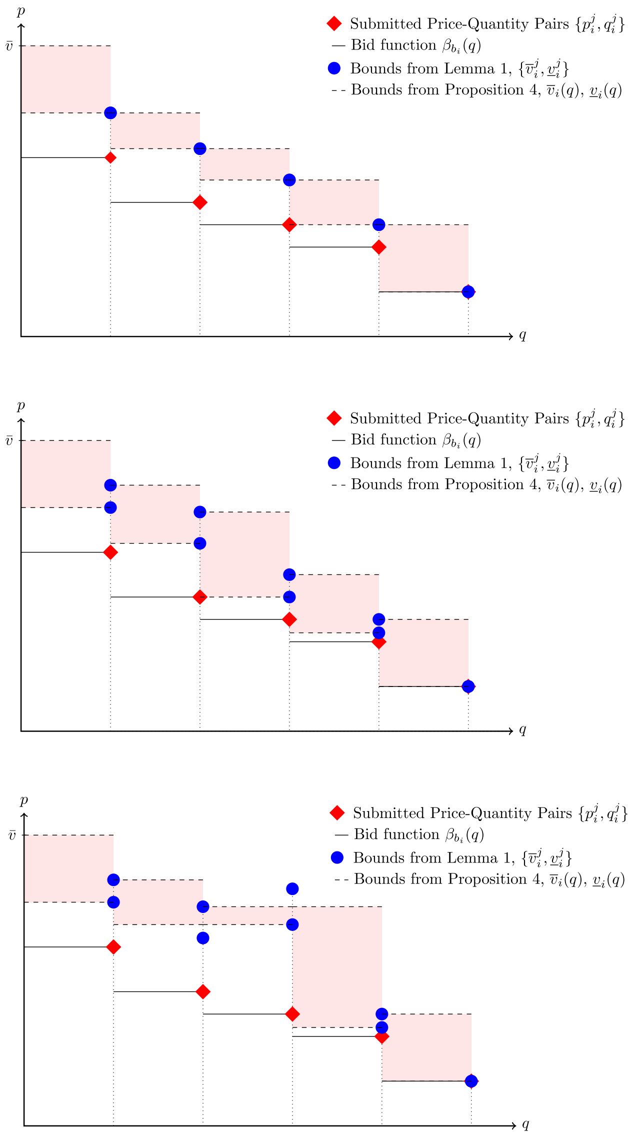


FIGURE 1. Each figure shows a bid function β_{b_i} (solid lines) constructed from the submitted price-quantity pairs $\{p_i^j, q_i^j\}$ depicted as diamonds. Further, each figure shows (hypothetical) upper and lower bounds $\overline{v}_i(q)$ and $\underline{v}_i(q)$ from Proposition 3 (dashed lines), constructed from bounds on the profit function at the submitted quantities, depicted as solid dots. The shaded area corresponds to the space of profit functions between $\overline{v}_i(q)$ and $\underline{v}_i(q)$. The top panel shows the risk-neutral case. Here, $\overline{v}_i^j = \underline{v}_i^j$ for all j as explained after Lemma 1. The middle panel depicts a case of risk aversion. Here, $\overline{v}_i^j > \underline{v}_i^j$ for all j. The bottom panel also shows a case of risk aversion. Here, neither $\overline{v}_i^j$ nor $\underline{v}_i^j$ are monotone, resulting in bounds $\overline{v}_i$ and $\underline{v}_i$ that are sometimes tighter than the bounds from Lemma 1.

inequalities were to fail for some j, there would be no (decreasing) marginal profit function that could rationalize this bidder's bid. More precisely, if the inequality in (15) were to fail for any $j < \ell_i$, then there would be no decreasing profit function consistent with the quantity points' optimality conditions. On the other hand, if one of the inequalities in (16) were to fail, there would be, among the monotone profit functions v_i that are consistent with quantity point optimality, no v_i that is also consistent with the respective price point's optimality condition.

The function $w_i^*(p_i^j, q)$, which affects F_i^j but not the bounds $\underline{v}_i$ and $\overline{v}_i$, is key to understanding violations of (16). The function $w_i^*(p_i^j, q)$ measures the marginal effect of raising p_i^j on winning at least a quantity q between q_i^{j-1} and q_i^j. Considering the case $\rho = 0$, we see from (14) that the right inequality in (16) will fail even for the lowest possible marginal profit function, $\underline{v}_i$, when the estimated values of $w_i^*(p_i^j, q)$ are large for all relevant q. In other words, if the marginal gain in winning probability from raising p_i^j is higher than the additional payment, the bidder would fare better by increasing p_i^j for any marginal profit function v_i between the bounds $\underline{v}_i$ and $\overline{v}_i$. A similar intuition applies for low values of $w_i^*(p_i^j, q)$, implying that the marginal gain in winning probability is lower than the associated additional cost when raising p_i^j, thus making it optimal to reduce p_i^j. For $\rho > 0$, these effects are not so clear-cut. Qualitatively, the impact of $w_i^*(p_i^j, q)$ on the value of F_i^j is the same, though.

On a technical note, observe that the function $F_i^j(b_i, v_i, \rho)$ always increases in v_i when $\rho = 0$. This can be directly seen from (14) because $\phi'(.)$ is constant under risk neutrality. Under risk aversion, $\rho > 0$, increasing differences at the submitted bids does not necessarily hold, though (and thus will have to be verified in the data); that the utility function under risk aversion might fail increasing differences is well known for multi-unit auctions (McAdams (2003), Reny (2011)) and the intuition in the share auction is similar. An increase in v_i at some q not only increases the value of winning that particular quantity but also decreases the weight put on winning higher quantities, thus potentially decreasing the marginal gain from increasing a price bid.

4.3 *Tighter bounds on marginal profits*

This section shows how the optimality conditions for the price points, (8), can be used to derive upper and lower bounds on the true marginal profit function v_i that are tighter than those obtained with Proposition 3.

To this end, let $\tilde{\mathcal{V}}_i$ be the set of all nonincreasing (but not necessarily Lipschitz-continuous) functions $v_i : [0, Q] \to [0, \overline{v}]$ lying between the upper and the lower envelopes $\overline{v}_i$ and $\underline{v}_i$ from (12)–(13); that is, $\underline{v}_i(q) \le v_i(q) \le \overline{v}_i(q)$ for all $q \in [0, Q]$. Then let

$$\mathcal{V}_i^F \equiv \left\{ v_i \in \tilde{\mathcal{V}}_i : F_i^j(b_i, v_i, \rho) = 0, \forall j \in \{1, \dots, \ell_i\} \right\} \tag{17}$$

be the set of nonincreasing functions that satisfy the optimality conditions (8) and lie between $\underline{v}_i$ and $\overline{v}_i$. The aim of the following is to characterize functions that lie weakly above the least upper bound $\overline{v}_i^F$ on $\mathcal{V}_i^F$ and weakly below the greatest lower bound $\underline{v}_i^F$ on

$\mathcal{V}_i^F$,

$$\overline{v}_i^F = \vee \mathcal{V}_i^F \quad \text{and} \quad \underline{v}_i^F = \wedge \mathcal{V}_i^F. \tag{18}$$

Crucially, because the type space $\mathcal{V}$ contains functions that are both nonincreasing and Lipschitz-continuous, the bounds $(\overline{v}_i^F, \underline{v}_i^F)$ defined in (18) are also upper and lower bounds on the marginal profit functions $v_i \in \mathcal{V}$ that satisfy the optimality conditions (8) and lie between $\underline{v}_i$ and $\overline{v}_i$.[17]

The aim of the following is to characterize a vector-valued function whose least fixed point is a nontrivial bound on $(\overline{v}_i^F, \underline{v}_i^F)$. I use the functions φ_u and φ_l, defined for $x \in [0, Q]$ with $q, v \in \mathbb{R}$ and $v_u, v_l \in \mathcal{V}$ as

$$\varphi_u(q, v, v_l)(x) = \begin{cases} \max\{v, v_l(x)\} & \text{if } x \leq q, \\ v_l(x) & \text{if } x > q, \end{cases} \tag{19}$$

$$\varphi_l(q, v, v_u)(x) = \begin{cases} v_u(x) & \text{if } x \leq q, \\ \min\{v, v_u(x)\} & \text{if } x > q, \end{cases} \tag{20}$$

to define

$$\theta_{i,u}(v_l)(q) \\ = \begin{cases} \inf\{v \in [\theta_{i,u}(v_l)(q_i^j), \overline{v}_i(q)] : F_i^j(b_i, \varphi_u(q, v, v_l), \rho) > 0\} & \text{if } q \in [q_i^{j-1}, q_i^j), j < \ell_i, \\ \inf\{v \in [\underline{v}_i(q), \overline{v}_i(q)] : F^{\ell_i}(b_i, \varphi_u(q, v, v_l), \rho) > 0\} & \text{if } q \in [q_i^{\ell_i - 1}, q_i^{\ell_i}], \\ \overline{v}_i(q_i^\ell) & \text{if } q \in (q_i^{\ell_i}, Q] \end{cases}$$

and

$$\theta_{i,l}(v_u)(q) \\ = \begin{cases} \sup\{v \in [\theta_{i,l}(v_u)(q_i^j), \overline{v}_i(q)] : F_i^j(b_i, \varphi_l(q, v, v_u), \rho) < 0\} & \text{if } q \in [q_i^{j-1}, q_i^j), j < \ell_i, \\ \sup\{v \in [\underline{v}_i(q), \overline{v}_i(q)] : F^{\ell_i}(b_i, \varphi_l(q, v, v_u), \rho) < 0\} & \text{if } q \in [q_i^{\ell_i - 1}, q_i^{\ell_i}], \\ 0 & \text{if } q \in (q_i^{\ell_i}, Q], \end{cases}$$

where, for given q, the inf of the empty set is $\overline{v}_i(q)$ and the sup of the empty set is the respective lower bound from which v is to be chosen.

Both φ_u and φ_l are nondecreasing in q and v. Further, both $F_i^j(b_i, \varphi_l(q, v, v_u), \rho)$ and $F_i^j(b_i, \varphi_u(q, v, v_l), \rho)$ are continuous in v. Hence, under the assumption that F_i^j is nondecreasing in v_i, the function $\theta_{i,u}$ returns for any given $v_l \in \tilde{\mathcal{V}}_i$ another, higher function in $\tilde{\mathcal{V}}_i$, which returns at any $q \in [0, Q]$ the highest point such that there is a nonincreasing function going through that point and satisfying the respective optimality condition (8).

[17]To verify this, fix some nonincreasing v_i and suppose it is not an upper bound on the set of nonincreasing and Lipschitz-continuous functions that satisfy the optimality conditions (8) and lie between $\underline{v}_i$ and $\overline{v}_i$. Then there must be $\hat{v}_i$ in that set such that $\hat{v}_i(q) > v_i(q)$ for some q. But then v_i is also not an upper bound on the set of functions that are merely nonincreasing, satisfy the optimality conditions (8), and lie between $\underline{v}_i$ and $\overline{v}_i$, because $\hat{v}_i$ is also a member of that set.

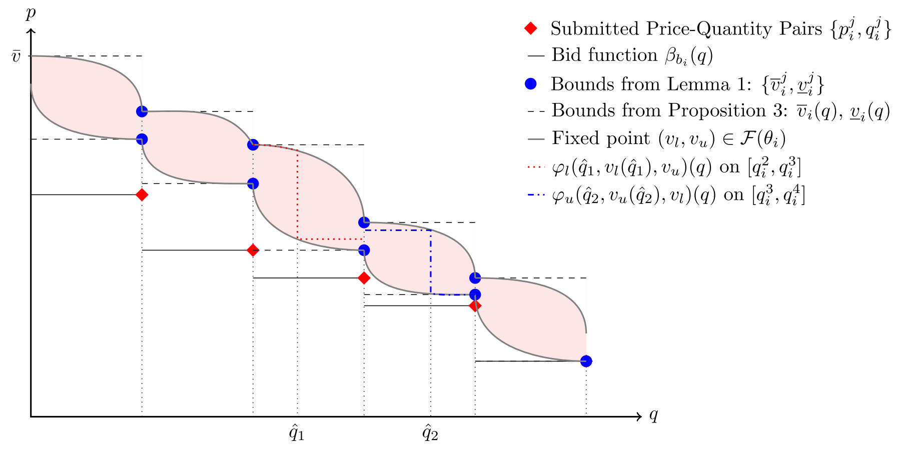


FIGURE 2. The figure schematically depicts a fixed point $(v_l, v_u) \in \mathcal{F}(\theta_i)$ for bidder i having submitted a bid function β_{b_i}. Loosely speaking, the function $\varphi_u(\hat{q}_2, v_u(\hat{q}_2), v_l)$, depicted on $[q_i^2, q_i^3]$, is among all nonincreasing functions above v_l that are equal to v_l for $q > \hat{q}_2$ the one with the highest value at $\hat{q}_2$ for which the left side of (8) evaluates to zero (taking $j = 3$). Analogously, the function $\varphi_l(\hat{q}_1, v_l(\hat{q}_1), v_u)$, depicted on $[q_i^2, q_i^3]$, is among all nonincreasing functions below v_u and equal to v_u for $q \le \hat{q}_1$, the one with the lowest value at $\hat{q}_1$ for which the left side of (8) evaluates to zero (taking $j = 2$). Because (v_l, v_u) is a fixed point, these characterizations of φ_u and φ_l are valid for all q. As shown in Proposition 5, if we consider the least element $\mathcal{F}(\theta_i)$, then the shaded area contains the set of all marginal profit functions between $\overline{v}_i$ and $\underline{v}_i$ that also satisfy the optimality conditions for the price points.

Conversely, the function $\theta_{i,l}$ returns for any given $v_u \in \tilde{\mathcal{V}}_i$, another lower function in $\tilde{\mathcal{V}}_i$, which returns at any $q \in [0, Q]$ the lowest point such that there is a nonincreasing function going through that point and satisfying the respective optimality condition (8). The caption of Figure 2 provides additional explanations.

Now, let $v = (v_l, v_u) \in \tilde{\mathcal{V}}_i \times \tilde{\mathcal{V}}_i$ and $\theta_i(v) = (\theta_{i,l}(v_u), \theta_{i,u}(v_l))$, and consider the set of fixed points

$$\mathcal{F}(\theta_i) \equiv \{v \in \tilde{\mathcal{V}}_i \times \tilde{\mathcal{V}}_i : v = \theta_i(v)\}. \tag{21}$$

By construction of $(\theta_{i,l}, \theta_{i,u})$, the set $\mathcal{F}(\theta_i)$ contains pairs of (nonincreasing) functions such that if we were to lower the higher of the two functions, call it $\overline{w}$, at some point $q \in [q_i^{j-1}, q_i^j]$, then there would be another nonincreasing function $\hat{v}_i$, which lies above the lower of the two functions, call it $\underline{w}$, and satisfies $F_i^j(b_i, \hat{v}_i, \rho) = 0$, but attains values that lie above $\overline{w}$ for some values of q. Conversely, if we were to increase the lower of the two functions, $\underline{w}$, at some point $q \in [q_i^{j-1}, q_i^j]$, then there would be another nonincreasing function $\hat{v}_i$, which lies below the higher of the two functions, $\overline{w}$, and satisfies $F_i^j(b_i, \hat{v}_i, \rho) = 0$, but attains values that lie below $\underline{w}$ for some values of q. Figure 2 schematically depicts a fixed point in the set of fixed points $\mathcal{F}(\theta_i)$.

One might expect that the least fixed point of θ_i, $\wedge\mathcal{F}(\theta_i)$ provides upper and lower bounds on the set $\mathcal{V}_i^F$. The ensuing result shows that this intuition is correct. The proof makes use of the fact that θ_i is continuous and order-preserving under the following partial order on the set $\tilde{\mathcal{V}}_i \times \tilde{\mathcal{V}}_i$.

Definition 4 (Order on $\tilde{\mathcal{V}}_i \times \tilde{\mathcal{V}}_i$). Let $v = (v_l, v_u) \in \tilde{\mathcal{V}}_i \times \tilde{\mathcal{V}}_i$ and $\tilde{v} = (\tilde{v}_l, \tilde{v}_u) \in \tilde{\mathcal{V}}_i \times \tilde{\mathcal{V}}_i$. We have $v \geq \tilde{v}$ iff both $v_u \leq \tilde{v}_u$ and $v_l \geq \tilde{v}_l$ hold.

