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Ellsberg meets Keynes at an urn

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Keynes (1921) and Ellsberg (1961) have articulated an aversion toward betting on an urn containing balls of two colors of unknown proportion to one with a 50–50 composition. Keynes views this as reflecting different preferences for bets arising from different sources of uncertainty. Ellsberg describes this as weighting the priors arising from the unknown urn pessimistically. In two experiments, we observe substantial links between attitude toward almost-objective uncertainty and attitudes toward multiple-prior uncertainties in terms of ambiguity and its corresponding compound risk. Our findings point to a shared component across domains of uncertainty and motivate the need for further theoretical development.

Keywords. Ambiguity, source preference, maxmin expected utility, recursive utility, experiment.

JEL classification. C91, D81.

1. Introduction

The proverbial urn has served a useful role as a canonical source of uncertainty since time immemorial. In probability and statistics, inferring the likelihood of drawing a ball from an urn with a known color composition is a basic problem, which has given rise to a rich range of studies spanning multiple disciplines. In an interesting twist evident in two seminal works 40 years apart, Keynes (1921) and Ellsberg (1961) have each come up

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with the idea of an urn containing colored balls of unknown proportions. Intriguingly, they offer distinct perspectives on the same unknown urn in a decision-making setting.

In *A Treatise on Probability* (1921, p. 83), Keynes proposes a thought experiment—between betting on an urn with equal proportions of white and black balls over betting on another urn in which the proportion of each color is unknown—and points out that the probability of drawing a white (black) ball from either urn is evidently 1/2, but the "weight of argument" is greater for the known urn leading possibly to a preference for the known bet. He asks, "if two probabilities are equal in degree, ought we, in choosing our course of action, to prefer that one which is based on a greater body of knowledge?" In contrast, Ellsberg (1961) observes that the unknown bet encompasses a range of possible priors and hypothesizes that the known bet is favored for those who are pessimistic in weighing the less favorable priors disproportionately. Ellsberg also coined the term ambiguity aversion to refer to a preference for the known bet over the unknown bet.

As it turns out, while the literature following Ellsberg (1961) has been vibrant, there has not been as much follow-up work on Keynes (1921). The Ellsbergian multiple-prior perspective gives rise to two directions in modeling ambiguity attitude. One direction is to assign all weights to the worst prior as axiomatized in Gilboa and Schmeidler's (1989) model, which delivers the lowest among the possible expected utilities.[1] Another direction can be traced to Becker and Brownson (1964) who offer a parallel view of the unknown bet as a subjective compound lottery over the range of possible priors (which can be further reduced to an even-chance simple lottery). As such, ambiguity aversion corresponds to a preference for the even-chance simple lottery from the known urn over the induced subjective compound lottery from the unknown urn in violation of the reduction of compound lottery axiom (RCLA).[2] Adopting this two-stage view, Segal (1987) models ambiguity aversion by applying a probability weighting function that overweights the less favored priors (in terms of delivering lower stage-2 utilities). This theoretical relation between the two different approaches to modeling ambiguity aversion, both involving multiple priors, motivates Halevy's (2007) experimental study on the link between attitudes toward ambiguity and objective compound lottery in which the distribution of priors is objectively given as uniform. Halevy's (2007) finding of a tight link between these two attitudes lends support to the compound lottery view and the corresponding recursive modeling approach of Segal (1987) and subsequently Klibanoff, Marinacci, and Mukerji (2005), Ergin and Gul (2009), Seo (2009).

Echoing Keynes, Smith (1969) arrives independently at the view that both the known and unknown bets are in effect 50–50 when subjects do not prefer one color over the other for balls drawn from either urn. He proposes to make use of two distinct von Neumann–Morgenstern utility functions so that a known bet can be worth more than

[1] Ghirardato, Maccheroni, and Marinacci (2004) offer a more moderate model of pessimism through a convex combination of the best and the worst prior. Gajdos, Hayashi, Tallon, and Vergnaud (2008) axiomatize the pessimism decision rule in Ellsberg (1961), which takes a convex combination of the worst prior and an "estimated' half/half prior. Schmeidler (1989) and Gilboa (1987) can capture the intuition of pessimism directly by assigning a greater decision weight than 1/2 to the losing event.

[2] RCLA requires that a compound lottery is indifferent to its reduction to a simple lottery with the same overall probabilities and outcomes.

an unknown bet. This can be accomplished as long as the utility function associated with risks arising from the unknown urn is more concave than that associated with risks from the known urn.[3] Building on Keynes' perspective, Fox and Tversky (1995) consider uncertainty arising from natural events such as the price of a stock and temperature of a city. In their experiment, subjects in the Bay Area value separately two pairs of bets with potentially similar likelihoods of winning: one pair is based on whether the temperature one week from today in San Francisco is above or below 60°F; another pair is based on whether the temperature one week from today in Istanbul is above or below 60°F. Surprisingly, they observe that the lower-valued bet on the San Francisco temperature is worth more than the higher-valued bet on the Istanbul temperature. This has led Fox and Tversky to originate the concept of source preference, referring to a preference for betting on sources of uncertainty, such as a known urn or a familiar city both involving greater knowledge. This source preference perspective prompts Chew and Sagi (2008) to offer an axiomatization more generally so that the decision maker can have distinct attitudes toward risks arising from different sources of uncertainty that induce the same distribution, whether they are from an urn, from a deck of cards, or from natural events such as the temperature of a city.

Following Halevy (2007), there has been several experimental studies of the link between ambiguity attitude and compound risk attitude (e.g., Abdellaoui, Klibanoff, and Placido (2015), Chew, Miao, and Zhong (2017), Gillen, Snowberg, and Yariv (2019), Dean and Ortoleva (2019)). This strand of findings reinforces the Ellsbergian multiple-prior perspective and at the same time points to the need to study whether these two attitudes relate to source preference arising from the Keynesian perspective. The present paper addresses this gap in the literature via Experiment 1, which examines attitudes toward three types of uncertainty besides attitude toward the benchmark 50–50 known risk from betting on the color of a card drawn from a deck of 20 cards with 10 red and 10 black.

Ambiguity: Betting on the color of a card drawn from a deck of 20 cards each being either red or black. The composition is (partially) unknown.

Compound risk: Betting on the color of a card drawn from a deck of 20 cards each being either red or black. The composition is objectively determined by a random device.

Natural-event uncertainty: Betting on the trailing decimal digit (odd versus even) of the highest temperature in a city on a pre-specified historical date.

Our experimental design encompasses three ambiguity lotteries, two compound lotteries, and two natural-event lotteries based on two different cities. All these lotteries deliver a half chance of winning in the spirit of Keynes. Notice that our choice of natural-event uncertainty is distinct from that in Fox and Tversky (1995) in that the events of odd/even in the temperature bets are 50–50 in an "almost objective" sense (see Section

[3] Smith (1969) attempts to model preference for the known urn bets by arguing that the "known" utility is cardinally higher than the "unknown" utility, but this is not legitimate given that vNM utility functions are unique up to affine transformations. In raising the possibility of adopting two distinct vNM utility functions for the two sources of uncertainty, Pratt's (1964) relative concavity condition can be applied in order to arrive at the result of the known bet being worth more than the unknown bet.

II for a detailed discussion). We adopt a within-subject design with more than 2000 undergraduate subjects in Singapore. Using a choice-list mechanism, we elicit the certainty equivalent (CE) for each lottery.

We infer subjects' attitude toward ambiguity (compound risk/natural event) by its premium, which is given by the difference between the CE of the benchmark 50–50 risky lottery and that of ambiguity (compound risk/natural event) lottery. In other words, there is neutrality (aversion) toward a certain type of uncertainty when the corresponding premium is zero (positive).[4] The majority of our subjects exhibit distinct attitudes toward the three types of uncertainty. Three-fourths of the subjects exhibit nonneutrality toward ambiguity as well as compound risk and two-thirds exhibit nonneutrality toward the natural-event uncertainty of a city temperature. Besides replicating the significant association between ambiguity attitude and compound risk attitude in the literature, we find that natural-event uncertainty attitude is closely linked to both attitudes toward ambiguity and compound risk. Subjects who are neutral toward the odd-even bet on temperature are more likely to be ambiguity neutral and more likely to reduce compound lottery. We observe significant correlations for premium across the three types of uncertainty and, as anticipated, the correlation coefficient between ambiguity and compound risk is substantially higher than the other two correlation coefficients (Table 1). In individual type analysis, we find that 65.3% of the subjects are nonneutral toward all three types of uncertainty, and 5.7% of the subjects are neutral toward all three. For the rest of subjects, 20.4% are nonneutral toward ambiguity and compound risk, and are neutral toward natural-event uncertainty. These three groups of subjects account for 91.3% of the subjects, with the rest of 8.7% exhibit other behavioral patterns. Moreover, the observed patterns do not differ substantially between subjects with different cognitive ability measured by Raven's progressive matrices. Finally, we show that the observed results are also robust with respect to replication using more than 1000 undergraduate subjects in Beijing.

In the literature of decision-making under risk and uncertainty, it is commonly observed that risk and ambiguity attitudes depend on both outcome and likelihood. A natural question arises as to whether the links observed in Experiment 1 may also exhibit

Table 1. Correlations across three types of uncertainty.

