Supplement to "Redistribution and the monetary-fiscal policy mix"

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Section A of this Online Appendix presents a tractable two-agent model that permits analytical solutions. The flexible-price model analyzed in Section 2 of the main text is introduced as a special case of this model. We give more details on the derivation of the results in that section. Section B details the quantitative model presented in Section 3 of the main text. Section C presents additional figures and tables.

Appendix A: The simple model

A.1 *Households*

A.1.1 *Ricardian household* There are Ricardian households of measure $1-\lambda$. These households, taking prices as given, choose $\{C_t^R, L_t^R, B_t^R\}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \left[\log C_t^R - \chi \frac{\left(L_t^R\right)^{1+\varphi}}{1+\varphi} \right]$$

subject to a standard No-Ponzi condition, $\lim_{t\to\infty}[\beta^t \frac{1}{C_t^R}(\frac{B_t^R}{P_t})] \geq 0$, and a sequence of flow budget constraints

$$C_t^R + \frac{B_t^R}{P_t} = R_{t-1}\frac{B_{t-1}^R}{P_t} + w_t L_t^R + \Psi_t^R - \tau_t^R,$$

where C_t^R, L_t^R, B_t^R, Ψ_t^R, τ_t^R, P_t, w_t, and R_t denote respectively consumption, hours, nominal government debt, real profits, lump-sum taxes, the price level, the real wage rate,

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and the nominal gross interest rate. The discount parameter and the inverse of the Frisch elasticity are denoted by $\beta \in (0, 1)$ and $\varphi \geq 0$. The superscript, R, represents "Ricardian." The flow constraints can be written as

$$C_t^R + b_t^R = R_{t-1}\frac{1}{\Pi_t}b_{t-1}^R + w_t L_t^R + \Psi_t^R - \tau_t^R,$$

where $b_t^R = \frac{B_t^R}{P_t}$ is the real value of debt, and $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross rate of inflation.

Optimality conditions are given by the Euler equation, labor supply condition, and transversality condition (TVC):

$$\frac{C_{t+1}^R}{C_t^R} = \beta\frac{R_t}{\Pi_{t+1}}, \tag{A.1}$$

$$\chi (L_t^R)^{\varphi} C_t^R = w_t, \tag{A.2}$$

$$\lim_{t\to\infty}\left[\beta^t \frac{1}{C_t^R}\left(\frac{B_t^R}{P_t}\right)\right] = 0. \tag{A.3}$$

A.1.2 *HTM household* The hand-to-mouth (HTM) households, of measure λ, simply consume government transfers, s_t^H, every period

$$C_t^H = s_t^H,$$

and has no optimization problem to solve.

A.2 *Firms*

A.2.1 *Final good producing firms* Perfectly competitive firms combine two types of intermediate composite goods $\{Y_{f,t}, Y_{s,t}\}$ to produce final consumption goods using a Cobb–Douglas production function

$$Y_t = (Y_{f,t})^{1-\gamma}(Y_{s,t})^{\gamma},$$

where the intermediate composites are given as

$$Y_{f,t} \equiv \left[\int_0^1 y_{f,t}(i)^{\frac{\theta-1}{\theta}}\,di\right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad Y_{s,t} \equiv \left[\int_0^1 y_{s,t}(i)^{\frac{\theta-1}{\theta}}\,di\right]^{\frac{\theta}{\theta-1}}.$$

Solving the standard cost minimization problems yield price indices of the form:

$$P_t = k^{-1}(P_{f,t})^{1-\gamma}(P_{s,t})^{\gamma},$$

$$P_{f,t} \equiv \left[\int_0^1 p_{f,t}(i)^{1-\theta}\,di\right]^{\frac{1}{1-\theta}} \quad \text{and} \quad P_{s,t} \equiv \left[\int_0^1 p_{s,t}(i)^{1-\theta}\,di\right]^{\frac{1}{1-\theta}},$$

where $k = (1-\gamma)^{1-\gamma}\gamma^{\gamma}$, and the demand functions for the intermediate goods:

$$Y_{f,t} = (1-\gamma)\left(\frac{P_{f,t}}{P_t}\right)^{-1} Y_t \quad \text{and} \quad Y_{s,t} = \gamma\left(\frac{P_{s,t}}{P_t}\right)^{-1} Y_t,$$

$$y_{f,t}(i) = \left(\frac{p_{f,t}(i)}{P_{f,t}}\right)^{-\theta} Y_{f,t} \quad \text{and} \quad y_{s,t}(i) = \left(\frac{p_{s,t}(i)}{P_{s,t}}\right)^{-\theta} Y_{s,t}.$$

A.2.2 *Intermediate good producing firms* These firms produce goods using the linear production function

$$y_{f,t}(i) = l_{f,t}(i) \quad \text{and} \quad y_{s,t}(i) = l_{s,t}(i),$$

where $l_{f,t}(j)$ and $l_{s,t}(j)$ are labor hours employed by the firms. Firm i's real profits are given as

$$\Psi_{j,t}(i) = \frac{p_{j,t}(i)}{P_t} y_{j,t}(i) - w_t y_{j,t}(i) \quad \text{for } j = f \text{ and } s.$$

Firms in sector f set prices every period flexibly. The first-order condition of these firms is given by

$$\frac{P_{f,t}}{P_t} = \frac{\theta}{\theta - 1} w_t = \mu w_t,$$

where $\mu \equiv \frac{\theta}{\theta-1}$. Firms in sector s, in contrast, set their prices to the previous period price index P_{t-1}:

$$\frac{P_{s,t}}{P_t} = \frac{P_{t-1}}{P_t} = \Pi_t^{-1}.$$

A.2.3 *Aggregation* First, we use the aggregate price index to obtain a Phillips curve relationship

$$1 = k^{-1} \left(\frac{P_{f,t}}{P_t}\right)^{1-\gamma} \left(\frac{P_{s,t}}{P_t}\right)^{\gamma} = k^{-1} (\mu w_t)^{1-\gamma} (\Pi_t^{-1})^{\gamma}.$$

Solve for w_t to get

$$w_t = \mu^{-1} k^{\frac{1}{1-\gamma}} \Pi_t^{\frac{\gamma}{1-\gamma}} \quad \text{(Phillips curve)}, \tag{A.4}$$

which shows the real wage depends positively on inflation, except for the flexible-price limit, $\gamma = 0$.

Aggregate hours are given as

$$L_t = \underbrace{\int l_{f,t}(j)\, dj}_{\equiv L_{f,t}} + \underbrace{\int l_{s,t}(j)\, dj}_{\equiv L_{s,t}}.$$

Since firms in each sector choose a common price, we have

$$y_{f,t}(j) = Y_{f,t} \quad \text{and} \quad y_{s,t}(j) = Y_{s,t},$$
$$l_{f,t}(j) = L_{f,t} \quad \text{and} \quad l_{s,t}(j) = L_{s,t}.$$

Aggregate profits are given by

$$\begin{aligned}
\Psi_t &\equiv \int \Psi_{f,t}(i)\,di + \int \Psi_{s,t}(i)\,di \\
&= \left(\frac{P_{f,t}}{P_t} Y_{f,t} - w_t Y_{f,t}\right) + \left(\frac{P_{s,t}}{P_t} Y_{s,t} - w_t Y_{s,t}\right) \\
&= Y_t - w_t(Y_{f,t} + Y_{s,t}) \\
&= Y_t - w_t(L_{f,t} + L_{s,t}) \\
&\implies \quad \Psi_t = Y_t - w_t L_t.
\end{aligned}$$

Finally, the aggregate production function can be obtained as

$$\begin{aligned}
L_t &= \int l_{f,t}(i)\,di + \int l_{s,t}(i)\,di = L_{f,t} + L_{s,t} \\
&= (1-\gamma)\left(\frac{P_{f,t}}{P_t}\right)^{-1} Y_t + \gamma\left(\frac{P_{s,t}}{P_t}\right)^{-1} Y_t \\
&= (1-\gamma)(\mu w_t)^{-1} Y_t + \gamma \Pi_t Y_t \\
&= \left(\gamma^{\frac{\gamma}{1-\gamma}} \Pi_t^{\frac{\gamma}{1-\gamma}}\right)^{-1} Y_t + \gamma \Pi_t Y_t \\
&= \left[\left(\frac{1}{\gamma \Pi_t}\right)^{\frac{\gamma}{1-\gamma}} + \gamma \Pi_t\right] Y_t \\
&\implies \quad L_t = \Xi(\Pi_t) Y_t. \qquad \text{(A.5)}
\end{aligned}$$

Notice that in the flexible-price limit, $\Xi(\Pi_t) = 1$, and output, Y_t, does not depend on inflation. Hours, L_t, therefore, is also independent from inflation in the absence of nominal rigidities. In general, however, inflation affects hours through Y_t and $\Xi(\Pi_t)$. Output Y_t is increasing in Π_t (as shown below).

A.3 *Government*

A.3.1 *Flow budget constraint* The government issues one-period nominal debt B_t. Its budget constraint (GBC) is

$$\frac{B_t}{P_t} = R_{t-1}\frac{B_{t-1}}{P_t} - \tau_t + s_t,$$

where τ_t is taxes and s_t is transfers. It can be rewritten as

$$b_t = \frac{R_{t-1}}{\Pi_t} b_{t-1} - \tau_t + s_t. \qquad \text{(A.6)}$$

Transfer, s_t, is exogenous and deterministic.

A.3.2 *Policy rules* Monetary and fiscal policy rules are

$$\frac{R_t}{\bar{R}} = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi}, \tag{A.7}$$

$$(\tau_t - \bar{\tau}) = \psi(b_{t-1} - \bar{b}), \tag{A.8}$$

where ϕ and ψ measure respectively the responsiveness of the policy instruments to inflation and government indebtedness. The steady-state value of inflation, debt, and the exogenous variable, $\{\bar{\Pi}, \bar{b}, \bar{s}\}$, are set by policymakers and given exogenously.

A.3.3 *Intertemporal budget constraint* For future use, we obtain the intertemporal GBC by combining the flow GBC and TVC. From the GBC (A.6), we have

$$b_t = R_{t-1} b_{t-1} \frac{1}{\Pi_t} - \tau_t + s_t \quad \Longrightarrow \quad b_{t-1} = \frac{\Pi_t}{R_{t-1}}(b_t + \tau_t - s_t).$$

Iterating it forward leads to

$$b_{t-1} = \left(\frac{\Pi_t}{R_{t-1}} \frac{\Pi_{t+1}}{R_t} \cdots \frac{\Pi_{t+k-1}}{R_{t+k-2}} \frac{\Pi_{t+k}}{R_{t+k-1}}\right) b_{t+k} + \sum_{k=0}^{\infty} \left[\prod_{j=0}^{k} \frac{\Pi_{t+j}}{R_{t-1+j}}\right] (\tau_{t+k} - s_{t+k}).$$

At $t = 0$,

$$b_{-1} = \underbrace{\left(\frac{\Pi_0}{R_{-1}} \frac{\Pi_1}{R_0} \cdots \frac{\Pi_{k-1}}{R_{k-2}} \frac{\Pi_k}{R_{k-1}}\right)}_{\beta^k \frac{C_0^R}{C_1^R} \frac{C_1^R}{C_2^R} \cdots \frac{C_{k-1}^R}{C_k^R}} b_k + \sum_{i=0}^{k} \left[\prod_{j=0}^{i} \frac{\Pi_j}{R_{-1+j}}\right] (\tau_i - s_i),$$

where the discount factor is given as

$$\left[\prod_{j=0}^{i} \frac{\Pi_j}{R_{-1+j}}\right] = \frac{\Pi_0}{R_{-1}} \frac{C_0^R}{C_1^R} \frac{C_1^R}{C_2^R} \cdots \frac{C_{i-1}^R}{C_i^R} = \frac{\Pi_0}{R_{-1}} \beta^i \frac{C_0^R}{C_i^R}.$$

In the limit, we have

$$b_{-1} = \underbrace{\frac{\Pi_0 C_0^R}{R_{-1}} \underbrace{\lim_{k \to \infty} \beta^k \frac{1}{C_k^R} b_k}_{\text{TVC}}}_{\to 0} + \frac{\Pi_0}{R_{-1}} \sum_{i=0}^{\infty} \beta^i \frac{C_0^R}{C_i^R} (\tau_i - s_i)$$

or

$$\frac{b_{-1} R_{-1}}{\Pi_0} = \sum_{i=0}^{\infty} \beta^i \frac{C_0^R}{C_i^R} (\tau_i - s_i). \tag{A.9}$$

The last equation is the intertemporal government budget constraint (IGBC).

A.4 *Aggregation and the resource constraint*

Aggregating the variables over the households yield

$$
\begin{aligned}
s_t &= \lambda s_t^H, \\
\tau_t &= (1-\lambda)\tau_t^R, \\
b_t &= (1-\lambda)b_t^R, \\
L_t &= (1-\lambda)L_t^R, \\
\Psi_t &= (1-\lambda)\Psi_t^R.
\end{aligned}
$$

Combining household and government budget constraints give

$$(1-\lambda)C_t^R + \lambda C_t^H = Y_t.$$

The resource constraint above, together with HTM household budget constraint, implies that output is simply divided between the two types of households as:

$$
\begin{aligned}
C_t^H &= \frac{1}{\lambda} s_t, \\
C_t^R &= \frac{1}{1-\lambda} Y_t - \frac{1}{1-\lambda} s_t.
\end{aligned}
\tag{A.10}
$$

A.5 *Solving the model*

As in the main text, we solve the model, considering a redistribution program in which $\{s_t\}_{t=0}^{\infty}$ can have arbitrary values greater than $\bar{s}$ until time period T, and then $s_t = \bar{s}$ for $t \geq T+1$.

A.5.1 *Output and consumption* As in the main text, we start with output. We use the household and firm optimality conditions to get

$$
\begin{aligned}
&\chi \big(L_t^R\big)^{\varphi} C_t^R = w_t \\
\implies \quad &\chi \Bigg(\frac{1}{1-\lambda} \underbrace{\Xi(\Pi_t) Y_t}_{L_t} \Bigg)^{\varphi} \bigg(\frac{1}{1-\lambda} Y_t - \frac{\omega}{1-\lambda} s_t \bigg) = \mu^{-1} k^{\frac{1}{1-\gamma}} \Pi_t^{\frac{\gamma}{1-\gamma}}.
\end{aligned}
\tag{A.11}
$$

Equation (A.11) implicitly defines output as a function of transfers and inflation, the latter of which in turn is also a function of the entire schedule of transfers $\{s_t\}_{t=0}^{\infty}$. Once output is determined, Ricardian consumption is determined by equation (A.10). We consider two special benchmarks, which helps us develop intuition for other in-between cases that are harder to solve.

A.5.1.1 *Flexible prices* First, as in the main text, we shut down any effects of nominal rigidities. A perfectly competitive and flexible-price economy can be obtained by setting $\gamma = 0$ and $\mu = 1$ (as $\theta \to \infty$).

Equation (A.11) then simplifies to

$$\chi\left(\frac{1}{1-\lambda}Y_t\right)^{\varphi}\left(\frac{1}{1-\lambda}Y_t-\frac{1}{1-\lambda}s_t\right)=1$$
$$\Longrightarrow \quad Y_t=\chi^{-1}(1-\lambda)^{1+\varphi}Y_t^{-\varphi}+s_t.$$

Output (and other real variables) are now independent from inflation.

We can obtain the "transfer multiplier" using the implicit function theorem. Let

$$F(Y,s)\equiv Y_t-\chi^{-1}(1-\lambda)^{1+\varphi}Y_t^{-\varphi}-s_t.$$

The derivative of Y with respect to s is

$$\frac{dY_t}{ds_t}=-\frac{F_s}{F_Y}=\frac{1}{1+(1-\lambda)^{1+\varphi}\dfrac{\varphi}{\chi}Y_t^{-(1+\varphi)}}.$$

Notice that

$$0\leq\frac{dY_t}{ds_t}\leq 1.$$

The Ricardian household consumption is

$$C_t^R=C^R(s_t)\equiv\frac{1}{1-\lambda}Y(s_t)-\frac{1}{1-\lambda}s_t.$$

The derivative is

$$\frac{dC^R(s_t)}{ds_t}=\frac{1}{1-\lambda}\left[\frac{dY(s_t)}{ds_t}-1\right]\leq 0.$$

These are the results presented in the main text.

A.5.1.2 *Sticky prices* We now consider the role of nominal rigidities. To this end, we assume perfectly elastic labor supply, $\varphi=0$, which is a typical assumption in the early RBC literature. This assumption allows for an analytical characterization of the solution. It maximizes the wealth effects on labor supply, and thus the multiplier. As a consequence, perfectly elastic labor supply eliminates the direct relationship between Ricardian consumption and transfers, which greatly simplifies the algebra.

We again use (A.11) to solve for output:

$$Y_t=(1-\lambda)(\chi\mu)^{-1}(\gamma\Pi_t)^{\frac{\gamma}{1-\gamma}}+s_t. \tag{A.12}$$

The last equation shows the output as a function of transfers and inflation. Unlike the case of flexible prices, the multiplier would in fact be greater if an increase in transfer generated inflation.

Ricardian consumption in this case is given as

$$C_t^R=C^R(\Pi_t)\equiv\frac{1}{1-\lambda}Y_t-\frac{1}{1-\lambda}s_t=(\chi\mu)^{-1}(\gamma\Pi_t)^{\frac{\gamma}{1-\gamma}},$$

which reveals that the Ricardian household consumption depends positively on inflation. Transfers no longer *directly* (and negatively) affect C_t^R. Consequently, and in contrast to the flexible-price case, an increase in s_t leads to an increase in C_t^R through the indirect channel (i.e., via Π_t) to the extent that transfers are inflationary.

A.5.1.3 *General case* A more general case is difficult to obtain an analytical solution. If labor supply were imperfectly elastic ($\varphi > 0$) and prices were sticky, Ricardian consumption would depend negatively on transfer—controlling for inflation. An increase in transfer, therefore, has opposing effects on Ricardian consumption. On one hand, it generates inflation, which raises C_t^R due to nominal rigidity. On the other hand, it lowers C_t^R due to the redistributive role of transfer. So, this is an intermediate case between the two benchmark setups above.

