Wage Bargaining with On-the-job Search: Theory and Evidence
Supplemented material: Bargaining in presence of a legal minimum wage

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Revised June 2005.

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In this Appendix we consider the existence of a wage floor, $w_{\min}$, such that no firm can make a wage offer of less than $w_{\min}$ at any stage of the bargaining process. For simplicity, we shall work under the assumption that workers are homogeneous conditional on their observed attributes, i.e. we shall assume away any dispersion in the workers' ε's (and consequently drop the dependence on ε of all functions in the analysis to come).[1] We otherwise take up the concepts and notation of the main text.

1 Basic outcomes

Because no firm can make a wage offer that falls short of the minimum wage $w_{\min}$ at any stage of the bargaining game(s) described in Appendix A.1, the entry wage negotiated by unemployed workers solves:

$$V\left[\phi_0\left(p\right), p\right] = \beta V\left(p, p\right) + \left(1-\beta\right) \cdot \max\left\{V_0; V\left(w_{\min}, p\right)\right\}. \tag{1}$$

And similarly the wage $\phi\left(p', p\right)$ resulting from a negotiation involving two firms of types $p \geq p'$ now solves:

$$V\left[\phi\left(p', p\right), p\right] = \beta V\left(p, p\right) + \left(1-\beta\right) \cdot \max\left\{V\left(p', p'\right); V\left(w_{\min}, p\right)\right\}. \tag{2}$$

2 Value functions

Following similar steps to Appendix A.2, we now derive the workers' value functions $V\left(\cdot\right)$ to substitute into the definitions above and obtain a more explicit definition of $\phi\left(\cdot\right)$. Before we proceed, however, a few preliminary definitions must be introduced.

A first definition is brought about by the following remark. Given our bargaining rules, all wages paid at a given type-p firm must be greater than a firm-specific lower bound $\phi_{\min}\left(p\right)$ defined by:

$$V\left[\phi_{\min}\left(p\right), p\right] = \beta V\left(p, p\right) + \left(1-\beta\right) V\left(w_{\min}, p\right). \tag{3}$$

Thus, because $V\left(w, p\right)$ is an increasing function of the wage w, it has to be the case that if $p > w_{\min}$ and $\beta > 0$, then all wages paid by that type-p firm will be strictly greater than the minimum wage $w_{\min}$. Hence unless $\beta = 0$, no worker in this economy will actually receive the minimum wage, except possibly those employed at firms with the minimum viable productivity $p = w_{\min}$.[2] This results from the assumption that the employers is not allowed to make wage offers below the minimum wage during the negotiation process. Hence, the payoff $V(w_{\min}, p)$ always acts as a lower bound to the worker's threat point. This property of the bargaining game may help explain why wage distributions have no evident atom at the minimum wage in our data.[3]

[1] Hence any residual individual heterogeneity in wages, once observed attributes and individual histories of job offers have been taken into account will be interpreted as measurement error. The additional complication brought about by "genuine" worker heterogeneity renders the model intractable. The main reason is that a common wage floor imposed on a market with heterogeneous firms and workers leads to *assortative matching* of firm-worker pairs, as a match between a type-p firm and a type-ε worker is then only profitable if $p\varepsilon \geq w_{\min}$ (which makes the lower bound of the set of worker types employable at a given type-p firm a function of p).

[2] Clearly, a firm with $p < w_{\min}$ cannot earn positive profits on our labor market. It thus has to be the case that $p_{\min} \geq w_{\min}$.

[3] This issue was also analyzed by Laroque and Salanié (2004) in a model in which the outcome of the wage negotiation is defined by the Kalai-Samorodinsky solution.

Second, we define the threshold $t(p)$ as:

$$V(p,p) = V[w_{\min}, t(p)]. \tag{4}$$

We shall assume from the outset that $t(p)$ exists, $t(p) \geq p$, and $t(p)$ is strictly increasing, a set of assumptions whose consistency will be confirmed later in the analysis. The threshold $t(p)$ is such that in a negotiation involving a firm of type p and some other firm type $p' \geq p$, the minimum wage constraint will be binding if and only if $p' > t(p)$. In other words, in the absence of a minimum wage, the wage that it would take a firm of type $t(p)$ to poach a worker away from a firm of type less than p would be below $w_{\min}$. This threshold $t(p)$ will play a central role in the analysis.