	Ambiguity	Compound risk
Compound risk	0.825 (0.012)	
Natural event	0.580 (0.008)	0.657 (0.011)

Note: The correlation is estimated using the obviously related instrumental variables (Gillen, Snowberg, and Yariv (2019)) with standard errors.

[4]We focus on the comparisons among the premia for three types of uncertainty in this current paper. When it comes to the comparison between uncertainty premium and risk premium, caution needs to be taken since one may need to first separate uncertainty premium into belief-related premium and curvature-related premium, which could be model specific. See Baillon, Bleichrodt, Li, and Wakker (2021) for a recent discussion.

dependence on outcome and likelihood. To address this question, we conduct Experiment 2 in which we examine four lotteries in line with Tversky and Kahneman's (1992) fourfold pattern of risk attitude—moderate prospect (50% chance of winning), longshot prospect (10% chance of winning), moderate hazard (50% chance of losing), longshot hazard (10% chance of losing)—in addition to a mixed lottery (50% chance of winning or losing). For each of the five lotteries, we consider again the three types of uncertainty in Experiment 1 besides known risk. To implement our four-plus-one design for known risk, we make use of a deck of 10 cards numbered 1 to 10. Even-chance (10% chance) corresponds to betting on parity (the number) of a card drawn. For ambiguity, the corresponding lotteries are generated by betting on a deck of 10 cards with unknown composition of numbers from 1 to 10. For compound risk, the deck of 10 cards to be bet on is constructed with each card independently drawn from a known deck of 10 cards numbered from 1 to 10. For natural-event uncertainty, subjects are to bet on the second decimal digit of a stock index at market closing. Overall, we find strong support for the robustness of the close links across the attitudes toward all three types of uncertainty in relation to risk—the correlations among attitudes toward ambiguity, compound risk, and natural-event uncertainty are on average 0.58.

Our overall results shed light on decision theoretic models under uncertainty. In particular, our inclusion of natural-event uncertainty helps link the two perspectives on ambiguity—multiple priors and source preference—which differentiate source-based models from those reflecting the multiple-prior perspective. At the same time, where natural-event uncertainty is concerned, the observed nonneutrality toward almost-objective events provides support for source preference in distinguishing among identically distributed risks. Together with the observed close links between attitude toward natural-event uncertainty and ambiguity attitude as well as compound risk attitude, our overall findings point to the need to extend the current recursive models by incorporating within-stage source preference.

Our results also contribute to the understanding on the experimental and empirical studies on risk attitudes across different domains. Weber, Blais, and Betz (2002) show that risk attitudes are not consistent across five domains including financial decision-making, health/safety, recreational, ethical and social decisions, and propose that risk attitudes are domain specific. Interestingly, the domains in Weber, Blais, and Betz (2002) differ in terms of whether risks are gain-oriented or loss-oriented, whether the odds are moderate or skewed as well as whether risks are arising from different sources of uncertainty. Barseghyan, Prince, and Teitelbaum (2011) examine deductible choices across domains of auto and home insurance and reject the null of fully domain-general risk aversion. Einav, Finkelstein, Pascu, and Cullen (2012) observe a strong correlation for five employer-provided insurance coverage decisions, and a considerably weaker relationship between insurance decisions and 401 (k) asset allocation. In an experimental study across 30 countries, Vieider et al. (2015) show that risk and uncertainty attitude correlate not only within decision contexts or measurement methods but also across contexts and methods. These findings are consonant with the results of our experimental study of attitudes toward risk across gains and losses and with different likelihoods arising from different sources of uncertainty, pointing to a shared component in decision-making across a range of domains of uncertainty.

The rest of the paper is organized as follows. Section 2 presents the theoretical background. Section 3 details our experimental design and Section 4 presents the main results. We discuss the implications of our findings in Section 5. Finally, Section 6 concludes. The Online Supplementary Material (Chew, Miao, and Zhong (2023)) include the supplementary tables and figures, experimental instructions, and the replication files (data and code).

2. Theoretical background

Denote by S the set of all possible states in the experiment. Consider four types of uncertainty: risk (R), ambiguity (A), compound risk (C), and almost-objective natural-event uncertainty (N), and each type $i \in \{R, A, C, N\}$ partitions the state space into two equally likely events: $\{E_i, E_i^c\}$. Let h_i be the bet on event E_i, which pays w if event E_i occurs and 0 otherwise. The two events are equally likely in the sense that a decision maker is indifferent between betting on E_i and betting on E_i^c. Denote by **R**, **A**, **C**, and **N**, the lotteries induced by the corresponding act h_i, and $\text{CE}_{\mathbf{R/A/C/N}}$ the certainty equivalent of each lottery.

Two points are noteworthy. First, the compound lottery **C** in fact corresponds to the "higher-order" act induced by h_C, which maps from $\Delta(S)$, the set of probability distributions on S (priors) to $\Delta(Z)$, the set of probability distributions on outcomes (simple lotteries). Second, the natural-event uncertainty considered here is "*almost-objective*" (Machina (2004)). For example, consider the uncertainty associated with the trailing digit of a city's temperature being either odd or even; it is almost objective in the sense that for any continuous random variable on the thermometer space with a smooth density function, the probability of odd approaches 0.5 asymptotically as the location of the decimal (e.g., 8th, 9th, etc.) goes to infinity. The intuition is clear. First, partition the state space into equal-length intervals, for example, $\{\ldots, (0, \frac{1}{m}], (\frac{1}{m}, \frac{2}{m}], \ldots\}$. It can be shown that the (subjective) probability of the union of all the left-half of each interval, $\bigcup_n (\frac{n}{m}, \frac{n+0.5}{m}]$, converges to 0.5 as m grows large regardless of the prior distribution as long as its density is smooth.

Using **R** as the benchmark, we infer decision-maker's attitudes toward **A**, **C**, and **N**, as well as the three pairwise correlations of these attitudes. Consider the benchmark subjective expected utility (SEU) model, the utilities for different lotteries are as follows:

$$U_{\text{SEU}}(\mathbf{R}/\mathbf{A}/\mathbf{C}/\mathbf{N}) = \mathbf{E}_p u \circ h_i,$$

where $p \in \Delta(S)$ denotes the (subjective) prior. SEU, or more generally probabilistic sophistication (Machina and Schmeidler (1992)), entails $p(E_i) = p(E_i^c) = 0.5$ in each lottery. Thus, SEU predicts that four lotteries **R**/**A**/**C**/**N** have the same CEs, and hence generate no correlation across attitudes toward ambiguity, compound risk, and natural-event uncertainty.

In the sequel, we discuss the predictions of a number of models under uncertainty in relation to the three attitudes as well as the interplay among these attitudes. All these models have been developed to accommodate ambiguity nonneutrality, and we differentiate among them according to whether they adopt an *multiple-prior perspective* by incorporating the possibility of multiple priors, or the *source preference perspective*, which insists on a single prior yet allows distinct attitudes toward different types of uncertainty.

2.1 *Ellsbergian perspective*

One-stage modeling incorporating multiple priors In his 1961 paper, Ellsberg advocates a "conservatism" rule in assigning a positive weight to the worst prior in the ambiguous bet. Gilboa and Schmeidler's (1989) *maxmin expected utility* model (MEU), widely acknowledged to be the representative model under the Ellsbergian perspective, represents the extreme case of assigning a weight of 1. Under MEU, the utility for ambiguity lottery **A** is given below:

$$U_{\text{MEU}}(\mathbf{A}) = \min_{p\in\Pi} \mathbf{E}_p u \circ h_A,$$

where $\Pi \subset \Delta(S)$ is a convex set of priors with $\min_{p\in\Pi} p(E_A) = \min_{p\in\Pi} p(E_A^c)$. MEU exhibits ambiguity aversion $\text{CE}_\mathbf{R} \geq \text{CE}_\mathbf{A}$ when $p(E_A)$ admits nonunique values in Π.

In relation to compound risk attitude, the MEU model axiomatized in Gilboa and Schmeidler (1989) predicts compound risk neutrality $\text{CE}_\mathbf{R} = \text{CE}_\mathbf{C}$, since it adopts the Anscombe–Aumann framework, which incorporates RCLA.[5] In contrast, the axiomatization of MEU in a Savagian domain (Casadesus-Masanell, Klibanoff, and Ozdenoren (2000), Alon and Schmeidler (2014)) makes no predictions concerning how compound lotteries may be evaluated. With regard to natural-event lottery **N**, MEU predicts neutrality toward almost-objective uncertainty given that $p(E_N)$ approaches 0.5 asymptotically.

In sum, MEU can generate ambiguity nonneutrality, but fails to predict nonneutral attitudes toward either compound risk or almost-objective natural event and, therefore, does not generate correlations among different uncertainty attitudes.[6]

Two-stage modeling incorporating multiple priors When incorporating multiple priors, an alternative modeling approach (Segal (1987)) views ambiguity as a distribution over the set of possible priors. Formally, such a two-stage modeling approach associates the ambiguity lottery **A** with a stage-1 prior μ_A, a distribution on $\Delta(S)$. It follows that for a given act h_A, μ_A induces a subjective compound lottery, which can be evaluated recursively. There are two ways to form a recursive evaluation in the literature.