A.5.2 *Inflation* We now turn to inflation determination given monetary, tax, and transfer policies. As shown in the main text, the equilibrium time path of $\{\Pi_t, R_t, b_t, \tau_t\}$ satisfies the following conditions:

- Difference equations

$$\Pi_{t+1} = \frac{C_t^R}{C_{t+1}^R}\beta R_t,$$

$$b_t = R_{t-1}b_{t-1}\frac{1}{\Pi_t} - \tau_t + s_t,$$

$$\frac{R_t}{\bar{R}} = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi},$$

$$(\tau_t - \bar{\tau}) = \psi(b_{t-1} - \bar{b}).$$

- Terminal condition (TVC)

$$\lim_{t\to\infty}\left[\beta^t\frac{1}{C_t^R}b_t\right] = 0.$$

- Initial conditions

$$b_{-1} \quad \text{and} \quad R_{-1}.$$

We first solve for a steady state. Assume $s = \bar{s}$. The system of the difference equation then simplifies to

$$\bar{R} = \beta^{-1}\bar{\Pi},$$

$$\bar{b} = \bar{b}\frac{\bar{R}}{\bar{\Pi}} - \bar{\tau} + \bar{s} \quad \Rightarrow \quad \bar{\tau} = (\beta^{-1} - 1)\bar{b} + \bar{s}.$$

So, $\bar{R}$ and $\bar{\tau}$ are determined given $\bar{s}$, $\bar{\Pi}$, and $\bar{b}$.

The system above can be simplified. First, as is well known in this simple set-up, the Euler equation and Taylor rule can be combined to yield a nonlinear difference equation in Π_t:

$$\Pi_{t+1} = \frac{C_t^R}{C_{t+1}^R}\beta R_t = \frac{C_t^R}{C_{t+1}^R}\beta \bar{R}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi}.$$

Using the steady-state relation, $\bar{R} = \beta^{-1}\bar{\Pi}$, we obtain

$$\frac{\Pi_{t+1}}{\bar{\Pi}} = \frac{C_t^R}{C_{t+1}^R}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi}.$$

This equation shows that, for given Π_t, an increase in r_t leads to a decrease in Π_{t+1}.

Second, we now simplify the GBC. Notice that the Euler equation implies

$$R_t = \beta^{-1}\frac{C_{t+1}^R}{C_t^R}\Pi_{t+1} \quad \text{for } t \geq 0$$

$$\implies \quad R_{t-1} = \beta^{-1}\frac{C_t^R}{C_{t-1}^R}\Pi_t \quad \text{for } t \geq 1.$$

Use the above equation, the fiscal rule, and the steady-state relation, $\bar{\tau} = (\beta^{-1} - 1)\bar{b} + \bar{s}$, to obtain the budget constraint of the form (for $t \geq 1$):

$$\begin{aligned}
b_t &= R_{t-1}b_{t-1}\frac{1}{\Pi_t} - \tau_t + s_t \\
&= \beta^{-1}\frac{C_t^R}{C_{t-1}^R}\Pi_t b_{t-1}\frac{1}{\Pi_t} - \tau_t + s_t \\
&= \beta^{-1}\frac{C_t^R}{C_{t-1}^R}b_{t-1} - \bar{\tau} - \psi(b_{t-1} - \bar{b}) + s_t \\
&= \beta^{-1}\frac{C_t^R}{C_{t-1}^R}b_{t-1} - \left(\beta^{-1} - 1\right)\bar{b} - \psi(b_{t-1} - \bar{b}) + (s_t - \bar{s}),
\end{aligned}$$

which can be written as

$$(b_t - \bar{b}) = \left[\beta^{-1}\frac{C_t^R}{C_{t-1}^R} - \psi\right](b_{t-1} - \bar{b}) + (s_t - \bar{s}) + \beta^{-1}\bar{b}\left[\frac{C_t^R}{C_{t-1}^R} - 1\right] \quad \text{for } t \geq 1.$$

Now consider time-0 GBC. At $t = 0$, the Euler equation does not apply. We therefore have

$$b_0 = R_{-1}b_{-1}\frac{1}{\Pi_0} - \left[\bar{\tau} + \psi(b_{-1} - \bar{b})\right] + s_0.$$

Again, use the steady-state relation, $\bar{\tau} = (\beta^{-1} - 1)\bar{b} + \bar{s}$, to obtain

$$b_0 = \left(\frac{R_{-1}}{\Pi_0} - \psi\right)b_{-1} - \left(\beta^{-1} - 1 - \psi\right)\bar{b} + (s_0 - \bar{s}).$$

Finally, for simplicity, we assume $R_{-1}=\bar{R}$ and $b_{-1}=\bar{b}$. The system then simplifies to

$$\left(\frac{\Pi_{t+1}}{\bar{\Pi}}\right)=\frac{C_t^R}{C_{t+1}^R}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi}, \tag{A.13}$$

$$(b_t-\bar{b})=\left[\beta^{-1}\frac{C_t^R}{C_{t-1}^R}-\psi\right](b_{t-1}-\bar{b})+(s_t-\bar{s})+\beta^{-1}\bar{b}\left[\frac{C_t^R}{C_{t-1}^R}-1\right] \quad \text{for } t\geq 1, \tag{A.14}$$

$$(b_0-\bar{b})=\beta^{-1}\left(\frac{\bar{\Pi}}{\Pi_0}-1\right)\bar{b}+(s_0-\bar{s}) \quad \text{at } t=0, \tag{A.15}$$

with the initial and terminal conditions.

A.5.2.1 *Inflation determination under flexible prices* We first solve the model under flexible prices. In this case, $C_t^R=C^R(s_t)$, as shown above.

A.5.2.1.1 *Monetary regime* Notice that, no matter what happens until time $T+1$, starting $T+2$, (A.14) becomes

$$(b_t-\bar{b})=\left(\beta^{-1}-\psi\right)(b_{t-1}-\bar{b}).$$

If $\psi>0$, debt b satisfies the TVC for all possible values of inflation (including Π_0) and regardless of monetary policy.

Inflation is solely determined by equation (A.13), which becomes

$$\left(\frac{\Pi_{t+1}}{\bar{\Pi}}\right)=\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi} \quad \text{for } t\geq T+1,$$

regardless of the history.

Suppose we are confined to find a bounded solution in the monetary regime ($\phi>1$). In this case, we must have

$$\frac{\Pi_{T+1}}{\bar{\Pi}}=1.$$

Otherwise, inflation would explode. Inflation before $T+1$ can then be solved backward using

$$\frac{\Pi_t}{\bar{\Pi}}=\left(\frac{\Pi_{t+1}}{\bar{\Pi}}\right)^{\frac{1}{\phi}}\left(\frac{C^R(s_{t+1})}{C^R(s_t)}\right)^{\frac{1}{\phi}}.$$

That is,

$$\frac{\Pi_T}{\bar{\Pi}}=\left(\frac{C^R(\bar{s})}{C^R(s_T)}\right)^{\frac{1}{\phi}},$$

$$\frac{\Pi_{T-1}}{\bar{\Pi}}=\left(\left(\frac{C^R(\bar{s})}{C^R(s_T)}\right)^{\frac{1}{\phi}}\right)^{\frac{1}{\phi}}\left(\frac{C^R(s_T)}{C^R(s_{T-1})}\right)^{\frac{1}{\phi}}=\left(\frac{C^R(\bar{s})}{C^R(s_T)}\right)^{\frac{1}{\phi^2}}\left(\frac{C^R(s_T)}{C^R(s_{T-1})}\right)^{\frac{1}{\phi}},$$

$$\frac{\Pi_{T-2}}{\bar{\Pi}}=\left(\left(\frac{C^R(\bar{s})}{C^R(s_T)}\right)^{\frac{1}{\phi^2}}\left(\frac{C^R(s_T)}{C^R(s_{T-1})}\right)^{\frac{1}{\phi}}\right)^{\frac{1}{\phi}}\left(\frac{C^R(s_{T-1})}{C^R(s_{T-2})}\right)^{\frac{1}{\phi}}$$

$$= \left(\frac{C^R(\bar{s})}{C^R(s_T)}\right)^{\frac{1}{\phi^3}} \left(\frac{C^R(s_T)}{C^R(s_{T-1})}\right)^{\frac{1}{\phi^2}} \left(\frac{C^R(s_{T-1})}{C^R(s_{T-2})}\right)^{\frac{1}{\phi}},$$

$$\vdots$$

$$\frac{\Pi_0}{\bar{\Pi}} = \left(\frac{C^R(\bar{s})}{C^R(s_T)}\right)^{\frac{1}{\phi^{T+1}}} \left(\frac{C^R(s_T)}{C^R(s_{T-1})}\right)^{\frac{1}{\phi^T}} \cdots \left(\frac{C^R(s_1)}{C^R(s_0)}\right)^{\frac{1}{\phi}}$$

$$= C^R(\bar{s})^{\frac{1}{\phi^{T+1}}} \left[\frac{1}{C^R(s_T) C^R(s_{T-1}) \cdots C^R(s_0)}\right]^{\frac{1}{\phi}}.$$

An interesting example is a one-time increase in transfer ($s_0 > \bar{s}$ and $s_t = \bar{s}$ afterwards). In the bounded solution, this raises the rate of inflation by

$$\frac{\Pi_0}{\bar{\Pi}} = \left(\frac{C^R(\bar{s})}{C^R(s_0)}\right)^{\frac{1}{\phi}},$$

and subsequently $\Pi_t = \bar{\Pi}$ (for $t \geq 1$). Notice that the effect of transfer on inflation is purely transitory in the monetary regime.

Given the time path of inflation, we can solve for debt. Debt at $t = 0$ is given by

$$b_0 = \left[\left(\frac{\bar{\Pi}}{\Pi_0} - 1\right)\beta^{-1} + 1\right]\bar{b} + (s_0 - \bar{s})$$

$$= \left[\left(\left(\frac{C^R(s_0)}{C^R(\bar{s})}\right)^{\frac{1}{\phi}} - 1\right)\beta^{-1} + 1\right]\bar{b} + (s_0 - \bar{s}).$$

An increase in s_0 has two opposing effects on b_0. On the one hand, it directly increases b_0 as reflected in the last term, $(s_0 - \bar{s})$. On the other hand, there exists an indirect effect, which lowers b_0 as an increase in s_0 raises inflation Π_0. The net effect depends on parameterization. In the following periods, $\{b_t\}$ is given by

$$(b_1 - \bar{b}) = \left[\beta^{-1}\frac{C^R(\bar{s})}{C^R(s_0)} - \psi\right](b_0 - \bar{b}) + \beta^{-1}\bar{b}\left[\frac{C^R(\bar{s})}{C^R(s_0)} - 1\right],$$

$$(b_t - \bar{b}) = \left[\beta^{-1} - \psi\right](b_{t-1} - \bar{b}) \quad \text{for } t \geq 2.$$

A.5.2.1.2 *Fiscal regime* We now consider the flip side of the policy space: $\psi \leq 0$ and $\phi < 1$. Consider the GBC at time $T + 2$:

$$(b_{T+2} - \bar{b}) = \left(\beta^{-1} - \psi\right)(b_{T+1} - \bar{b}).$$

Suppose $b_{T+1} \neq \bar{b}$. This violates the TVC, and thus cannot be an equilibrium because $(\beta^{-1} - \psi) \geq \beta^{-1}$. It thus has to be that $b_{T+1} = \bar{b}$—if a solution exists.

Now look at the GBC at time $T + 1$,

$$(b_{T+1} - \bar{b}) = \left[\beta^{-1}\underbrace{\frac{C^R(s_{T+1})}{C^R(s_T)}}_{\frac{C^R(\bar{s})}{C^R(s_T)}} - \psi\right](b_T - \bar{b}) + \underbrace{(s_{T+1} - \bar{s})}_{=0} + \beta^{-1}\bar{b}\left[\underbrace{\frac{C^R(s_{T+1})}{C^R(s_T)}}_{\frac{C^R(\bar{s})}{C^R(s_T)}} - 1\right]. \quad \text{(A.16)}$$

Substituting out debt backwards yields

$$
\begin{aligned}
(b_{T+1}-\bar{b}) = (b_0-\bar{b})\prod_{j=1}^{T+1}\left[\beta^{-1}\frac{C^R(s_j)}{C^R(s_{j-1})}-\psi\right] \\
+\sum_{k=1}^{T}(s_k-\bar{s})\prod_{j=k+1}^{T+1}\left[\beta^{-1}\frac{C^R(s_j)}{C^R(s_{j-1})}-\psi\right] \\
+\sum_{k=1}^{T}\beta^{-1}\bar{b}\left[\frac{C^R(s_k)}{C^R(s_{k-1})}-1\right]\prod_{j=k+1}^{T+1}\left[\beta^{-1}\frac{C^R(s_j)}{C^R(s_{j-1})}-\psi\right]+\beta^{-1}\bar{b}\left[\frac{C^R(\bar{s})}{C^R(s_T)}-1\right].
\end{aligned}
$$

Using the equilibrium property that $b_{T+1}=\bar{b}$, we can solve for b_0:

$$
\begin{aligned}
-(b_0-\bar{b}) = \sum_{k=1}^{T}(s_k-\bar{s})\frac{\prod_{j=k+1}^{T+1}\left[\beta^{-1}\frac{C^R(s_j)}{C^R(s_{j-1})}-\psi\right]}{\prod_{j=1}^{T+1}\left[\beta^{-1}\frac{C^R(s_j)}{C^R(s_{j-1})}-\psi\right]} \\
+\sum_{k=1}^{T}\beta^{-1}\bar{b}\left[\frac{C^R(s_k)}{C^R(s_{k-1})}-1\right]\frac{\prod_{j=k+1}^{T+1}\left[\beta^{-1}\frac{C^R(s_j)}{C^R(s_{j-1})}-\psi\right]}{\prod_{j=1}^{T+1}\left[\beta^{-1}\frac{C^R(s_j)}{C^R(s_{j-1})}-\psi\right]} \\
+\frac{\beta^{-1}\bar{b}\left[\frac{C^R(\bar{s})}{C^R(s_T)}-1\right]}{\prod_{j=1}^{T+1}\left[\beta^{-1}\frac{C^R(s_j)}{C^R(s_{j-1})}-\psi\right]}.
\end{aligned}
$$

Let

$$
\Omega_k \equiv \left\{\prod_{j=1}^{k}\left[\beta^{-1}\frac{C^R(s_j)}{C^R(s_{j-1})}-\psi\right]\right\}^{-1}, \quad \text{and} \quad \Omega_0 \equiv 1.
$$

We can then rewrite the equation above as

$$
(b_0-\bar{b}) = -\sum_{k=1}^{T}\Omega_k(s_k-\bar{s})-\beta^{-1}\bar{b}\sum_{k=1}^{T+1}\Omega_k\left[\frac{C^R(s_k)}{C^R(s_{k-1})}-1\right], \tag{A.17}
$$

which shows the value of b_0 required to generate $b_t=\bar{b}$ for $t \geq T+1$. Given b_0, debt in the ensuing periods is then determined by (A.14).

Let us now turn to inflation. In order to obtain Π_0 necessary to generate b_0 in (A.17), we look at the GBC at $t = 0$:

$$b_0 - \bar{b} = \left(\frac{\bar{\Pi}}{\Pi_0} - 1\right)\beta^{-1}\bar{b} + (s_0 - \bar{s}).$$

Substitute out $(b_0 - \bar{b})$ using (A.17), and solve for Π_0 to obtain

$$-\sum_{k=1}^{T}\Omega_k(s_k - \bar{s}) - \beta^{-1}\bar{b}\sum_{k=1}^{T+1}\Omega_k\left[\frac{C^R(s_k)}{C^R(s_{k-1})} - 1\right] = \left(\frac{\bar{\Pi}}{\Pi_0} - 1\right)\beta^{-1}\bar{b} + (s_0 - \bar{s})$$

$$\implies \quad \frac{\Pi_0}{\bar{\Pi}} = \frac{1}{1 - \dfrac{\beta}{\bar{b}}\displaystyle\sum_{k=0}^{T}\Omega_k(s_k - \bar{s}) - \sum_{k=1}^{T+1}\Omega_k\left[\frac{C^R(s_k)}{C^R(s_{k-1})} - 1\right]}, \tag{A.18}$$

which shows that Π_0 rises when current and/or future transfers increase. Subsequently, inflation follows (A.13), converging to $\bar{\Pi}$.

The solution equation (A.18) reveals that the interest rate channel can in principle, work in both directions. On the one hand, as shown in the one-period transfer increase case, a redistribution program that raises the real interest rate leads to an increase in interest payments and a larger rise in inflation—as captured by the last term in the denominator. On the other hand, such redistribution decreases the discount factor Ω_k. The economy thus discounts future primary surplus/deficits more heavily, which causes inflation to adjust by less when *future* transfers rise.[1] Therefore, generally, the net effect on inflation through the interest rate channel of a multiperiod redistribution program is difficult to isolate analytically, without further restrictions on the path of transfers.[2]

As before, consider the case of a one-time increase in s_0. Then inflation at time 0 is given by

$$\frac{\Pi_0}{\bar{\Pi}} = \frac{1}{1 - \dfrac{\beta}{\bar{b}}(s_0 - \bar{s}) - \Omega_1\left[\dfrac{C^R(\bar{s})}{C^R(s_0)} - 1\right]}$$

$$= \left\{1 - \frac{\beta}{\bar{b}}(s_0 - \bar{s}) - \frac{\left[\dfrac{C^R(\bar{s})}{C^R(s_0)} - 1\right]}{\left[\beta^{-1}\dfrac{C^R(\bar{s})}{C^R(s_0)} - \psi\right]}\right\}^{-1}. \tag{A.19}$$

[1]Equation (A.17) also provides intuition: To achieve a target level of b_1, b_0 needs not decrease as much when the coefficient (which is increasing in the real rate) is greater; consequently, inflation increases by less.