The order defined with Definition 4 orders bounds by their tightness. That is, $v = (v_l, v_i) \geq (\tilde{v}_l, \tilde{v}_i) = \tilde{v}$ means that v is tighter than $\tilde{v}$ in the sense that both the lower bound v_l is higher than $\tilde{v}_l$ in the pointwise order and the upper bound v_u is lower than $\tilde{v}_u$.

The first two parts of the following statement are a consequence of what is alternatively referred to as the Kleene fixed-point theorem or the Tarski–Kantorovitch fixed-point theorem (cf. Baranga (1991), Jachymski, Gajek, and Pokarowski (2000), respectively).

Proposition 5. *Assume that, for bidder $i \in \{1, \dots, n\}$ having submitted an equilibrium bid b_i with distinct price-quantity pairs $\{p_i^j, q_i^j\}_{j=1}^{\ell_i}$, the function $F_i^j(b_i, v_i, \rho)$ increases in v_i for $\underline{v}_i \leq v_i \leq \overline{v}_i$ and all $j \in \{1, \dots, \ell_i\}$. Then*:

(i) *The set $\mathcal{F}(\theta_i)$ is nonempty, and there is a least fixed point, $\wedge\mathcal{F}(\theta_i)$.*

(ii) *Let the sequence (x_m, y_m) for $m = 1, 2, \dots$ be recursively defined as $(x_m, y_m) = \theta_i(x_{m-1}, y_{m-1})$ with $(x_0, y_0) = (\underline{v}_i, \overline{v}_i)$. Then $\lim_{m\to\infty}(x_m, y_m) = \wedge\mathcal{F}(\theta_i)$.*

(iii) $\wedge\mathcal{F}(\theta_i) \leq (\underline{v}_i^F, \overline{v}_i^F)$.

The central results for the empirical application in the next section are stated in parts (ii)–(iii). Part (ii) establishes that we can find the least fixed point $\wedge\mathcal{F}(\theta_i)$ by a simple fixed-point iteration, which takes the bounds $(\underline{v}_i, \overline{v}_i)$ obtained in (12)–(13) from the characterization in Proposition 3 as the initial condition. Part (iii) asserts that the bounds given by the least fixed point $\wedge\mathcal{F}(\theta_i)$ are indeed less tight than $(\underline{v}_i^F, \overline{v}_i^F)$. That is, $\wedge\mathcal{F}(\theta_i)$ provides upper and lower bounds on the set $\mathcal{V}_i^F$.

In Appendix D, I present an algorithm for the fixed-point iteration. The algorithm moves backward through the steps $j = 1, \dots, \ell_i$ of a bid, computing iterates of $\theta_{i,l} \circ \theta_{i,u}$.[18] It starts with the last step, ℓ_i, discretizes the interval $[q_i^{\ell_i - 1}, q_i^{\ell_i}]$, and finds, for each q in the discretized interval, the value v such that the (increasing) function $F^{\ell_i}(b_i, \varphi_u(q, v, \underline{v}_i), \rho)$ evaluates to zero. Denoting by $\tilde{v}_i$ the upper envelope of the functions $\varphi_u(q, v, \underline{v}_i)$ thus obtained, the algorithm then seeks, for each q in the discretized interval, the value v such that the function $F^{\ell_i}(b_i, \varphi_l(q, v, \tilde{v}_i), \rho)$ evaluates to zero. Taking the lower envelope of all functions $\varphi_l(q, v, \tilde{v}_i)$ thus obtained, the algorithm then repeats with this new lower bound, and so on. In other words, the algorithm iteratively seeks, for every relevant quantity, the decreasing function with the highest (lowest) feasible value at that quantity, given the lower (upper) bound from the previous round.

[18]Whether we iterate θ_i or $\theta_{i,l} \circ \theta_{i,u}$ yields the same outcome. Take any fixed point of $\theta_{i,l} \circ \theta_{i,u}$; that is, $v_l = \theta_{i,l} \circ \theta_{i,u}(v_l)$, together with $v_u = \theta_{i,u}(v_l)$. Then $v = (v_l, v_u)$ is a fixed point of $\theta_i(v) = (\theta_{i,l}(v_u), \theta_{i,u}(v_l))$. The converse is also true.

Once convergence is reached, the algorithm moves to the $(\ell_i - 1)$-th step and repeats the procedure, taking the bounds obtained on $[q_i^{\ell_i - 1}, q_i^{\ell_i}]$ as given (in case $\rho = 0$ they are irrelevant). Once convergence on all steps is reached the algorithm stops. From Point (ii) in Proposition 5 and footnote 18 above, together with the fact that, for every $j \in \{1, \dots, \ell_i\}$, the right side of the equality in (7) only depends on the segment of v_i that is on $[q_i^j, Q]$, we conclude that the outcome (v_ℓ, v_u) of this procedure corresponds to the least fixed point of θ_i.

5. Application: The TRQ auctions

This section applies the results from Section 4 to the Swiss TRQ auction data described in Section 2. The data set covers a series of $T = 39$ auctions indexed by $t = 1, \dots, T$. There is a total of 123 registered bidders, taken together in the set N. For every auction t, the data set contains the quota Q_t, the identity of the bidders $N_t \subseteq N$, and their submitted bid functions $B_t \equiv \{\beta_{i,t}\}_{i \in N_t}$. The total number of submitted price-quantity pairs in all 39 auctions is 12,398.

In the following, I first discuss estimating the residual supply distribution. Second, I elaborate on determining risk preferences. And third, I present estimates of import rents and surplus extraction. I conclude with some discussion.

5.1 *Estimating W_i^* and w_i^**

I make two assumptions about the data generating process. Part (i) of the following first assumption is standard. It is necessary for the resampling procedure described below. With Part (ii), I follow Kastl (2011) and consider potential bidder heterogeneity.[19]

(A5.i) Observed bids are mutually independent both within and across auctions.

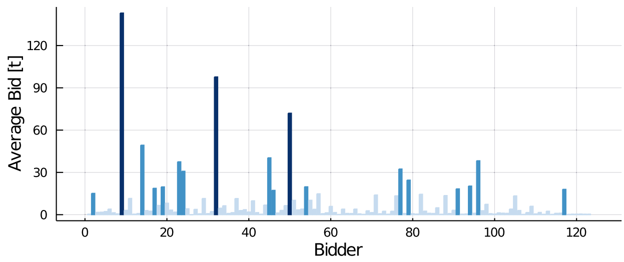
(A5.ii) The bidders in N come in m groups $g = 1, \dots, m$ so that, conditional on an auction t, bids within a group are identically distributed.

Grouping bidders entails a tradeoff. On the one hand, assuming a high number of groups allows capturing a high proportion of bidder heterogeneity. On the other hand, having an increased number of groups reduces the number of bids we can use for estimation (cf. below).

For the data at hand, I resolve this tradeoff by assigning the registered bidders to $m = 3$ groups $g = 1, 2, 3$, based on the average quantities for which they submitted a positive price. We can read off the group assignments in Figure 3. The left panel depicts the average quantity bids for all the 123 registered bidders. The right panel gives an overview of the average bids in the respective groups.[20] Reflecting the structure of the

[19] For a recent discussion of how bidder asymmetry affects equilibrium behavior in uniform price multi-unit auctions both theoretically and in the laboratory, see Hefti and Shen (2019). An experimental paper studying the effect of asymmetric capacity constraints is Sade, Schnitzlein, and Zender (2006).

[20] Other specifications do not seem to alter the results. A systematic analysis of different specifications is computationally too burdensome, though. See Appendix B for more details.



Bidder Groups

Group	#	Average Bid (in kg) Min	Mean	Max
1	105	30	3269	14846
2	15	15144	26712	49324
3	3	72013	104280	143070

FIGURE 3. The chart on the left depicts the average quantity bids of each of the 123 registered bidders. Based on these average quantity bids, I divide the bidders in three groups, where the three different shades indicate the respective group assignment. The table on the right gives an overview of the average bids in the respective groups.

Swiss meat market, there are a few large bidders (3 in total), some medium-sized bidders (15), and a multitude of smaller ones (105). The average number of active bidders in each of the groups across all auctions are 58 for group $g = 1$, 11 for group $g = 2$, and 3 for group $g = 3$.

Assumption (A5.ii) gives us that, in any auction t, we can divide the bidder set N_t into m groups $M_{g,t} \subseteq N_t$, whose bids are identically distributed. Together with Assumptions (A5.i), this implies that any bidder i in a given group g faces the same distribution of opponent demand, $\sum_{j\in N_t\setminus i} \beta_{j,t}^{-1}(p) \geq 0$. I write $D_{g,t}(p)$ for the random opponent demand faced by a bidder $i \in M_{g,t}$ in auction t and make the following assumption on the distribution of $D_{g,t}(p)$.

(A6) For any $p \in (0, \bar{p})$, any group g, and any bidder $i \in M_{g,t}$, the distribution of the aggregate opponent demand $D_{g,t}(p)$ follows a gamma distribution on $\mathbb{R}_+$.

A parametric approach to estimating residual demand is not standard in the literature. The main reason I chose it over the more standard nonparametric approach is that it guarantees full support of $D_{g,t}(p)$ on $\mathbb{R}_+$. This is crucial when it comes to detecting best-response violations. A quantity point q at a price p for which the estimated probability of winning is one is dominated by price-quantity pairs $(\hat{p}, q)$ with $\hat{p} < p$ for which the probability of winning is still one. As I discuss in Remark 4 in Appendix B, such estimates frequently result when using the empirical CDF. The specific choice of the distribution is motivated by the observation that the distribution of $D_{g,t}(p)$ is unimodal for all relevant p. To verify the robustness of (A6), I compare the gamma specification to a log-normal specification in the Supplementary Appendix.

Assumptions (A5.i), (A5.ii), and (A6) allow us to estimate $W_i^*(p, q)$ as follows. For simplicity, suppose for a moment that we only have bidding data from a single auction t. Fix some group g and a bidder $i \in M_{g,t}$ and let $B_{g,t}$ collect all the bids that were submitted by bidders in group g. Then let $B_{g,t}^{|M_{g,t}|}$ be the set of all $|M_{g,t}|$-tuples of $B_{g,t}$, and write $\beta = (\beta_1, \ldots, \beta_{|N_t|-1})$ for an element of $B_{-g} \equiv B_{1,t}^{|M_{1,t}|} \times \cdots \times B_{g-1,t}^{|M_{g-1,t}|} \times B_{g,t}^{|M_{g,t}|-1} \times B_{g+1,t}^{|M_{g+1,t}|} \times \cdots \times B_{m,t}^{|M_{m,t}|}$. Now let $D_\beta(p) = \sum_j \beta_j^{-1}(p)$ be the aggregate opponent demand at a given $p \in (0, \bar{p})$ for some $\beta \in B_{-g}$ and write $\mathbf{D}(p) = \{D_\beta(p)\}_{\beta\in B_{-g}}$ for the set of all aggregate opponent demands at price p. As is well known, the parameters, and hence, the CDF of the gamma distribution can be consistently estimated (e.g., Forbes et al. (2011)).[21] Writing $F(\cdot; \mathbf{D}(p))$ for the estimate of the gamma CDF, we then obtain an estimate of $W_i^*(p, q)$ by computing $\hat{W}_{g,t}(p, q) = F(Q_t - q; \mathbf{D}(p))$. Further, we can estimate w_i^* with

$$\hat{w}^*_{g,t}(p, q) = \frac{\hat{W}^*_{g,t}(p + h, q) - \hat{W}^*_{g,t}(p, q)}{h}, \tag{22}$$

by choosing some small increment $h > 0$, where the data constrain the particular choice of h (again, for more details, cf. Appendix B). Now, if—rather than having a single auction—we have a set of auctions $\mathbf{T} \subset \{1, \dots, T\}$ with identically distributed bid functions within the respective groups for all auctions in $\mathbf{T}$, then this approach can easily be extended to computing the set of opponent demands, $\mathbf{D}(p)$, from all the bid functions submitted in any of the auctions in $\mathbf{T}$.

As is well known, a direct calculation of the estimator $\hat{W}^*_{g,t}$ is computationally infeasible already for a small number of bid functions because the cardinality of $\mathbf{D}(p)$ grows very fast. I thus employ a resampling procedure to approximate $\hat{W}^*_{g,t}$ along the lines of the resampling procedures used in Hortaçsu and McAdams (2010) and Kastl (2011). For every bidder group, the procedure samples bid functions with replacement from a set of available bids. It constructs these sets by collecting bids from auctions with a similar quota, which I take as a proxy for similar covariates.[22] The number of bid functions drawn from a bidder group is equal to the average number of active bidders of that group across all auctions (minus one if bidder i in question belongs to that particular group). Doing so yields one instance of the opponent demand function. Iterating this procedure multiple times then allows to compute a resampled estimate of W_i^* that approaches $\hat{W}_{g,t}$ as the number of iterations grows large (see Kastl (2011) and Appendix B for more explanations).

As regards standard errors, I follow the literature and report bootstrap standard errors (e.g., Kastl (2011)). Other than the literature, however, I resort to *bagging* of the estimates. That is, the reported estimates are the average of the bootstrapped estimates. This also gives a consistent estimate (Breiman (1996)) and is motivated by the fact that the variance of the resampled estimator not only stems from sample variance but also from the sampling procedure itself.

5.2 *Determining risk preferences*

I now discuss how Proposition 4 allows determining risk preferences from the data. Technically, I treat this as a model selection problem. The models between which we want to select differ in their assumptions about the bidders' risk preferences. The number of inequality violations from Proposition 4 serves as a metric for model fit.

[21] To fit the distribution, I use the *Distributions.jl* package (Lin et al. (2019)); see Besançon et al. (2019) for an overview.

[22] This follows an analogous logic to that in Kastl (2011), who collects bids from temporally close auctions. Alternatively, one could use a kernel that computes weights on the auctions from which bid functions are sampled based on the quotas and possibly additional covariates, as in Hortaçsu and McAdams (2010).

Recall that the inequalities (15)–(16) in Proposition 4 provide necessary conditions for best-response behavior. The inequalities in (15) follow from the quantity points' optimality conditions. If one of the inequalities were to fail for a given risk preference ρ and a bid b_i, the set of decreasing profit functions v_i for which the quantity points in b_i are optimal would be empty. The inequalities in (16) follow from the optimality conditions for the price points. As discussed after Proposition 4, if one of these inequalities were to fail for some price-quantity pair j, the bidder could gain by changing p_i^j for any v_i that is consistent with the quantity points' optimality conditions.

To make the inequalities in (15)–(16) operable as a metric for model fit, I assume that the bidders within each group $g = 1, 2, 3$ have the same, group-specific and time-invariant risk-aversion parameter, ρ_g. For a given risk parameter ρ and a bidder $i \in M_{g,t}$ in group g and auction t, let $\hat{\overline{v}}_i^j(\rho)$ and $\hat{\underline{v}}_i^j(\rho)$ be estimates of $\overline{v}_i^j$ and $\underline{v}_i^j$ from Lemma 1, computed by inserting the estimates $\hat{W}_{g,t}^*$ and $\hat{w}_{g,t}^*$ into the respective expressions. Using these estimates, let then $\hat{\overline{v}}_i(\rho)$ and $\hat{\underline{v}}_i(\rho)$ be estimates of the upper and lower bounds $\overline{v}_i$ and $\underline{v}_i$ given in (12) and (13) of Proposition 3.[23]

Then, letting $\ell_{i,t}$ be the number of distinct price-quantity pairs of bidder i in auction t, I am interested in the function

$$\mathrm{BRV}_{Q,g,t}(\rho) \equiv \sum_{i \in M_{g,t}} \sum_{j=1}^{\ell_{i,t}-1} \mathbf{1}\Big\{\min_{m \leq j}\big\{\hat{\overline{v}}_i^m(\rho)\big\} < \max_{m \geq j+1}\big\{\hat{\underline{v}}_i^m(\rho)\big\}\Big\},$$

which returns the total number of violations of the inequality in (15) in group g, given we assume the risk preference in that group to be ρ.