Worker's value: $V(w,p)$ for $w \geq \phi_{\min}(p)$. We can now turn to the workers' value functions per se. We begin by considering $V(w,p)$ for a wage w which will effectively be observed in a type-p firm in equilibrium, i.e. a wage $w \geq \phi_{\min}(p)$. (As is clear from the various equations above, we will also need to consider $V(w_{\min}, p)$—even though no worker receives exactly $w_{\min}$ in equilibrium—which has a slightly different definition.)

Taking up equation (A12) in Appendix A.2, and amending it according to (2) and (1), we obtain:

$$\begin{aligned}\left[\rho + \delta + \lambda_1 \overline{F}(q(w,p))\right] V(w,p) &= w + \delta V_0 \\ &+ \lambda_1 \int_{q(w,p)}^{p} [\beta V(p,p) + (1-\beta) \cdot \max\{V(x,x); V(w_{\min},p)\}]\, dF(x) \\ &+ \lambda_1 \int_{p}^{p_{\max}} [\beta V(x,x) + (1-\beta) \cdot \max\{V(p,p); V(w_{\min},x)\}]\, dF(x),\end{aligned} \tag{5}$$

where $q(w,p)$ is still defined by equation (A8):

$$V(w,p) = \beta V(p,p) + (1-\beta) V[q(w,p), q(w,p)].$$

This, together with the definition (3) of $\phi_{\min}(p)$ immediately implies that for any w in the set of wages observed at a type p firm, $V[q(w,p), q(w,p)] \geq V(w_{\min}, p)$. Hence the "$\max\{\cdot\}$" term in the first integral of (5) is unambiguously equal to $V(x,x)$ for all x in the range of integration. Hence turning back to (5), we have:

$$\begin{aligned}\left[\rho + \delta + \lambda_1 \overline{F}(q(w,p))\right] V(w,p) &= w + \delta V_0 + \lambda_1 \int_{q(w,p)}^{p} [\beta V(p,p) + (1-\beta) V(x,x)]\, dF(x) \\ +\lambda_1 \int_{p}^{p_{\max}} [\beta V(x,x) + (1-\beta) V(p,p)]\, dF(x) &+ \lambda_1 (1-\beta) \int_{t(p)}^{p_{\max}} [V(w_{\min},x) - V(p,p)]\, dF(x).\end{aligned} \tag{6}$$

Comparing this to the corresponding expression (A12) of the worker's value function in the absence of a minimum wage, we see that the last term in (6) reflects the extra amount of rent granted to the worker by the presence of a wage floor. Next imposing $w = p$ and differentiating, one gets:

$$\frac{dV}{dp}(p,p) = \frac{1}{\rho + \delta + \lambda_1 \beta \left[\overline{F}(p) - \overline{F}(t(p))\right] + \lambda_1 \overline{F}(t(p))}. \tag{7}$$

Then integrating by parts in (6) and rearranging:

$$(\rho+\delta)V(w,p) = w + \delta V_0 + \lambda_1(1-\beta)\int_{q(w,p)}^{p} \frac{\overline{F}(x)}{\rho+\delta+\lambda_1\beta\left[\overline{F}(x)-\overline{F}(t(x))\right]+\lambda_1\overline{F}(t(x))}dx$$
$$+\lambda_1\beta\int_{p}^{p_{\max}} \frac{\overline{F}(x)}{\rho+\delta+\lambda_1\beta\left[\overline{F}(x)-\overline{F}(t(x))\right]+\lambda_1\overline{F}(t(x))}dx + \lambda_1(1-\beta)\int_{t(p)}^{p_{\max}} \overline{F}(x)\frac{\partial V}{\partial x}(w_{\min},x)\,dx. \tag{8}$$

As a final manipulation we use the change of variables $x = t(z)$ in the last integral of this latter equation. Since by definition $V(w_{\min},x) = V(z,z)$, we have $\frac{\partial V}{\partial x}(w_{\min},x)\,dx = \frac{dV}{dz}(z,z)\,dz$ and (8) finally becomes:[4]

$$(\rho+\delta)V(w,p) = w + \delta V_0 + \lambda_1(1-\beta)\int_{q(w,p)}^{p} \frac{\overline{F}(x)}{\rho+\delta+\lambda_1\beta\left[\overline{F}(x)-\overline{F}(t(x))\right]+\lambda_1\overline{F}(t(x))}dx$$
$$+\lambda_1\int_{p}^{p_{\max}} \frac{\beta\overline{F}(x)+(1-\beta)\overline{F}(t(x))}{\rho+\delta+\lambda_1\beta\left[\overline{F}(x)-\overline{F}(t(x))\right]+\lambda_1\overline{F}(t(x))}dx. \tag{9}$$