[2]Moreover, there is a significant flexibility in the schedule of transfer payments when studying a multiperiod redistribution program. The time path of transfers $\{s_t\}_{t=0}^{T}$ can be constant, (weakly) monotonic, or neither. Depending on the time path, the real interest rate, $\beta^{-1}\frac{C^R(s_t)}{C^R(s_{t-1})}$, need not be greater than or equal to its steady-state value β^{-1} for the entire duration of a redistribution program. Interest payments thus can be lower than the preprogram level in some periods. Generally, different transfer schedules would result in different dynamics of the real interest rate. A constant or monotonic schedule is however, most commonly used in quantitative models.

One can easily show that Π_0 is increasing in s_0. A *sufficient* condition is that

$$g(s_0) \equiv \frac{\left[\dfrac{C^R(\bar{s})}{C^R(s_0)} - 1\right]}{\left[\beta^{-1}\dfrac{C^R(\bar{s})}{C^R(s_0)} - \psi\right]}$$

is increasing in s_0. Consider the derivative:

$$\frac{dg(s_0)}{ds_0} \equiv \frac{-\dfrac{C^R(\bar{s})C^{R\prime}(s_0)}{C^R(s_0)^2}\left[\dfrac{C^R(\bar{s})}{C^R(s_0)} - \psi\beta\right] + \left[\dfrac{C^R(\bar{s})}{C^R(s_0)} - 1\right]\dfrac{C^R(\bar{s})C^{R\prime}(s_0)}{C^R(s_0)^2}}{\beta\left[\beta^{-1}\dfrac{C^R(\bar{s})}{C^R(s_0)} - \psi\right]^2}$$

$$= \frac{-\dfrac{C^R(\bar{s})C^{R\prime}(s_0)}{C^R(s_0)^2}[1 - \psi\beta]}{\beta\left[\beta^{-1}\dfrac{C^R(\bar{s})}{C^R(s_0)} - \psi\right]^2},$$

which is positive when $C^{R\prime}(s_0) < 0$.

Alternatively, one can solve the model using the IGBC. Equation (A.9) implies

$$\Pi_0 = \frac{b_{-1}R_{-1}}{\sum_{i=0}^{\infty}\beta^i \dfrac{C^R(s_0)}{C^R(s_i)}(\tau_i - s_i)}.$$

We consider a plausible case where $\psi = 0$.[3] We then have

$$\frac{\Pi_0}{\bar{\Pi}} = \frac{\bar{b}\beta^{-1}}{\sum_{i=0}^{\infty}\beta^i \dfrac{C^R(s_0)}{C^R(s_i)}\left(\beta^{-1} - 1\right)\bar{b} - \sum_{i=0}^{\infty}\beta^i \dfrac{C^R(s_0)}{C^R(s_i)}(s_i - \bar{s})}$$

$$= \frac{1}{(1-\beta)\sum_{i=0}^{\infty}\beta^i \dfrac{C^R(s_0)}{C^R(s_i)} - \dfrac{\beta}{\bar{b}}(s_0 - \bar{s})}$$

$$= \frac{1}{1 - \dfrac{\beta}{\bar{b}}(s_0 - \bar{s}) - \beta\left[1 - \dfrac{C^R(s_0)}{C^R(\bar{s})}\right]}. \qquad \text{(A.20)}$$

This coincides with (A.19) when $\psi = 0$.

A.5.2.2 *Inflationary effects of the redistribution policy* In Proposition 1, we show that under a mild sufficient condition, the redistribution policy is more inflationary under the fiscal regime than under the monetary regime.

[3] Cases in which $\psi < 0$ are implausible and difficult to solve using IGBC as τ_i in the equation is endogenous.

PROPOSITION 1. *The redistribution policy is more inflationary on impact under the fiscal regime than under the monetary regime if the debt-to-GDP ratio is sufficiently low.*

PROOF. Let us consider the case that transfers increase only for one period: $s_0 > \bar{s}$ and $s_t = \bar{s}$ for $t \geq 1$. First, using equation (A.16) at $T = 0$, we can obtain the initial debt level under the fiscal regime, b_0^F, ensuring that $b_1 = \bar{b}$:

$$\frac{b_0^F - \bar{b}}{\bar{b}} = -\frac{\frac{1}{\beta}\frac{\bar{C}^R}{C_0^R} - \frac{1}{\beta}}{\frac{1}{\beta}\frac{\bar{C}^R}{C_0^R} - \psi} < 0.$$

We can also obtain the initial debt level under the monetary regime, b_0^M, using equations (A.13) and (A.15):

$$\begin{aligned}\frac{b_0^M - \bar{b}}{\bar{b}} &= \left(\frac{\bar{\Pi}}{\Pi_0} - 1\right)\frac{1}{\beta} + \frac{s_0 - \bar{s}}{\bar{b}} \\ &= \left(\left(\frac{C_0^R}{\bar{C}^R}\right)^{\frac{1}{\phi}} - 1\right)\frac{1}{\beta} + \frac{s_0 - \bar{s}}{\bar{b}} \geq \left(\frac{C_0^R - \bar{C}^R}{\bar{C}^R}\right)\frac{1}{\beta} + \frac{s_0 - \bar{s}}{\bar{b}}.\end{aligned}$$

Here, the second equality holds since $C_1^R = \bar{C}^R$ and $\Pi_1 = \bar{\Pi}$ under the monetary regime. Notice that equation (A.15) implies that if $\frac{b_0^M - \bar{b}}{\bar{b}} > 0$, then $b_0^M > b_0^F$, and thus $\Pi_0^F > \Pi_0^M$. We want to find a sufficient condition for $\frac{b_0^M - \bar{b}}{\bar{b}} > 0$. Note that from the solution of C_0^R and $\bar{C}^R$, we can derive

$$\frac{C_0^R - \bar{C}^R}{\bar{C}^R} = \frac{Y_0 - \bar{Y} - (s_0 - \bar{s})}{\bar{Y} - \bar{s}}.$$

Then

$$\begin{aligned}\frac{b_0^M - \bar{b}}{\bar{b}} &\geq \left(\frac{C_0^R - \bar{C}^R}{\bar{C}^R}\right)\frac{1}{\beta} + \frac{s_0 - \bar{s}}{\bar{b}} \\ &= \left(\frac{Y_0 - \bar{Y}}{\bar{Y} - \bar{s}}\right)\frac{1}{\beta} + (s_0 - \bar{s})\left(\frac{1}{\bar{b}} - \frac{1}{\beta}\frac{1}{\bar{Y} - \bar{s}}\right).\end{aligned}$$

Here, the first term is positive since $Y_0 > \bar{Y}$ and $\bar{Y} > \bar{s}$. Thus, $\frac{b_0^M - \bar{b}}{\bar{b}} > 0$ if the second term is positive, that is,

$$\frac{\bar{b}}{\bar{Y}} < \beta\left(1 - \frac{\bar{s}}{\bar{Y}}\right).$$ □

A.5.2.3 *Inflation determination under sticky prices* We now solve the model under sticky prices. In this case, $C_t^R = C^R(\Pi_t)$ rather than $C_t^R = C^R(s_t)$.[4]

[4]In the general case (which we do not consider here), $C_t^R = C^R(\Pi_t, s_t)$.

A.5.2.3.1 *Monetary regime* As in the flexible-price case, we focus on a bounded solution. Notice that the inverse of consumption growth is given by

$$\frac{C^R(\Pi_t)}{C^R(\Pi_{t+1})} = \left(\frac{\Pi_t}{\Pi_{t+1}}\right)^{\frac{\gamma}{1-\gamma}}.$$

Equation (A.13) thus can be written as

$$\left(\frac{\Pi_{t+1}}{\bar{\Pi}}\right) = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi(1-\gamma)+\gamma}. \tag{A.21}$$

When $\tilde{\phi} = \phi(1-\gamma) + \gamma > 1$ ($\Longleftrightarrow \phi > 1$), the solution for nonexplosive gross inflation is

$$\frac{\Pi_t}{\bar{\Pi}} = 1 \quad \text{for all } t \geq 0.$$

In other words, transfers does not generate inflation in the monetary regime.

Given the constant rate of inflation, (A.14) and (A.15) become

$$\begin{aligned}(b_t - \bar{b}) &= \left[\beta^{-1} - \psi\right](b_{t-1} - \bar{b}) + (s_t - \bar{s}),\\ (b_0 - \bar{b}) &= (s_0 - \bar{s}).\end{aligned}$$

If $\psi > 0$, debt b satisfies the TVC for all possible values of inflation and regardless of monetary policy.

A.5.2.3.2 *Fiscal regime* We let $\tilde{\phi} \equiv \phi(1-\gamma) + \gamma < 1$ (or $\phi < 1$). This condition generates bounded inflation for any given Π_0—as indicated by (A.21). To pin down Π_0, it is easier to use the IGBC (A.9) in this case; we obtain

$$\frac{\Pi_0}{\bar{\Pi}} = \frac{\beta^{-1}\bar{b}}{\sum_{i=0}^{\infty} \beta^i \left(\frac{\Pi_0}{\bar{\Pi}}\right)^{\frac{\gamma}{1-\gamma}(1-\tilde{\phi}^i)} (\tau_i - s_i)}.$$

Once again, we consider the plausible case where $\psi = 0$. We then obtain

$$\begin{aligned}\frac{\Pi_0}{\bar{\Pi}} &= \frac{\beta^{-1}\bar{b}}{\sum_{i=0}^{\infty} \beta^i \left(\frac{\Pi_0}{\bar{\Pi}}\right)^{\frac{\gamma}{1-\gamma}(1-\tilde{\phi}^i)} (\bar{\tau} - s_i)}\\ &= \frac{1}{\sum_{i=0}^{\infty} \beta^i \left(\frac{\Pi_0}{\bar{\Pi}}\right)^{\frac{\gamma}{1-\gamma}(1-\tilde{\phi}^i)} \left[(1-\beta) - \frac{\beta}{\bar{b}}(s_i - \bar{s})\right]}.\end{aligned} \tag{A.22}$$

Equation (A.22) implicitly defines Π_0 as a function of transfers. Equilibrium Π_0 can be obtained as a fixed point of the equation.

For intuition, consider a one-time increase in transfer. Equation (A.22) then can be written as

$$\frac{\Pi_0}{\bar{\Pi}} = \frac{1}{(1-\beta)\sum_{i=0}^{\infty}\beta^i\left(\frac{\Pi_0}{\bar{\Pi}}\right)^{\frac{\gamma}{1-\gamma}(1-\tilde{\phi}^i)} - \frac{\beta}{\bar{b}}(s_0-\bar{s})}. \tag{A.23}$$

It is easy to show that Π_0 is increasing in s_0. Compared to the flexible-price case, however, inflation does not increase as much in this sticky-price case. The reason is that the real interest rate

$$r_t = \beta^{-1}\frac{C^R(\Pi_{t+1})}{C^R(\Pi_t)} = \beta^{-1}\left(\frac{\Pi_0}{\bar{\Pi}}\right)^{-\frac{\gamma(1-\tilde{\phi})}{1-\gamma}\tilde{\phi}^t}$$

is decreasing in Π_0. Therefore, an increase in Π_0 now exerts a downward pressure on real value of debt in the ensuing periods, which implies that a smaller increase in inflation is necessary to stabilize debt.

We now formally show the claim that Π_0 is increasing in s_0 using the implicit function theorem. Let

$$F(\Pi_0, s_0) \equiv f(\Pi_0) - g(\Pi_0, s_0) = 0,$$

where

$$f(\Pi_0) = \frac{\Pi_0}{\bar{\Pi}} \quad \text{and} \quad g(\Pi_0, s_0) = \left((1-\beta)\sum_{i=0}^{\infty}\beta^i\left(\frac{\Pi_0}{\bar{\Pi}}\right)^{\frac{\gamma}{1-\gamma}(1-\tilde{\phi}^i)} - \frac{\beta}{\bar{b}}(s_0-\bar{s})\right)^{-1}.$$

Then the derivative is given by

$$\frac{d\Pi_0}{ds_0} = -\frac{F_s}{F_{\Pi_0}} = \frac{\overset{+}{g_{s_0}}}{\underset{+}{f_{\Pi_0}} - \underset{-}{g_{\Pi_0}}} > 0.$$

In the flexible-price limit ($\gamma = 0$), the function g does not depend on inflation. Inflation at time 0 responds more as $g_{\Pi_0} = 0$; it is given by

$$\frac{\Pi_0}{\bar{\Pi}} = \left(1 - \frac{\beta}{\bar{b}}(s_0-\bar{s})\right)^{-1},$$

which coincides with the previous solution in (A.20) under perfectly elastic labor supply.

A.5.3 *Comparison of the two regimes under sticky prices* The results on inflation are qualitatively similar to those obtained in the flexible-price case. The fiscal regime produces more persistent and greater inflation, compared to the monetary regime. In fact, the latter regime does not generate inflation at all.

A.6 *Simple model extension*

In this Appendix, we extend our simple model presented in Section 2 with preference shocks (Appendix A.6.1) and government spending (Appendix A.6.2).

A.6.1 *Simple model with preference shocks* Consider the simple model with a preference shock, ξ_t. The system of equilibrium equations can be summarized as

$$\frac{C_{t+1}^R}{C_t^R} = \beta \frac{1+\xi_{t+1}}{1+\xi_t} \frac{1+i_t}{\Pi_{t+1}}, \qquad 1 = \chi\left(C_t^R + \frac{s_t}{1-\lambda}\right)^{\varphi} C_t^R,$$

$$b_t = \frac{1+i_{t-1}}{\Pi_t} b_{t-1} - \tau_t + s_t, \qquad \frac{1+i_t}{1+\bar{i}} = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi}, \qquad \tau_t - \bar{\tau} = \psi(b_{t-1} - \bar{b}).$$

We first consider the case of infinite Frisch elasticity. Appendix Figure A.1 shows the IRFs to transfer shocks and Appendix Figure A.2 shows the variable responses to transfer shocks under the different sizes of preference shocks. Next, we consider the case of $\varphi = 2$. Appendix Figure A.3 shows the IRFs and Appendix Figure A.4 shows the variable responses to transfer shocks under the different sizes of preference shocks with $\varphi = 2$. Appendix Table A.1 shows the sum of inflation responses to a transfer increase with the preference shocks that lead to different horizons of negative real interest rates.[5] They show that the fiscal regime leads to higher inflation (in total, even if not for both periods in all cases) than the monetary regime under transfer increases when such shocks hit that drive the interest rate to negative temporarily. In fact, for infinite Frisch elasticity, Proposition 2 shows that total inflation is higher in the fiscal regime compared to the monetary regime.

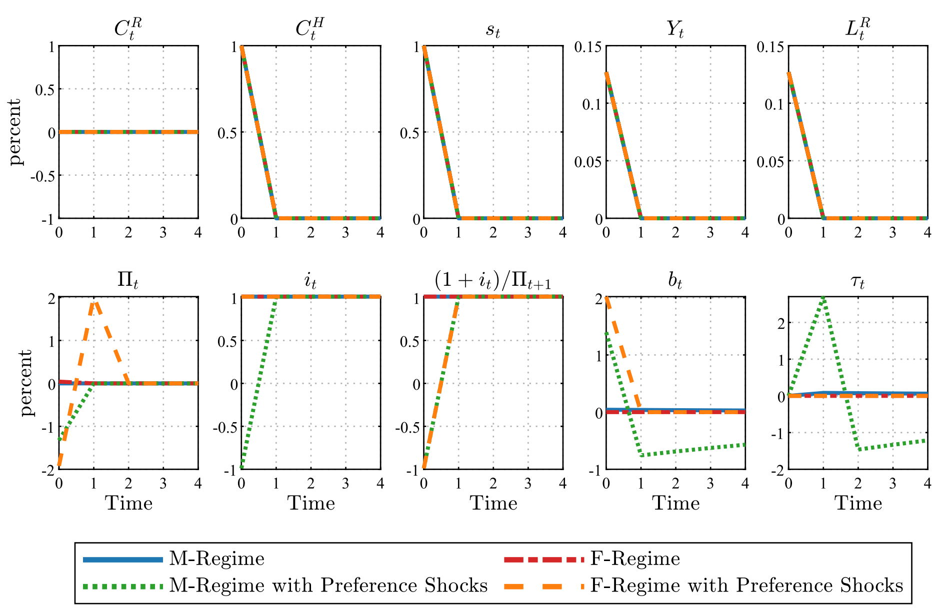

FIGURE A.1. IRFs in the Simple Model with $\varphi = 0$.

[5]For the numerical exercises, we set the similar parameterization used in the baseline quantitative model: $\beta = 0.99$, $\lambda = 0.23$, $\frac{\bar{s}}{\bar{Y}} = 0.127$, and $\frac{\bar{b}}{6\bar{Y}} = 0.509$. We set $\phi = 1.5$ and $\psi = 0.1$ for the monetary regime and $\phi = 0.0$ and $\psi = 0.0$ for the fiscal regime.