For all "last" price-quantity pairs $j = \ell_{i,t}$ and whenever the inequality in (15) holds for a price-quantity pair $j < \ell_{i,t}$, I additionally check the inequalities in (16) for that j. To this end, I let the set of such price-quantity pairs j be $\hat{L}_{i,t}$ and define

$$\mathrm{BRV}_{P,g,t}(\rho) \equiv \sum_{i \in M_{g,t}} \sum_{j \in \hat{L}_{i,t}} \big[\mathbf{1}\big\{\big(F_i^j\big(b_i, \hat{\underline{v}}_i(\rho), \rho\big) > 0\big) \text{ or } \big(F_i^j\big(b_i, \hat{\overline{v}}_i(\rho), \rho\big) < 0\big)\big\}\big],$$

which returns the total number of violations of the inequalities in (16) in group g, given we assume the risk preference in that group to be ρ.[24]

From these two functions, I want to determine the value of ρ that minimizes the average ratio of violations to the total number of submitted price-quantity pairs by the bidders of group g across all auctions. More precisely, I am interested to find

$$\rho_g^* \in \arg\min_{\rho \geq 0} \ \Theta_{Q,g}(\rho) + \Theta_{P,g}(\rho),$$

[23]Because there are hardly any imports at the over-quota tariff (Loi, Esposti, and Gentile (2016)) and bidders compete to import at the in-quota tariff, I assume that the estimates of the bounds $\overline{v}_i^j$ and $\underline{v}_i^j$ lie below the difference in the over-quota tariff and the in-quota tariff. Specifically, I use $\overline{v} = 20.53$ CHF, which corresponds to the highest spread between the two tariffs across the different subcategories subsuming high-quality beef (cf. footnote 9). Further, the estimated bounds should lie above the price bid p_i^j (cf. Corollary 1). Of course, either $\hat{\overline{v}}_i^j, \hat{\underline{v}}_i^j \leq \overline{v}$ or $\hat{\overline{v}}_i^j, \hat{\underline{v}}_i^j \geq p_i^j$ might fail if the value of ρ that is used for estimation does not correspond to the true risk preference. For this reason, I set the estimate equal to $\overline{v}$ if the value that I obtain with the construction in Lemma 1 exceeds $\overline{v}$ and I set it equal to the price bid if it lies below the price bid.

[24]Here and throughout the following, I compute F_i^j by inserting the estimates $\hat{W}_{g,t}^*$ and $\hat{w}_{g,t}^*$ into (14).

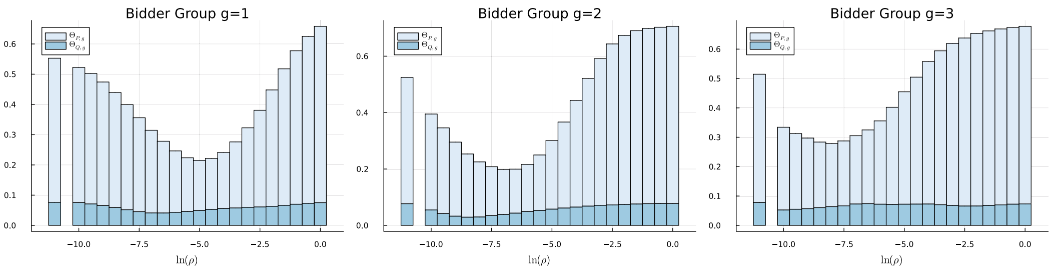


FIGURE 4. The figure shows the estimated shares of violations of the inequalities in (15), $\Theta_{Q,g}$, and (16), $\Theta_{P,g}$, among the bidders of the three groups $g = 1, 2, 3$ for given risk preferences $\ln(\rho)$. Table 2 reports all estimates including standard errors.

where the functions

$$\Theta_{x,g}(\rho) \equiv \frac{1}{\sum_{t=1}^{T} \sum_{i \in M_{g,t}} \ell_{i,t}} \sum_{t=1}^{T} \mathrm{BRV}_{x,g,t}(\rho) \quad \text{for } x \in \{Q, P\}$$

return the average violation ratios for the respective inequality conditions in (15) and (16).

The estimated values of $\Theta_{Q,g}(\rho)$ and $\Theta_{P,g}(\rho)$ from 200 bootstrap estimates for $\rho = 0$ and $\log(\rho) \in \{-10, -9.5, -9, -8.5, \dots, 0\}$ are depicted in Figure 4; the full list of estimates including standard errors can be found in Table 2.

REMARK 3 (Interpretation of ρ_g^*). Strictly speaking, because (15)–(16) are necessary conditions for best response behavior, we cannot take a bid to result from best response behavior as soon as one of the inequalities fails for a bid. Nevertheless, the violations that we observe in reality may also be due to violations of other model assumptions. Possible violations include alternative risk preferences, nonmonotonicities in v_i, misspecified beliefs, and other cognitive limitations.[25] After all, checking for best response violations always amounts to a joint test of the maintained model assumptions. In that sense, we may take the values ρ_g^* as the CARA risk preferences that minimize the need to resort to such alternative explanations.[26]

We can make three main observations. First, risk neutrality gives a bad model fit. For each group, we obtain $\Theta_{Q,g}(0) + \Theta_{P,q}(0) \geq 0.5$; for the largest group, $g = 1$, it is even above 0.55. Second, the violations of (15) as measured by $\Theta_{Q,g}(\rho)$ are relatively few across all groups and risk-aversion parameters, lending support to the assumption

[25]Indeed, if the marginal profits failed to be monotone, then Lemma 1 would not be valid, and the expressions in (12)–(13) would not provide bounds in the first place.

[26]If the maintained model assumptions are correct (this includes the risk preference and that the beliefs of the bidders correspond to the estimated distribution of residual supply in the auctions), then we should not observe any best-response violations at all.

TABLE 2. Best-response violations in the three bidder groups. For each bidder group g, the table depicts the estimates of $\Theta_{Q,g}$ and $\Theta_{P,g}$ as well as their sums together with the sums' bootstrap standard error.

	Bidder Group 1				Bidder Group 2				Bidder Group 3			
ρ	$\Theta_{Q,1}$	$\Theta_{P,1}$	$\Theta_{Q,1}+\Theta_{P,1}$	s.e.	$\Theta_{Q,2}$	$\Theta_{P,1}$	$\Theta_{Q,2}+\Theta_{P,2}$	s.e.	$\Theta_{Q,3}$	$\Theta_{P,3}$	$\Theta_{Q,3}+\Theta_{P,3}$	s.e.
0.0	0.0761	0.477	0.553	0.015	0.0768	0.448	0.525	0.0152	0.0778	0.437	0.515	0.0213
4.54e−5	0.0752	0.447	0.522	0.0141	0.055	0.34	0.395	0.0153	0.0526	0.281	0.334	0.0195
7.49e−5	0.0711	0.431	0.502	0.0139	0.0417	0.304	0.346	0.0152	0.0548	0.258	0.313	0.0198
0.000123	0.0655	0.409	0.474	0.0137	0.0326	0.263	0.295	0.0138	0.0568	0.241	0.297	0.0198
0.000203	0.0589	0.381	0.439	0.0131	0.029	0.224	0.253	0.0134	0.0603	0.223	0.284	0.0209
0.000335	0.0518	0.348	0.399	0.0126	0.0299	0.195	0.225	0.0141	0.0637	0.215	0.279	0.0213
0.000553	0.0453	0.311	0.356	0.0119	0.0345	0.174	0.208	0.0152	0.0667	0.221	0.288	0.0206
0.000912	0.0413	0.273	0.315	0.0115	0.0384	0.16	0.198	0.0163	0.0729	0.232	0.305	0.0187
0.0015	0.0411	0.237	0.278	0.0118	0.0434	0.156	0.2	0.0166	0.0739	0.251	0.325	0.0191
0.00248	0.0432	0.203	0.246	0.012	0.0489	0.168	0.217	0.0161	0.0721	0.284	0.356	0.0187
0.00409	0.0458	0.178	0.224	0.0125	0.0529	0.197	0.25	0.0162	0.0712	0.331	0.402	0.0193
0.00674	0.0489	0.166	0.215	0.013	0.0576	0.243	0.301	0.0163	0.0721	0.383	0.455	0.0181
0.0111	0.0529	0.169	0.221	0.0131	0.0616	0.305	0.367	0.0165	0.0725	0.432	0.505	0.0174
0.0183	0.0556	0.186	0.241	0.0134	0.0649	0.378	0.443	0.0173	0.0729	0.485	0.557	0.0192
0.0302	0.0578	0.218	0.276	0.0132	0.0683	0.452	0.521	0.0189	0.0703	0.524	0.594	0.0178
0.0498	0.0597	0.263	0.323	0.0132	0.071	0.52	0.591	0.0199	0.068	0.552	0.62	0.0169
0.0821	0.0611	0.32	0.381	0.0141	0.0728	0.571	0.644	0.0173	0.066	0.572	0.638	0.0176
0.135	0.063	0.385	0.448	0.0154	0.0744	0.6	0.674	0.0152	0.0661	0.587	0.654	0.0171
0.223	0.0662	0.452	0.518	0.0161	0.0759	0.614	0.69	0.0131	0.0679	0.594	0.662	0.016
0.368	0.0698	0.508	0.577	0.0156	0.0769	0.622	0.698	0.0112	0.07	0.599	0.669	0.0148
0.607	0.0729	0.552	0.625	0.0148	0.0774	0.625	0.703	0.0104	0.0722	0.601	0.673	0.0135
1.0	0.075	0.583	0.658	0.0134	0.0773	0.629	0.706	0.00982	0.0733	0.604	0.678	0.0128

of decreasing marginal profit functions. And third, the functions $\Theta_{P,g}(\rho)$ appear to be U-shaped for all groups.[27]

The estimates give $\rho_3^* = 0.0003 < \rho_2^* = 0.0009 < \rho_1^* = 0.0067$. At these values, we have $\Theta_{Q,1}(\rho_1^*) + \Theta_{P,1}(\rho_1^*) = 0.215$, $\Theta_{Q,2}(\rho_2^*) + \Theta_{P,2}(\rho_2^*) = 0.198$, and $\Theta_{Q,3}(\rho_3^*) + \Theta_{P,3}(\rho_3^*) = 0.279$. That is, the fractions of inequality violations at the respective minima are much lower than those obtained under risk neutrality. The average fraction of violations under the selected risk-aversion parameters across groups can be computed by weighting the minima of $\Theta_{Q,g} + \Theta_{P,g}$ with the average fraction of active bidders from the respective groups; that is, $0.215 \times 58/72 + 0.198 \times 11/72 + 0.279 \times 3/72 = 0.215$. This corresponds to less than one price-quantity pair per bidder (the average number of price-quantity pairs per bid is 4.42; cf. Section 2).

From Table 2, we obtain that the 95% confidence band around $\Theta_g(\rho_g^*) = \Theta_{Q,g}(\rho_g^*) + \Theta_{P,g}(\rho_g^*)$ does not overlap with that around $\Theta_g(\rho_1^*)$ for either of the groups $g = 2, 3$. Moreover, the confidence band around $\Theta_1(\rho_g^*)$ does not overlap with that around $\Theta_1(\rho_1^*)$. We may thus safely conclude that the risk aversion best explaining the behavior of the smallest bidders from group $g = 1$ is higher than the risk aversion best explaining the behavior of the (larger) bidders from groups $g = 2, 3$.[28] This is not surprising. The large bidders in the auctions correspond to larger firms (they are, in some cases, retailers that operate on a national level). They thus have potentially more means to mitigate

[27]As we increase ρ from zero, the estimated bounds $\hat{\bar{v}}_i$ and $\hat{\underline{v}}_i$ will be less tight because the optimality conditions only set identify v_i at the submitted quantity points. Yet, observing a decreasing $\Theta_g(\rho)$ at $\rho = 0$ is not a foregone conclusion. The reason is that both $\hat{\bar{v}}_i$ and $\hat{\underline{v}}_i$ decrease at least over some range (as can be inferred from the lower bid-shading under risk aversion reported in Table 3). On the other hand, it is not surprising that $\Theta_g(\rho)$ eventually grows for large ρ. To see this, pick a bidder i having submitted b_i, a distinct price-quantity pair $j \in \{1, \dots, \ell_i\}$, and suppose $v_i(q) > \beta_{b_i}(q)$ for some nonmeasure-zero set of $q \in [q_i^{j-1}, q_i^j]$. (Because v_i is drawn from a set of Lipschitz-continuous monotone functions, if the number of submitted price-quantity pairs is at least two, $\ell_i \geq 2$, then there always exists at least one price-quantity pair for which this holds.) Then, because $\rho\bar{\Pi}_i^j(b_i, v_i, \rho) \geq 0$, the left side of (8) diverges to infinity as $\rho \to \infty$. But this implies that, for any given uniform Lipschitz constant on the functions in $\mathcal{V}$, no v_i can rationalize b_i as $\rho \to \infty$.

A related question is whether it could be the case that bidders are so risk averse that they bid their true valuation; that is, $\beta_{b_i}(q) = v_i(q)$. Even though discontinuous profit functions v_i are not covered by the assumptions on the type space $\mathcal{V}$, we may still use parts of our equilibrium characterization to safely answer this question in the negative for the data at hand. To see this, pick any $\rho \geq 0$ and observe that the equilibrium interim utility (4) is continuously differentiable in $p_i^j \in (0, \bar{p})$ whenever $w_i^*(p, q)$ exists for the quantities $q \in [q_i^{j-1}, q_i^j]$ and is continuous in p (as all characterization results assume), even if v_i is a decreasing step function. Consequently, the optimality condition (8) holds, from which it must be true that

$$\int_{q_i^{j-1}}^{q_i^j} W_i^*\big(p_i^j, q\big)\,\mathrm{d}q = \big(q_i^j - q_i^{j-1}\big)\rho\overline{\Pi}_i^j(b_i, v_i, \rho),$$

whenever $\beta_{b_i}(q) = v_i(q)$ for all q. However, when $\beta_{b_i}(q) = v_i(q)$ for all q then clearly $\rho\overline{\Pi}_i^j(b_i, v_i, \rho) = 0$. So, for any positive ρ, a marginal profit function being equal to an observed bid can only ever rationalize that bid if the left side of above equality is zero for all relevant j, which is refuted for a given bidder whenever the estimated probability to win at least some nonnegative share is positive.

[28]One idea to obtain proper estimates and confidence intervals of the minimizers ρ_g^* of the respective functions $\Theta_g(\rho)$ is to extend the sampling idea in De Haan (1981) by taking into account that the function to be minimized, $\Theta_g(\rho)$, is itself stochastic. However, this is beyond the scope of the current application in which I am primarily interested in whether and how risk preferences affect rent estimates.

auction-specific risks. Smaller bidders correspond to smaller butcheries that lack these means. Moreover, among the smaller butcheries, the bidders are often also the owners of the firms whose very (economic) existence depends on how their firms fare.

To put the values $(\rho_1^*, \rho_2^*, \rho_3^*)$ into perspective, it is instructive to compute the certainty equivalent for a lottery that pays either zero or CHF $x \geq 0$ with equal probability, $CE(x) = -\frac{1}{\rho_g^*}\ln(\frac{1}{2}(1+\exp(-\rho_g^* x)))$. Using the value $\rho_1^* = 0.0067$ for the smallest bidders (which make up the majority of all bidders), this gives $CE(50) = 23$, $CE(100) = 42$, and $CE(200) = 69$. These certainty equivalents are roughly in line with the average certainty equivalents that Tversky and Kahneman (1992, Table 3) elicited for the corresponding lotteries in laboratory experiments (which were 21, 36, and 76, respectively).

Naturally, the stakes of the meat importers in the auctions are higher than those in these experiments. As we will see in the next section, the estimated net per-kg profit is between 1 and 4 CHF. In bidder group 1, we have an average bid of 3200 kg and a risk preference of $\rho_1^* = 0.0067$. So, suppose a bidder faces a lottery between winning $x = 2000$ CHF and $y = 4000$ CHF with probability one half each. The certainty equivalent is computed as $CE = -\frac{1}{\rho_1^*}\ln(\frac{1}{2}(e^{-\rho_1^* x} + e^{-\rho_1^* y})) = 2103$. A bidder from group 3 with a risk preference of $\rho_3^* = 0.0003$, would have $CE = 2852$. On the other hand, in bidder group 3, we have an average bid of 104,200 kg. Assuming such a bidder faces a lottery between winning $x = 80{,}000$ CHF and $y = 120{,}000$ CHF with probability one-half each, we get $CE = 82{,}310$. For comparison, a bidder from group 1 would have $CE = 80{,}103$. So, risk aversion among bidders is substantial, which will also be reflected in the rent estimates to be discussed in Section 5.3 below.