[5]RCLA is implicit in the assumption of reversal of order in Anscombe and Aumann (1963). See Seo (2009) for a detailed discussion.

[6]The CEU model axiomatized in Gilboa (1987) and Schmeidler (1989) generalizes SEU with a capacity function ν, a nonadditive extension of a probability measure that maps events into the unit interval and is monotonic in terms of inclusion. In relaxing additivity, CEU is compatible with both ambiguity aversion and ambiguity tolerance without restrictions on the capacities $\nu(E_\mathbf{A})$ and $\nu(E_\mathbf{A}^c)$. It, in fact, accommodates MEU given a convex ν.

For compound risk **C**, the observations of MEU also apply to CEU models including those adopting the Anscombe–Aumann framework (Schmeidler (1989)) and those adopting a Savagian domain (Gilboa (1987), Wakker (1987)). In addition, the almost objectivity of events odd/even in **N** again entails $\text{CE}_\mathbf{R} \simeq \text{CE}_\mathbf{N}$ in the CEU model.

Segal (1987, 1990) applies a *recursive rank-dependent utility* specification (RRDU) to evaluate **A** as follows:

$$U_{\text{RRDU}}(\mathbf{A}) = \int_{\Delta(S)} \int_S (u \circ h_A)\, df(\mathbf{P})\, df(\mathbf{M}_A),$$

where $\mathbf{M}_A$ ($\mathbf{P}$) is the cumulative distribution function derived from μ_A (p), and f (u) a common probability weighting function (utility function) applied to both stages.

An alternative model is to apply expected utility preference recursively in evaluating the compound lottery, which is accomplished in the *recursive expected utility* model (REU) of Klibanoff, Marinacci, and Mukerji (2005) and Seo (2009) as follows:

$$U_{\text{REU}}(\mathbf{A}) = \mathbf{E}_{\mu_\mathbf{A}} v(\mathbf{E}_p u \circ h_A),$$

where v and u are stage-1 and stage-2 utility functions, respectively.

Both RRDU and REU can exhibit ambiguity nonneutrality in violating RCLA for the induced subjective compound lottery. In particular, Segal (1987) shows that a convex f in RRDU can be compatible with ambiguity aversion $\text{CE}_\mathbf{R} \geq \text{CE}_\mathbf{A}$, while a concave v in REU is shown to be compatible with ambiguity aversion in Klibanoff, Marinacci, and Mukerji (2005).[7] For compound lottery **C**, μ_C is objective and both models can predict compound-risk aversion: $\text{CE}_\mathbf{R} \geq \text{CE}_\mathbf{C}$, under the same conditions as that for ambiguity aversion. Hence, both models are compatible with positive correlation between compound risk attitude and ambiguity attitude.[8] When considering almost objective natural-event lottery **N**, μ_N collapses to an atom, and it follows that both models predict natural-event uncertainty neutrality, that is, $\text{CE}_\mathbf{R} \simeq \text{CE}_\mathbf{N}$, and cannot generate any correlation between ambiguity/compound risk attitude and natural-event uncertainty attitude.

2.2 *Keynesian perspective*

One-stage source Fox and Tversky (1995) provide evidence in support of Keynes' source perspective, prompting Chew and Sagi (2008) to axiomatize source preference directly in terms of probabilistic sophistication on endogenous smaller families of events. In their model, ambiguity lottery **A** is viewed as an alternative source of uncertainty in addition to **R** and **N**. Note that we shall be using the term "*source of uncertainty*" only when referring to source preference models in the sequel. The Chew–Sagi model displays flexibility

[7] Halevy and Feltkamp (2005) link ambiguity aversion with aversion to mean-preserving spreads in an environment of bundled risks and deliver similar predictions as those of REU.

[8] Note that we focus on the functional forms of recursive models when discussing their predictions on ambiguity attitude, compound risk attitude, as well as the correlation between the two. Formally, the REU model axiomatized in Klibanoff, Marinacci, and Mukerji (2005) adopts a Savagian domain and makes no predictions on its behavior in the domain of compound risks. In contrast, Seo's (2009) approach of distinguishing distributions over acts from the statewise mixture of acts gives rise to ambiguity and compound risk in a unified domain and is able to link ambiguity nonneutrality with non-RCLA behavior for compound risk. See Halevy (2007, pp. 515–517) for a related discussion on how various recursive models can(not) differentiate between objective and subjective stage-1 priors.

in evaluating **R**, **A**, and **N** with local source-specific utility functionals:

$$U_{\text{CS}}(\mathbf{R}/\mathbf{A}/\mathbf{N}) = \mathbf{U}^i(p, h_i),$$

where $\mathbf{U}^i$ can admit different (expected or nonexpected) specifications for different sources of uncertainty even when the prior p assigns the same probabilities to events $E_R/E_A/E_N$. In adopting a source-dependent probability weighting function using rank-dependent utility (Quiggin (1982)), Abdellaoui, Baillon, Placido, and Wakker (2011) offer a more specific form of the Chew–Sagi model:

$$\mathbf{U}^i(p, h_i) = \int_S (u \circ h_i)\, df^i(\mathbf{P}),$$

where f^i is a source-dependent probability weighting function, and **P** the cumulative distribution function derived from p. Notably, Chew–Sagi admits RCLA for compound risk and predicts $\text{CE}_{\mathbf{R}} = \text{CE}_{\mathbf{C}}$. Given the flexibility in choosing $\mathbf{U}^i$ for different sources of uncertainty, Chew–Sagi can produce correlation between ambiguity attitude and natural-event uncertainty attitude, but fails to link compound risk attitude with the attitude toward ambiguity or with attitude toward natural-event uncertainty.

Two-stage source Nau (2006) distinguishes the uncertainty arising from two exogenously given "issues" (sources) and applies distinct expected utility preferences across the two different issues. By interpreting one issue as the first-order prior and the other as the second-order prior, Nau's representation coincides with REU. Ergin and Gul (2009) axiomatize a more general second-order probabilistic sophistication representation (SPS), which incorporates possibly distinct NEU preferences across different "issues."[9]

Specifically, the utility for ambiguity lottery **A** under SPS is as follows:

$$U_{\text{SPS}}(\mathbf{A}) = \mathbf{V}\big(\mu_A, \mathbf{U}(p, h_A)\big),$$

where **U** is the stage-2 utility functional evaluating act h_A under a given stage-2 prior p, and **V** the stage-1 utility functional that aggregates different stage-2 utilities according to the stage-1 prior μ_A. In SPS, **V** and **U** both admit probabilistic sophistication, and hence SPS incorporates REU and RRDU as special cases. It follows that both Nau (2006) and SPS can exhibit ambiguity nonneutrality and non-RCLA, as well as positive correlation between ambiguity attitude and compound risk attitude.

When it comes to **N**, Nau (2006) and SPS in general can treat risk and almost-objective uncertainty as two "issues" and deliver distinct evaluations between **R** and **N**. This notwithstanding, both models are more restrictive than Chew–Sagi in that they can deal with up to two exogenous sources of uncertainty. Therefore, upon interpreting the two sources as priors at different stages, Nau (2006) and SPS fail to generate natural-event nonneutrality and deliver the same predictions as RRDU and REU.

[9]Both models work on a product state space $S_1 \times S_2$, where S_1 and S_2 are assumed to be two exogenous issues of uncertainty and are free to admit the interpretation of priors at different stages. Broadly speaking, the REU model as in Klibanoff, Marinacci, and Mukerji (2005) applies distinct expected utility preferences for priors at different stages, and hence can also be regarded as a two-stage source model.

Table 2. Theoretical predictions.

		A	**C**	**N**	**A–C**	**A–N**	**C–N**
Ellsbergian	Pessimism	Nonneutral	Neutral	Neutral	N	N	N
	Recursive	Nonneutral	Nonneutral	Neutral	Y	N	N
Keynesian	One-stage source	Nonneutral	Neutral	Nonneutral	N	Y	N
	Two-stage source	Nonneutral	Nonneutral	Neutral	Y	N	N

Note: This table summarizes the qualitative predictions of different modeling approaches regarding attitude toward each type of uncertainty, as well as the pairwise correlations among those attitudes. "Neutral" ("Nonneutral") for a specific modeling approach means it is incompatible (compatible) with nonneutral attitude toward certain type of uncertainty. When marked in grey, "Neutral"/''Nonneutral" includes the case that some specific model within the approach makes no predictions concerning the corresponding uncertainty attitude, depending on the axiomatization framework. Similarly, a "Y" for a specific approach means it can generate correlations between two corresponding uncertainty attitudes, while a "N" means it either makes no prediction concerning or is incompatible with correlations between different uncertainty attitudes.

2.3 *Summary*

Table 2 summarizes the theoretical predictions of different models in terms of attitudes toward the three types of uncertainty (ambiguity/compound risk/nature-event uncertainty), as well as pairwise correlations among these attitudes.