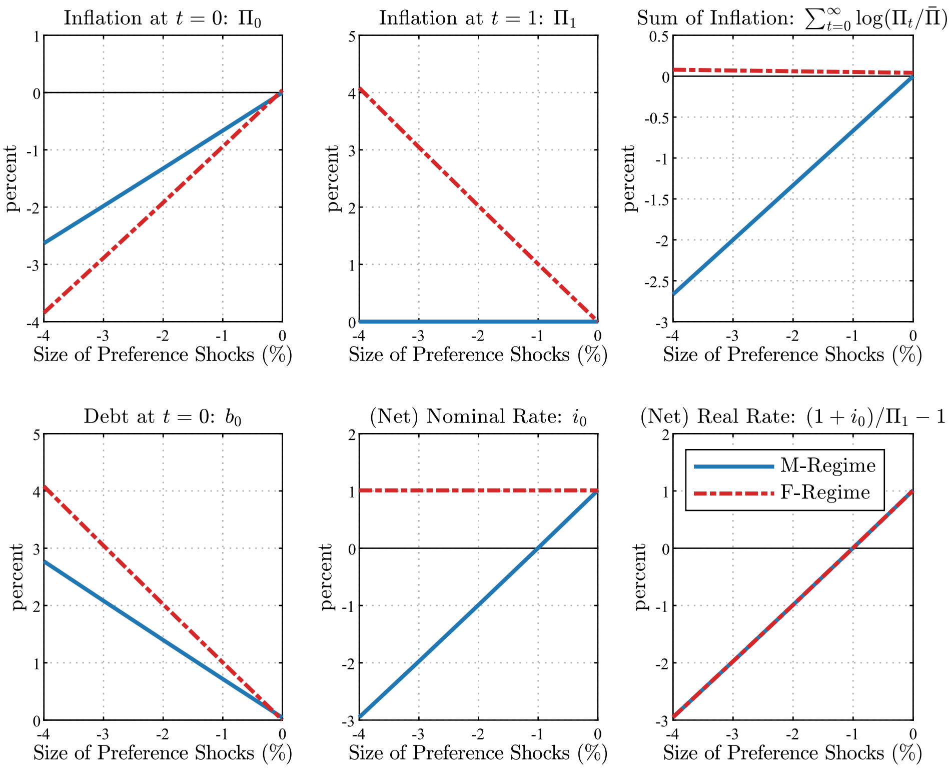

FIGURE A.2. Variable Responses by Different Size of Preference Shocks with $\varphi = 0$.

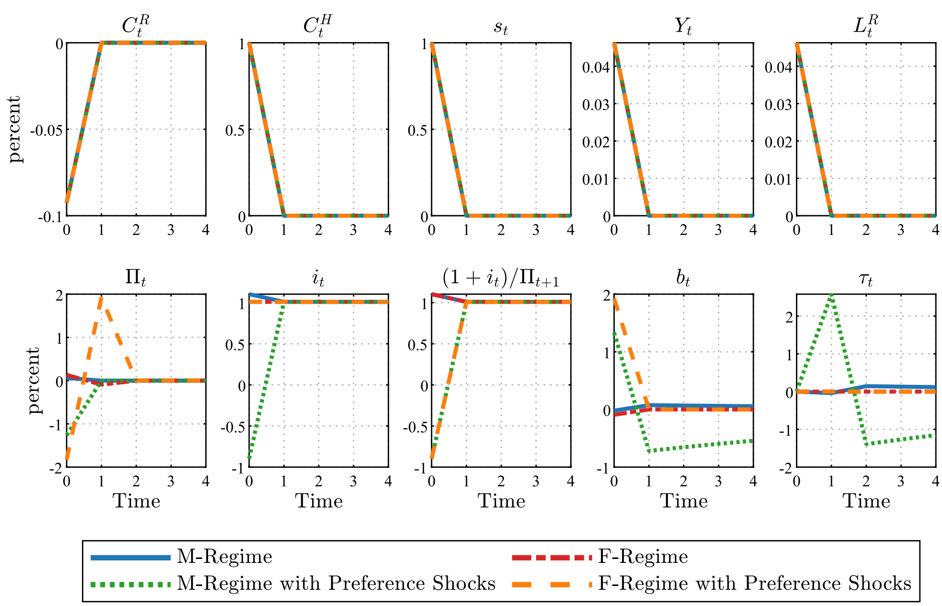

FIGURE A.3. IRFs in the Simple Model with $\varphi = 2$.

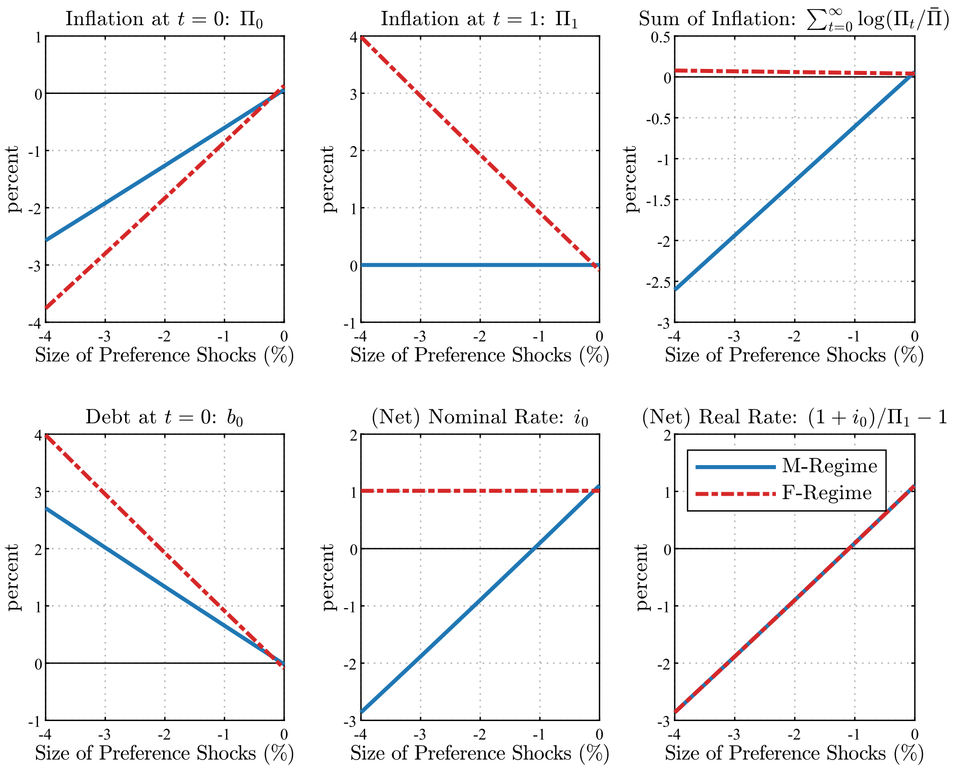

FIGURE A.4. Variable Responses by a Different Size of Preference Shocks with $\varphi = 2$.

TABLE A.1. Sum of inflation responses ($\sum_{t=0}^{\infty} \log(\Pi_t/\bar{\Pi})$).

	1-Period (-) Real Rate	3-Period (-) Real Rate	5-Period (-) Real Rate
Panel A: Infinite Frisch Elasticity ($\varphi = 0$)			
M-Regime without Beta shocks	0.00	0.00	0.00
F-Regime without Beta shocks	0.04	0.04	0.04
M-Regime with Beta shocks	−1.33	−6.37	−13.05
F-Regime with Beta shocks	0.06	0.16	0.32
Panel B: Finite Frisch Elasticity ($\varphi = 2$)			
M-Regime without Beta shocks	0.06	0.06	0.06
F-Regime without Beta shocks	0.04	0.04	0.04
M-Regime with Beta shocks	−1.27	−6.31	−12.99
F-Regime with Beta shocks	0.06	0.15	0.32

Note: This table shows the sum of inflation responses to a one-time transfer increase under the different horizon of preference shocks. Panel A shows the results with an infinite Frisch elasticity ($\varphi = 0$) and Panel B shows the results with a finite Frisch elasticity ($\varphi = 2$).

PROPOSITION 2. $\log \frac{\Pi_0^M}{\bar{\Pi}} + \log \frac{\Pi_1^M}{\bar{\Pi}} < \log \frac{\Pi_0^F}{\bar{\Pi}} + \log \frac{\Pi_1^F}{\bar{\Pi}}$ *with infinite Frisch elasticity.*

PROOF. Consider the system of equilibrium conditions:

$$\frac{\Pi_{t+1}}{\bar{\Pi}} = \frac{C_t^R}{C_{t+1}^R} \frac{1+\xi_{t+1}^\beta}{1+\xi_t^\beta} \left(\frac{\Pi_t}{\bar{\Pi}}\right)^\phi,$$

$$b_t - \bar{b} = \left[\frac{1}{\beta} \frac{C_t^R}{C_{t-1}^R} \frac{1+\xi_{t-1}^\beta}{1+\xi_t^\beta} - \psi\right](b_{t-1} - \bar{b}) + (s_t - \bar{s}) + \frac{1}{\beta}\bar{b}\left[\frac{C_t^R}{C_{t-1}^R} \frac{1+\xi_{t-1}^\beta}{1+\xi_t^\beta} - 1\right],$$

$$b_0 - \bar{b} = \frac{1}{\beta}\left(\frac{\bar{\Pi}}{\Pi_0} - 1\right)\bar{b} + (s_0 - \bar{s}).$$

Note that with infinite Frisch ($\varphi = 0$), $C_t^R = \bar{C}^R$ for all t. Under M-regime with one-time shock ($s_0 = (1+\xi_0^s)\bar{s}$, $\xi_{t>0}^\beta = 0$, $s_{t>0} = \bar{s}$):

$$\frac{\Pi_0^M}{\bar{\Pi}} = \left(1+\xi_0^\beta\right)^{\frac{1}{\phi}} \quad \text{and} \quad \frac{\Pi_1^M}{\bar{\Pi}} = 1,$$

$$\log \frac{\Pi_0^M}{\bar{\Pi}} + \log \frac{\Pi_1^M}{\bar{\Pi}} = \frac{1}{\phi}\log\left(1+\xi_0^\beta\right) \approx \frac{1}{\phi}\xi_0^\beta < 0.$$

Under the F-regime with one-time shock ($s_0 = (1+\xi_0^s)\bar{s}$, $\xi_{t>0}^\beta = 0$, $s_{t>0} = \bar{s}$) and $\phi = 0$, $\psi = 0$: then $b_{t>0} = \bar{b}$ and

$$\frac{\Pi_1^F}{\bar{\Pi}} = \frac{1}{1+\xi_0} \quad \text{and} \quad \frac{\Pi_0^F}{\bar{\Pi}} = \frac{1+\xi_0^\beta}{1+(1+\beta)\xi_0^\beta - \beta\frac{\bar{s}}{\bar{b}}\xi_0^s\left(1+\xi_0^\beta\right)},$$

$$b_0 - \bar{b} = \xi_0^s\bar{s} + (s_0 - \bar{s}).$$

Then

$$\log \frac{\Pi_0^F}{\bar{\Pi}} + \log \frac{\Pi_1^F}{\bar{\Pi}} = -\log\left(1+\xi_0^\beta\right) + \log\left(\frac{1+\xi_0^\beta}{1+(1+\beta)\xi_0^\beta - \beta\frac{\bar{s}}{\bar{b}}\xi_0^s\left(1+\xi_0^\beta\right)}\right)$$

$$\approx -(1+\beta)\xi_0^\beta + \beta\frac{\bar{s}}{\bar{b}}\xi_0^s\left(1+\xi_0^\beta\right).$$

Then $-1 < \xi_0^\beta < 0$ and $\xi_0^s > 0$, $\log \frac{\Pi_0^F}{\bar{\Pi}} + \log \frac{\Pi_1^F}{\bar{\Pi}} > 0$. Thus,

$$\log \frac{\Pi_0^F}{\bar{\Pi}} + \log \frac{\Pi_1^F}{\bar{\Pi}} > 0 > \log \frac{\Pi_0^M}{\bar{\Pi}} + \log \frac{\Pi_1^M}{\bar{\Pi}}.$$ □

A.6.2 *Government spending shocks in the simple model* In this subsection, we point out how transfer and government spending changes are isomorphic in the simple model. The system of equilibrium equations is

$$\frac{C_{t+1}^R}{C_t^R} = \beta \frac{1+i_t}{\Pi_{t+1}}, \qquad \chi\left(C_t^R + \frac{s_t + G_t}{1-\lambda}\right)^{\varphi} C_t^R = 1,$$

$$b_t = \frac{1+i_{t-1}}{\Pi_t} b_{t-1} - \tau_t + s_t + G_t, \qquad \frac{1+i_t}{1+\bar{i}} = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi},$$

$$\tau_t - \bar{\tau} = \psi(b_{t-1} - \bar{b}).$$

Note that changes in s_t and G_t have identical effects on the model dynamics.

A.6.3 *Government spending feedback rule in the simple model* We consider endogenous feedback rules for government spending and present numerical results below for a few parameterizations. The government spending rule then is

$$G_t - \bar{G} = \psi_G(b_{t-1} - \bar{b}).$$

Under the fiscal regime, $\psi_G = 0$ by definition (i.e., no primary surplus adjustment in this regime), so whether government spending or taxes adjust (or more precisely, do not adjust at all) in the model does not matter.

Under the monetary regime, $\psi_G < 0$. That is, although an increase in the transfer is *not* met by a decrease in government spending of *the equal size in all periods* (like in the previous bullet point), government spending does decrease gradually. So we should expect to see a qualitatively similar result as before. Appendix Figure A.5 illustrates the result in the simple model. We can see that inflation and output increase by less in the government spending adjustment case than in the tax adjustment case, broadly confirming our statement above and your conjecture. For a comparison, Appendix Figure A.6 shows the IRFs with the infinite Frisch elasticity ($\varphi = 0$).

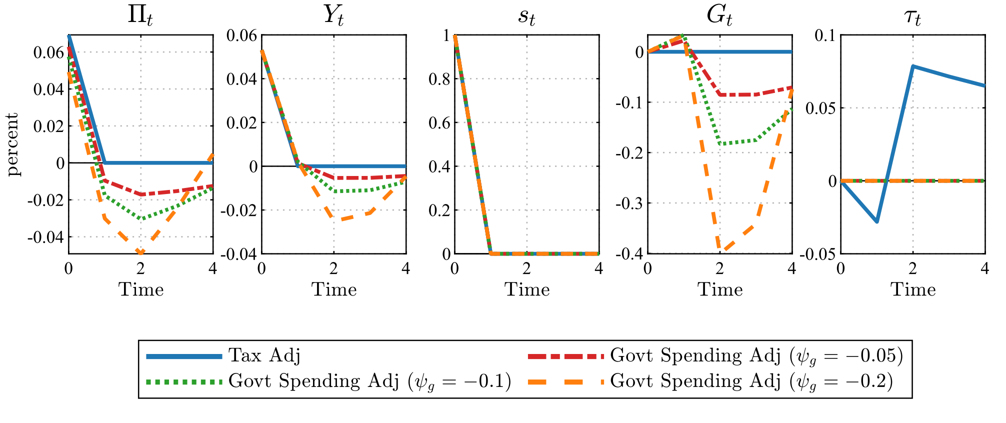

FIGURE A.5. IRFs with Government Spending Adjustment with $\varphi = 2$.

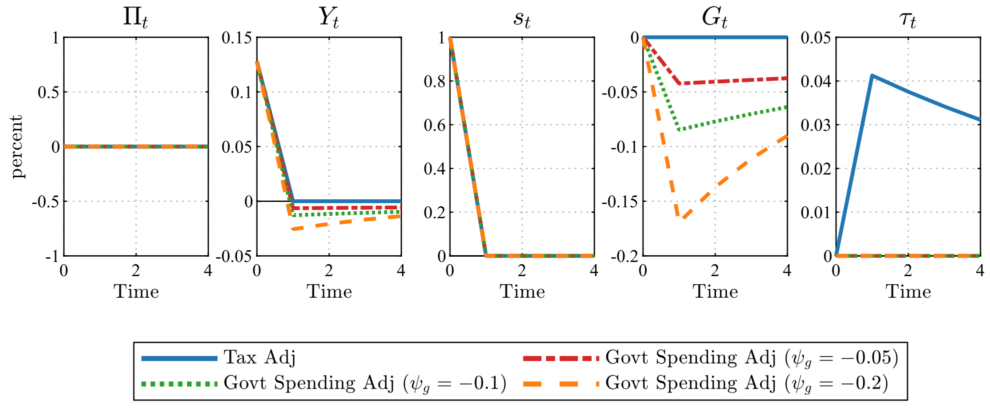

FIGURE A.6. IRFs with Government Spending Adjustment with $\varphi = 0$.

APPENDIX B: QUANTITATIVE MODEL

B.1 *Model setup*

There are two-sectors: Ricardian and hand to mouth. Labor is immobile across these two sectors. Each sector produces a distinct good, which is in turn produced in differentiated varieties. Firms in both sectors are owned by the Ricardian household.

B.1.1 *Ricardian sector*

B.1.1.1 *Households* There are Ricardian (R) households of measure $1 - \lambda$. The optimization problem of this type households is to

$$\max_{\{C_t^R, L_t^R, \frac{B_t^R}{P_t^R}\}} \sum_{t=0}^{\infty} \beta^t \exp(\eta_t^\xi) \left[\frac{(C_t^R)^{1-\sigma}}{1-\sigma} - \chi \frac{(L_t^R)^{1+\varphi}}{1+\varphi} \right]$$

subject to a standard No-Ponzi-game constraint and sequence of flow budget constraints

$$C_t^R + b_t^R = R_{t-1} \frac{1}{\Pi_t^R} b_{t-1}^R + (1 - \tau_{L,t}^R) w_t^R L_t^R + \Psi_t^R,$$

where σ is the coefficient of relative risk aversion, η_t^ξ is a preference shock, C_t^R is consumption, L_t^R is labor supply, $b_t^R = \frac{B_t^R}{P_t^R}$ is the real value of government issued debt, Π_t^R is inflation, R_{t-1} is the nominal interest rate, w_t^R is the real wage, and Ψ_t^R is real profits (this household owns firms in both sectors). We introduce a labor tax, $(1 - \tau_{L,t}^R)$, which constitutes one way in which the government finances transfers to the hand-to-mouth household.

Note that as we make clear below, we set up the model generally so that there could be two "CPI" indices in the economy, due to different baskets. So here, we are deflating nominal variables by the "CPI" index of the Ricardian household (defined as P_t^R).