In any case, the number of violations that I find under the risk-preference parameters $(\rho_1^*, \rho_2^*, \rho_3^*)$ is comparable to the number of best response violations found in Chapman, McAdams, and Paarsch (2006) for Canadian term auctions under risk neutrality. These term auctions are multiunit auctions in which bidders can submit up to four price-quantity bids, yet most bidders just submit one price-quantity bid. Using a discrete bid-space Chapman, McAdams, and Paarsch (2006) check for profitable local deviations and find that 34% of all bids (resp., price-quantity pairs) violate best-response behavior. When using smoothed kernel estimates for the winning probabilities, this number drops to 9%.

Finally, we need to verify that the monotonicity condition in Proposition 4 holds (which is also a prerequisite for the results in the subsequent section). To do so, I first compute, for all bidders and any potential risk preference parameter ρ, estimates of the upper and lower bounds $\overline{v}_i$ and $\underline{v}_i$ given in (12) and (13) as above. Then I employ a simple algorithm that repeatedly checks for every price-quantity point (p_i^j, q_i^j) submitted by that bidder whether $F_i^j(b_i, v_i, \rho) \geq F_i^j(b_i, v_i', \rho)$ holds for two randomly drawn profit function $v_i > v_i'$ between the estimated bounds. Across all the values for ρ that I considered a potential risk preference above, I find that monotonicity holds for $95 - 99\%$ of all the submitted price-quantity pairs. Appendix C provides more details.

5.3 *Risk aversion and tighter bounds*

Accounting for risk aversion is a two-edged sword. As discussed after Corollary 1, equation (7) suggests that we underestimate the firms' profits if we wrongly assume risk neutrality because risk-averse bidders bid closer to their marginal profit function at the submitted quantity points than risk-neutral bidders do. Yet, once we appropriately account for risk aversion, the set of marginal profits between the bounds from Proposition 3 likely becomes larger because the marginal profit function at the submitted quantity points is only set-identified under risk aversion. In the following, I show that this loss in precision is quite substantial. Nevertheless, using the bounds that additionally take the optimality conditions for the price points into account (Proposition 5) fully compensates for this loss.

To this end, I estimate upper and lower bounds on each firm's indirect profits, v_i, in three different ways. First, I estimate the upper and lower bounds (12)–(13) in Proposition 3 under the assumption that the bidders in all three groups are risk neutral, $\vec{\rho} = (\rho_1, \rho_2, \rho_3) = (0, 0, 0)$. Second, I estimate the same bounds under the assumption that all bidders have group-specific risk-aversion parameters $\vec{\rho} = (\rho_1^*, \rho_2^*, \rho_3^*) \equiv \vec{\rho}^*$ from above. And third, I use these estimates as initial condition to calculate the least fixed point, $\wedge \mathcal{F}(\theta_i)$, from Proposition 5.[29] I will refer to the bounds obtained from Proposition 3 as the *standard bounds*, because (as observed above) they reduce to the bounds that are currently used in the literature when $\vec{\rho} = (0, 0, 0)$. On the other hand, I refer to the bounds obtained from Proposition 5 as the *tighter bounds* because they make use of the additional information from the optimality conditions for the price points.

For each of these three types of upper and lower bound estimates, I compute three statistics of interest. Writing $\hat{\bar{v}}_i$ and $\hat{\underline{v}}_i$ for generic upper and lower bound estimates, I am first interested in upper and lower bounds on the (ex post) average per-kg profits before payments,

$$AvP_u^{\text{pre}} = \frac{1}{Q} \sum_{i=1}^{n} \int_0^{q_i^c} \hat{\bar{v}}_i(q)\,\mathrm{d}q \quad \text{and} \quad AvP_l^{\text{pre}} = \frac{1}{Q} \sum_{i=1}^{n} \int_0^{q_i^c} \hat{\underline{v}}_i(q)\,\mathrm{d}q, \tag{23}$$

where Q is the quota in the respective auction, q_i^c is the allocation of bidder i, and n is the number of active bidders. Second, I am interested in bounds on the average per-kg profits after payments,

$$\begin{aligned} AvP_u^{\text{post}} &= \frac{1}{Q} \sum_{i=1}^{n} \int_0^{q_i^c} \left[\hat{\bar{v}}_i(q) - \beta_{b_i}(q)\right] \mathrm{d}q \quad \text{and} \\ AvP_l^{\text{post}} &= \frac{1}{Q} \sum_{i=1}^{n} \int_0^{q_i^c} \left[\hat{\underline{v}}_i(q) - \beta_{b_i}(q)\right] \mathrm{d}q. \end{aligned} \tag{24}$$

And third, I compute bounds on the average shading-to-profit ratios among the bidders. Letting $\overline{n}$ be the number of bidders that have obtained a strictly positive amount for a

[29]To estimate the bounds from Proposition 3, I proceed as in the last section. There is a question of handling estimates when at least one of the necessary conditions in (15) or (16) fails. If the estimates fail (15) for a bid b_i, I take a conservative approach and set $\hat{\bar{v}}_i(q) = \bar{v}$ for all $q \in [0, Q]$ and $\hat{\underline{v}}_i = \beta_{b_i}$. Similarly, the algorithm implementing the bounds from Proposition 5 returns the lowest bounds on $[q_i^{j-1}, q_i^j]$ that are consistent with the initial conditions $(\hat{\underline{v}}_i, \hat{\bar{v}}_i)$ if the right inequality in (16) fails, and the highest consistent bounds if the left inequality in (16) fails. I provide additional explanations in Appendix D. Figure 5 also provides more details about the algorithm.

given auction, this ratio is

$$AvP_u^{\text{rat}} = \frac{1}{\overline{n}} \sum_{i=1}^{\overline{n}} \frac{\int_0^{q_i^c} \left[\hat{\overline{v}}_i(q) - \beta_{b_i}(q)\right] \mathrm{d}q}{\int_0^{q_i^c} \hat{\overline{v}}_i(q)\, \mathrm{d}q} \quad \text{and}$$

$$AvP_l^{\text{rat}} = \frac{1}{\overline{n}} \sum_{i=1}^{\overline{n}} \frac{\int_0^{q_i^c} \left[\hat{\underline{v}}_i(q) - \beta_{b_i}(q)\right] \mathrm{d}q}{\int_0^{q_i^c} \hat{\underline{v}}_i(q)\, \mathrm{d}q}, \tag{25}$$

measuring the average fraction of the import rent across bidders not extracted in the auction.

The bounds on the profits before payments, AvP_u^{pre} and AvP_l^{pre}, give an idea of how competitive the market for imported high-quality beef is in a given quota period. The bounds on the other measures provide an idea of how competitive the respective auctions are in absolute and relative terms. Table 3 gives a summary of my estimates for the individual auctions. The entire set of estimates, including bootstrap standard errors, can be found in the Supplementary Appendix.

I begin by comparing the estimates obtained under risk neutrality to those obtained under risk aversion when using the standard bounds from Proposition 3 (cf. the rows

Table 3. Summary of the estimates for the individual auctions obtained under $\vec{\rho} = 0$ and $\vec{\rho} = \vec{\rho}^*$, the latter both using the standard bounds from Proposition 3 (tight is *no*) and using the tighter bounds from Proposition 5 (tight is *yes*).

Estimate		Tight	$\vec{\rho}$	Mean	Min	Median	Max
Average per-kg profits before auction payments (in CHF)	Upper bounds (AvP_u^{pre})	no	0	16.85	11.43	16.44	20.53
		no	$\vec{\rho}^*$	15.41	10.57	14.07	20.53
		yes	$\vec{\rho}^*$	12.75	7.364	11.45	20.21
	Lower bounds (AvP_l^{pre})	no	0	10.73	5.072	9.636	18.28
		no	$\vec{\rho}^*$	9.459	4.928	8.601	16.43
		yes	$\vec{\rho}^*$	9.715	5.734	8.74	16.65
Average per-kg profits after auction payments (in CHF)	Upper bounds (AvP_u^{post})	no	0	7.931	5.346	7.522	12.59
		no	$\vec{\rho}^*$	6.491	4.166	6.049	10.69
		yes	$\vec{\rho}^*$	3.834	1.284	3.723	8.616
	Lower bounds (AvP_l^{post})	no	0	1.816	0.1077	0.7469	8.24
		no	$\vec{\rho}^*$	0.5449	0.01277	0.09349	4.236
		yes	$\vec{\rho}^*$	0.8009	0.03401	0.3044	5.087
Average shading-to-profit ratios	Upper bounds (AvP_u^{ratio})	no	0	0.4853	0.2838	0.5023	0.6541
		no	$\vec{\rho}^*$	0.4478	0.2838	0.4496	0.5713
		yes	$\vec{\rho}^*$	0.2811	0.1721	0.2806	0.4437
	Lower bounds (AvP_l^{ratio})	no	0	0.1176	0.0137	0.06406	0.4365
		no	$\vec{\rho}^*$	0.03209	0.003632	0.01285	0.1944
		yes	$\vec{\rho}^*$	0.0611	0.006088	0.03721	0.2973

with *Tight* set to *no*). Taking risk aversion into account has a substantial effect on the lower bounds of all three statistics yet only a small effect on the upper bound. Specifically, the mean estimate of AvP_l^{post} falls from 1.82 CHF/kg to 0.54 CHF/kg, corresponding to a drop of more than 70%. The estimates of AvP_l^{rat} fall from 0.12 to 0.032. Moreover, the estimates of the earnings before payment, AvP_l^{pre}, fall from 10.73 CHF/kg to 9.46 CHF/kg.

Once we use the tighter bounds from Proposition 5, though, we do see a substantial effect on the upper bounds, too. For $\vec{\rho} = \vec{\rho}^*$, the average tighter upper bounds for all three measures are well below the standard bounds from Proposition 3 (cf. the rows where *Tight* is *yes* vs. those where *Tight* is *no*). For example, the average earnings after payments are estimated at 6.49 CHF/kg with the standard bounds, while they are estimated at 3.83 CHF/kg with the tighter bounds. A similar decrease, by roughly 37%, is reported for the shading-to-profit ratio.

Moreover, the range of possible profits and shading-to-profit ratios is considerably smaller under the tighter than the standard bounds. For example, the difference in the mean estimates of the upper and lower bounds on the average profits before payments, AvP_u^{pre} and AvP_l^{pre}, is CHF 5.95 under the standard bounds, yet only CHF 3 under the tighter bounds. This corresponds to a drop of almost 50%. Considering the additional information from the optimality conditions for the price points thus compensates for the loss in precision when accounting for risk aversion.

Figure 5 shows how using the optimality conditions for the price points shrinks the set of possible marginal profit functions. The figure depicts upper and lower bounds estimates under risk aversion, $\vec{\rho} = \vec{\rho}^*$, for two different bidders in two separate auctions. Each graph shows the estimates of both the standard bounds from Proposition 3 and the tighter bounds from Proposition 5, including the range between the 5th and the 95th percentile of the respective bootstrap estimates in a lighter color. For either bidder, the tighter upper bounds lie well below the standard upper bounds, and the tighter lower bounds are considerably higher than the standard lower bounds.

Finally, the profit and shading estimates do not differ between bidder groups (cf. Appendix E for more details). Specifically, small butcheries earn roughly the same per allocated kg of high-quality beef imports as the large retailers do. Moreover, the fraction of the import rent captured by the auctions does not depend on bidder size, either.

5.4 *Discussion of the empirical results*

The import rents created by the TRQ are considerable. The average estimated net margin on imported beef lies between CHF 9.72 and 12.75 per kg (cf. Table 3). If we weigh these numbers against the average retail price of sirloin steak during the auctions (which was CHF 60.67, as mentioned above), then we obtain an average net profit margin of 16–21%. As a comparison, the average profit margin in the global retailing industry is generally assumed to lie between 2.5–3.5%.[30] Moreover, the gross margin on domestically produced beef in Switzerland is roughly CHF 10 per kg (Bokusheva et al. (2019), Figure 6.2),

[30]Compare the Global Power of Retailing Reports by Deloitte, 2015–2019; https://www2.deloitte.com/global/en/pages/consumer-business/articles/global-powers-of-retailing.html

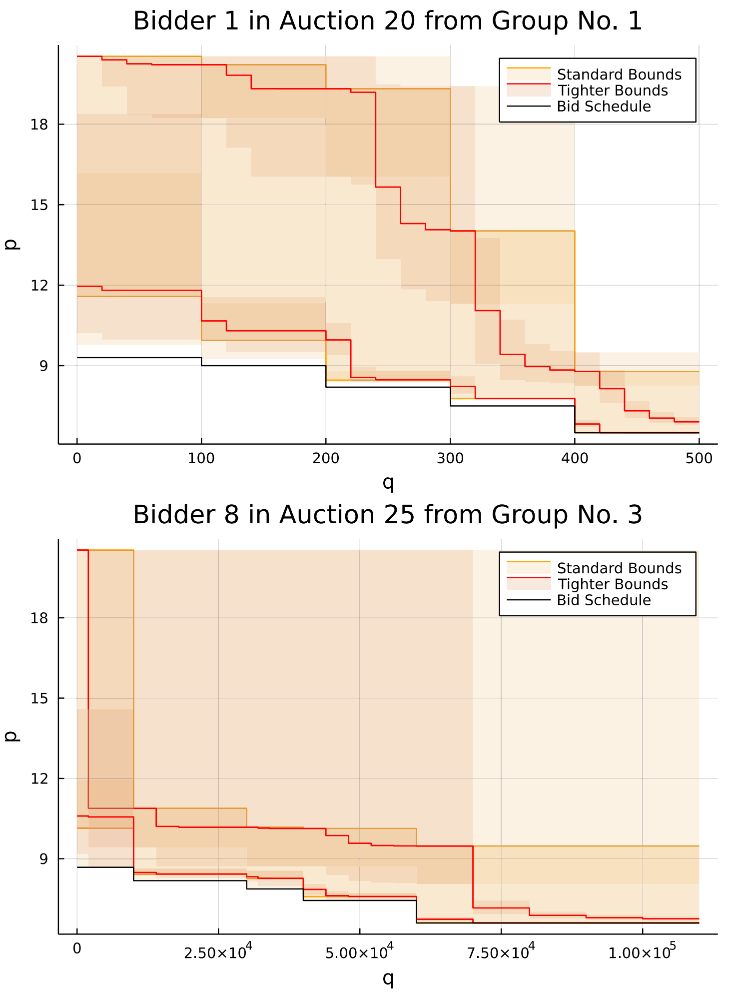


FIGURE 5. The two graphs show estimated upper and lower bounds on the marginal profits v_i for two different bidders under risk aversion, $\vec{\rho} = \vec{\rho}^*$. The top panel shows a bidder from bidder group $g = 1$ in Auction no. 20, and the bottom panel shows a bidder from group $g = 3$ in Auction no. 25. The tighter bounds correspond to the least fixed point $\wedge\mathcal{F}(\theta_i)$ from Proposition 5, while the standard bounds correspond to the bounds from Proposition 3. To obtain an idea of the estimates' variance, the shaded areas depict the range between the 5th and the 95th percentile of 200 bootstrap estimates. The algorithm to compute the tighter bounds divides the segment between the submitted quantity points in 5 (clearly visible) subintervals in which it takes the bounds as constant. The algorithm is constructed such that the computed fixed point is a lower bound on the actual least fixed point, $\wedge\mathcal{F}(\theta_i)$, and converges to that fixed point as the number of subintervals grows large; for details, see Appendix D.

giving a gross profit margin on domestic beef of 16%. Given that this equals the lower bound on the net profit margin on imported beef, it should not be surprising that the interest in importing high-quality beef is so considerable.