Except for SEU, all models considered can exhibit nonneutrality toward **A**. Taking compound risk **C** into consideration, the observed "*known*'' link (Halevy (2007), Abdellaoui, Klibanoff, and Placido (2015), Chew, Miao, and Zhong (2017)) between ambiguity attitude and compound risk attitude (**A–C**) helps distinguish between the two approaches both adopting the multiple-prior perspective—those with built-in pessimism by overweighting the worst prior to accommodate ambiguity aversion, and those assuming higher-order distributions on the set of priors coupled with non-RCLA to deliver ambiguity nonneutrality. Specifically, MEU, as a representative of the former, permits RCLA (or makes no predictions concerning how compound lottery should be evaluated, depending on the axiomatization framework), and thus is unable to generate such a known link. In contrast, recursive models, including RRDU and REU, view ambiguity lottery as subjective compound lottery, and hence are compatible with the known link.

Considering additionally almost-objective natural-event uncertainty helps distinguish further models adopting the *multiple-prior* or *source preference* perspective. In particular, all multiple-prior models fail to generate (nonnegligible) difference between an objective risk **R** and an "almost" objective lottery **N**. It follows that they cannot generate links between natural-event uncertainty attitude and either ambiguity attitude (**A–N**) or compound risk attitude (**C–N**). In contrast, the Chew–Sagi source model, by treating different equally-likely events distinctly, is able to generate a link between ambiguity attitude and natural-event uncertainty attitude (**A–N**), but fails to deliver the other two possible links (**A–C** and **C–N**), given that it admits RCLA. Notably, the Nau or SPS model differentiates stage-1 and stage-2 priors as distinct sources of uncertainty. Consequently, their predictions coincide with those of the recursive multiple-prior models.

Beyond moderate prospect The preceding summary focuses on the qualitative predictions of different models, which can be extended naturally to loss outcomes as well as skewed winning and losing events. For example, consider a general setting in which each type of uncertainty $i \in \{R, A, C, N\}$ partitions the state space into m equally-likely events $\{E_i^1, E_i^2, \ldots, E_i^m\}$. Here, a bet on a particular event $\bigcup_{k\in K\subset\{1,2,\ldots,m\}} E_i^k$ can induce different kinds of lotteries involving moderate (or skewed) chance of winning w (or losing w'). No matter which kind of lottery we consider, it remains valid that models adopting the Ellsbergian perspective fail to deliver nonneutrality toward almost-objective natural-event uncertainty, and thus are unable to generate links between natural-event uncertainty attitude and ambiguity attitude or compound risk attitude. In contrast, models adopting the Keynesian perspective, including the Chew–Sagi model, treat ambiguity and natural-event uncertainty as distinct sources of uncertainty in parallel with risk. While they are compatible with nonneutrality toward natural-event uncertainty, they fail to accommodate compound risk nonneutrality as well as the links between compound risk attitude and ambiguity attitude or natural-event uncertainty attitude.[10]

In sum, none of the existing models can generate nonneutrality toward all three types of uncertainty, nor pairwise links between attitudes toward the three types of uncertainty, regardless of the underlying lottery considered.[11] This observation provides the foundation of our experimental design, as illustrated in the subsequent section.

3. Experimental design

3.1 *Experiment 1*

Experiment 1 includes one risk lottery, three ambiguity lotteries, two compound lotteries, and two natural-event lotteries as summarized in Table 3. For risk lottery **R**, subjects bet on the color of a card randomly drawn from 20 cards with 10 red and 10 black, and they are explicitly told that the winning probability is 50%. The full ambiguity lottery **A** is the same as that in the Ellsberg paradox, and we include two more partial ambiguity lotteries, that is, interval ambiguity **A1** and disjoint ambiguity **A2** in which the partial information about the composition of the deck is given to the subjects. The full compound lottery **C** is implemented as follows. One ticket is drawn randomly from a bag containing 21 tickets numbered from 0 to 20, and the number drawn subsequently determines the number of red cards in the deck with the remaining cards black. Similarly, for the p-q compound lottery **C1**, one number is drawn from an envelope containing 8 tickets

[10] While it is straightforward that the current predictions for even-chance lottery can be extended to incorporate other kinds of lotteries, it remains a question whether or not different models are compatible with ambiguity/compound risk/natural-event aversion or seeking behavior. For example, consider skewed compound risk attitude under two-stage models, Segal (1987) and Dillenberger and Segal (2017) show that RRDU can generate either skewed compound-risk aversion or seeking, depending on the functional forms at each stage as well as the distribution of stage-1 priors.

The question is also related to the well-known fourfold pattern for risk—risk aversion for moderate prospect and longshot hazard as well as risk seeking for longshot prospect and moderate hazard. While fourfold pattern of risk has been extensively explored since the seminal works of Kahneman and Tversky (1979) and Tversky and Kahneman (1992), the experimental evidence of fourfold pattern for attitude toward ambiguity (or more generally attitudes toward different types of uncertainty), has been limited and mixed. As the current paper focuses on the links among attitudes toward different types of uncertainty, we would like to leave the question for future study.

[11] In Section 5, we discuss extensions of existing source models, in terms of delivering nonneutral attitude toward all three sources of uncertainty as well as pairwise correlations among them.

Table 3. Summary of lotteries used in Experiment 1.

	Lottery	Implementation
R	Risk	A deck of 20 cards with 10 red and 10 black.
A	Full ambiguity	A deck of 20 cards with no information about the composition of red and black.
A1	Interval ambiguity	A deck of 20 cards with the number of red (black) cards between 5 and 15.
A2	Disjoint ambiguity	A deck of 20 cards with the number of red (black) cards either between 0 and 5 or between 15 and 20.
C	Full compound risk	A deck of 20 cards with 21 possible compositions of red and black being equally likely.
C1	p-q compound risk	A deck of 20 cards with 5/8 (3/8) chance of containing 16 (0) red cards and 4 (20) black cards.
N	Natural event (home)	Trailing digit of home city highest temperature on a given historical date.
N1	Natural event (foreign)	Trailing digit of foreign city highest temperature on a given historical date.

numbered 1 to 8. If the number drawn from is 1 to 5 (6 to 8), the deck will have 16 (0) red cards and 4 (20) black cards. In both compound lotteries, subjects bet on the color of a card drawn from the deck before drawing the ticket, and the winning probability of betting on red (black) can be reduced to 50%. In natural-event lottery **N**, subjects bet on the trailing decimal digit (odd/even) of the temperature in their home city (Singapore) on a randomly selected historical date. Correspondingly, lottery **N1** is based on the temperature of a foreign city (Istanbul). We have included multiple lotteries for the three types of uncertainty besides the baseline known risk. This enables us to apply the method of Obviously Related Instrumental Variables (ORIV, Gillen, Snowberg, and Yariv (2019)) to conduct more efficient correlation analyses, which we elaborate further in Section IV.

In our experiment, subjects are allowed to choose the color of the card or parity of city temperature to bet, and the psychology of suspicion is less of a concern.[12] Subjects receive a winning prize SGD60 (about USD43) for a correct bet (guess) in each lottery, and nothing otherwise. To elicit the certainty equivalent (henceforth CE) of each lottery, we use a choice-list design with subjects choosing between a given lottery and a sure amount ranging from SGD15 to SGD35 (see Appendix B for experimental instructions). In general, subjects will have a single switching point, that is, choosing the lottery when its CE is higher than the sure amount and switch to the sure amount as it increases to the point that is higher than the CE of the lottery. Thus, the switch point serves as a proxy for CE of the lottery.

The eight choice-lists are implemented along with the other ten tasks examining risk preference and time preference. Subjects complete the decision-making tasks without feedback. To incentivize participation, in addition to a SGD35 show-up fee, we adopt the random incentive mechanism (RIM), paying each subject based on one of her randomly selected decisions in the experiment. The use of RIM to elicit valuations of lotteries has

[12]Subjects may suspect that the experimenter manipulates the composition of the cards against them when they are not allowed to choose the event to bet. Such suspicion may increase the observed ambiguity aversion (Berger and Tymula (2022)).

triggered debates from of both theoretical and experimental perspectives.[13] We adopt RIM with the price list as it enables us to examine the potential links across different types of uncertainty.

In addition to the decision-making tasks, we include a test of cognitive ability using Raven's standard progressive matrices, one of the most popular measures of analytic intelligence. The test consists of five increasingly difficult parts of 12 questions each. In each question, subjects are asked to identify a missing element to complete a visual pattern. While we do not impose a time limit, we state that people tend to finish the test in 20–45 minutes. In the literature, higher performance on Raven's test has been found to be associated with fewer Bayesian updating errors (Charness, Rustichini, and Van de Ven (2011)), more accurate beliefs (Burks, Carpenter, Goette, and Rustichini (2009)), and increased likelihood of choosing strategies that are closer to Nash equilibrium and converge faster to equilibrium play (Gill and Prowse (2016)). The inclusion of Raven's test enables us to examine a possible relationship between cognitive ability and attitudes toward different types of uncertainty.

We have recruited a cohort of 2066 undergraduate students from the National University of Singapore who participated in a study on the biological basis of decision-making (53.0% female; mean age: 21.4). Following the Singapore cohort, an additional cohort of 1181 Han Chinese are recruited from several universities in Beijing (48.4% female; mean age: 21.5). The instructions and procedures are the same for the Singapore and Beijing experiments, except for the following points. First, the prize of the lotteries is changed to RMB240 (about SGD50). Second, for the natural-event lotteries, the home city is Beijing and the foreign city is Tokyo. Last, the written and oral instructions and decision-making tasks are in Chinese instead of English. All subjects provide written informed consent and the experiment was approved by the Institutional Review Board at the National University of Singapore.