Three optimality conditions are given by the Euler equation, (distorted) labor supply condition, and TVC:

$$\left(\frac{\exp(\eta_t^{\xi})C_t^R}{\exp(\eta_{t+1}^{\xi})C_{t+1}^R}\right)^{-\sigma} = \beta\frac{R_t}{\Pi_{t+1}^R},$$
$$\chi(L_t^R)^{\varphi}(C_t^R)^{\sigma} = (1-\tau_{L,t}^R)w_t^R,$$
$$\lim_{t\to\infty}\left[\beta^t(C_t^R)^{-\sigma}\left(\frac{B_t^R}{P_t^R}\right)\right] = 0.$$

Here, C_t^R is a CES/Armington-type aggregator ($\varepsilon > 0$) of the consumption good produced in the R and HTM sectors:

$$C_t^R = \left[(\alpha_R)^{\frac{1}{\varepsilon}}(C_{R,t}^R)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\alpha_R)^{\frac{1}{\varepsilon}}(\exp(\zeta_{H,t})C_{H,t}^R)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $C_{R,t}^R$ and $C_{H,t}^R$ are R-household's demand for R-sector and for HTM-sector goods, respectively. $\zeta_{H,t}$ is demand shocks for HTM goods. This gives the following optimal price index and demand functions from a standard static expenditure minimization problem

$$P_t^R = \left[\alpha_R(P_{R,t}^R)^{1-\varepsilon} + (1-\alpha_R)\left(\frac{P_{H,t}^R}{\exp(\zeta_{H,t})}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}},$$
$$\frac{C_{R,t}^R}{C_t^R} = \alpha_R\left(\frac{P_{R,t}^R}{P_t^R}\right)^{-\varepsilon}, \qquad \frac{C_{H,t}^R}{C_t^R} = (1-\alpha_R)(\exp(\zeta_{H,t}))^{\varepsilon-1}\left(\frac{P_{H,t}^R}{P_t^R}\right)^{-\varepsilon}.$$

Let us define for future use one of the relative prices

$$X_{R,t} \equiv \left(\frac{P_{R,t}^R}{P_t^R}\right).$$

Within each sector, there is monopolistic competition, as we make clear with the firm's problem. Thus, $C_{R,t}^R$ and $C_{H,t}^R$ in turn are Dixit–Stiglitz aggregators of a continuum of varieties. That is, with $\theta > 1$,

$$C_{R,t}^R = \left[\int_0^1 (C_{R,t}^R(i))^{\frac{\theta-1}{\theta}}\,di\right]^{\frac{\theta}{\theta-1}}, \qquad C_{H,t}^R = \left[\int_0^1 (C_{H,t}^R(i))^{\frac{\theta-1}{\theta}}\,di\right]^{\frac{\theta}{\theta-1}}$$

and

$$P_{R,t}^R = \left[\int_0^1 (P_{R,t}^R(i))^{1-\theta}\,di\right]^{1-\theta}, \qquad P_{H,t}^R = \left[\int_0^1 (P_{H,t}^R(i))^{1-\theta}\,di\right]^{1-\theta},$$

where

$$\frac{C_{R,t}^R(i)}{C_{R,t}^R} = \left(\frac{P_{R,t}^R(i)}{P_{R,t}^R}\right)^{-\theta}, \qquad \frac{C_{H,t}^R(i)}{C_{H,t}^R} = \left(\frac{P_{H,t}^R(i)}{P_{H,t}^R}\right)^{-\theta}.$$

There is no price discrimination across sectors for varieties, and we will impose the law of one price later.

B.1.1.2 *Firms* Firms in the R-sector produce differentiated varieties using the linear production function

$$Y_{R,t}(i) = L_{R,t}(i)$$

and set prices according to Calvo friction. Flow (real) profts are given by

$$\Psi_{R,t}(i) = \frac{P^{R*}_{R,t}(i) Y_{R,t}(i)}{P^R_t} - w^R_t L_{R,t}(i).$$

Profit maximization problem of firms that get to adjust prices is given by

$$\max \sum_{s=0}^{\infty} (\omega^R \beta)^s \left(\frac{C^R_{t+s}}{C^R_t}\right)^{-\sigma} \left[\left(\frac{P^{R*}_{R,t}(i)}{P^R_{R,t+s}}\right) X_{R,t+s} - w^R_{t+s}\right] \left(\frac{P^{R*}_{R,t}(i)}{P^R_{R,t+s}}\right)^{-\theta} Y_{R,t+s}.$$

Notice that no price discrimination (with notation introduced later, $P^R_{R,t}(i) = P^H_{R,t}(i)$) allows us to write the demand directly in terms of $Y_{R,t}(i) = (\frac{P^R_{R,t}(i)}{P^R_{R,t}})^{-\theta} Y_{R,t}$. Relative prices, $X_{R,t}$, show up here, because of a different price levels of the good and CPI of this sector, where we use CPI to deflate wages in the household problem. This is clear from the flow profit expression above. Moreover, the linearity of the production function gives marginal cost as w^R_t.

Optimal first-order conditions are given by

$$P^{R*}_{R,t}(i) = \left(\frac{\theta}{\theta - 1}\right) \frac{\sum_{s=0}^{\infty} (\omega^R \beta)^s \left(\frac{C^R_{t+s}}{C^R_t}\right)^{-\sigma} \left[w^R_{t+s} \left(\frac{1}{P^R_{R,t+s}}\right)^{-\theta}\right] Y_{R,t+s}}{\sum_{s=0}^{\infty} (\omega^R \beta)^s \left(\frac{C^R_{t+s}}{C^R_t}\right)^{-\sigma} \left[\left(\frac{1}{P^R_{R,t+s}}\right)^{1-\theta} X_{R,t+s}\right] Y_{R,t+s}}.$$

We can rewrite this optimal condition in terms of the law of motions of prices as follows:

$$P^{R*}_{R,t}(i) = \left(\frac{\theta}{\theta - 1}\right) \frac{Z^R_{1,t}}{Z^R_{2,t}},$$

$$Z^R_{1,t} = w^R_t (P^R_{R,t})^\theta Y_{R,t} + \omega^R \beta \left(\frac{C^R_{t+1}}{C^R_t}\right)^{-\sigma} Z^R_{1,t+1},$$

$$Z^R_{2,t} = X_{R,t} (P^R_{R,t})^{\theta-1} Y_{R,t} + \omega^R \beta \left(\frac{C^R_{t+1}}{C^R_t}\right)^{-\sigma} Z^R_{2,t+1}.$$

B.1.2 *Hand-to-mouth sector*

B.1.2.1 *Households* HTM households, of measure λ, solve the problem

$$\max_{\{C^H_t, L^H_t\}} \frac{(C^H_t)^{1-\sigma}}{1-\sigma} - \chi^H \frac{((1+\eta^\xi_t) L^H_t)^{1+\varphi}}{1+\varphi}$$

subject to the flow budget constraint

$$C_t^H = w_t^H L_t^H + \left(\frac{P_t^R}{P_t^H}\right) s_t^H,$$

where η_t^ξ is a labor supply shock, s_t^H is government transfer, w_t^H is the real wage, L_t^H is labor supply, and C_t^H is consumption. Note that relative price appears in transfers as for transfers/government variables we use the Ricardian household CPI as the deflator. We define the "real exchange rate" across sectors as, $Q_t \equiv (P_t^H/P_t^R)$. Then the intra-temporal optimality condition is

$$\chi^H (1+\eta_t^\xi)^{1+\varphi} (L_t^H)^\varphi (C_t^H)^\sigma = w_t^H.$$

C_t^H is a CES aggregator of the consumption goods produced in the two sectors

$$C_t^H = \left[(1-\alpha)^{\frac{1}{\varepsilon}} \left(\exp(\zeta_{H,t}) C_{H,t}^H\right)^{\frac{\varepsilon-1}{\varepsilon}} + (\alpha)^{\frac{1}{\varepsilon}} \left(C_{R,t}^H\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $1-\alpha$ is HTM households' consumption weight on the HTM-sector goods and $\zeta_{H,t}$ is a demand shock specific for HTM-sector goods.[6] Let us define for future use one of the relative prices, $X_{H,t} \equiv P_{H,t}^H/P_t^H$, where $P_{H,t}^H$ is the HTM sector's good price while P_t^H is the CPI price index of the HTM household. This implies that $Q_t X_{H,t} = P_{H,t}^H/P_t^R$, which will be useful later. The optimal price index and demand functions from a standard static expenditure minimization problem are given by

$$P_t^H = \left[(\alpha_H)\left(\frac{P_{H,t}^H}{\exp(\zeta_{H,t})}\right)^{1-\varepsilon} + (1-\alpha_H)\left(P_{R,t}^H\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}},$$

$$\frac{C_{H,t}^H}{C_t^H} = \alpha_H \left(\exp(\zeta_{H,t})\right)^{\varepsilon-1} \left(\frac{P_{H,t}^H}{P_t^H}\right)^{-\varepsilon}, \qquad \frac{C_{R,t}^H}{C_t^H} = (1-\alpha_H)\left(\frac{P_{R,t}^H}{P_t^H}\right)^{-\varepsilon}.$$

Within each sector, there is monopolistic competition, as we make clear with the firm's problem. Thus, $C_{H,t}^H$ and $C_{R,t}^H$ in turn are Dixit–Stiglitz aggregators of a continuum of varieties. That is, with $\theta > 1$,

$$C_{H,t}^H = \left(\int_0^1 \left(C_{H,t}^H(i)\right)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}, \qquad C_{R,t}^H = \left(\int_0^1 \left(C_{R,t}^H(i)\right)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}},$$

$$P_{H,t}^H = \left(\int_0^1 \left(P_{H,t}^H(i)\right)^{1-\theta} di\right)^{\frac{1}{1-\theta}}, \qquad P_{R,t}^H = \left(\int_0^1 \left(P_{R,t}^H(i)\right)^{1-\theta} di\right)^{\frac{1}{1-\theta}},$$

$$C_{H,t}^H(i) = \left(\frac{P_{H,t}^H(i)}{P_{H,t}^H}\right)^{-\theta} C_{H,t}^H, \qquad C_{R,t}^H(i) = \left(\frac{P_{R,t}^H(i)}{P_{R,t}^H}\right)^{-\theta} C_{R,t}^H.$$

There is no price discrimination across sectors for varieties, and we will impose the law of one price later.

[6]Our modeling choice of the same consumption basket for the two types of households is driven by the data, as we discuss later. This implies that the CPI of the two households is the same.

B.1.2.2 *Firms* Firms in the HTM sector produce differentiated varieties using the linear production function

$$Y_{H,t}(i) = L_{H,t}(i)$$

and set prices according to Calvo friction. Flow (real, in terms of CPI of Ricardian household) profits are given by

$$\Psi_{H,t}(i) = \frac{P^*_{HH,t}(i) Y_{H,t}(i)}{P^R_t} - \frac{P^H_t}{P^R_t} w^H_t L_{H,t}(i).$$

The profit maximization problem of firms that get to adjust prices is given by (they are owned by R households)

$$\max \sum_{s=0}^{\infty} (\omega^H \beta)^s \left(\frac{C^R_{t+s}}{C^R_t}\right)^{-\sigma} \left[\left(\frac{P^{H*}_{H,t}(i)}{P^H_{H,t+s}}\right) Q_{t+s} X_{H,t+s} - Q_{t+s} w^H_{t+s}\right] \left(\frac{P^{H*}_{H,t}(i)}{P^H_{H,t+s}}\right)^{-\theta} Y_{H,t+s}.$$

Relative prices, $Q_t X_{H,t} = \frac{P^H_{H,t}}{P^R_t}$, show up here, because of different price levels of the good and CPI of this sector, where we use CPI to deflate wages in the household problem. Moreover, a real exchange rate also shows up as we deflate the real profits by the Ricardian household's CPI as they own the firms. This is clear from the flow profit expression above. Moreover, the linearity of the production function gives marginal cost as w^R_t. Firms' optimal first-order condition is given by

$$P^{H*}_{H,t}(i) = \left(\frac{\theta}{\theta - 1}\right) \frac{\sum_{s=0}^{\infty} (\omega^H \beta)^s \left(\frac{C^R_{t+s}}{C^R_t}\right)^{-\sigma} \left[Q_{t+s} w^H_{t+s} \left(\frac{1}{P^H_{H,t+s}}\right)^{-\theta}\right] Y_{H,t+s}}{\sum_{s=0}^{\infty} (\omega^H \beta)^s \left(\frac{C^R_{t+s}}{C^R_t}\right)^{-\sigma} \left[\left(\frac{1}{P^H_{H,t+s}}\right)^{1-\theta} Q_{t+s} X_{H,t+s}\right] Y_{H,t+s}}.$$

We can rewrite it in terms of the law of motions of prices as follows:

$$P^{H*}_{H,t}(i) = \left(\frac{\theta}{\theta - 1}\right) \frac{Z^H_{1,t}}{Z^H_{2,t}},$$

$$Z^H_{1,t} = Q_t w^H_t (P^H_{H,t})^\theta Y_{H,t} + \omega^H \beta \left(\frac{C^R_{t+1}}{C^R_t}\right)^{-\sigma} Z^H_{1,t+1},$$

$$Z^H_{2,t} = Q_t X_{H,t} (P^H_{H,t})^{\theta-1} Y_{H,t} + \omega^H \beta \left(\frac{C^R_{t+1}}{C^R_t}\right)^{-\sigma} Z^H_{2,t+1}.$$

B.1.3 *Law of one price* There is no pricing to market on varieties across sectors. Thus, the law of one price holds for each variety. This is implicitly already imposed while writing the price-setting problem of the firms. This means

$$P^R_{R,t}(i) = P^H_{R,t}(i), \qquad P^H_{H,t}(i) = P^R_{H,t}(i)$$

and correspondingly the various sector-specific prices (but not the CPI prices) are also equalized:

$$P^R_{R,t} = P^H_{R,t}, \qquad P^H_{H,t} = P^R_{H,t}.$$

B.1.4 *Government* Government budget constraint is (deflating by CPI of the Ricardian household)

$$B_t + T^L_t = R_{t-1}B_{t-1} + P^R_t s_t \quad \text{and} \quad T^L_t = (1-\lambda)\tau^R_{L,t}P^R_t w^R_t L^R_t.$$

Transfer, s_t, is exogenous and deterministic.

Monetary and tax policy rules are of the feedback types with "smoothing," given by

$$\frac{R_t}{\bar{R}} = \max\left\{\frac{1}{\bar{R}}, \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_1}\left(\frac{R_{t-2}}{\bar{R}}\right)^{\rho_2}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_x}\left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_{\Delta y}}\right]^{(1-\rho_1-\rho_2)}\right\},$$

$$\tau^R_{L,t} - \bar{\tau}^R_L = \rho_L\left(\tau^R_{L,t-1} - \bar{\tau}^R_L\right) + (1-\rho_L)\psi_L\left(\frac{b_{t-1}-\bar{b}}{\bar{b}}\right).$$

We use the parameter $\omega \in [0, 1]$ to measure the fraction of transfers given to the HTM households. We therefore have

$$s^H_t = \frac{\omega}{\lambda}s_t \quad \text{and} \quad s^R_t = \frac{(1-\omega)}{(1-\lambda)}s_t,$$

that is, each HTM household receives $\frac{\omega}{\lambda}s_t$.

B.1.5 *Market clearing, aggregation, resource constraints* Notice that

$$s_t = (1-\lambda)s^R_t + \lambda s^H_t \quad \text{and} \quad b_t = (1-\lambda)b^R_t + \lambda b^H_t,$$
$$L_t = (1-\lambda)L^R_t + \lambda L^H_t \quad \text{and} \quad \Psi_t = (1-\lambda)\Psi^R_t + \lambda\Psi^H_t.$$

In our benchmark model, $b^H_t = \Psi^H_t = 0$.