Nevertheless, the shading-to-profit ratio estimates suggest that between 72% and 94% of the average profit per kg is captured by the auctions, indicating that the auctions work pretty well in distributing import rents back to the general public. Such an assessment is close but not as impressive as the results from treasury bill auctions, where the ex post surplus of the bidders (i.e., the ex post monetary gain from the auction net of payment) tends to be very small. For example, Kastl (2011) reports for Czech treasury bill auctions that all but 3 basis points of bidder surplus are captured. Similar results also hold for other treasury bill auctions studied in the literature (Kastl (2017)).

A natural question is whether the uniform payment rule would be superior to the discriminatory payment rule. Unfortunately, this is an open question both theoretically (e.g., Pycia and Woodward (2020)) and empirically (e.g., Chapman, McAdams, and Paarsch (2007)). Answering this question in the current context would require computing counterfactual bids, necessitating an analytical solution of the equilibrium bidding strategies under the uniform payment rule. This is a tough problem, especially because risk aversion is involved. I leave this question for further research.

6. Concluding remarks

Despite being a convenient assumption, risk neutrality cannot always be taken for granted in real-world auctions (Li, Lu, and Zhao (2015), Bolotnyy and Vasserman (2019), Luo and Takahashi (2019), Aryal et al. (2022), Kong (2020)). In this paper, I analyzed share auctions used by the Swiss government to sell tariff-rate quotas on meat imports. I found that assuming (constant absolute) risk aversion rather than risk neutrality yields a better explanation for the data and that accounting for risk aversion considerably affects profit estimates. Having accurate estimates is important to assess how much rent a given tariff-rate quota generates and how well the auctions perform in distributing these rents to the general public.

For my analysis, I introduced risk aversion to a discriminatory k-step share auction *á la* Kastl (2012). I showed that the optimality conditions of the bidders allow (1) to determine the bidders' CARA parameter from the data, (2) to set identify the bidders' marginal profits at the submitted quantity points, and (3) to provide tighter bounds on the marginal profits between the quantity points. The key insight is that the optimality conditions for the price points contain valuable information when determining risk preferences and obtaining estimates of the marginal profits.

Risk-averse bidders choose their price points closer to the marginal profit than risk-neutral bidders. Thus, not accounting for risk aversion results in positively biased estimates of the bidders' marginal profits. For the Swiss meat tariff-rate quota auctions, this bias is substantial. For example, properly accounting for risk aversion reduces average profits per kg estimates after auction payments by as much as 55%, namely the estimates obtained under risk neutrality. Moreover, I showed that using the information from the optimality conditions for the price points substantially reduces the set of marginal profits, and hence, the range of rents that can rationalize the data.

My findings are likely to be important for further empirical studies on share auctions, too, especially when risk aversion is a concern. Of course, the focus on CARA preference is quite restrictive, as is the focus on the discriminatory payment rule. Together, these assumptions imply that inframarginal quantities do not play a role in the optimality of a given price-quantity pair. This was the key observation for formulating the bounds in Propositions 3 and Propositions 5. Further research is required to extend these results to other risk preferences and payment rules.

Appendix A: Proofs

A.1 *Proofs of Section* 3

Proof of Proposition 1. See the Supplementary Appendix. □

Proof of Proposition 2. Because ties happen with zero probability in equilibrium, a bidder's interim utility $\Pi_i(b_i, v_i, \mu^*_{-i})$ when submitting a bid b_i and all other bidders play according to their equilibrium strategies in μ^* can be written as

$$\Pi_i\big(b_i, v_i, \mu^*_{-i}\big) = \int_0^Q \phi\big(V_i(q) - B_i(q)\big)\, \mathrm{d}\big[1 - W_i^*\big(\beta_{b_i}(q), q\big)\big]. \tag{26}$$

Because $\phi(V_i(q) - B_i(q))$ is continuous and $[1 - W_i^*(\beta_{b_i}(q), q)]$ is increasing in q, the inner integral of the right-hand side in (26) can be integrated by parts (cf. Apostol (1974), Theorem 7.6), yielding

$$\begin{aligned}\Pi_i\big(b_i, v_i, \mu^*_{-i}\big) = &-\int_0^Q \phi'\big(V_i(q) - B_i(q)\big)\big[v_i(q) - \beta_{b_i}(q)\big]\big[1 - W_i^*\big(\beta_{b_i}(q), q\big)\big]\,\mathrm{d}q \\ &+ \phi\big(V_i(q) - B_i(q)\big)\big[1 - W_i^*\big(\beta_{b_i}(q), q\big)\big]\big|_0^Q,\end{aligned} \tag{27}$$

which, using $\phi(0) = 0$, is equal to

$$\Pi_i\big(b_i, v_i, \mu^*_{-i}\big) = \sum_{j=1}^{k+1} \int_{q_i^{j-1}}^{q_i^j} \phi'\big(V_i(q) - B_i(q)\big)\big[v_i(q) - \beta_{b_i}(q)\big]W_i^*\big(p_i^j, q\big)\,\mathrm{d}q. \tag{28}$$

Then, putting multiplier λ_j on the constraint that $p_i^j \le p_i^{j-1}$ and multiplier φ_j on the constraint that $q_i^j \ge q_i^{j-1}$ for $j = 1, \dots, k+1$, the Lagrangian associated with agent i's optimizing problem is given by

$$\begin{aligned}L = &\sum_{j=1}^{k+1} \int_{q^{j-1}}^{q^j} \phi'\big(V_i(q) - B_i(q)\big)\big[v_i(q) - p_i^j\big]W_i^*\big(p_i^j, q\big)\,\mathrm{d}q \\ &- \sum_{j=1}^{k+1} \lambda_j\big(p_i^j - p_i^{j-1}\big) - \sum_{j=1}^{k+1} \varphi_j\big(q_i^{j-1} - q_i^j\big),\end{aligned} \tag{29}$$

where $p_i^{k+1} = q_i^0 = 0$, $p_i^0 = \overline{p}$, and $q_i^{k+1} = Q$. Because (i) $v_i(q)$ is continuous, (ii) $W_i^*(p_i^j, q_i^j)$ is continuous at q_i^j by the absence of ties in equilibrium, and (iii) w_i^* is continuous in p by assumption, L is continuously differentiable. Then the optimality conditions obtained by setting $\partial L/\partial q_i^j = 0$ for some $j = 1, \dots, k$ are given by

$$\begin{aligned}
&\phi'\big(V_i(q_i^j) - B_i(q_i^j)\big)\big[[v_i(q_i^j) - p_i^j]W_i^*(p_i^j, q_i^j) - [v_i(q_i^j) - p_i^{j+1}]W_i^*(p_i^{j+1}, q_i^j)\big] \\
&\quad - \sum_{m=j}^{k} \int_{q_i^m}^{q_i^{m+1}} \phi''\big(V_i(q) - B_i(q)\big)\big[p_i^j - p_i^{j+1}\big]\big[v_i(q) - p_i^{m+1}\big]W_i^*\big(p_i^{m+1}, q\big)\, \mathrm{d}q \\
&\quad - (\varphi_{j+1} - \varphi_j) = 0.
\end{aligned} \tag{30}$$

If neither of the constraints $q_i^{j+1} \geq q_i^j$ and $q_i^j \geq q_i^{j-1}$ binds, then condition (30) holds with $\varphi_{j+1} = \varphi_j = 0$. If, however, the constraint $q_i^j \geq q_i^{j-1}$ binds (the argument for the other constraint, $q_i^{j+1} \geq q_i^j$, is analogous), then the optimality condition (30) for step j is given by

$$\begin{aligned}
&\phi'\big(V_i(q_i^j) - B_i(q_i^j)\big)\big[[v_i(q_i^j) - p_i^j]W_i^*(p_i^j, q_i^j) - [v_i(q_i^j) - p_i^{j+1}]W_i^*(p_i^{j+1}, q_i^j)\big] \\
&\quad - \sum_{m=j}^{k} \int_{q_i^m}^{q_i^{m+1}} \phi''\big(V_i(q) - B_i(q)\big)\big[p_i^j - p_i^{j+1}\big]\big[v_i(q) - p_i^{m+1}\big]W_i^*\big(p_i^{m+1}, q\big)\, \mathrm{d}q \\
&\quad + \varphi_j = 0.
\end{aligned} \tag{31}$$

and the optimality condition (30) for step $j-1$ is given by

$$\begin{aligned}
&\phi'\big(V_i(q_i^{j-1}) - B_i(q_i^{j-1})\big) \\
&\quad \times \big[[v_i(q_i^{j-1}) - p_i^j]W_i^*(p_i^{j-1}, q_i^{j-1}) - [v_i(q_i^{j-1}) - p_i^j]W_i^*(p_i^j, q_i^{j-1})\big] \\
&\quad - \sum_{m=j-1}^{k} \int_{q_i^m}^{q_i^{m+1}} \phi''\big(V_i(q) - B_i(q)\big)\big[p_i^{j-1} - p_i^j\big]\big[v_i(q) - p_i^{m+1}\big]W_i^*\big(p_i^{m+1}, q\big)\, \mathrm{d}q \\
&\quad - \varphi_j = 0.
\end{aligned} \tag{32}$$

Adding (31) and (32) and taking into account that $q_i^{j-1} = q_i^j$, we arrive at

$$\begin{aligned}
&\phi'\big(V_i(q_i^{j-1}) - B_i(q_i^{j-1})\big) \\
&\quad \times \big[[v_i(q_i^{j-1}) - p_i^j]W_i^*(p_i^{j-1}, q_i^{j-1}) - [v_i(q_i^{j-1}) - p_i^{j+1}]W_i^*(p_i^{j+1}, q_i^{j-1})\big] \\
&\quad - \sum_{m=j-1}^{k} \int_{q_i^m}^{q_i^{m+1}} \phi''\big(V_i(q) - B_i(q)\big)\big[p_i^{j-1} - p_i^{j+1}\big]\big[v_i(q) - p_i^{m+1}\big]W_i^*\big(p_i^{m+1}, q\big)\, \mathrm{d}q \\
&= 0.
\end{aligned} \tag{33}$$

Analogous operations can be performed when more than one adjacent constraint binds. Also, observe that above equation trivially holds when $p_i^{j-1} = p_i^{j+1}$; that is, whenever the inequality constraint w.r.t. the price points binds. Finally, we only consider price-quantity pairs with $q_i^j \in (0, Q)$. Hence, (2) follows by appropriately relabeling the observed bid points, as discussed in Definition 2.

Turning to the inequality constraint w.r.t. the price points we see, because the derivative of W_i^* with respect to p is assumed to exist whenever necessary, that $\partial L/\partial p_i^j = 0$ is equivalent to

$$\begin{aligned}&\int_{q^{j-1}}^{q^j} \left[\phi'\big(V_i(q) - B_i(q)\big)\left[\left[v_i(q) - p_i^j\right] w_i^*\big(p_i^j, q\big) - W_i^*\big(p_i^j, q\big)\right]\right. \\ &\quad \left. - \phi''\big(V_i(q) - B_i(q)\big)\left[q - q_i^{j-1}\right]\left[v_i(q) - p_i^j\right] W_i^*\big(p_i^j, q\big)\right] \mathrm{d}q \\ &\quad - \sum_{m=j}^{k} \int_{q_i^m}^{q_i^{m+1}} \phi''\big(V_i(q) - B_i(q)\big)\left[q_i^j - q_i^{j-1}\right]\left[v_i(q) - p_i^{m+1}\right] W_i^*\big(p_i^m, q\big)\, \mathrm{d}q \\ &\quad - (\lambda_j - \lambda_{j+1}) = 0. \end{aligned} \tag{34}$$

If neither of the constraints $p_i^{j+1} \le p_i^j$ and $p_i^j \le p_i^{j-1}$ binds, then the condition (34) with $\lambda_{j+1} = \lambda_j = 0$. If, however, the constraint $p_i^j \le p_i^{j-1}$ binds (the argument when constraint $p_i^{j+1} \le p_i^j$ binds is equivalent), then by a similar manipulation as for the quantity points above to get (33), we get

$$\begin{aligned}&\int_{q^{j-1}}^{q^{j+1}} \left[\phi'\big(V_i(q) - B_i(q)\big)\left[\left[v_i(q) - p_i^{j+1}\right] w_i^*\big(p_i^{j+1}, q\big) - W_i^*\big(p_i^{j+1}, q\big)\right]\right. \\ &\quad \left. - \phi''\big(V_i(q) - B_i(q)\big)\left[q - q_i^{j-1}\right]\left[v_i(q) - p_i^{j+1}\right] W_i^*\big(p_i^{j+1}, q\big)\right] \mathrm{d}q \\ &\quad - \sum_{m=j+1}^{k} \int_{q_i^m}^{q_i^{m+1}} \phi''\big(V_i(q) - B_i(q)\big)\left[q_i^{j+1} - q_i^{j-1}\right]\left[v_i(q) - p_i^{m+1}\right] W_i^*\big(p_i^m, q\big)\, \mathrm{d}q = 0. \end{aligned}$$

where, again, analogous operations can be performed when more than one adjacent constraint binds. Moreover, observe that the equality above trivially holds when $q_i^{j-1} = q_i^{j+1}$; that is, when the constraint on the quantity points binds. Finally, we only consider price-quantity points with $p_i^j \in (0, \overline{p})$. Hence, (3) follows by appropriately relabeling the observed bid points, as discussed in Definition 2. □

A.2 *Proofs of Section* 4

Proof of Corollary 1. For $\rho = 0$, ϕ is linear, and hence, both (7)–(8) follow trivially from (2)–(3). For $\rho > 0$, the equality in (7) follows by rearranging (2), appreciating the fact that the assumption of CARA utility, (5), yields

$$\frac{\phi''\big(V_i(q) - B_i(q)\big)}{\phi'\big(V_i\big(q_i^j\big) - B_i\big(q_i^j\big)\big)} = -\rho\phi'\Big(V_i(q) - B_i(q) - \big(V_i\big(q_i^j\big) - B_i\big(q_i^j\big)\big)\Big),$$

and using the definition of $\overline{\Pi}_i^j$ in (6). The equality in (8) follows analogously by additionally appreciating that under (5), we have

$$\frac{\phi'\big(V_i(q) - B_i(q)\big)}{\phi'\big(V_i\big(q_i^j\big) - B_i\big(q_i^j\big)\big)} = \phi'\Big(V_i(q) - B_i(q) - \big(V_i\big(q_i^j\big) - B_i\big(q_i^j\big)\big)\Big).$$

Last, the inequality in (7) holds, because for $j = \ell_i$, we have $\overline{\Pi}_i^j = 0$, and, for $j < \ell_i$ we have

$$
\begin{aligned}
& W_i^*\left(p_i^{j+1}, q_i^j\right) - \rho \overline{\Pi}_i^j(b_i, v_i) \\
& \quad = W_i^*\left(p_i^{j+1}, q_i^j\right) - \sum_{m=j}^{\ell_i} \int_{q_i^m}^{q_i^{m+1}} \rho \exp\left(-\rho\left(\int_{q_i^j}^{q} \left[v_i(x) - \beta_{b_i}(x)\right] \mathrm{d}x\right)\right)\left[v_i(q) - p_i^{m+1}\right] \\
& \qquad \times W\left(p_i^{m+1}, q\right) \mathrm{d}q \\
& \quad \geq W_i^*\left(p_i^{j+1}, q_i^j\right)\left[1 - \sum_{m=j}^{\ell_i - 1} \int_{q_i^m}^{q_i^{m+1}} \rho \exp\left(-\rho\left(\int_{q_i^j}^{q} \left[v_i(x) - \beta_{b_i}(x)\right] \mathrm{d}x\right)\right)\right. \\
& \qquad \left. \times \left[v_i(q) - p_i^{m+1}\right] \mathrm{d}q\right] \\
& \quad = W_i^*\left(p_i^{j+1}, q_i^j\right)\left[1 - \sum_{m=j}^{\ell_i - 1}\left[-\exp\left(-\rho\left(\int_{q_i^j}^{q} \left[v_i(x) - \beta_{b_i}(x)\right] \mathrm{d}x\right)\right)\Big|_{q_i^m}^{q_i^{m+1}}\right]\right] \\
& \quad = W_i^*\left(p_i^{j+1}, q_i^j\right)\left[1 - \left[1 - \exp\left(-\rho\left(\int_{q_i^j}^{q^{\ell_i}} \left[v_i(x) - \beta_{b_i}(x)\right] \mathrm{d}x\right)\right)\right]\right] \geq 0,
\end{aligned}
$$

where the first inequality follows from the fact that $W_i(p, q)$ is increasing in p and decreasing in q, $W_i(p_i^{\ell_i+1}, q) = 0$, and it must be that $p_i^j > p_i^{j+1}$ for all steps j. □

Proof of Lemma 1. To begin, observe that we can use the integration-by-parts argument from the proof of Proposition 2 to obtain

$$
\overline{\Pi}_i^j(b_i, v_i, \rho) = \int_{q_i^j}^{Q} \phi\left(\int_{q_i^j}^{q} \left[v_i(x) - \beta_{b_i}(x)\right] \mathrm{d}x\right) \mathrm{d}\left[1 - W_i^*\left(\beta_{b_i}(q), q\right)\right],
$$

which is increasing in v_i. Consequently, the claim follows from the following observations:

1. As a consequence of $\overline{\Pi}^{\ell_i}(b_i, v_i, \rho) = W(p_i^{\ell_i+1}, q_i^{\ell_i}) = 0$, the equation in (7) gives $v_i(q_i^{\ell_i}) = p_i^{\ell_i}$, implying that the statement for $j = \ell_i$, $\underline{v}_i^{\ell_i} = \overline{v}_i^{\ell_i} = p_i^j$, is correct.