3.2 *Experiment 2*

Experiment 2 goes beyond even-chance prospects and considers lotteries that vary in both outcome (gains versus losses) and likelihood (moderate versus longshot). Specifically, we consider five baseline risk lotteries: (1) moderate prospect: 50% chance of winning RMB100 (about USD14.54), otherwise 0; (2) moderate hazard: 50% chance of losing RMB50, otherwise 0; (3) longshot prospect: 10% chance of winning RMB100, otherwise

[13]RIM generates an objective compound lottery and may contaminate inferences of risk and ambiguity attitudes under certain circumstances. A sufficient condition for the validity of RIM is isolation (Kahneman and Tversky (1979)). See, for example, Wakker (2007) for a general discussion. In the domain of ambiguity, Bade (2015), Azrieli Y., Chambers, and Healy (2018), Baillon, Halevy, and Li (2022a), among others argue that RIM may not be incentive compatible in eliciting ambiguity attitude due to the concern of ambiguity hedge (Raiffa (1961)). Baillon, Halevy, and Li (2022b) find experimental evidence of using the RIM to hedge ambiguity. Johnson, Baillon, Bleichrodt, Li, Van Dolder, and Wakker (2021), and Baillon, Halevy and Li (2022a, 2022b) further suggest that randomization needs to precede the resolution of uncertainty so that incentive compatibility can be partially restored. In the current study, we adopt RIM as it facilitates within-subject comparison. Moreover, in both experiments, the subjects are only allowed to choose one color to bet on for all the lotteries in one choice list, making ambiguity hedge more difficult.

0; (4) longshot hazard: 10% chance of losing RMB50, otherwise 0; (5) mixed lottery: 50% chance of winning RMB100, otherwise losing RMB50. For each of the three types of uncertainty, the corresponding five lotteries are included, which results in a total number of 20 lotteries. The detailed implementation of each lottery is as follows.

For the three risk lotteries involving 50% chance, that is, moderate prospect, moderate hazard, and mixed lottery, subjects bet on the parity (odd versus even) of the number on a card randomly drawn from a deck of 10 cards of 5 odd and 5 even numbers. The corresponding three ambiguity lotteries concern betting on a deck of 10 cards with unknown composition of odd and even numbers. The three respective compound lotteries involve betting on a deck of 10 cards constructed with each of the 10 cards independently drawn from another deck of 10 cards of 5 odd and 5 even numbers. For the two risk lotteries involving 10% chance, that is, longshot prospect and longshot hazard, subjects bet on the exact number of a card randomly drawn from a deck of 10 cards numbered from 1 to 10. The corresponding two ambiguity lotteries are implemented by betting on a deck of 10 cards with unknown composition of numbers from 1 to 10. The two respective compound lotteries are implemented by betting on a deck of 10 cards constructed with each of the 10 cards independently drawn from another deck of 10 cards numbered from 1 to 10. At last, the five natural-event lotteries are based on market closing of the Tehran Stock Exchange Index on a given day. Subjects bet on the parity or the exact number of the second decimal digit of the close price for the lotteries with moderate or longshot likelihoods, respectively. Similar to Experiment 1, to ameliorate potential effects of suspicion, we allow subjects to choose freely the parity or exact number to bet on in all of the related lotteries.

To elicit the CE of each lottery, we again adopt the price-list design with subjects choosing between a given lottery and a sure amount from a list of 21 amounts. The list of sure amounts are arranged in an ascending order with the expected payoff of the lottery in the middle. Some of the sure amounts are chosen to first order stochastically dominate or be dominated by the lottery. More specifically, for moderate prospect (50% chance of winning RMB100, otherwise 0), we include 0 and RMB100 as two of the sure amounts. For moderate hazard (50% chance of losing RMB50, otherwise 0), we include RMB50 and 0 as two of the sure amounts. For both longshot prospect (10% chance of winning RMB100, otherwise 0), and longshot hazard (10% chance of losing RMB50, otherwise 0), we include RMB0 as one of the sure amounts (see Appendix B for details).

In order to avoid a substantial increase in the overall cognitive effort demanded from our subjects, we do not include duplicate elicitations for each of 20 lotteries as in Experiment 1. As a result, we cannot implement the ORIV for the data analysis in Experiment 2. In each price list, subjects make 21 binary choices, resulting in a total of 420 choices for 20 lotteries. To further simplify the choice tasks, we enforce a single switching point for each choice list and subjects only need to click on one sure amount in each list indicating that they would choose the lottery over the sure amounts that are strictly lower than their chosen amount and switch to the lottery for the rest. The sure amount chosen serves as the switch point and provides a proxy for CE of the lottery.

We randomize the order of the 20 lists in a two-step manner. First, within each of the five types of lotteries varying in outcomes and likelihoods, the risk lotteries always appear first, and the three types of uncertainty appear later in random order. Second, we randomize the order of the five lotteries. As with Experiment 1, we adopt the random incentive mechanism, paying each subject based on one of her randomly selected decisions in the experiment. To incentivize participation and to avoid subjects losing money out of their own pocket, each subject receives a show-up fee of RMB60. If the randomly selected decision leads to a loss of RMB50, 50 will be deducted from the show-up fee. The experiment is implemented in o-tree, an open-source platform for behavioral research. We have recruited 208 undergraduate students from Wuhan University as participants using e-advertisement (see Appendix B for experimental instructions).

4. Results

In this section, we first present the results from Experiment 1 on the links among attitudes toward the three types of uncertainty based on even-chance gain bets, including summary statistics, the effects of cognitive ability, and replication in Beijing. We end the section with the generalizability of the results of Experiment 1 to include small probability and loss outcome in Experiment 2.

The summary statistics of Experiment 1 is presented in Table 4 for the eight lotteries (see also Figure A.1 and Figure A.2 in Appendix A in the Online Supplementary Material (Chew, Miao, and Zhong (2023)). The data is coded as the number of choices in which subjects choose the lottery over the sure amount. A higher number means a higher CE with number 7 indicating risk neutrality. We summarize the number of observations, the mean and standard deviation of CE, and the percentage of subjects who are neutral, averse, and seeking for the baseline risk lottery.[14] We also compare the CE of the risk lottery to that of the rest of the lotteries, and report the proportion of subjects who are neutral, averse and seeking toward ambiguity, compound risk, and natural-event uncertainty.

We observe that the majority of the subjects are ambiguity averse. Among the ambiguity lotteries, the CE of the full ambiguity lottery: $CE_{\mathbf{A}}$ is significantly lower than that of the interval ambiguity lottery: $CE_{\mathbf{A1}}$ (paired t-test, $p < 0.001$), as well as that of the

Table 4. Summary statistics of attitudes toward uncertainty.

	Lottery	N	Mean	SD	Neutral	Aversion	Seeking
R	Risk	1956	5.239	2.595	9.4%	76.0%	14.7%
A	Full ambiguity	2031	2.910	3.041	22.0%	65.0%	13.0%
A1	Interval ambiguity	2014	3.683	3.003	25.2%	55.8%	19.1%
A2	Disjoint ambiguity	2011	3.989	3.364	23.2%	53.6%	23.2%
C	Full compound risk	2016	3.672	3.246	23.1%	56.3%	20.6%
C1	p-q compound risk	2018	4.679	3.383	23.8%	44.3%	32.0%
N	Natural event (home)	2025	4.962	2.771	35.9%	35.4%	28.7%
N1	Natural event (foreign)	2022	4.659	2.788	35.3%	39.8%	24.9%

[14]We observe 2.4% to 6% of choice lists with multiple switch points and exclude these data from the analysis. This results in the difference in number of observations.

disjoint ambiguity: $CE_{\mathbf{A2}}$ (paired t-test, $p < 0.001$). These findings replicate the corresponding results in Chew, Miao, and Zhong (2017).

For the compound lotteries, consistent with the findings in previous experimental studies (e.g., Abdellaoui, Klibanoff, and Placido (2015)), we observe that the majority of the subjects exhibit compound-risk aversion. Between the two compound lotteries, the CE of the full compound lottery: $CE_{\mathbf{C}}$, is significantly lower than that of the p-q compound lottery: $CE_{\mathbf{C1}}$ (paired t-test, $p < 0.001$). This may reflect the sense that p-q compound lottery is less complex in the sense of having less states in the stage-1 risk compared to the full compound lottery.

For natural-event lotteries, while about one-third of the subjects exhibit neutrality toward the two temperature lotteries, more subjects are averse to natural-event uncertainty, especially for the foreign temperature lottery. Between these two lotteries, the CE of home temperature lottery $CE_{\mathbf{N}}$ is significantly higher than that of the foreign temperature lottery: $CE_{\mathbf{N1}}$ (paired t-test, $p < 0.001$). Using natural-event lotteries with almost objective uncertainty, we demonstrate a preference for familiarity in a within-subject design. This extends the findings of previous studies on preference for familiarity whereby source preference can also be attributable to potential difference in terms of knowledge (Fox and Tversky (1995), Abdellaoui et al. (2011)).[15]

4.1 *New links: Ellsberg meets Keynes*

We examine the links among attitudes toward ambiguity, compound risk, and almost-objective natural events. Table 5. A reveals a significant association between ambiguity nonneutrality and non-RCLA behavior (Pearson's chi-squared test, $p < 0.001$). Of 203 subjects who are ambiguity neutral, 124 of them exhibit compound risk neutrality. This is more than six times the expected frequency under the null hypothesis of independence and replicates the studies on the link between ambiguity and compound lottery since Halevy (2007).