Labor market clear conditions are

$$(1-\lambda)L^R_t = \int L_{R,t}(i)\,di, \qquad \lambda L^H_t = \int L_{H,t}(i)\,di.$$

To derive an aggregate resource constraint, we combine households' budget constraints and government budget constraint:

$$(1-\lambda)C^R_t + \lambda Q_t C^H_t = \int\left(\frac{P_{H,t}(i)}{P^R_t}Y_{H,t}(i) + \frac{P_{R,t}(i)}{P^R_t}Y_{R,t}(i)\right)di.$$

Define an aggregate consumption, C_t, as

$$C_t = (1-\lambda)C^R_t + \lambda Q_t C^H_t = \int\frac{P_{R,t}(i)}{P^R_t}Y_{R,t}(i)\,di + \int\frac{P_{H,t}(i)}{P^R_t}Y_{H,t}(i)\,di.$$

Note that from the law of one price,

$$Y_{R,t}(i) = (1-\lambda)C^R_{R,t}(i) + \lambda C^H_{R,t}(i) = \left(\frac{P_{R,t}(i)}{P_{R,t}}\right)^{-\theta} Y_{R,t},$$

$$Y_{H,t}(i) = (1-\lambda)C^R_{H,t}(i) + \lambda C^H_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\theta} Y_{H,t},$$

where

$$Y_{R,t} = (1-\lambda)C^R_{R,t} + \lambda C^H_{R,t} \quad \text{and} \quad Y_{H,t} = (1-\lambda)C^R_{H,t} + \lambda C^H_{H,t}.$$

Then

$$\begin{aligned} C_t &= \int \frac{P_{R,t}(i)}{P^R_t} Y_{R,t}(i)\,di + \int \frac{P_{H,t}(i)}{P^R_t} Y_{H,t}(i)\,di \\ &= \frac{P_{R,t}}{P^R_t} \int \left(\frac{P_{R,t}(i)}{P_{R,t}}\right)^{1-\theta} Y_{R,t}\,di + \left(\exp(\zeta_{H,t})\right)^{\theta-1} \frac{P_{H,t}}{P^R_t} \int \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\theta} Y_{H,t}\,di \\ &= X_{R,t} Y_{R,t} + X_{H,t} Q_t Y_{H,t}. \end{aligned}$$

To derive an aggregate sectoral output, we aggregate firms' product function:

$$\int L^R_t(i)\,di = Y_{R,t} \int \left(\frac{P_{R,t}(i)}{P_{R,t}}\right)^{-\theta} di \quad \text{and} \quad \int L^H_t(i)\,di = Y_{H,t} \int \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\theta} di.$$

Each sectoral market clears

$$(1-\lambda)L^R_t = Y_{R,t}\Xi_{R,t}, \qquad \lambda L^H_t = Y_{H,t}\Xi_{H,t},$$

where $\Xi_{R,t}$ and $\Xi_{H,t}$ are price dispersion terms which are given by

$$\Xi_{R,t} = \left(1-\omega^R\right)\left(\frac{P^*_{R,t}}{P_{R,t}}\right)^{-\theta} + \omega^R (\pi_{R,t})^{\theta} \Xi_{R,t-1},$$

$$\Xi_{H,t} = \left(1-\omega^H\right)\left(\frac{P^*_{H,t}}{P_{H,t}}\right)^{-\theta} + \omega^H (\pi_{H,t})^{\theta} \Xi_{H,t-1}.$$

Lastly, we derive law of motions of each sector's inflation:

$$(P_{H,t})^{1-\theta} = \left(\int_0^1 \left(P_{H,t}(i)\right)^{1-\theta} di\right),$$

$$(\pi_{H,t})^{1-\theta} = \left(1-\omega^H\right)\left(\frac{P^*_{H,t}}{P_{H,t}}\right)^{1-\theta} (\pi_{H,t})^{1-\theta} + \omega^H,$$

$$(\pi_{R,t})^{1-\theta} = \left(1-\omega^R\right)\left(\frac{P^*_{R,t}}{P_{R,t}}\right)^{1-\theta} (\pi_{R,t})^{1-\theta} + \omega^R.$$

B.2 *System of equilibrium conditions*

- Ricardian HH—Intertemporal EE

$$\exp(\eta_t^{\xi})(C_t^R)^{-\sigma} = \beta \frac{R_t}{\pi_{t+1}^R} \exp(\eta_{t+1}^{\xi})(C_{t+1}^R)^{-\sigma}. \tag{B.1}$$

- Ricardian HH—Intratemporal EE

$$\chi(L_t^R)^{\varphi}(C_t^R)^{\sigma} = (1 - \tau_{L,t}^R) w_t^R. \tag{B.2}$$

- Ricardian HH—Phillips curve 1

$$\frac{P_{R,t}^*}{P_{R,t}} = \left(\frac{\theta}{\theta - 1}\right) \frac{\tilde{Z}_{1,t}^R}{\tilde{Z}_{2,t}^R}. \tag{B.3}$$

- Ricardian HH—Phillips curve 2

$$\tilde{Z}_{1,t}^R = w_t^R Y_{R,t} + \omega^R \beta \left(\frac{C_{t+1}^R}{C_t^R}\right)^{-\sigma} \tilde{Z}_{1,t+1}^R (\pi_{R,t+1})^{\theta}. \tag{B.4}$$

- Ricardian HH—Phillips curve 3

$$\tilde{Z}_{2,t}^R = X_{R,t} Y_{R,t} + \omega^R \beta \left(\frac{C_{t+1}^R}{C_t^R}\right)^{-\sigma} \tilde{Z}_{2,t+1}^R (\pi_{R,t+1})^{\theta-1}. \tag{B.5}$$

- HTM HH—Intratemporal EE

$$\chi^H (\eta_t^{\xi})^{1+\varphi} (L_t^H)^{\varphi} (C_t^H)^{\sigma} = w_t^H. \tag{B.6}$$

- HTM HH—Budget constraint

$$C_t^H = w_t^H L_t^H + \left(\frac{1}{Q_t}\right) s_t^H. \tag{B.7}$$

- HTM HH—Phillips curve 1

$$\frac{P_{H,t}^*}{P_{H,t}} = \left(\frac{\theta}{\theta - 1}\right) \frac{\tilde{Z}_{1,t}^H}{\tilde{Z}_{2,t}^H}. \tag{B.8}$$

- HTM HH—Phillips curve 2

$$\tilde{Z}_{1,t}^H = Q_t w_t^H Y_{H,t} + \omega^H \beta \left(\frac{C_{t+1}^R}{C_t^R}\right)^{-\sigma} \tilde{Z}_{1,t+1}^H (\pi_{H,t+1})^{\theta}. \tag{B.9}$$

- HTM HH—Phillips curve 3

$$\tilde{Z}_{2,t}^H = Q_t X_{H,t} Y_{H,t} + \omega^H \beta \left(\frac{C_{t+1}^R}{C_t^R}\right)^{-\sigma} \tilde{Z}_{2,t+1}^H (\pi_{H,t+1})^{\theta-1}. \tag{B.10}$$

- Output R sector

$$Y_{R,t} = (1-\lambda)C^{R}_{R,t} + \lambda C^{H}_{R,t}. \tag{B.11}$$

- Output H sector

$$Y_{H,t} = (1-\lambda)C^{R}_{H,t} + \lambda C^{H}_{H,t}. \tag{B.12}$$

- Consumption 1

$$C^{R}_{R,t} = \alpha_R (X_{R,t})^{-\varepsilon} C^{R}_{t}. \tag{B.13}$$

- Consumption 2

$$C^{R}_{H,t} = \big(\exp(\zeta_{H,t})\big)^{\varepsilon-1}(1-\alpha_R)(X_{H,t}Q_t)^{-\varepsilon} C^{R}_{t}. \tag{B.14}$$

- Consumption 3

$$C^{H}_{H,t} = \big(\exp(\zeta_{H,t})\big)^{\varepsilon-1}\alpha_H (X_{H,t})^{-\varepsilon} C^{H}_{t}. \tag{B.15}$$

- Consumption 4

$$C^{H}_{R,t} = (1-\alpha_H)\left(X_{R,t}\frac{1}{Q_t}\right)^{-\varepsilon} C^{H}_{t}. \tag{B.16}$$

- Resource constraint

$$C_t = X_{R,t}Y_{R,t} + Q_t X_{H,t} Y_{H,t}. \tag{B.17}$$

- Aggregate output 1

$$(1-\lambda)L^{R}_{t} = Y_{R,t}\Xi_{R,t}. \tag{B.18}$$

- Price dispersion 1

$$\Xi_{R,t} = \big(1-\omega^R\big)\left(\frac{P^{*}_{R,t}}{P_{R,t}}\right)^{-\theta} + \omega^R (\pi_{R,t})^{\theta}\Xi_{R,t-1}. \tag{B.19}$$

- Aggregate output 2

$$\lambda L^{H}_{t} = Y_{H,t}\Xi_{H,t}. \tag{B.20}$$

- Price dispersion 2

$$\Xi_{H,t} = \big(1-\omega^H\big)\left(\frac{P^{*}_{H,t}}{P_{H,t}}\right)^{-\theta} + \omega^H (\pi_{H,t})^{\theta}\Xi_{H,t-1}. \tag{B.21}$$

- Aggregate price index 1

$$(\pi_{R,t})^{1-\theta} = \big(1-\omega^R\big)\left(\frac{P^{*}_{R,t}}{P_{R,t}}\right)^{1-\theta}(\pi_{R,t})^{1-\theta} + \omega^R. \tag{B.22}$$

- Aggregate price index 2

$$(\pi_{H,t})^{1-\theta} = \left(1-\omega^H\right)\left(\frac{P^*_{H,t}}{P_{H,t}}\right)^{1-\theta}(\pi_{H,t})^{1-\theta} + \omega^H. \tag{B.23}$$

- GBC

$$b_t + T^L_t = R_{t-1}\frac{b_{t-1}}{\pi^R_t} + s_t. \tag{B.24}$$

- Labor income tax

$$T^L_t = (1-\lambda)\tau^R_{L,t}w^R_t L^R_t. \tag{B.25}$$

- Transfer

$$s_t : \text{exogenous}. \tag{B.26}$$

- MP rule

$$\frac{R_t}{\bar{R}} = \max\left\{\frac{1}{\bar{R}}, \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_1}\left(\frac{R_{t-2}}{\bar{R}}\right)^{\rho_2}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_x}\left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_{\Delta y}}\right]^{(1-\rho_1-\rho_2)}\right\}, \tag{B.27}$$

where $\Pi_t = (1-\lambda)\Pi^R_t + \lambda\Pi^H_t$.

- Relative prices relationship

$$1 = \left(\alpha_R - \left(\frac{1-\alpha_R}{\alpha_H}\right)(1-\alpha_H)\right)(X_{R,t})^{1-\varepsilon} + \left(\frac{1-\alpha_R}{\alpha_H}\right)(Q_t)^{1-\varepsilon}, \tag{B.28}$$

$$X_{H,t} = \exp(\zeta_{H,t})\left(\frac{1-\alpha_R(X_{R,t})^{1-\varepsilon}}{1-\alpha_R}\left(\frac{1}{Q_t}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}. \tag{B.29}$$

If symmetry: $(1-\alpha_R = \alpha_H)$, then

$$Q_t = 1,$$

$$X_{H,t} = \exp(\zeta_{H,t})\left(\frac{1-\alpha_R(X_{R,t})^{1-\varepsilon}}{(1-\alpha_R)}\right)^{\frac{1}{1-\varepsilon}}.$$

- Inflation relationship

$$\pi^H_t = \frac{Q_t}{Q_{t-1}}\pi^R_t, \tag{B.30}$$

$$\left(\pi^R_t\right)^{1-\varepsilon} = \frac{(\pi_{R,t}\pi_{H,t})^{1-\varepsilon}}{\alpha_R(X_{R,t})^{1-\varepsilon}(\pi_{H,t})^{1-\varepsilon} + \left(1-\alpha_R(X_{R,t})^{1-\varepsilon}\right)(\pi_{R,t})^{1-\varepsilon}}, \tag{B.31}$$

$$\left(\pi^H_t\right)^{1-\varepsilon} = \frac{(\pi_{R,t}\pi_{H,t})^{1-\varepsilon}}{\alpha_H(X_{H,t})^{1-\varepsilon}(\pi_{R,t})^{1-\varepsilon} + \left(1-\alpha_H(X_{H,t})^{1-\varepsilon}\right)(\pi_{H,t})^{1-\varepsilon}}. \tag{B.32}$$

- Tax rules

$$\tau^R_{L,t} - \bar{\tau}^R_L = \rho_L\left(\tau^R_{L,t-1} - \bar{\tau}^R_L\right) + (1-\rho_L)\psi_L\left(\frac{b_{t-1}-\bar{b}}{\bar{b}}\right). \tag{B.33}$$

- Transfer sharing rule

$$s_t^H = \frac{\xi}{\lambda} s_t, \tag{B.34}$$

$$s_t^R = \frac{1-\xi}{1-\lambda} s_t. \tag{B.35}$$

B.3 *Model extensions*

In this subsection, we present our setup with the extended models, discussed in Section 3.4.3 in the paper.

B.3.1 *Adding government spending* As one model extension, we consider government spending on goods in the model, which does not enter utility. Under this setup, both households' and firms' problems are identical to the baseline model. Now, we introduce the government sector, which consumes G_t, the CES aggregator of the consumption good produced in the Ricardian and HTM sectors:

$$G_t = \left[(\alpha_G)^{\frac{1}{\varepsilon}} (G_{R,t})^{\frac{\varepsilon-1}{\varepsilon}} + (1-\alpha_G)^{\frac{1}{\varepsilon}} \left(\exp(\zeta_{H,t}) G_{H,t}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

This gives the following optimal price index and demand functions from a standard static expenditure minimization problem

$$P_t^G = \left[\alpha_G (P_{R,t})^{1-\varepsilon} + (1-\alpha_G)\left(\frac{P_{H,t}}{\exp(\zeta_{H,t})}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}},$$

$$G_{R,t} = \alpha_G \left(\frac{P_{R,t}^G}{P_t^G}\right)^{-\varepsilon} G_t, \qquad G_{H,t} = \left(\exp(\zeta_{H,t})\right)^{\varepsilon-1} (1-\alpha_G)\left(\frac{P_{H,t}^G}{P_t^G}\right)^{-\varepsilon} G_t,$$

$G_{R,t}$ and $G_{H,t}$ are Dixit–Stiglitz aggregators of a continuum of varieties. That is, with $\theta > 1$,

$$G_{R,t} = \left(\int_0^1 \left(G_{R,t}(i)\right)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}, \qquad G_{H,t} = \left(\int_0^1 \left(G_{H,t}(i)\right)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}},$$

$$P_{R,t}^G = \left(\int_0^1 \left(P_{R,t}^G(i)\right)^{1-\theta} di\right)^{\frac{1}{1-\theta}}, \qquad P_{H,t}^G = \left(\int_0^1 \left(P_{H,t}^G(i)\right)^{1-\theta} di\right)^{\frac{1}{1-\theta}},$$

$$G_{R,t}(i) = \left(\frac{P_{R,t}^G(i)}{P_{R,t}^G}\right)^{-\theta} G_{R,t}, \qquad G_{H,t}(i) = \left(\frac{P_{H,t}^G(i)}{P_{H,t}^G}\right)^{-\theta} G_{H,t}.$$

Now, we can rewrite the government budget constraint:

$$B_t + T_t^L = R_{t-1} B_{t-1} + P_t^R G_t + P_t^R s_t.$$

The law of one price implies that

$$P_{R,t}(i) = P_{R,t}^R(i) = P_{R,t}^H(i) = P_{R,t}^G(i), \qquad P_{H,t}(i) = P_{H,t}^H(i) = P_{H,t}^R(i) = P_{H,t}^G(i),$$

$$P_{R,t} = P_{R,t}^R = P_{R,t}^H = P_{R,t}^G, \qquad P_{H,t} = P_{H,t}^R = P_{H,t}^H = P_{H,t}^G.$$

Market clearing condition is given by

$$C_t + Q_t^G G_t = \int \frac{P_{R,t}(i)}{P_t^R} Y_{R,t}(i)\, di + \int \frac{P_{H,t}(i)}{P_t^R} Y_{H,t}(i)\, di,$$

where $C_t = (1-\lambda)C_t^R + \lambda Q_t C_t^H$ and $Q_t^G = \frac{P_t^G}{P_t^R}$. Note that from the law of one price,

$$Y_{R,t}(i) = (1-\lambda)C_{R,t}^R(i) + \lambda C_{R,t}^H(i) + G_{R,t}(i) = \left(\frac{P_{R,t}(i)}{P_{R,t}}\right)^{-\theta} Y_{R,t},$$

$$Y_{H,t}(i) = (1-\lambda)C_{H,t}^R(i) + \lambda C_{H,t}^H(i) + G_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\theta} Y_{H,t},$$

and

$$Y_{R,t} = (1-\lambda)C_{R,t}^R + \lambda C_{R,t}^H + G_{R,t}, \qquad Y_{H,t} = (1-\lambda)C_{H,t}^R + \lambda C_{H,t}^H + G_{H,t}.$$

Then we have

$$\begin{aligned} C_t + Q_t^G G_t &= \int \frac{P_{R,t}(i)}{P_t^R} Y_{R,t}(i)\, di + \int \frac{P_{H,t}(i)}{P_t^R} Y_{H,t}(i)\, di \\ &= \frac{P_{R,t}}{P_t^R} \int \left(\frac{P_{R,t}(i)}{P_{R,t}}\right)^{1-\theta} Y_{R,t}\, di + \frac{P_{H,t}}{P_t^R} \int \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\theta} Y_{H,t}\, di \\ &= S_{R,t} Y_{R,t} + S_{H,t} Q_t Y_{H,t}. \end{aligned}$$

We have two experiments regarding government spending. First, we simply introduce steady-state government spending in the model, where we set the steady-state government spending to output ratio to be 0.15, in line with the U.S. data average from 1990Q1–2020Q1. In this case, the modified equilibrium equations are the following:

- Output R sector

$$Y_{R,t} = (1-\lambda)C_{R,t}^R + \lambda C_{R,t}^H + G_{R,t}. \tag{B.11$'$}$$

- Output H sector

$$Y_{H,t} = (1-\lambda)C_{H,t}^R + \lambda C_{H,t}^H + G_{H,t}. \tag{B.12$'$}$$

- Resource constraint

$$C_t + Q_t^G G_t = S_{R,t} Y_{R,t} + Q_t S_{H,t} Y_{H,t}. \tag{B.17$'$}$$

- GBC

$$b_t + T_t^L = R_{t-1} \frac{b_{t-1}}{\pi_t^R} + Q_t^G G_t - \tau_t + s_t. \tag{B.24$'$}$$

- Government R-consumption

$$G_{R,t} = \alpha_G \left(\frac{S_{R,t}}{Q_t^G}\right)^{-\varepsilon} G_t. \tag{new}$$

- Government HTM-consumption

$$G_{H,t} = \big(\exp(\zeta_{H,t})\big)^{\varepsilon-1}(1-\alpha_G)\left(\frac{Q_t S_{H,t}}{Q_t^G}\right)^{-\varepsilon} G_t. \tag{new}$$

Second, we consider the endogenous government spending rules, which respond to the debt dynamics. In this case, we need a new rule for government spending instead of the tax adjustment rule:

$$\frac{G_t - \bar{G}}{\bar{G}} = \rho_G\left(\frac{G_{t-1} - \bar{G}}{\bar{G}}\right) + (1-\rho_G)\psi_G\left(\frac{b_{t-1} - \bar{b}}{\bar{b}}\right) + \varepsilon_{G,t}, \tag{B.33$'$}$$

where $\varepsilon_{G,t}$ is the government spending shock used when we calculate government spending multipliers. We calibrated the parameters of this rule at the same values as for our baseline labor tax rate rule.

B.3.2 *Money-in-the-utility function* Our quantitative model is cashless. As an extension, we now introduce (noninterest bearing) cash into the economy, where we follow Chari, Kehoe, and McGrattan (2002) by introducing a money-in-the-utility function for Ricardian households. The motivation is that this allows us to consider a classical channel through which inflation can affect model dynamics and welfare via real balances.