2. Next, consider $j = \ell_i - 1$. The construction of $v_l(q)$ together with $\underline{v}_i^{\ell_i}$ obtained above guarantees that $v_l(q)$ is a pointwise lower bound on the profit v_i on $[q_i^{\ell_i-1}, q_i^{\ell_i}]$. Thus, if we insert this lower bound v_l in $\overline{\Pi}^{\ell_i-1}$, then by the fact that $\overline{\Pi}^j(b_i, v_i, \rho)$ is increasing in v_i, the equation in (7) provides us with an upper bound $\overline{v}_i^{\ell_i-1}$ on $v_i(q_i^{\ell_i-1})$, giving us $\overline{v}_i^{\ell_i-1} \geq v_i(q_i^{\ell_i-1})$ as desired.

3. Because $\overline{v}_i^{\ell_i-1}$ is an upper bound on $v_i(q_i^{\ell_i-1})$, $v_u(q)$ is a pointwise upper bound on the profit v_i on $[q_i^{\ell_i-1}, q_i^{\ell_i}]$. Thus, inserting v_u in the right-hand side of the equation in (7) yields a lower bound $\underline{v}_i^{\ell_i-1}$ on $v_i(q_i^{\ell_i-1})$, giving us $\underline{v}_i^{\ell_i-1} \leq v_i(q_i^{\ell_i-1})$ as desired.

4. Repeating the steps 2 – 3 until the first price-quantity pair, $j = 1$, then yields upper and lower bounds for any $v_i(q_i^j)$; that is, $\underline{v}_i^j \leq v_i(q_i^j) \leq \overline{v}_i^j$ for all $j = 1, \dots, \ell_i - 2$, as desired. □

Proof of Proposition 5. I proceed in two steps. First, I prove parts (i)–(ii) and then part (iii).

Parts (i)–(ii): As $\tilde{\mathcal{V}}_i$ is a complete lattice both under the partial order given in Definition 3 as well as under the corresponding reversed order, the product set $\tilde{\mathcal{V}}_i \times \tilde{\mathcal{V}}_i$ is also a complete lattice under the partial order given in Definition 4. Further, $\theta_{i,u}$ maps $\tilde{\mathcal{V}}_i$ into $\tilde{\mathcal{V}}_i$, $\theta_{i,l}$ maps $\tilde{\mathcal{V}}_i$ into $\tilde{\mathcal{V}}_i$ as discussed in the text. Moreover, both $\theta_{i,u}$ and $\theta_{i,l}$ are order-reversing under the order on $\tilde{\mathcal{V}}_i$. To see this, it is enough to appreciate that both $\theta_{i,u}$ and $\theta_{i,l}$ are nonincreasing under the pointwise partial order (cf. Definition 3), which follows from the facts that both φ_u and φ_l are nondecreasing in each of their three arguments and that F_i^j is nondecreasing in its second argument. Hence, by the order on $\tilde{\mathcal{V}}_i \times \tilde{\mathcal{V}}_i$, the function $\theta_i = (\theta_{i,l}, \theta_{i,u}) : \tilde{\mathcal{V}}_i \times \tilde{\mathcal{V}}_i \to \tilde{\mathcal{V}}_i \times \tilde{\mathcal{V}}_i$ is order-preserving. Then, because θ_i is continuous it is ω-continuous in the sense of Baranga (1991), and thus the statement of Kleene's fixed-point theorem therein applies, giving us parts (i) and (ii) directly.

Part (iii): It suffices to establish that $(\underline{v}_i^F, \overline{v}_i^F) \in \mathcal{F}(\theta_i)$. To do so, I report three observations that together give us the claim.

Observation I: It must hold for all $j \in \{1, \dots, \ell_i\}$ and $\tilde{q} \in [q_i^{j-1}, q_i^j]$ that the functions $v_{\tilde{q}}^u : [0, Q] \to \mathbb{R}$ and $v_{\tilde{q}}^l : [0, Q] \to \mathbb{R}$ defined as

$$v_{\tilde{q}}^u(v)(q) = \begin{cases} \max\{v, \underline{v}_i^F(q)\} & \text{if } q \leq \tilde{q}, \\ \underline{v}_i^F(q) & \text{if } q > \tilde{q} \end{cases} \quad \text{and} \quad v_{\tilde{q}}^l(v)(q) = \begin{cases} \overline{v}_i^F(q) & \text{if } q \leq \tilde{q}, \\ \min\{v, \overline{v}_i^F(q)\} & \text{if } q > \tilde{q} \end{cases}$$

satisfy

$$F_i^j\big(b_i, v_{\tilde{q}}^u\big(\overline{v}_i^F(\tilde{q})\big), \rho\big) \leq 0 \quad \text{and} \quad F_i^j\big(b_i, v_{\tilde{q}}^l\big(\underline{v}_i^F(\tilde{q})\big), \rho\big) \geq 0. \tag{35}$$

To see this, suppose not; that is, suppose there is $\tilde{q} \in [q_i^{j-1}, q_i^j]$ such that either or both of above inequalities is violated. Specifically, assume that $F_i^j(b_i, v_{\tilde{q}}^u(\overline{v}_i^F(\tilde{q})), \rho) > 0$ holds (the argument for the other inequality is analogous). Because F_i^j is increasing and continuous in its second argument, there is an alternative decreasing function $\hat{v}$ satisfying $\underline{v}_i^F \leq \hat{v} \leq \overline{v}_i$, $\hat{v}(\tilde{q}) < \overline{v}_i^F(\tilde{q})$ and $F_i^j(b_i, \hat{v}, \rho) > 0$. Together with the fact that $v_{\tilde{q}}^u(\overline{v}_i^F(\tilde{q}))$ is the lowest decreasing function above $\underline{v}_i^F$ that goes through the point $(\tilde{q}, \overline{v}_i^F(\tilde{q}))$ this contradicts the assumption that $\overline{v}_i^F$ is a pointwise least upper bound on the decreasing functions $\overline{v}_i \geq v_i \geq \underline{v}_i$ that satisfy $F_i^j(b_i, v_i, \rho) = 0$ for all $j = 1, \dots, \ell_i$.

Observation II: For all $j \in \{1, \dots, \ell_i\}$ and any $\tilde{q} \in [q_i^{j-1}, q_i^j]$, the left inequality in (35) is strict only if $\overline{v}_i^F(\tilde{q}) = \overline{v}_i(\tilde{q})$. By contradiction, suppose the left inequality (35) holds with strict inequality and we have $\overline{v}_i^F(\tilde{q}) < \overline{v}_i(\tilde{q})$. Because F_i^j is increasing and continuous in its second argument, there is an alternative decreasing function $\hat{v}$ satisfying $\underline{v}_i^F \leq \hat{v} \leq \overline{v}_i$, $\hat{v}(\tilde{q}) > \overline{v}_i^F(\tilde{q})$ and $F_i^j(b_i, \hat{v}, \rho) < 0$. Together with the fact that $v_{\tilde{q}}^u(\overline{v}_i^F(\tilde{q}))$ is the lowest decreasing function above $\underline{v}_i^F$ that goes through the point $(\tilde{q}, \overline{v}_i^F(\tilde{q}))$ this contradicts the assumption that $\overline{v}_i^F$ is a pointwise least upper bound on the decreasing functions $\overline{v}_i \geq$

$v_i \geq \underline{v}_i$ that satisfy $F_i^j(b_i, v_i, \rho) = 0$ for all $j = 1, \ldots, \ell_i$. By an analogous observation for $v_{\tilde{q}}^l(\underline{v}_i^F(\tilde{q}))$, we get that the right inequality in (35) can only be strict if $\underline{v}_i^F(\tilde{q}) = \underline{v}_i(\tilde{q})$.

Observation III: If, for any $j \in \{1, \ldots, \ell_i\}$ and $\tilde{q} \in [q_i^{j-1}, q_i^j]$, the left inequality in (35) holds with equality, then it must hold

$$\overline{v}_i^F(\tilde{q}) = \inf\{v \in [\overline{v}_i^F(q_i^j), \overline{v}_i(\tilde{q})] : F_i^j(b_i, v_{\tilde{q}}^u(v), \rho) > 0\}.$$

To see this, suppose the left inequality of (35) holds with equality but the equality above is not true. This implies that there is a value $\hat{v} > \overline{v}_i^F(\tilde{q})$ such that $F_i^j(b_i, \hat{v}_{\tilde{q}}^u(\hat{v}), \rho) = 0$. But this contradicts the assumption that $\overline{v}_i^F$ is a pointwise least upper bound on the decreasing functions $\overline{v}_i \geq v_i \geq \underline{v}_i$ that satisfy $F_i^j(b_i, v_i, \rho) = 0$ for all $j = 1, \ldots, \ell_i$. By an analogous argument, we obtain that, if for any $\tilde{q} \in [q_i^{j-1}, q_i^j]$, the right inequality in (35) holds with equality then it must hold $\underline{v}_i^F(\tilde{q}) = \sup\{v \in [\underline{v}_i^F(q_i^j), \overline{v}_i(\tilde{q})] : F_i^j(b_i, v_{\tilde{q}}^l(v), \rho) < 0\}$.

In view of these three observations, inspection of the constructions of $\theta_{i,u}$ and $\theta_{i,l}$ in (21)–(21) then gives us that $(\underline{v}_i^F, \overline{v}_i^F) = (\theta_{i,l}(\overline{v}_i^F), \theta_{i,u}(\underline{v}_i^F))$, or that $(\underline{v}_i^F, \overline{v}_i^F) \in \mathcal{F}(\theta_i)$, as desired. □

Appendix B: Implementation of the estimators $\hat{W}_{g,t}^*$ and $\hat{w}_{g,t}^*$

This appendix describes the procedure to approximate the estimators $\hat{W}_{g,t}^*$ and $\hat{w}_{g,t}^*$ (i.e., the resampling algorithm) and elaborates on the specific assumptions employed.

Resampling algorithm The procedure to approximate $\hat{W}_{g,t}^*$ repeatedly draws at random a sample of size $|N_t| - 1$ from the set B_t of available bid-functions, where $|M_{j,t}|$ functions are drawn from $B_{j,t} \subseteq B_t$ when $j \neq g$ and $|M_{g,t}| - 1$ functions are drawn from $B_{g,t} \subseteq B_t$. The procedure then constructs the respective aggregate opponent demand functions from these random bid profiles. This yields a set of demand functions from which we can estimate the distribution of $D_{g,t}(p)$. The procedure details are in Algorithm 1.

Algorithm 1 requires the price of interest, $p \in (0, \bar{p})$, the number of resampled supply functions, R, the sets of bid functions, $\{B_{j,t}\}_{j=1}^m$, the number of bidders in any of the groups, $\{n_{j,t}\}_{j=1}^m$, and the quota, Q. The algorithm returns the approximated estimator

Algorithm 1 Approximation of $\hat{W}_{g,t}^*$.

Require: $p, R, \{B_{j,t}\}_{g=1}^m, \{n_{j,t}\}_{g=1}^m, Q$.

for $r = 1$ to R **do**

  Construct a set $\{\beta_j\}$ of bid functions: For all $j \neq g$, randomly draw (with replacement) a set of $n_{j,t}$ bid functions from $B_{j,t}$ and from $B_{g,t}$ draw a set of $n_{g,t} - 1$ bid functions.

  From $\{\beta_j\}$, compute aggregate opponent demand $D_r(p) = \sum_j \beta_j^{-1}(p)$.

end for

return $\hat{W}_{g,t}^{*,R}(p, q) = F(Q - q; \{D_r(p)\}_{r=1}^R)$.

$\hat{W}^{*,R}_{g,t}(p,q)$. Throughout, I use $R = 500$. As in the text, $F(\cdot; X)$ corresponds to the estimate of the gamma CDF from a given set of points, X.

Regarding the approximation of $\hat{w}^{*}_{g,t}$, we have to remember that the number of submitted price bids is finite. Hence, an estimate of w^*_i that uses the resampling estimator $\hat{W}^{*,R}_{g,t}$ will be zero when the increment h in (22) is too small. A remedy for this problem is as follows: Let $\{p_m\}_{m=1}^{M}$ be the set of all the different submitted price points in auction t; that is, $p_m > p_{m+1}$ holds for all $m \in \{1, \dots, M-1\}$. Then I approximate $\hat{w}^{*}_{g,t}$ with

$$\hat{w}^{*,R}_{g,t}(p_m, q) = \frac{\hat{W}^{*,R}_{g,t}(p_m, q) - \hat{W}^{*,R}_{g,t}(p_{m+1}, q)}{p_m - p_{m+1}},$$

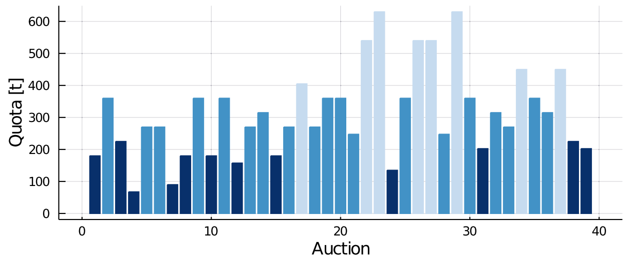
for every price point in the set $\{p_m\}_{m=1}^{M}$, where I take p_{M+1} to be zero.

Specific assumptions I not only use the bids from the current auction t when computing $\hat{W}^{*,R}_{g,t}$ but also from auctions with a similar quota. Since the quotas have to be set by the Federal Office for Agriculture with an eye on the current market conditions, similar quotas can be taken to reflect similar market conditions, thus justifying such an approach.

Specifically, I divide the quotas into three clusters from which I resample. The auction clusters are depicted in Figure B.1, showing the quotas in the auctions along with a description of the groups. The Supplementary Appendix shows that the risk preferences are invariant across groups.

As discussed in Section 5, I assume that there are $m = 3$ groups of identical bidders. For all auctions t, the number of bidders $\{n_{j,t}\}_{j=1}^{m}$ that I pass to Algorithm 1, correspond to the average number of active bidders in each of the groups across all auctions, which are 58 for group $g = 1$, 11 for group $g = 2$, and 3 for group $g = 3$. This amounts to assuming that each bidder believes the number of participants and their composition to be invariant over time.

Remark 4 (The advantage of a parametric approach). Using a parametric approach to estimating the distribution of $D_{g,t}(p)$ has a distinct advantage over employing the usually used, nonparametric strategy in the literature when it comes to determining best-



Auction Groups

		Quota (in t)		
Group	#	Min	Mean	Max
1	8	405000	523120	630000
2	19	247500	312630	360000
3	12	67500	168750	225000

Figure B.1. The chart on the left shows the quotas for the 39 auctions. Based on the quotas, I divided the auctions into three groups. The three shades indicate the respective group assignment. The table on the right gives an overview of the quotas in the respective groups.