Table 5.B reveals a significant association between ambiguity neutrality and neutrality toward natural events (Pearson's chi-squared test, $p < 0.001$). Specifically, of 202 ambiguity neutral subjects, 153 subjects are neutral toward natural events, which is more than 2.5 times the expected frequency under the null hypothesis of independence. This unveils a missing link between ambiguity attitude and attitude toward natural-event uncertainty. Moreover, Table 5.C shows a significant association between RCLA behavior and neutrality toward natural-event uncertainty (Pearson's chi-squared test, $p < 0.001$). Specifically, of 192 subjects exhibiting RCLA, 148 subjects are neutral toward natural-event uncertainty, which is about 3 times the expected frequency under the null hypothesis of independence. This reveals another missing link between attitude toward compound risk and attitude toward natural-event uncertainty.

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[15]We also examine the effect of background characteristics including gender, age, parental education, number of siblings, and family income (Table A.1 for Singapore subjects and A.2 for Beijing subjects). We find that female subjects are more risk averse, compared to the male subjects, and subjects from lower income families are more risk averse, relative to those from higher income families. We do not observe significant and consistent effects of other characteristics on attitudes toward risk, ambiguity, compound risk, and natural-event uncertainty.

TABLE 5. Linking attitudes toward three types of uncertainty.

A. Ambiguity and compound risk

	Compound risk		
Ambiguity	Reduction	Nonreduction	Total
Neutral	124 (20.7)	79 (182.3)	203
Nonneutral	67 (170.3)	1607 (1503.7)	1674
Total	191	1686	1877

Pearson chi2(1) $= 645.38$, $p < 0.001$

B. Ambiguity and natural events

	Natural events		
Ambiguity	Neutral	Nonneutral	Total
Neutral	153 (62.2)	49 (139.8)	202
Nonneutral	426 (516.80)	1253 (1162.2)	1679
Total	579	1302	1881

Pearson chi2(1) $= 214.71$, $p < 0.001$

C. Natural events and compound risk

	Natural events		
Compound risk	Reduction	Nonreduction	Total
Neutral	148 (58.7)	44 (133.3)	192
Nonneutral	430 (519.3)	1267 (1177.7)	1697
Total	578	1311	1889

Pearson chi2(1) $= 217.48$, $p < 0.001$

Note: This two-way table presents the number of subjects by (non)neutrality toward ambiguity, compound risk, and natural-event uncertainty. Each cell indicates the number of subjects with the expected number displayed in parentheses (Pearson's chi-squared test, $p < 0.001$).

We further investigate the association among the three uncertainty attitudes by assessing the correlations among the ambiguity premium, the compound risk premium and the natural-event uncertainty premium. The respective premiums are measured by the difference in CEs between the risk lottery and the ambiguity (compound/natural-event) lottery. We find that the correlations between premiums across each individual lottery mostly lie between 0.40 and 0.60 (Table A.3 in Appendix A). The pairwise scatter plots of the average premium in each of the three types of uncertainty are presented in Figure 1. We observe significant correlations between average ambiguity premium and average compound risk premium (0.62), between average ambiguity premium and average natural-event uncertainty premium (0.49), and between average compound risk premium and average natural-event uncertainty premium (0.48).

A common concern in preference elicitation is measurement error. Note that the observed correlations are highly significant among the three ambiguity lotteries (0.75, 0.56, and 0.63), between the two compound lotteries (0.49), and between the two natural-event lotteries (0.87), This suggests common underpinnings within each type of uncertainty in conjunction with measurement errors. In subsequent analysis, we make use of the method of Obviously Related Instrumental Variables (ORIV) developed in Gillen,

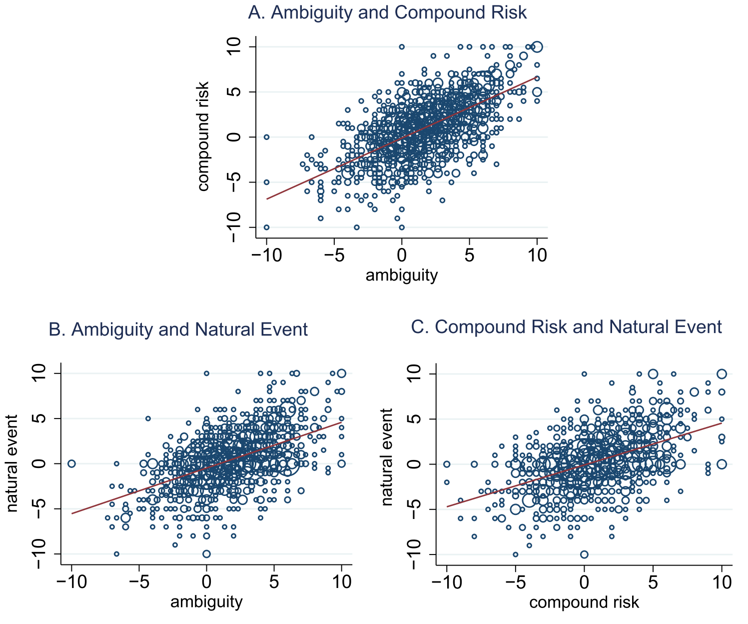

FIGURE 1. Pairwise scatter plots of average premium across three types of uncertainty.

Snowberg, and Yariv (2019) to correct for measurement error given that we have multiple elicitations of closely related attitudes.[16] Specifically, we have three ambiguity lotteries, two compound lotteries, and two natural-event lotteries that can serve as multiple elicitations. As reported in Table 1 in the Introduction, the correlations are substantially improved after adopting ORIV: correlation between ambiguity premium and compound risk premium increases to 0.83, correlation between ambiguity premium and natural-event uncertainty premium increases to 0.58, and correlation between compound risk premium and natural-event uncertainty premium increases to 0.66.[17] This is in line with the observations in Gillen, Snowberg, and Yariv (2019), which reports an increase in the correlation between ambiguity premium and compound risk premium from 0.44 to 0.85 using ORIV. Overall, these analyses further strengthen the links among these three uncertainty attitudes.

Finally, we examine the behavior at the individual level (Table 6). A subject is classified as being neutral toward a particular type of uncertainty if the CEs for lotteries in that type of uncertainty are the same as the CE for the risk lottery. We find that 65.3% of the subjects are nonneutral toward all three types of uncertainty, while 5.7% of the subjects are neutral toward all three. For the rest of subjects, 20.4% are nonneutral toward ambiguity and compound risk, and neutral toward natural-event uncertainty. These three

[16]ORIV adopts an errors-in-variable instrumental variable approach to analyze the data when there are multiple elicitations of the behavioral proxies.

[17]In addition, we also use principal component analysis within each source of uncertainty and use the first component to conduct the correlation analysis. The correlation is 0.64 between ambiguity premium and compound risk premium, 0.54 between ambiguity premium and natural-event uncertainty premium, and 0.51 between compound risk premium and natural-event uncertainty premium.

TABLE 6. Individual type analysis.

Ambiguity	Compound risk	Natural event	Percent
Nonneutral	Nonneutral	Nonneutral	65.3%
Nonneutral	Nonneutral	Neutral	20.4%
Neutral	Neutral	Neutral	5.7%
Nonneutral	Neutral	Nonneutral	1.4%
Neutral	Nonneutral	Nonneutral	1.7%
Nonneutral	Neutral	Neutral	2.2%
Neutral	Nonneutral	Neutral	2.5%
Neutral	Neutral	Nonneutral	0.9%

Note: This table presents the percentage of individual type classified by attitudes toward three types of uncertainty. Neutrality toward certain type of uncertainty corresponds to the same CE between the risky lottery and each of the lotteries in that type of uncertainty.

groups account for 91% of the subjects, with the rest of 9% exhibiting other behavioral patterns.

4.2 *Cognitive ability*

In this subsection, we explore the role of cognitive ability measured by subjects' scores in Raven's test (mean = 56.02, median = 57; SD = 4.60) by comparing subjects with high scores and those with low scores (1059 subjects with scores higher than 57; 1022 subjects with scores lower than or equal to 57). We report the statistics in Tables A.4 to A.6 in Appendix A, and summarize the results here. Overall, our results are robust with respect to cognitive ability. For both high ability and low ability groups, we find: (1) correlation remains high across all three types of uncertainty and is higher between attitudes toward ambiguity and compound risk; (2) a strong majority of the subjects are nonneutral toward the three types of uncertainty, followed by subjects who are nonneutral toward ambiguity and compound risk, but neutral toward natural-event uncertainty, with neutrality toward all three types coming in at a distant third.