In this model extension, Ricardian (R) households solve the problem

$$\max_{\{C_t^R, L_t^R, b_t^R, \frac{M_t}{P_t^R}\}} \sum_{t=0}^{\infty} \beta^t \exp(\eta_t^{\xi}) \left[\frac{\left(\nu (C_t^R)^{\frac{\eta-1}{\eta}} + (1-\nu)\left(\frac{M_t}{P_t}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta(1-\sigma)}{\eta-1}}}{1-\sigma} - \chi \frac{(L_t^R)^{1+\varphi}}{1+\varphi} \right]$$

subject to a standard No-Ponzi-game constraint and a sequence of flow budget constraints

$$C_t^R + b_t^R + \frac{M_t}{P_t} = R_{t-1}\frac{1}{\Pi_t^R} b_{t-1}^R + \frac{M_{t-1}}{P_t} + (1-\tau_{L,t}^R) w_t^R L_t^R + \Psi_t^R.$$

The optimal first-order conditions are given by

$$P_t U_{M,t} = U_{C,t} - \beta \frac{1}{\Pi_{t+1}^R} U_{C,t+1},$$

$$U_{C,t} = \beta \frac{R_t}{\Pi_{t+1}^R} U_{C,t+1},$$

$$\frac{U_{L,t}}{U_{C,t}} = (1-\tau_{L,t}^R) w_t^R,$$

where

$$U_{C,t} = \exp\big(\eta_t^{\xi}\big)\nu\big(C_t^R\big)^{\frac{-1}{\eta}}\left\{\nu\big(C_t^R\big)^{\frac{\eta-1}{\eta}} + (1-\nu)\left(\frac{M_t}{P_t}\right)^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}(1-\sigma)-1},$$

$$P_t U_{M,t} = \exp\big(\eta_t^{\xi}\big)(1-\nu)\left(\frac{M_t}{P_t}\right)^{\frac{-1}{\eta}}\left\{\nu\big(C_t^R\big)^{\frac{\eta-1}{\eta}} + (1-\nu)\left(\frac{M_t}{P_t}\right)^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}(1-\sigma)-1},$$

$$U_{L,t} = \exp\big(\eta_t^{\xi}\big)\chi\big(L_t^R\big)^{\varphi}.$$

Here, the optimality condition over real balances, $m_t^R = \frac{M_t^R}{P_t}$, gives rise to the following money-demand equation:

$$\left(\frac{M_t}{P_t}\right)^{\frac{-1}{\eta}} = \frac{\nu}{1-\nu}\big(C_t^R\big)^{\frac{-1}{\eta}}\left(\frac{R_t-1}{R_t}\right).$$

In this case, the modified equilibrium equations are the following:

- Ricardian HH—Intertemporal EE

$$U_{C,t} = \beta\frac{R_t}{\Pi_{t+1}^R}U_{C,t+1}. \tag{B.1$''$}$$

- Ricardian HH—Intratemporal EE

$$\frac{\xi_t\chi\big(L_t^R\big)^{\varphi}}{U_{C,t}} = \big(1-\tau_{L,t}^R\big)w_t^R. \tag{B.2$''$}$$

- Ricardian HH—Money-demand equation

$$\left(\frac{M_t}{P_t}\right)^{\frac{-1}{\eta}} = \frac{\nu}{1-\nu}\big(C_t^R\big)^{\frac{-1}{\eta}}\left(\frac{R_t-1}{R_t}\right). \tag{new}$$

- Ricardian HH MU

$$U_{C,t} = \xi_t\nu\big(C_t^R\big)^{\frac{-1}{\eta}}\left\{\nu\big(C_t^R\big)^{\frac{\eta-1}{\eta}} + (1-\nu)\left(\frac{M_t}{P_t}\right)^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}(1-\sigma)-1}. \tag{new}$$

B.3.3 *Inflationary cost-push shocks* An important caveat to our quantitative results is the assumption that other than COVID shocks, there are no other shocks in the economy. To address this shortcoming partially, and to make our analysis more relevant for current events, we now introduce an inflationary shock ξ_t^{π} directly into the firm's optimal prices. To be specific, we assume that Ricardian-sector firms' optimal reset price is given by

$$P_{R,t}^{R*}(i) = \exp(\xi_t^\pi)\left(\frac{\theta}{\theta-1}\right)\frac{\sum_{s=0}^{\infty}(\omega^R\beta)^s\left(\frac{C_{t+s}^R}{C_t^R}\right)^{-\sigma}\left[w_{t+s}^R\left(\frac{1}{P_{R,t+s}^R}\right)^{-\theta}\right]Y_{R,t+s}}{\sum_{s=0}^{\infty}(\omega^R\beta)^s\left(\frac{C_{t+s}^R}{C_t^R}\right)^{-\sigma}\left[\left(\frac{1}{P_{R,t+s}^R}\right)^{1-\theta}X_{R,t+s}\right]Y_{R,t+s}}.$$

Similarly, HTM-sector firms' optimal reset price is

$$P_{H,t}^{H*}(i) = \exp(\xi_t^\pi)\left(\frac{\theta}{\theta-1}\right)\frac{\sum_{s=0}^{\infty}(\omega^H\beta)^s\left(\frac{C_{t+s}^R}{C_t^R}\right)^{-\sigma}\left[Q_{t+s}w_{t+s}^H\left(\frac{1}{P_{H,t+s}^H}\right)^{-\theta}\right]Y_{H,t+s}}{\sum_{s=0}^{\infty}(\omega^H\beta)^s\left(\frac{C_{t+s}^R}{C_t^R}\right)^{-\sigma}\left[\left(\frac{1}{P_{H,t+s}^H}\right)^{1-\theta}Q_{t+s}X_{H,t+s}\right]Y_{H,t+s}}.$$

This is akin to cost-push shocks in standard sticky-price models in the literature. We assume that the inflationary shock follows an AR(1) process:

$$\xi_t^\pi = \rho_\pi \xi_{t-1}^\pi + \varepsilon_{\pi,t}.$$

In this case, the modified equilibrium equations are the following:

- Ricardian HH—Phillips curve 1

$$\frac{P_{R,t}^*}{P_{R,t}} = \exp(\xi_t^\pi)\left(\frac{\theta}{\theta-1}\right)\frac{\tilde{Z}_{1,t}^R}{\tilde{Z}_{2,t}^R}. \tag{B.3'''}$$

- HTM HH—Phillips curve 1

$$\frac{P_{H,t}^*}{P_{H,t}} = \exp(\xi_t^\pi)\left(\frac{\theta}{\theta-1}\right)\frac{\tilde{Z}_{1,t}^H}{\tilde{Z}_{2,t}^H}. \tag{B.8'''}$$

Appendix C: Additional tables and figures

Table C.1. Data and model moments.

	Time	Data	Model
Panel A: Targeted Moments			
Total Hours for retail, transportation, leisure/hospitality	April	−16.36%	−16.35%
	June	−18.67%	−18.67%
	August	−12.91%	−12.91%
Total Hours excluding retail, transportation, leisure/hospitality	April	−6.62%	−6.62%
	June	−8.64%	−8.64%
	August	−6.26%	−6.26%
PCE Inflation for recreation, transportation, food services	April	−0.95%	−0.95%
	June	−0.20%	−0.20%
	August	0.08%	0.08%
Panel B: Nontargeted Moments			
PCE Inflation excluding recreation, transportation, food services	April	−0.15%	−2.81%
	June	−0.10%	−4.96%
	August	0.56%	−5.37%
Real PCE for recreation, transportation, food services	April	−40.72%	−23.37%
	June	−38.06%	−0.46%
	August	−27.68%	12.06%
Real PCE excluding recreation, transportation, food services	April	−7.79%	−4.37%
	June	−3.75%	−16.64%
	August	−0.44%	−16.35%
Real PCE	April	−12.35%	−10.20%
	June	−8.50%	−11.68%
	August	−4.21%	−7.64%
Real GDP (percent deviation from Q1)	Q2	−8.94%	−8.06%
	Q3	−2.06%	−2.12%

Note: This table shows moments of the data and simulated series from the baseline model. Panel A shows targeted moments and Panel B shows nontargeted moments. Data moments are expressed as the percent deviation from the average values of outcome variables in January and February 2020.

TABLE C.2. Transfer multipliers under alternative calibrations.

	Monetary Regime				Fiscal Regime			
	$\mathcal{M}_t^M(Y)$	$\mathcal{M}_t^M(Y_R)$	$\mathcal{M}_t^M(C^R)$	$\mathcal{M}_t^M(C^H)$	$\mathcal{M}_t^F(Y)$	$\mathcal{M}_t^F(Y_R)$	$\mathcal{M}_t^F(C^R)$	$\mathcal{M}_t^F(C^H)$
Panel A: Alternative Calibration Excluding One-Time Tax Rebates (15.7% Transfer Increases)								
Impact Multipliers	1.957	1.901	0.120	7.967	3.371	3.101	1.579	9.238
4-Year Cumulative Multiplier	1.785	2.107	−0.015	7.678	7.459	7.167	4.565	16.932
Panel B: Alternative Calibration Excluding Unemployment Benefit Components (16.7% Transfer Increases)								
Impact Multipliers	1.953	1.898	0.120	7.954	3.312	3.049	1.519	9.180
4-Year Cumulative Multiplier	1.780	2.099	−0.014	7.652	7.186	6.920	4.350	16.470
Panel C: Alternative Calibration With Tax Rebates to Both Ricardian and HTM Households								
Impact Multipliers	1.332	1.294	0.078	5.435	2.167	2.001	0.938	6.190
4-Year Cumulative Multiplier	1.236	1.453	0.020	5.217	4.582	4.436	2.722	10.672
Panel D: Alternative Calibration With Transfer Distribution Starting From April 2020								
Impact Multipliers	1.774	1.959	0.255	6.748	3.500	3.410	2.011	8.374
4-Year Cumulative Multiplier	1.723	2.105	0.029	7.267	5.538	5.503	3.109	13.491

Note: This table shows the transfer multipliers for the models under monetary and fiscal regimes when we re-calibrate the baseline model. $\mathcal{M}_t^i(X)$ represent the cumulative transfer multiplier of variable X at t-horizon under i regime. We report impact multipliers and 4-year cumulative multipliers when the government distributes transfers equally over 6 months.

TABLE C.3. Welfare gains under alternative calibrations.

	Monetary Regime		Fiscal Regime	
	Long-Run	Short-Run	Long-Run	Short-Run
Panel A: Excluding One-Time Tax Rebates (15.7% Transfer Increases)				
Ricardian Household	−0.009	−0.897	0.013	−0.693
HTM Household	0.046	3.752	0.083	5.010
Panel B: Excluding Unemployment Benefit Components (16.7% Transfer Increases)				
Ricardian Household	−0.009	−0.950	0.012	−0.742
HTM Household	0.048	3.983	0.086	5.263
Panel C: Tax Rebates to Both Ricardian and HTM Households				
Ricardian Household	−0.010	−1.039	0.012	−0.831
HTM Household	0.053	4.365	0.091	5.630
Panel D: Alternative Calibration With Transfer Distribution Starting From April 2020				
Ricardian Household	−0.014	−1.493	0.012	−1.236
HTM Household	0.073	6.183	0.115	7.657

Note: This table shows long- and short-run ($t = 4$) welfare gains resulting from the redistribution, compared to the models without redistribution. The values are the difference in the welfare measure ($\mu_{t,k}^i$) between the transfer cases (under the two regimes) and the benchmark case (the monetary regime without transfers).

TABLE C.4. Transfer and government spending multipliers with tax adjustment.

	Monetary Regime				Fiscal Regime			
	$\mathcal{M}_t^M(Y)$	$\mathcal{M}_t^M(Y_R)$	$\mathcal{M}_t^M(C^R)$	$\mathcal{M}_t^M(C^H)$	$\mathcal{M}_t^F(Y)$	$\mathcal{M}_t^F(Y_R)$	$\mathcal{M}_t^F(C^R)$	$\mathcal{M}_t^F(C^H)$
Panel A: Transfer Multipliers With Steady-State Government Spending								
Impact Multipliers	1.875	1.836	0.079	7.757	2.915	2.689	1.108	8.829
4-Year Cumulative Multiplier	1.669	2.039	−0.010	7.165	5.655	5.575	3.032	14.243
Panel B: Government Spending Multipliers								
Impact Multipliers	1.218	1.068	0.026	0.847	2.386	2.027	1.251	1.826
4-Year Cumulative Multiplier	1.138	1.068	−0.182	1.186	5.414	4.814	3.261	8.185

Note: This table shows the transfer multipliers for the models under monetary and fiscal regimes when we re-calibrate the baseline model. $\mathcal{M}_t^i(X)$ represent the cumulative transfer multiplier of variable X at t-horizon under i regime. We report impact multipliers and 4-year cumulative multipliers when the government distributes transfers equally over 6 months.

TABLE C.5. Welfare gains with tax adjustment.

	Monetary Regime		Fiscal Regime	
	Long-Run	Short-Run	Long-Run	Short-Run
Panel A: Welfare Gains With Transfer Shocks and Steady-State Government Spending				
Ricardian Household	−0.017	−1.954	0.015	−1.618
HTM Household	0.073	6.111	0.119	7.939
Panel B: Welfare Gains With Government Spending Shocks				
Ricardian Household	−0.015	−1.138	0.024	−0.504
HTM Household	0.006	0.779	0.055	2.456

Note: This table shows long- and short-run ($t = 4$) welfare gains resulting from the redistribution, compared to the models without redistribution. The values are the difference in the welfare measure ($\mu_{t,k}^i$) between the transfer cases (under the two regimes) and the benchmark case (the monetary regime without transfers).

TABLE C.6. Transfer multipliers and welfare gains with government spending adjustment in the monetary regime.

	$\mathcal{M}_t^M(Y)$	$\mathcal{M}_t^M(Y_R)$	$\mathcal{M}_t^M(C^R)$	$\mathcal{M}_t^M(C^H)$
Panel A: Transfer Multipliers				
Impact Multipliers	1.866	1.833	0.066	7.759
4-Year Cumulative Multiplier	1.655	2.054	−0.022	7.143
		Long-run		Short-run
Panel B: Welfare Gains				
Ricardian Household		−0.015		−1.973
HTM Household		0.072		6.050

Note: This table shows the transfer multipliers and welfare gains for the model with government spending adjustment under the monetary regime. Panel A reports impact multipliers and 4-year cumulative multipliers when the government distributes transfers equally over 6 months. Panel B shows long- and short-run ($t = 4$) welfare gains resulting from the redistribution, compared to the model without redistribution. The values are the difference in the welfare measures ($\mu_{t,k}^i$) between the with-transfer case and the without-transfer case under the monetary regime.

TABLE C.7. Government spending multipliers with government spending adjustment.

	Monetary Regime				Fiscal Regime			
	$\mathcal{M}_t^M(Y)$	$\mathcal{M}_t^M(Y_R)$	$\mathcal{M}_t^M(C^R)$	$\mathcal{M}_t^M(C^H)$	$\mathcal{M}_t^F(Y)$	$\mathcal{M}_t^F(Y_R)$	$\mathcal{M}_t^F(C^R)$	$\mathcal{M}_t^F(C^H)$
Impact Multipliers	1.194	1.051	0.001	0.828	2.464	2.100	1.338	1.878
4-Year Cumulative Multiplier	1.275	1.226	−0.013	1.221	5.299	4.620	1.904	9.497

Note: This table shows the government spending multipliers for the models under monetary and fiscal regimes when we recalibrate the baseline model.

TABLE C.8. Transfer multipliers with money-in-the-utility.

	Monetary Regime				Fiscal Regime			
	$\mathcal{M}_t^M(Y)$	$\mathcal{M}_t^M(Y_R)$	$\mathcal{M}_t^M(C^R)$	$\mathcal{M}_t^M(C^H)$	$\mathcal{M}_t^F(Y)$	$\mathcal{M}_t^F(Y_R)$	$\mathcal{M}_t^F(C^R)$	$\mathcal{M}_t^F(C^H)$
Impact Multipliers	2.211	2.067	−1.203	13.388	4.640	4.083	−0.028	19.920
4-Year Cumulative Multiplier	1.043	1.284	−1.463	9.246	2.696	2.805	−0.256	12.359

Note: This table shows the transfer multipliers for the models under monetary and fiscal regimes when we re-calibrate the baseline model.

TABLE C.9. Transfer multipliers with inflationary cost-push shocks.

	Monetary Regime				Fiscal Regime			
	$\mathcal{M}_t^M(Y)$	$\mathcal{M}_t^M(Y_R)$	$\mathcal{M}_t^M(C^R)$	$\mathcal{M}_t^M(C^H)$	$\mathcal{M}_t^F(Y)$	$\mathcal{M}_t^F(Y_R)$	$\mathcal{M}_t^F(C^R)$	$\mathcal{M}_t^F(C^H)$
Panel A: 10% Shock								
Impact Multipliers	1.947	1.874	0.158	7.803	2.915	2.691	1.160	8.662
4-Year Cumulative Multiplier	1.795	2.033	0.102	7.337	5.364	5.197	2.824	13.678
Panel B: 20% Shock								
Impact Multipliers	1.977	1.882	0.197	7.802	2.857	2.629	1.122	8.537
4-Year Cumulative Multiplier	1.865	2.025	0.203	7.307	5.089	4.863	2.510	13.528

Note: This table shows the transfer multipliers for the models under monetary and fiscal regimes when we re-calibrate the baseline model.

TABLE C.10. Welfare gains with inflationary cost-push shocks.

	Monetary Regime		Fiscal Regime	
	Long-Run	Short-Run	Long-Run	Short-Run
Panel A: Welfare Gains With 10% Inflationary Shocks				
Ricardian Household	−0.012	−1.450	0.011	−1.248
HTM Household	0.075	6.372	0.119	7.825
Panel B: Welfare Gains With 20% Inflationary Shocks				
Ricardian Household	−0.011	−1.413	0.010	−1.243
HTM Household	0.076	6.496	0.120	7.823

Note: This table shows long- and short-run ($t = 4$) welfare gains resulting from the redistribution, compared to the models without redistribution. The values are the difference in the welfare measure ($\mu_{t,k}^i$) between the transfer cases (under the two regimes) and the benchmark case (the monetary regime without transfers).