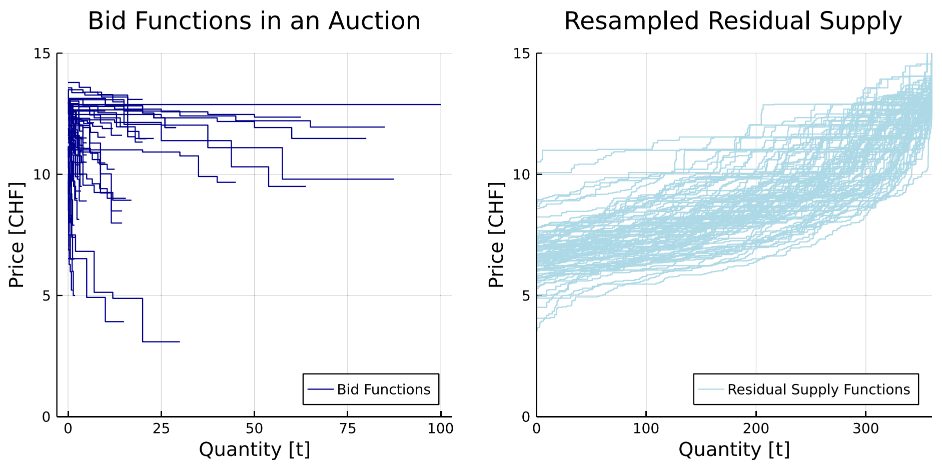


FIGURE B.2. The left figure shows the submitted bid functions in auction no. 36. The right figure depicts 100 redraws of the residual supply function $Q - D_\beta(p)$, using bid functions from the group of auctions containing auction no. 36.

response behavior. Consider some of the highest bid functions on the left side of Figure B.2. For the depicted residual supply functions on the right, the empirical CDF of D_β would yield a winning probability of one for many of the quantity points in those bid functions. Yet, such bids can never be best responses because the bidders could always lower their price bids without winning less. Assuming a particular functional form for the distribution of D_β that has full support on the positive reals avoids this problem.

REMARK 5 (Ties in the data). The approach to estimate W_i^* assumes that ties happen with zero probability. As in Kastl (2011), ties do occur in my data, yet they do so even less frequently than in Kastl (2011). The ex ante probability of tying on a submitted price point in my data is 0.004 as opposed to the likelihood of 0.116 reported in footnote 21 in Kastl (2011). This suggests that we may safely assume that bidders ignore the possibility of a tie. The likelihood is computed by the probability of a tie in a given auction (0.74) times the number of tying bidders conditional on a tie (1.8) divided by the product of the average number of bidders (71.6) and the average number of submitted price-quantity pairs (4.42). The (unconditional) expected number of tying bidders is much lower in my data than in Kastl (2011), and both the expected number of bidders and the average number of submitted price-quantity pairs are much higher.

APPENDIX C: TESTING FOR INCREASING DIFFERENCES

This appendix describes the procedure to test for increasing differences as required in Propositions 4 and 5. The procedure repeatedly and randomly picks, for every bidder i and every price-quantity point j in the data, two decreasing, possible profit functions v_i and v_i' satisfying $v_i > v_i'$ in the pointwise partial order and then checks whether $F_i^j(b_i, v_i, \rho) \geq F_i^j(b_i, v_i', \rho)$ indeed holds. Specifically, I proceed as follows:

TABLE C.1. Fraction of price-quantity points j for which monotonicity of F_i^j in v_i was violated.

$R=50$			$R=100$			$R=200$		
ρ	mean	se	ρ	mean	se	ρ	mean	se
4.54e−5	0.01093	0.0072	4.54e−5	0.01149	0.007543	4.54e−5	0.01149	0.007543
7.485e−5	0.01028	0.007011	7.485e−5	0.01101	0.007091	7.485e−5	0.01157	0.007572
0.0001234	0.0102	0.006888	0.0001234	0.0106	0.007223	0.0001234	0.01109	0.007728
0.0002035	0.009227	0.007469	0.0002035	0.009874	0.007356	0.0002035	0.01101	0.007829
0.0003355	0.008498	0.006876	0.0003355	0.009713	0.007399	0.0003355	0.01109	0.007521
0.0005531	0.008741	0.006772	0.0005531	0.0102	0.007882	0.0005531	0.01101	0.007381
0.0009119	0.008418	0.006116	0.0009119	0.01044	0.007496	0.0009119	0.01149	0.007916
0.001503	0.009146	0.005918	0.001503	0.01044	0.006109	0.001503	0.01174	0.00713
0.002479	0.00947	0.006313	0.002479	0.01246	0.007019	0.002479	0.01538	0.01019
0.004087	0.01182	0.006343	0.004087	0.01295	0.007992	0.004087	0.01886	0.01224
0.006738	0.01311	0.00775	0.006738	0.0187	0.0127	0.006738	0.02282	0.01589
0.01111	0.01481	0.01027	0.01111	0.02121	0.01464	0.01111	0.02873	0.01943
0.01832	0.01837	0.01239	0.01832	0.02711	0.01827	0.01832	0.03302	0.02233
0.0302	0.02104	0.01257	0.0302	0.03068	0.0185	0.0302	0.03731	0.02379
0.04979	0.02331	0.01483	0.04979	0.03416	0.02171	0.04979	0.04508	0.0256
0.08208	0.02452	0.0154	0.08208	0.03634	0.01942	0.08208	0.04678	0.0245
0.1353	0.02687	0.01303	0.1353	0.04015	0.01826	0.1353	0.05123	0.0238
0.2231	0.02703	0.0111	0.2231	0.03812	0.01757	0.2231	0.04864	0.01798
0.3679	0.02331	0.01083	0.3679	0.03472	0.01214	0.3679	0.04767	0.01586
0.6065	0.02161	0.01097	0.6065	0.031	0.01333	0.6065	0.04176	0.01545

1. Fix ρ and an auction $t \in \{1, \dots, T\}$.

2. For every submitted bid schedule b_i compute the corresponding estimates $\hat{\overline{v}}_i$ and $\hat{\underline{v}}_i$ of the bounds $\overline{v}_i$ and $\underline{v}_i$ given in (12)–(13), respectively.

3. For every submitted price-quantity pair (p_i^j, q_i^j) in the data, draw R pairs of decreasing functions $v_i, \tilde{v}_i$ satisfying $\hat{\overline{v}}_i \geq v_i > \tilde{v}_i \geq \hat{\underline{v}}_i$ (in the pointwise partial order) that coincide with $\hat{\underline{v}}_i$ on $[q_i^j, Q]$.[31] Specifically, I proceed as follows:

   (a) Draw a random number m of equidistant values $q_1, \dots, q_m \in [q_i^{j-1}, q_i^j]$; that is, $q_\kappa = q_i^{j-1} + \kappa \frac{q_i^j - q_i^{j-1}}{m+1}$ for $\kappa = 1, \dots m$. Draw $m+1$ random rational numbers $\tilde{y}_1 \geq \tilde{y}_2 \geq \dots \geq \tilde{y}_{m+1} \geq v_i(q_i^j)$ satisfying $\hat{\overline{v}}_i(q_\kappa) \geq \tilde{y}_\kappa \geq \hat{\overline{v}}_i(q_{\kappa-1})$, where $q_0 = q_i^{j-1}$ and $q_{m+1} = q_i^j$. These values are then used to define a decreasing step function $\tilde{v}_i(q)$ for $q \in [q_i^{j-1}, q_i^j]$; that is, $\tilde{v}_i(q) = \tilde{y}_\kappa$ whenever $q \in (q_{\kappa-1}, q_\kappa]$ for $\kappa = 1, \dots, m+1$.

   (b) Draw another set of random values $y_1 \geq y_2 \geq \dots \geq y_{m+1}$ satisfying $\hat{\overline{v}}_i(q_\kappa) \geq y_\kappa \geq \tilde{y}_\kappa$ for all $\kappa \in \{1, \dots, m+1\}$ to define, using the same values $q_1, \dots, q_m$ as above, a decreasing step function $v_i(q) \geq \tilde{v}_i(q)$ for $q \in [q_i^{j-1}, q_i^j]$.

[31] Because the term involving $\overline{\Pi}_i^j$ in the expression (14) of F_i^j is increasing in v_i, it is sufficient to compare marginal profit functions $v_i, \tilde{v}_i$ that only differ on the segment $[q_i^{j-1}, q_i^j]$.

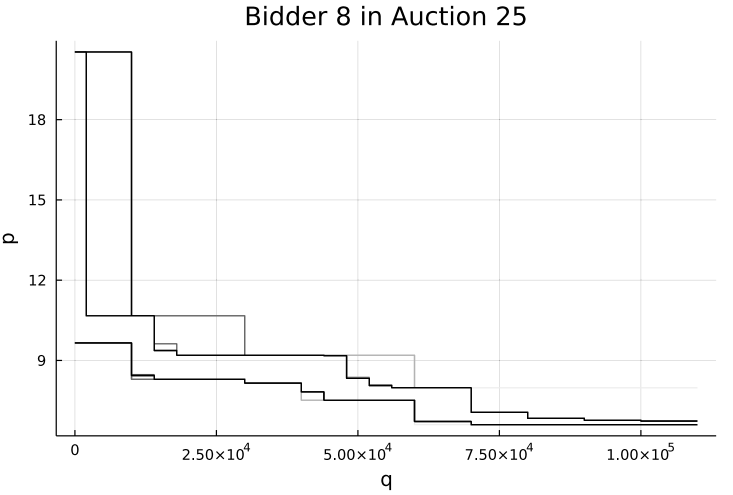


FIGURE D.1. The figure depicts the iterations of Algorithm 2. In this particular instance, the algorithm took 13 iterations. The least tight bounds in the picture are the initial conditions, and the tightest bounds are the algorithm's outcome. There are 5 price-quantity pairs to go through, so it took on average 2.6 iterations per pair. Particular iterations are in light gray, implying that the darker a bound segment appears, the less often that particular segment changed during the iterations.

4. Go through all submitted price-quantity pairs (p_i^j, q_i^j) in the data and increase the counter of monotonicity violations by one whenever at least one of the corresponding R pairs v_i, $\tilde{v}_i$ drawn in step 3 above violates $F_i^j(b_i, v_i, \rho) \geq F_i^j(b_i, \tilde{v}_i, \rho)$.

I repeat this procedure for values $\log(\rho) \in \{-12, -11.5, -11, \ldots, -0.5\}$ (the same values for which I compute Θ, except for $\rho = 0$) and for different values of $R \in \{50, 100, 200\}$. Table C.1 reports for every value of ρ, the average fraction of price-quantity pairs across all auctions for which I detected a monotonicity violation. These fractions are computed by the total number of violations in an auction divided by the product of the average number of bidders (72) and the average number of price-quantity pairs per bidder (4.4). Unsurprisingly, the numbers increase in R, yet they do not do so proportionate to R. For $R = 200$, the fraction of violations are between 1% and 5%, depending on ρ.

APPENDIX D: AN ALGORITHM TO COMPUTE $\wedge\mathcal{F}(\theta_i)$

This Appendix describes the algorithm to compute the least fixed point of the function $\theta_i = (\theta_{i,l}, \theta_{i,u})$ constructed in (21)–(21). As established in point (ii) of Proposition 5, it is possible to compute the least fixed point by a simple fixed-point iteration procedure that takes the estimated standard bounds $\hat{\bar{v}}_i$ and $\hat{\underline{v}}_i$ from (12)–(13) as initial conditions and then iteratively applies θ_i until convergence is reached. The algorithm computes the least fixed point by iterating $\theta_{i,l} \circ \theta_{i,u}$ directly (as observed in footnote 18, this yields the same result). The details are in Algorithm 2.

On a general level, the computation in Algorithm 2 makes use of the fact that for any j the function F_i^j only depends on the segment of the marginal profit function for

Algorithm 2 Computing $\wedge\mathcal{F}(\theta_i)$.

Require: $\epsilon, h, b_i, \rho, (\hat{\bar{v}}_i, \hat{\underline{v}}^i)$.

Use b_i and h to define $\{\mathcal{Q}^j\}_{j=1}^{\ell_i}$ as in (36).
$u_1 \leftarrow \hat{\bar{v}}_i$
$l_1 \leftarrow \hat{\underline{v}}_i$
$r \leftarrow 1$
for $j = \ell_i$ to 1 **do**
  repeat
    for $m = 1$ to $h - 1$ **do**
      $u_{r+1,m} \leftarrow \inf\{x \in [\lim_{q \downarrow q_i^j} u_r(q), \hat{\bar{v}}_i((\mathcal{Q}^j)_m)] : F_i^j(b_i, \varphi_u((\mathcal{Q}^j)_m, x, l_r), \rho) > 0\}$
    end for
    $u_{r+1}(q) \leftarrow u_r(q) + \sum_{s=1}^{h-1}[u_{r+1,s} - u_r(q)] \cdot \mathbf{1}\{q \in ((\mathcal{Q}^j)_s, (\mathcal{Q}^j)_{s+1}]\}$
    for $m = 1$ to h **do**
      $l_{r+1,m} \leftarrow$
        $\sup\{x \in [\lim_{q \downarrow q_i^j} l_r(q), \hat{\bar{v}}_i((\mathcal{Q}^j)_m)] : F_i^j(b_i, \varphi_l((\mathcal{Q}^j)_m, x, u_{r+1}), \rho) < 0\}$
    end for
    $l_{r+1}(q) \leftarrow l_r(q) + \sum_{s=1}^{h}[l_{r+1,s} - l_r(q)] \cdot \mathbf{1}\{q \in ((\mathcal{Q}^j)_{s-1}, (\mathcal{Q}^j)_s]\}$
    $r \leftarrow r + 1$
  until $\sup_{q \in \mathcal{Q}^j} |l_r(q) - l_{r-1}(q)| + \sup_{q \in \mathcal{Q}^j} |u_r(q) - u_{r-1}(q)| \leq \epsilon$
end for
return (l, u)

quantities beyond the $(j-1)$-th quantity point. This allows to first compute the least fixed point of the function θ_i restricted to the segment $[q_i^{\ell_i - 1}, q_i^{\ell_i}]$, because the values for v_u and v_l on the segment $[q_i^{\ell_i - 1}, q_i^{\ell_i}]$ thus obtained must be part of the least fixed point (v_u, v_l) on the whole interval $[0, Q]$. Consequently, by keeping these values fixed, we can then continue by computing the least fixed point of the function θ_i restricted to the segment $[q_i^{\ell_i - 2}, q_i^{\ell_i}]$. Having bounds for the segments after the third-to-last quantity point then allows to compute the values of v_u and v_l for the segment after the fourth-to-last quantity point, and so on, proceeding backward through all quantity points.

The algorithm requires a number $h \in \mathbb{N}_+$ which is used to partition every line segment $[q_i^{j-1}, q_i^j]$ into

$$\mathcal{Q}^j \equiv \left\{q_i^{j-1} + \frac{q_i^j - q_i^{j-1}}{h}, q_i^{j-1} + 2\frac{q_i^j - q_i^{j-1}}{h}, \dots, q_i^j\right\}, \tag{36}$$

with the mth element denoted by $(\mathcal{Q}^j)_m$, where we define $(\mathcal{Q}^j)_0 = q_i^{j-1}$. The algorithm further requires the risk parameter ρ and a pair of bounds $(\hat{\bar{v}}_i, \hat{\underline{v}}_i)$ on the values of the (true) marginal profit, which are used as initial conditions. Throughout, I use the risk-preference parameter determined in Section 5.2; that is, if a bidder is in bidder group g, then I set $\rho = \rho_g^*$. For $(\hat{\bar{v}}_i, \hat{\underline{v}}_i)$ I use the estimates of the individual upper and lower bounds in (12)–(13) from Proposition 3.

Using $\{\mathcal{Q}^j\}_{j=0}^{\ell_i}$, the pseudo-code in Algorithm 2 iterates discrete analogues of $\theta_{i,l} \circ \theta_{i,u}$ with initial condition $l_1 = \underline{\hat{v}}_i$. Starting with the segment after the second-to-last quantity point, the iteration produces sequences of bound candidates $l_r \in \mathcal{V}$ and $u_r \in \mathcal{V}$, $r = 2, \dots$ until the sum of the differences between two iterations of l and u is lower than some prespecified $\epsilon > 0$ (cf. steps 6–16). The resulting bounds are then again used to compute the bounds on the segment after the third-to-last quantity point, and so on, until the bounds on the whole codomain are computed. Figure D.1 exemplarily shows the iterations of the algorithm for Bidder 8 in Auction 25.