We further examine the role of cognitive ability. First examining how cognitive ability is related to attitude toward ambiguity, compound risk, and natural-event uncertainty, we find one out of seven lotteries exhibiting significant difference. More specifically, subjects with high scores tend to have a higher premium for full ambiguity, compared with those with low scores (OLS, $p = 0.033$). We then examine the pairwise correlations across the three types of uncertainty, and find that above-median subjects are linked to higher correlation between ambiguity attitude and compound risk attitude, and lower correlation between ambiguity attitude and natural-event uncertainty attitude, as well as between compound risk attitude and natural-event uncertainty attitude. Lastly, in individual type analysis, among subjects exhibiting nonneutrality toward all three types of uncertainty, the percentage of 63.4% for above-median subjects is significantly lower than the 67.4% for below-median subjects (proportion test, $p < 0.067$). For subjects exhibiting neutrality toward all three types of uncertainty, the percentage is 6.6% for above-median subjects and 4.6% for below-median subjects (proportion test, $p < 0.070$). In sum, close links are observed across types of uncertainty with respect to cognitive ability. Moreover, subjects with high scores are less likely to be nonneutral toward all three types of uncertainty. We would also like to add a cautious note that none of the correlations between IQ and attitudes toward the seven lotteries remains statistically significant after we correct for the multiple comparisons conducted when analyzing the seven lotteries.

4.3 *Replicability*

In this subsection, we examine the replicability of our findings using the Beijing sample. We display the results of our analysis in Tables A.7 to A.9 in Appendix A and summarize them here. Overall, the high correlations among attitudes toward the three types of uncertainty, and the similarity in proportions of individual types, are replicated in the Beijing sample. To explore potential differences between the Beijing and Singapore samples, we first examine subjects' attitudes toward ambiguity, compound risk, and natural-event uncertainty. Despite statistical significance, the differences in proportion between Singapore and Beijing samples of being averse are generally less than 3%. One exception is with the p-q compound lottery, in which the proportion of being averse is 44.3% among Singapore subjects compared to 31.5% among Beijing subjects. We then examine the pairwise correlations across the three types of uncertainty, and find that the correlation is 0.78 between ambiguity attitude and compound risk attitude, 0.60 between ambiguity attitude and natural-event uncertainty attitude, and 0.62 between compound risk attitude and natural-event uncertainty attitude. Furthermore, in individual type analysis, 68.5% of Beijing subjects exhibit nonneutrality toward all three types of uncertainty, 16.5% exhibit nonneutrality toward ambiguity and compound risk, and neutrality toward natural-event uncertainty, and 4.0% are neutral toward all three types of uncertainty. Lastly, the observed patterns with regard to cognitive ability are also replicated in Beijing sample (see Tables A.10 to A.12 in Appendix A). Overall, the choice behavior of Beijing subjects and Singapore subjects are remarkably similar. This said, Beijing subjects are less averse to p-q compound lottery, exhibit lower correlation between ambiguity attitude and compound risk attitude, and less likely to exhibit nonneutrality toward ambiguity and compound risk together with neutrality toward natural-event uncertainty.

4.4 *Extendibility to fourfold and mixed lotteries*

This subsection examines whether the links among the attitudes toward the three types of uncertainty observed in Experiment 1 can be generalized via Experiment 2 to lotteries in the fourfold pattern and additionally a mixed lottery involving both a gain and a loss outcome. The summary statistics for the 20 lotteries in Experiment 2 is presented in Table A.13.[18] When comparing the CEs of risk lotteries with their expected values,

[18]We report the results after excluding 59 subjects who violate first-order stochastic dominance at least once. In the Appendix, we report the corresponding tables with the full sample of 208 subjects (Table A.14 and A.15). The results are quantitatively similar.

TABLE 7. Correlations across three types of uncertainty in Experiment 2.

	A–C	**A–N**	**C–N**
Moderate prospect	0.556	0.398	0.504
Moderate hazard	0.669	0.492	0.548
Longshot prospect	0.687	0.602	0.637
Longshot hazard	0.613	0.584	0.627
Mixed lottery	0.588	0.617	0.615
Average	0.623	0.538	0.586

Note: This table reports the Spearman correlations after dropping those subjects violating FOSD. N = 149.

we find that the majority of the subjects are risk averse for moderate prospect, longshot hazard, and mixed lottery, and are risk-seeking for moderate hazard and longshot prospect. In this regard, we replicate the observed fourfold pattern in risk attitude and aversion to mixed risk (Kahneman and Tversky (1979), Tversky and Kahneman (1992)). When comparing the CEs of risk lotteries with those of the other three types of uncertainty, we observe that subjects are, on average, averse to ambiguity, compound risk, and natural-event uncertainty except for longshot hazard. While it has been hypothesized that ambiguity attitudes exhibit a fourfold pattern, namely ambiguity aversion for moderate prospect and longshot hazard, and ambiguity seeking for moderate hazard and longshot prospect, experimental evidence has been mixed (see Trautmann and Van De Kuilen (2015) for a comprehensive review).[19]

Table 7 presents the correlations among the ambiguity premium, the compound risk premium and the natural-event uncertainty premium. Similarly, the respective premiums are measured by the difference in CEs between the risk lottery and the ambiguity (compound/natural-event) lottery. Notice that we do not include multiple elicitations of closely related lotteries in Experiment 2, so we cannot apply ORIV to correct measurement errors. Nevertheless, we find that the correlations of premiums across each individual lottery and different types of uncertainty range between 0.40 and 0.69 with an average of 0.58. In particular, for all the five lotteries considered, the average correlation is 0.62 between ambiguity premium and compound risk premium, 0.54 between ambiguity premium and natural-event uncertainty premium, and 0.59 between compound risk premium and natural-event uncertainty premium. Overall, these results provide strong support for the existing link in Halevy (2007) and the new links discovered in Experiment 1.

5. Discussion

Related experimental studies

Our results have bearing on recent experimental studies testing ambiguity models both qualitatively and quantitatively. Some experimental studies focus exclusively on mod-

[19]For example, while Baillon and Bleichrodt (2015) provide experimental support for the fourfold pattern, they also observe ambiguity neutrality for unlikely events. Relatedly, Kocher, Lahno, and Trautmann (2018) observe ambiguity neutrality or ambiguity seeking for moderate hazard and longshot prospect, and ambiguity seeking for moderate hazard.

els involving multiple priors. For example, Conte and Hey (2013) examine the predictive power of multiple-prior models through estimating a mixture model, and find that recursive expected utility explains the overall patterns better than α-maxmin expected utility. Cubitt, Van de Kuilen, and Mukerji (2020) test preference for hedging under ambiguity to discriminate between α-maxmin expected utility and recursive expected utility, and find support for the latter. When different outcomes and different levels of likelihoods are considered, Baillon and Bleichrodt (2015) find different ambiguity attitudes toward gains and losses, and toward moderate and longshot likelihoods. They suggest that their findings are more consistent with prospect theory (adapted to the domain of ambiguity), but to a lesser extent, with α-maxmin expected utility or recursive expected utility.

Building on the works of Tversky and his colleagues (Heath and Tversky (1991), Fox and Tversky (1995)), a number of recent studies examine attitude toward natural-event uncertainty. de Lara Resende and Wu (2010) show that subjects' preference for betting on a natural event as well as its complement, such as the temperature of a particular city being higher or lower than a given level, may depend on their level of knowledge of the specific source of uncertainty. Adopting the definition of event exchangeability in Chew and Sagi (2008), Abdellaoui et al. (2011) use a bisection procedure for subjects to partition the state space, for example, the range of home (foreign) city temperature, into disjoint intervals with equal likelihoods. They observe that a substantial proportion of subjects exhibit a preference for betting on the temperature in their home city rather than a foreign city. Relating to the opening quote of Keynes, subjects in their study may possess different degrees of confidence or affinity with the source—home versus foreign—of the underlying temperature uncertainty, despite having assessed their likelihoods as being equal. In this design, models with multiple priors can still account for the observed preference for the familiar with different (distributions on the) sets of priors based on the degree of confidence in the assessed probability. By contrast, the current study is the first to investigate source preference using almost-objective uncertainty and to examine its interaction with compound risk attitude and ambiguity attitude.

Almost objective uncertainty

Machina (2004) proposes the notion of "almost objective" uncertainty, and argues that most randomization devices, including a simple toss of a coin, may actually generate almost-objective uncertainty. In Experiment 1, the uncertainty associated with the trailing digit of a city temperature being odd or even is almost-objective. Yet, we observe a preference for the risk lottery to these two almost-objective lotteries, as well as a significant but weaker preference to bet on home temperature rather than on foreign temperature. In Experiment 2, when we use the second decimal digit at market closing of a stock exchange index, we observe similar preference for the risk lottery to the almost-objective lottery. These observations challenge ambiguity models adopting the multiple-prior perspective, whose preferences over almost-objective acts approach probabilistic sophistication asymptotically. In contrast, the source preference perspective is compatible with the wide range of phenomena mentioned here through decision makers having preference over distinct sources of uncertainty.