TABLE C.11. Transfer multipliers under two alternative calibrations.

	Monetary Regime				Fiscal Regime			
	$\mathcal{M}_t^M(Y)$	$\mathcal{M}_t^M(Y_R)$	$\mathcal{M}_t^M(C^R)$	$\mathcal{M}_t^M(C^H)$	$\mathcal{M}_t^F(Y)$	$\mathcal{M}_t^F(Y_R)$	$\mathcal{M}_t^F(C^R)$	$\mathcal{M}_t^F(C^H)$
Panel A: Alternative Calibration With Above Steady-State Initial Debt (50.9%)								
Impact Multipliers	1.938	1.860	0.133	7.849	6.759	5.988	4.921	12.777
4-Year Cumulative Multiplier	1.800	2.012	0.065	7.478	15.638	14.768	10.319	33.049
Panel B: Alternative Calibration With Above Steady-State Initial Debt (71.3%)								
Impact Multipliers	1.824	1.732	0.113	7.426	5.916	5.168	4.187	11.576
4-Year Cumulative Multiplier	1.732	1.913	0.080	7.141	13.325	12.329	8.747	28.311

Note: This table shows the transfer multipliers for the models under monetary and fiscal regimes when we re-calibrate the baseline model. In Panel A, we calibrate the COVID shocks in the baseline model under the monetary regime with time-0 government debt which is 10% higher than the steady state (50.9% of debt-to-GDP). In Panel B, we calibrate the COVID shocks in the baseline model under the monetary regime with time-0 government debt, which is 10% higher than the alternative steady state (71.3% of debt-to-GDP which matches the average U.S. debt-to-GDP ratio from 2010Q1 through 2020Q1). $\mathcal{M}_t^i(X)$ represent the cumulative transfer multiplier of variable X at t-horizon under i regime. We report impact multipliers and 4-year cumulative multipliers when the government distributes transfers equally over 6 months.

TABLE C.12. Welfare gains under two alternative calibrations.

	Monetary Regime		Fiscal Regime	
	Long-Run	Short-Run	Long-Run	Short-Run
Panel A: Alternative Calibration With Above Steady-State Initial Debt (50.9%)				
Ricardian Household	−0.013	−1.436	0.066	−1.498
HTM Household	0.078	6.365	0.250	14.015
Panel B: Alternative Calibration With Above Steady-State Initial Debt (71.3%)				
Ricardian Household	−0.014	−1.646	0.094	−1.359
HTM Household	0.080	6.478	0.241	12.776

Note: This table shows long- and short-run ($t = 4$) welfare gains resulting from the redistribution, compared to the models without redistribution. The values are the difference in the welfare measure ($\mu^i_{t,k}$) between the transfer cases (under the two regimes) and the benchmark case (the monetary regime without transfers).

TABLE C.13. Transfer multipliers with above steady-state initial debt (without COVID shocks).

	Monetary Regime				Fiscal Regime			
	$\mathcal{M}_t^M(Y)$	$\mathcal{M}_t^M(Y_R)$	$\mathcal{M}_t^M(C^R)$	$\mathcal{M}_t^M(C^H)$	$\mathcal{M}_t^F(Y)$	$\mathcal{M}_t^F(Y_R)$	$\mathcal{M}_t^F(C^R)$	$\mathcal{M}_t^F(C^H)$
Panel A: Impact Multipliers								
Baseline	2.670	2.464	−0.911	14.394	4.640	4.083	−0.028	19.920
Above steady-state initial debt	2.385	2.190	−0.808	12.836	3.903	3.428	−0.027	16.770
Panel B: 4-Year Cumulative Multipliers								
Baseline	1.490	1.703	−1.107	9.991	2.696	2.805	−0.256	12.359
Above steady-state initial debt	1.426	1.608	−0.974	9.285	2.403	2.492	−0.246	11.075

Note: This table shows the transfer multipliers for aggregate output, Ricardian sector output, Ricardian consumption, and HTM consumption. $\mathcal{M}_t^i(X)$ represent the cumulative transfer multiplier of variable X at t-horizon under i regime. We report impact multipliers ($t = 0$) as well as 4-year ($t = 24$) cumulative multipliers.

TABLE C.14. Transfer multipliers with different duration of binding ZLB periods.

	Monetary Regime				Fiscal Regime			
	$\mathcal{M}_t^M(Y)$	$\mathcal{M}_t^M(Y_R)$	$\mathcal{M}_t^M(C^R)$	$\mathcal{M}_t^M(C^H)$	$\mathcal{M}_t^F(Y)$	$\mathcal{M}_t^F(Y_R)$	$\mathcal{M}_t^F(C^R)$	$\mathcal{M}_t^F(C^H)$
Panel A: ZLB Duration: 4 Periods (Baseline)								
Impact Multipliers	1.923	1.863	0.119	7.828	2.949	2.726	1.166	8.788
4-Year Cumulative Multiplier	1.732	2.023	−0.002	7.409	5.552	5.429	3.078	13.652
Panel B: ZLB Duration: 5 Periods								
Impact Multipliers	1.850	1.800	0.059	7.710	3.461	3.134	1.703	9.218
4-Year Cumulative Multiplier	1.529	1.773	−0.052	6.705	6.570	6.207	4.263	14.124
Panel C: ZLB Duration: 6 Periods								
Impact Multipliers	1.759	1.733	0.000	7.514	4.100	3.656	2.408	9.639
4-Year Cumulative Multiplier	1.337	1.569	−0.118	6.098	7.927	7.325	5.826	14.805
Panel D: ZLB Duration: 7 Periods								
Impact Multipliers	1.628	1.648	−0.063	7.165	5.071	4.461	3.537	10.091
4-Year Cumulative Multiplier	1.125	1.388	−0.202	5.469	10.079	9.189	8.366	15.684
Panel E: ZLB Duration: 8 Periods								
Impact Multipliers	1.567	1.607	−0.099	7.019	5.419	4.751	3.955	10.212
4-Year Cumulative Multiplier	1.027	1.315	−0.264	5.253	10.87	9.896	9.323	15.935

Note: This table shows the transfer multipliers for the models under monetary and fiscal regimes with different periods of ZLB. We introduce different degrees of persistence in preference shocks to generate different ZLB duration (persistence of preference shocks in Panel A: 0.0, in Panel B: 0.2, in Panel C: 0.4, in Panel D: 0.6, in Panel E: 0.65). $\mathcal{M}_t^i(X)$ represent the cumulative transfer multiplier of variable X at t-horizon under i regime. We report impact multipliers and 4-year cumulative multipliers when the government distributes transfers equally over 6 months.

TABLE C.15. Transfer multipliers with only preference shocks.

	Monetary Regime				Fiscal Regime			
	$\mathcal{M}_t^M(Y)$	$\mathcal{M}_t^M(Y_R)$	$\mathcal{M}_t^M(C^R)$	$\mathcal{M}_t^M(C^H)$	$\mathcal{M}_t^F(Y)$	$\mathcal{M}_t^F(Y_R)$	$\mathcal{M}_t^F(C^R)$	$\mathcal{M}_t^F(C^H)$
Panel A: Only Preference Shocks (Calibrated Baseline Preference Shocks: $\rho_\beta = 0.0$)								
Impact Multipliers	3.083	2.746	0.066	12.961	5.518	4.691	1.629	18.250
4-Year Cumulative Multiplier	1.791	1.703	0.094	7.348	6.453	5.768	4.085	14.205
Panel B: Only Preference Shocks (Calibrated Baseline Preference Shocks: $\rho_\beta = 0.8$)								
Impact Multipliers	1.672	1.738	−0.207	7.821	10.664	8.877	5.755	26.734
4-Year Cumulative Multiplier	0.909	1.131	−0.358	5.059	14.993	13.557	13.912	18.532
Panel C: Only Preference Shocks (Shock to Initial Period: −50%) ($\rho_\beta = 0.0$)								
Impact Multipliers	1.423	1.326	−0.156	6.591	2.288	2.013	0.773	7.248
4-Year Cumulative Multiplier	1.348	1.509	−0.476	7.319	5.582	5.265	2.865	14.478
Panel D: Only Preference Shocks (Shock to Initial Period: −50%) ($\rho_\beta = 0.8$)								
Impact Multipliers	1.437	1.408	−0.088	6.430	4.328	3.457	3.293	7.719
4-Year Cumulative Multiplier	0.882	0.950	−0.201	4.428	13.038	11.587	12.953	13.316

Note: This table shows the transfer multipliers for the models under monetary and fiscal regimes when we only have preference shocks. $\mathcal{M}_t^i(X)$ represent the cumulative transfer multiplier of variable X at t-horizon under i regime. We report impact multipliers and 4-year cumulative multipliers when the government distributes transfers equally over 6 months.

TABLE C.16. Welfare gains with only preference shocks.

	Monetary Regime		Fiscal Regime	
	Long-Run	Short-Run	Long-Run	Short-Run
Panel A: Only Preference Shocks Calibrated Baseline Preference Shocks: $\rho_\beta = 0.0$)				
Ricardian Household	−0.011	−1.221	0.025	−0.610
HTM Household	0.081	6.308	0.130	7.758
Panel B: Only Preference Shocks Calibrated Baseline Preference Shocks: $\rho_\beta = 0.8$)				
Ricardian Household	−0.014	−1.453	0.139	2.377
HTM Household	0.064	5.153	0.172	7.727
Panel C: Only Preference Shocks (Shock to Initial Period: −50%: $\rho_\beta = 0.0$)				
Ricardian Household	−0.022	−1.703	0.008	−1.556
HTM Household	0.074	6.205	0.123	8.316
Panel D: Only Preference Shocks (Shock to Initial Period: −50%: $\rho_\beta = 0.8$)				
Ricardian Household	−0.009	−1.213	0.147	3.720
HTM Household	0.067	5.217	0.151	5.612

Note: This table shows long- and short-run ($t = 4$) welfare gains resulting from the redistribution, compared to the models without redistribution. The values are the difference in the welfare measure ($\mu^i_{t,k}$) between the transfer cases (under the two regimes) and the benchmark case (the monetary regime without transfers).

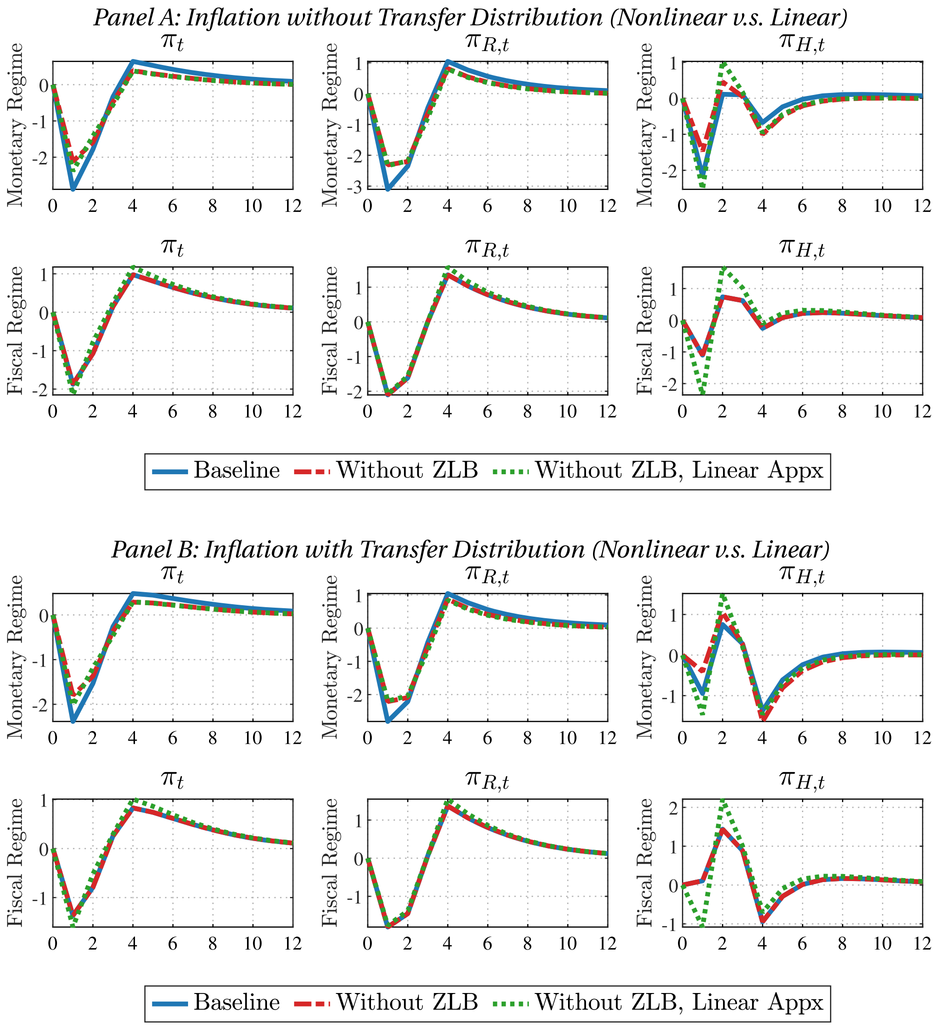

FIGURE C.1. Inflation Dynamics: Comparison between Nonlinear and Linear Solutions.

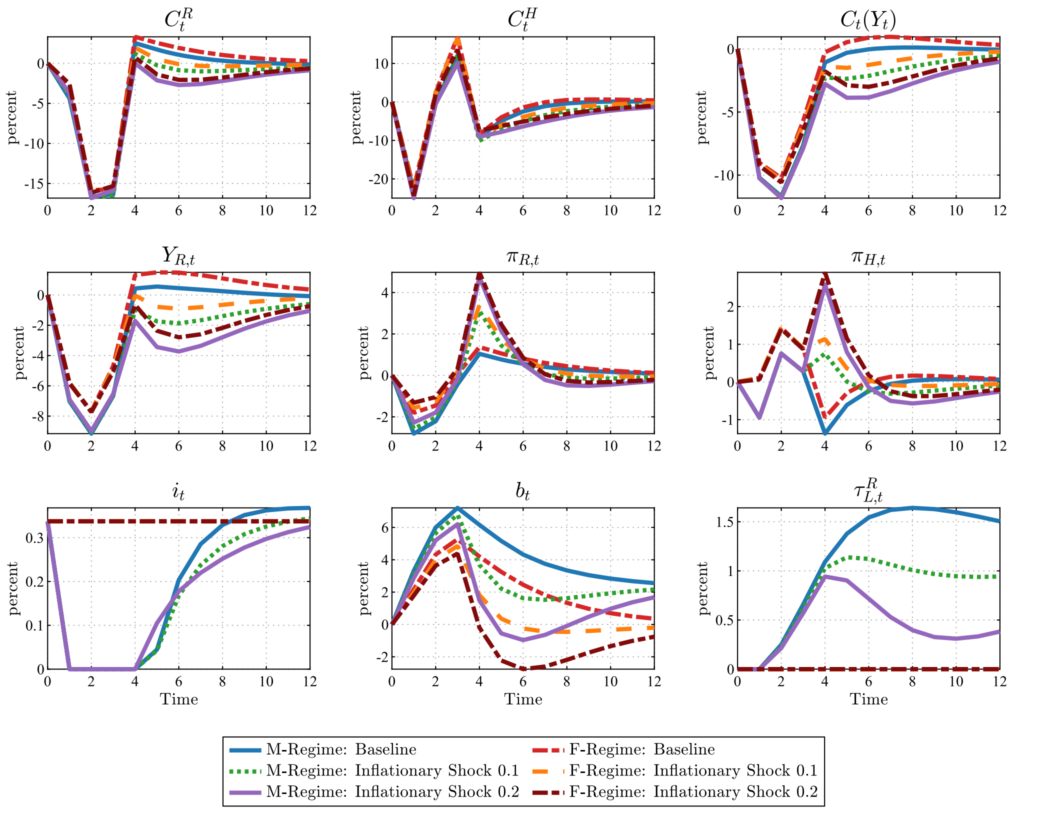

FIGURE C.2. Redistribution Policy with Inflationary Shocks.

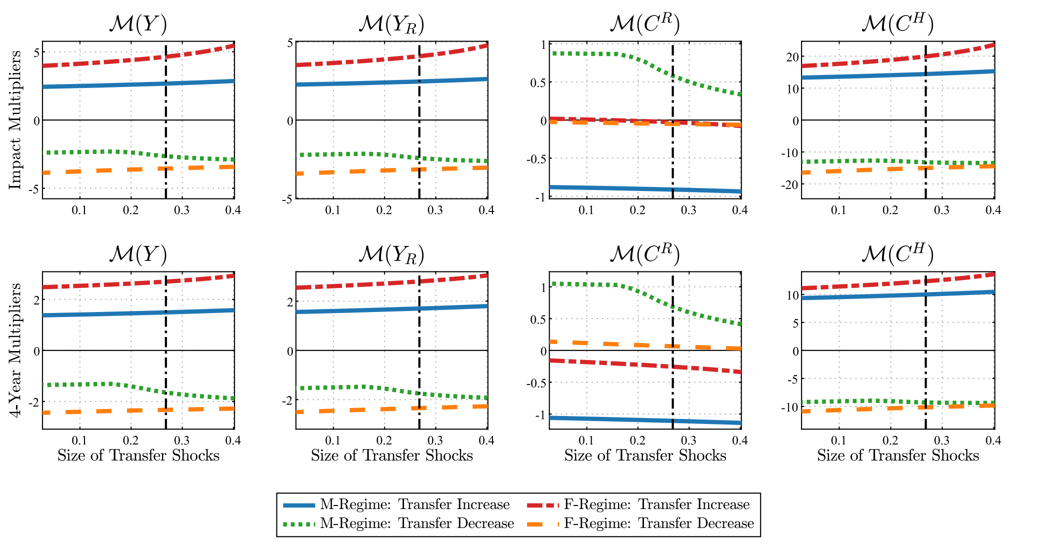

FIGURE C.3. Impact and Cumulative Multipliers by Different Transfer Size/Sign without COVID Shocks.

References

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