Observe that the iterated bounds l_r and u_r are constructed such that—by the monotonicity of $\theta_{i,l}$ and $\theta_{i,u}$—the resulting fixed point is below the fixed point that would obtain with continuous $(\theta_{i,l}, \theta_{i,u})$. This can be seen in steps 8 and 10 as well as in steps 12 and 14, which take an upper estimate for the upper bound and a lower estimate for the lower bound, respectively. From this, it also follows that the resulting fixed point increases in h. In other words, the estimate obtained for any finite h is a conservative estimate of the continuous limit case $h \to \infty$. Throughout, I use $h = 5$ for the estimation.

As noted in footnote 29, if the initial conditions $(\underline{\hat{v}}_i, \hat{\overline{v}}_i)$ violate the left inequality in (16) on a segment $[q_i^j, q_i^{j+1}]$, then the algorithm returns the highest bounds that are consistent with the initial conditions and the tighter bounds obtained for the segments with higher q's. This is motivated by the fact that, because F_i^j is increasing in v_i, all v_i that would be consistent with price optimality in such a case must lie above $\hat{\overline{v}}_i$. On the other hand, the algorithm returns the lowest consistent bounds if the right inequality in (16) is violated. This is motivated by the fact that in this case, any feasible v_i must lie below $\underline{\hat{v}}_i$.

To see the corresponding mechanics in Algorithm 2, suppose the right inequality fails, $F_i^j(b_i, \underline{\hat{v}}_i, \rho) > 0$. Recall that $F_i^j(b_i, v, \rho)$ is increasing in v. Because $\varphi_u(q, x, v)$ is bounded below by v for all (q, x) and l_r is bounded below by $\underline{\hat{v}}_i$, the set in the square brackets in step 8 is equal to $[\lim_{q \downarrow q_i^j} u_r(q), \hat{\overline{v}}_i((\mathcal{Q}^j)_m)]$ for all m, giving $u_{r+1,m} = \lim_{q \downarrow q_i^j} u_r(q)$ for all m. On the other hand, the functions $\varphi_l((\mathcal{Q}^j)_m, x, u_{r+1})$ used to evaluate F_i^j in step 12 are also bounded below by $\underline{\hat{v}}_i$. Hence, the set in the square brackets in step 12 is empty for all m, giving $l_{r+1,m} = \lim_{q \downarrow q_i^j} l_r(q)$ for all m. Consequently, the algorithm will run steps 6 to 16 exactly once. The situation is reversed if $F_i^j(b_i, \hat{\overline{v}}_i, \rho) < 0$, returning $u_{r+1,m} = l_{r+1,m} = \hat{\overline{v}}_i((\mathcal{Q}^j)_m)$ for all m.

Appendix E: Bidder group-specific estimates

To compute the bidder group-specific AvP^{pre} and AvP^{post} for a given group $g = 1, 2, 3$ in an auction t, I proceed as in (23)–(24) but I only sum over the bidders in that group. I divide the sum by the aggregate amount obtained within that group, rather than by the respective quota. To compute AvP^{ratio}, I proceed as in (25), but I only sum over those bidders in the group that have obtained a positive quantity in the auction. Also, rather than dividing by $\overline{n}$, I divide by the number of bidders within the group that have obtained a positive quantity.

Table E.1. Bidder group-specific means and medians of the profit and shading-to-profit ratio estimates. Bidder groups are indexed by $g = 1, 2, 3$.

			$g=1$		$g=2$		$g=3$	
Estimate	Tight	$\vec{\rho}$	Mean	Median	Mean	Median	Mean	Median
AvP_u^{pre}	no	0	17.1	16.8	16.82	16.67	16.3	17.49
	no	$\vec{\rho}^*$	15.85	14.88	15.02	13.42	15.25	14.39
	yes	$\vec{\rho}^*$	12.91	11.05	12.58	10.74	12.91	11.47
AvP_l^{pre}	no	0	10.81	9.489	10.82	9.316	10.62	9.695
	no	$\vec{\rho}^*$	9.568	8.694	9.437	8.623	9.317	8.472
	yes	$\vec{\rho}^*$	9.737	8.789	9.707	8.749	9.77	8.542
AvP_u^{post}	no	0	8.204	8.221	7.935	7.584	7.476	7.775
	no	$\vec{\rho}^*$	6.955	6.891	6.135	5.707	6.429	6.375
	yes	$\vec{\rho}^*$	4.014	3.138	3.694	2.506	4.085	3.385
AvP_l^{post}	no	0	1.912	1.012	1.933	1.001	1.793	0.4783
	no	$\vec{\rho}^*$	0.6733	0.09508	0.5517	0.1034	0.4935	0.07064
	yes	$\vec{\rho}^*$	0.8427	0.2134	0.8218	0.3026	0.9467	0.3031
AvP_u^{ratio}	no	0	0.488	0.5028	0.4708	0.4795	0.4682	0.4766
	no	$\vec{\rho}^*$	0.4523	0.4548	0.4184	0.4229	0.4366	0.4337
	yes	$\vec{\rho}^*$	0.2839	0.2747	0.2647	0.2478	0.2914	0.2993
AvP_l^{ratio}	no	0	0.1198	0.06965	0.1251	0.07998	0.1119	0.06052
	no	$\vec{\rho}^*$	0.03297	0.01398	0.03257	0.01118	0.03356	0.01172
	yes	$\vec{\rho}^*$	0.06146	0.03831	0.06119	0.0432	0.08026	0.04187

Table E.1 provides an overview of the results. There is no substantial difference between the three groups in terms of the range of possible profits and shading-to-profit ratios. The intervals between the respective upper and lower mean estimates and those between the respective upper and lower median estimates exhibit substantial overlap for all three measures. Moreover, the median and the mean estimates do not differ much between groups.

References

Anderson, Edward J., Pär Holmberg, and Andrew B. Philpott (2013), "Mixed strategies in discriminatory divisible-good auctions." *The RAND Journal of Economics*, 44, 1–32. [422]

Apostol, Tom M. (1974), *Mathematical Analysis*. Addison Wesley Publishing Company. [454]

Aryal, Gaurab, Hanna Charankevich, Seungwon Jeong, and Dong-Hyuk Kim (2022), "Procurements with bidder asymmetry in cost and risk-aversion." Journal of Business & Economic Statistics, 1–37. [420, 453]

Back, Kerry and Jaime F. Zender (1993), "Auctions of divisible goods: On the rationale for the treasury experiment." *Review of Financial Studies*, 6, 733–764. [422]

Back, Kerry and Jaime F. Zender (2001), "Auctions of divisible goods with endogenous supply." *Economics Letters*, 73, 29–34. [422]

Baranga, Andrei (1991), "The contraction principle as a particular case of kleene's fixed point theorem." *Discrete Mathematics*, 98, 75–79. [440, 458]

Barseghyan, Levon, Francesca Molinari, Ted O'Donoghue, and Joshua C. Teitelbaum (2018), "Estimating risk preferences in the field." *Journal of Economic Literature*, 56, 501–564. [423]

Besançon, Mathieu, David Anthoff, Alex Arslan, Simon Byrne, Dahua Lin, Theodore Papamarkou, and John Pearson (2019), "Distributions.jl: Definition and modeling of probability distributions in the JuliaStats ecosystem." arXiv:1907.08611. [443]

Bokusheva, Raushan, Silvan Fischer, Michael Grass, and Aaron Grau (2019), "Eine Analyse von Food-Wertschöpfungsketten auf Basis internationaler Vergleichsdaten und Fallstudien." In *Strukturberichtserstattung Nr.*, Vol. 60, Swiss State Secretariat for Economic Affairs. [451]

Bolotnyy, Valentin and Shoshana Vasserman (2019), "Scaling auctions as insurance: A case study in infrastructure procurement." Working Paper. [420, 453]

Boughner, Devry S., Harry de Gorter, and Ian M. Sheldon (2000), "The economics of two-tier tariff-rate import quotas in agriculture." *Agricultural and Resource Economics Review*, 29, 58–69. [423]

Breiman, Leo (1996), "Bagging predictors." *Machine Learning*, 24, 123–140. [443]

Campo, Sandra, Emmanuel Guerre, Isabelle Perrigne, and Quang Vuong (2011), "Semiparametric estimation of first-price auctions with risk-averse bidders." *The Review of Economic Studies*, 78, 112–147. [423]

Chapman, James T. E., David McAdams, and Harry J. Paarsch (2006), "Bounding best-response violations in discriminatory auctions with private values." Working Paper. [421, 448]

Chapman, James T. E., David McAdams, and Harry J. Paarsch (2007), "Bounding revenue comparisons across multi-unit auction formats under ε-best response." *American Economic Review*, 97, 455–458. [453]

De Haan, Laurens (1981), "Estimation of the minimum of a function using order statistics." *Journal of the American Statistical Association*, 76, 467–469. [447]

Elsinger, Helmut, Philipp Schmidt-Dengler, and Christine Zulehner (2019), "Competition in treasury auctions." *American Economic Journal: Microeconomics*, 11, 1–29. [422]

Fabra, Natalia and Mar Reguant (2014), "Pass-through of emissions costs in electricity markets." *American Economic Review*, 104, 2872–2899. [422]

Forbes, Catherine, Merran Evans, Nicholas Hastings, and Brian Peacock (2011), *Statistical Distributions*. John Wiley & Sons. [443]

Guerre, Emmanuel, Isabelle Perrigne, and Quang Vuong (2009), "Nonparametric identification of risk aversion in first-price auctions under exclusion restrictions." *Econometrica*, 77, 1193–1227. [423]

Häfner, Samuel (2023), "Supplement to 'Risk aversion in share auctions: Estimating import rents from TRQs in Switzerland'." *Quantitative Economics Supplemental Material*, 14, https://doi.org/10.3982/QE1907. [419]

Hefti, Andreas and Peiyao Shen (2019), "Supply function competition with asymmetric costs: Theory and experiment." *Economics Letters*, 178, 24–27. [441]

Hortaçsu, Ali, Jakub Kastl, and Allen Zhang (2018), "Bid shading and bidder surplus in the US treasury auction system." *American Economic Review*, 108, 147–169. [422]

Hortaçsu, Ali, Fernando Luco, Steven L. Puller, and Dongni Zhu (2019), "Does strategic ability affect efficiency? Evidence from electricity markets." *American Economic Review*, 109, 4302–4342. [423]

Hortaçsu, Ali and David McAdams (2010), "Mechanism choice and strategic bidding in divisible good auctions: An empirical analysis of the Turkish treasury auction market." *Journal of Political Economy*, 118, 833–865. [421, 422, 443]

Hortaçsu, Ali and Steven L. Puller (2008), "Understanding strategic bidding in multi-unit auctions: A case study of the Texas electricity spot market." *The RAND Journal of Economics*, 39, 86–114. [422]

Jachymski, Jacek, Leslaw Gajek, and Piotr Pokarowski (2000), "The Tarski–Kantorovitch prinicple and the theory of iterated function systems." *Bulletin of the Australian Mathematical Society*, 61, 247–261. [440]

Kastl, Jakub (2011), "Discrete bids and empirical inference in divisible good auctions." *The Review of Economic Studies*, 78, 974–1014. [419, 421, 422, 428, 429, 435, 441, 443, 453, 461]

Kastl, Jakub (2012), "On the properties of equilibria in private value divisible good auctions with constrained bidding." *Journal of Mathematical Economics*, 48, 339–352. [419, 420, 422, 427, 429, 431, 433, 434, 453]

Kastl, Jakub (2017), "Recent advances in empirical analysis of financial markets: Industrial organization meets finance." In *Advances in Economics and Econometrics: Volume 2: Eleventh World Congress*, Vol. 2, 231. Cambridge University Press. [420, 422, 453]

Kastl, Jakub (2020), "Auctions in financial markets." *International Journal of Industrial Organization*, 70, 102559. http://www.sciencedirect.com/science/article/pii/S0167718719300876. [420, 422]

Kong, Yunmi (2020), "Not knowing the competition: Evidence and implications for auction design." *The RAND Journal of Economics*, 51, 840–867. [420, 453]

Li, Tong, Jingfeng Lu, and Li Zhao (2015), "Auctions with selective entry and risk averse bidders: Theory and evidence." *The RAND Journal of Economics*, 46, 524–545. [420, 453]

Lin, Dahua, John Myles White, Simon Byrne, Douglas Bates, Andreas Noack, John Pearson, Alex Arslan, Kevin Squire, David Anthoff, Theodore Papamarkou, Mathieu Besançon, Jan Drugowitsch, Moritz Schauer, and other contributors (2019), "JuliaStats/Distributions.jl: A Julia package for probability distributions and associated functions." https://doi.org/10.5281/zenodo.2647458. [443]

Loi, Alberico, Roberto Esposti, and Mario Gentile (2016), "Policy evaluation of tariff rate quotas." Report mandated by the Swiss Federal Office of Agriculture. [424, 425, 444]

Lu, Jingfeng and Isabelle Perrigne (2008), "Estimating risk aversion from ascending and sealed-bid auctions: The case of timber auction data." *Journal of Applied Econometrics*, 23, 871–896. [423]

Luo, Yao and Hidenori Takahashi (2019), "Bidding for contracts under uncertain demand: Skewed bidding and risk sharing." Working Paper. [420, 453]

Marszalec, Daniel (2017), "The impact of auction choice on revenue in treasury bill auctions – an empirical evaluation." *International Journal of Industrial Organization*, 53, 215–239. [422]

McAdams, David (2003), "Isotone equilibrium in games of incomplete information." *Econometrica*, 71, 1191–1214. [437]

Milgrom, Paul R. and Robert J. Weber (1985), "Distributional strategies for games with incomplete information." *Mathematics of Operations Research*, 10, 619–632. [429]

Nyborg, Kjell G., Kristian Rydqvist, and Suresh M. Sundaresan (2002), "Bidder behavior in multiunit auctions: Evidence from Swedish treasury auctions." *Journal of Political Economy*, 110, 394–424. [420]

Olszewski, Wojciech and Ron Siegel (2019), "Equilibrium existence in games with ties." Working Paper. [429]

Pycia, Marek and Kyle Woodward (2020), "Auctions of homogeneous goods: A case for pay-as-bid." Working Paper. [422, 453]

Reny, Philip J. (1999), "On the existence of pure and mixed strategy equilibria in discontinuous games." *Econometrica*, 67, 1029–1056. [429]

Reny, Philip J. (2011), "On the existence of monotone pure-strategy equilibria in Bayesian games." *Econometrica*, 79, 499–553. [429, 437]

Reny, Philip J. and Shmuel Zamir (2004), "On the existence of pure strategy monotone equilibria in asymmetric first-price auctions." *Econometrica*, 72, 1105–1125. [429]

Sade, Orly, Charles Schnitzlein, and Jaime F. Zender (2006), "When less (potential demand) is more (revenue): Asymmetric bidding capacities in divisible good auctions." *Review of Finance*, 10, 389–416. [441]

Tversky, Amos and Daniel Kahneman (1992), "Advances in prospect theory: Cumulative representation of uncertainty." *Journal of Risk and uncertainty*, 5, 297–323. [448]

Wang, James J. D. and Jaime F. Zender (2002), "Auctioning divisible goods." *Economic Theory*, 19, 673–705. [422]

Wilson, Robert (1979), "Auctions of shares." *The Quarterly Journal of Economics*, 93, 675–689. [422]

Wolak, Frank A. (2000), "An empirical analysis of the impact of hedge contracts on bidding behavior in a competitive electricity market." *International Economic Journal*, 14, 1–39. [422]

Wolak, Frank A. (2003), "Identification and estimation of cost functions using observed bid data: An application to electricity markets." In *Advances in Economics and Econometrics: Theory and Applications* (Mathias Dewatripont, Lars Hansen, and Stephen Turnovsky, eds.), 115–149. [422]

WTO Committee on Agriculture (2018), "Tariff quota administration methods and fill rates 2007–2016." Technical Report G/AG/W/183, WTO. [423]

Co-editor Limor Golan handled this manuscript.

Manuscript received 30 April, 2021; final version accepted 8 December, 2022; available online 20 December, 2022.