In the meantime, we cannot rule out the possibility of some subjects having different degrees of confidence in assessing the likelihood in terms of home versus foreign city temperature. A Singaporean subject may have heard nothing (prior) bizarre about local temperature and updates her belief and treats the (posterior) probability of the trailing digit being odd/even as almost objectively 50–50. She may not be as confident when updating her posterior about the foreign city temperature given a weaker prior. This reasoning may rely on an implicit assumption that the natural event is "perceived" as embracing a certain degree of ambiguity despite its almost-objective nature. In the example in Section II, while the subjective probability of the union of left-half of each interval $\bigcup_n (\frac{n}{m}, \frac{n+0.5}{m}]$ approaches 0.5 asymptotically, a subject may think of such a way of partitioning the state space cognitively demanding and simply partition the state space in to {odd, even}, thereby generating a certain degree of ambiguity. An interesting question concerns whether this *as-if* view should be modeled through the lens of multiple priors or source preference. While either way is admissible, we note that our theoretical predictions in Section II need to be revised should we adopt this *as-if* view in conjunction with the multiple-prior perspective. In particular, all of the models considered there, especially those adopting the multiple-prior perspective, would be compatible with natural-event nonneutrality, and can then generate a correlation between almost-objective uncertainty attitude and ambiguity attitude, or even compound risk attitude, depending on whether a two-stage approach is adopted.

Rich domains of uncertainty

A number of recent studies can be interpreted as investigating the commonalities of attitudes across types of uncertainty. Armantier and Treich (2015) consider a "complex risk" treatment, in which the outcome of the bet depends on the color of two balls simultaneously drawn from two risky urns. They observe a tight association between attitude toward this complex risk and the attitude toward ambiguity, and suggest that their finding may hint at the link between ambiguity attitude and complexity attitude. Epstein and Halevy (2018) examine bets, which depend on the color of two balls drawn from two urns, where there is ambiguity in the compositions of each urn as well as the correlation between them. They find that attitude toward such ambiguous correlation between two unknown urns is related to attitude toward the standard Ellsbergian ambiguity. Li, Muller, Wakker, and Wang (2018) examine uncertainty arising from betting on the home district of an Indian rural child. Given that subjects have no prior knowledge on different Indian districts, they observe a significant correlation between attitudes toward betting on the districts and betting on an unknown urn.

In their seminal work of prospect theory, Kahneman and Tversky (1979) posit that attitudes toward risk are sensitive to outcomes and likelihoods, and arrive at the so-called fourfold pattern in risk attitude: risk aversion toward moderate prospect or longshot hazard, and risk seeking toward longshot prospect or moderate hazard. The fourfold pattern in risk attitude has been widely observed in experimental studies on decision-making under risk. Correspondingly, in the literature on decision-making under uncertainty, a number of experimental studies investigate ambiguity attitude beyond the even-chance prospect and find mixed evidence regarding how ambiguity attitude may depend on outcome and likelihood (i.e., de Lara Resende and Wu (2010), Baillon and Bleichrodt (2015), Kocher, Lahno, and Trautmann (2018)). In Experiment 2, we observe a reassuring fourfold pattern in risk attitude as well as an aversion to even-chance mixed risk. In addition, we observe (weak) aversion toward all three types of uncertainty except for longshot hazard. While attitudes toward risk and ambiguity depend on outcome and likelihood, our study shows that the pairwise correlations among attitudes toward the three types of uncertainty do not exhibit such sensitivity. The robustness in the observed links suggests a shared component in decision-making underpinning different attitudes toward the three different types of uncertainty.

Cognitive and consequentialist considerations

Implicit in the formulation of the expected utility hypothesis is an exclusive focus on what may be considered as consequences in an uncertain alternative, chiefly its outcomes and their associated likelihoods. This consequentialist focus is also evident in the development of nonexpected utility models, culminating in the definition of probabilistic sophistication in Machina and Schmeidler (1992). It has been suggested that departures from the prescriptions of expected utility, such as Allais and Ellsbergian behavior, exemplify nonconsequentialist behavior and may have their roots in limitations in cognitive ability or attentiveness (see, e.g., Morgenstern (1979)). To address this cognitive limitation hypothesis in conjunction with ambiguity attitude elicited using the two-urn problem, subjects in Chew, Ratchford, and Sagi (2017) are first screened for comprehension and attentiveness before being presented with a matrix version of the two-urn problem intended to minimize the difference in complexity between the known and the unknown bets. They find that low-comprehension subjects are seemingly ambiguity neutral in choosing randomly while high-comprehension subjects continue to favor the known bet.[20]

As discussed earlier, 57 out of 60 is the median score of our subjects in Ravens IQ test following the choice tasks. This suggests that the bulk of our subjects are high-comprehension and attentive. At the same time, we continue to observe systematic departures from consequentialist behavior in attitudes toward the three types of uncertainty, including the strong pairwise correlations, among the subsample of above-median subjects.[21] In conjunction with the literature, our findings reveal that decision-making under uncertainty can depend on how choice is perceived even when there is sufficient attention and comprehension.

[20]It is instructive to revisit Raiffa's (1961) reflection on his "error" in favoring the known bet when responding to Ellsberg's request to provide his choice in the two-urn problem "without any pencil pushing." He realized afterwards that he could fashion from the unknown bet a two-stage lottery with overall winning probability of 1/2 by betting red if a coin flip turns out head and black otherwise. Consistent with RCLA, Raiffa would be indifferent between the two-stage bet and the known bet. Raiffa's reflection points to an innate tendency to avoid ambiguity when he is not particularly attentive while there is little doubt about his ability to behave in a consequentialist manner with sufficient attention.

[21]In this regard, the finding in Abdellaoui, Klibanoff, and Placido (2015) of the ambiguity-compound risk link being stronger for nonengineers than engineers may be due to greater engagement and attentiveness.

Toward an integrative theory

Despite the flexibility in viewing each type of uncertainty as a distinct source, a closer examination of existing models adopting the source preference perspective reveals that they do not deliver a joint account of our experimental findings. In particular, the Chew–Sagi approach admits RCLA and cannot distinguish compound risk from its reduced simple risk, thereby failing to generate the observed links between compound risk attitude and ambiguity attitude or natural-event attitude. On the other hand, Nau (2006) and SPS limit their domain to two exogenous sources of uncertainty and coincide with recursive models upon interpreting the two sources as priors at different stages. As such, they fail to generate nonneutral attitude toward almost-objective natural event as well as the observed links between natural-event attitude and ambiguity attitude or compound risk attitude. Overall, the observed differences in attitudes toward three types of uncertainty as well as the pairwise links among these uncertainty attitudes point to the possibility of having an integrative theory across the multiple-prior and the source perspectives. One may consider for simplicity a Chew–Sagi type of source model without RCLA that treats compound risk as yet another source of uncertainty, which is different from its reduced simple risk. However, such a model lacks tractability, and more importantly, cannot offer a plausible account for the stronger correlation in attitudes between ambiguity and compound risk than that between ambiguity and natural-event uncertainty as well as between compound risk and natural-event uncertainty. This stronger correlation between ambiguity attitude and compound risk attitude tends to support the recursive approach to modeling ambiguity attitude.[22] Taken together, our findings point to an integrative theory, which extends the current recursive models by incorporating within-stage source preferences.

6. Conclusion

The consideration of the unknown urn by Keynes and Ellsberg has spawned a voluminous literature on decision-making under uncertainty encompassing theory, experiment, and application, and spanning multiple disciplines. Subsequently, distinct perspectives concerning betting on the unknown urn has given rise to two parallel strands of research. In one strand of research, ambiguity attitude reflects Ellsberg's multiple-prior perspective and can be linked to attitude toward objective compound risk. In the other strand, ambiguity attitude arises from having preference over the sources of uncertainty underpinning a lottery, which can range from urns and balls to city temperatures and market indices.

[22]The observed link between attitudes toward full ambiguity and full compound risk in Experiment 1 admits an alternative interpretation that the compound lottery may be viewed as ambiguity due to the similarity in presentation or its complexity. We observe an equally significant correlation in attitudes between full ambiguity and p-q compound risk as that between full ambiguity and full compound risk. As it is unlikely for the subjects to view the p-q compound lottery as ambiguity, our results favor the view that subjects view ambiguity as compound lottery. In this regard, our findings lend support to the recursive modelling approach, and corroborate Halevy's (2007) observed close link between ambiguity attitude and compound risk attitude.

The present paper examines experimentally attitudes toward multiple-prior uncertainty in terms of ambiguity and its corresponding objective compound risk and attitude toward almost-objective uncertainty. We observe a general tendency to be nonneutral toward each of the three types of uncertainty and each that pair of these attitudes tend to be highly correlated. We further find that this tendency is robust with respect to the valence of the outcomes and to the degree of likelihood. In this regard, attitude toward one type of uncertainty may be informative in predicting attitude toward another type of uncertainty.

Our findings contribute to an emerging sense of the role of perception in decision-making under uncertainty. Whether a bet on the unknown urn is perceived as being probabilistic (as with Keynes) or having multiple priors (as with Ellsberg), can materially impact how it is evaluated. Taken as a whole, our findings point to the need for an integrative model of decision-making under uncertainty to incorporate how decision makers may perceive a lottery not only in terms of its likelihoods and the valence of its outcomes, but also its underlying sources of uncertainty as well as whether and how attitudes toward such uncertainties may be linked.

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