Worker's value: $V(w_{\min},p)$**.** As we already noticed, we also need to consider $V(w_{\min},p)$, even though $w_{\min}$ lies outside the support of a typical firm's wage distribution. Starting again from (6), we first notice that, if paid $w_{\min}$, an employee of a type-p firm is in a position to use *any* outside job offer to renegotiate his/her wage up to at least $\phi_{\min}(p)$. More precisely, because $q[\phi_{\min}(p),p] = t^{-1}(p)$, the worker's wage will be raised to $\phi_{\min}(p)$ by any contact with a firm of type $p' \in \left[p_{\min}, t^{-1}(p)\right]$, and to a wage $\phi(p',p)$ defined by $V[\phi(p',p),p] = \beta V(p,p) + (1-\beta)V(p',p')$ and which is strictly greater than $\phi_{\min}(p)$ upon contacting a firm of type $p' \in \left[t^{-1}(p), p\right]$. Inserting these considerations into (6), we obtain:

$$\left[\rho+\delta+\lambda_1\overline{F}\left(t^{-1}(p)\right)\right]V(w_{\min},p) = w_{\min}+\delta V_0+\lambda_1\int_{t^{-1}(p)}^{p}\left[\beta V(p,p)+(1-\beta)V(x,x)\right]dF(x)$$
$$+\lambda_1\int_{p}^{p_{\max}}\left[\beta V(x,x)+(1-\beta)V(p,p)\right]dF(x)+\lambda_1(1-\beta)\int_{t(p)}^{p_{\max}}\left[V(w_{\min},x)-V(p,p)\right]dF(x)$$
$$+\lambda_1 F\left(t^{-1}(p)\right)\cdot\left(V[\phi_{\min}(p),p]-V(w_{\min},p)\right). \tag{10}$$

Then again integrating by parts and rearranging as we did for the previous case, we arrive at:

$$(\rho+\delta)V(w_{\min},p) = w_{\min}+\delta V_0+\lambda_1(1-\beta)\int_{t^{-1}(p)}^{p} \frac{\overline{F}(x)}{\rho+\delta+\lambda_1\beta\left[\overline{F}(x)-\overline{F}(t(x))\right]+\lambda_1\overline{F}(t(x))}dx$$
$$+\lambda_1\int_{p}^{p_{\max}} \frac{\beta\overline{F}(x)+(1-\beta)\overline{F}(t(x))}{\rho+\delta+\lambda_1\beta\left[\overline{F}(x)-\overline{F}(t(x))\right]+\lambda_1\overline{F}(t(x))}dx+\lambda_1\left(V[\phi_{\min}(p),p]-V(w_{\min},p)\right). \tag{11}$$

As a final step, we can apply (9) to the case $w = \phi_{\min}(p)$ to retrieve $V[\phi_{\min}(p),p]$, then take the difference with (11) to show that:

$$V[\phi_{\min}(p),p]-V(w_{\min},p) = \frac{\phi_{\min}(p)-w_{\min}}{\rho+\delta+\lambda_1}. \tag{12}$$

[4]Note that we write the upper bound of the last integral in (8) as $p_{\max}$, whereas it is in fact $t^{-1}(p_{\max})$. This is merely for convenience—and of course it is licit as the integrand is identically equal to zero for $x \in \left[t^{-1}(p_{\max}), p_{\max}\right]$.

Hence:

$$(\rho+\delta)V(w_{\min},p) = w_{\min}+\delta V_0+\lambda_1(1-\beta)\int_{t^{-1}(p)}^{p} \frac{\overline{F}(x)}{\rho+\delta+\lambda_1\beta\left[\overline{F}(x)-\overline{F}(t(x))\right]+\lambda_1\overline{F}(t(x))}dx$$
$$+\lambda_1\int_{p}^{p_{\max}} \frac{\beta\overline{F}(x)+(1-\beta)\overline{F}(t(x))}{\rho+\delta+\lambda_1\beta\left[\overline{F}(x)-\overline{F}(t(x))\right]+\lambda_1\overline{F}(t(x))}dx+\lambda_1\frac{\phi_{\min}(p)-w_{\min}}{\rho+\delta+\lambda_1}. \quad (13)$$

3 Wages

Generic wage values. Using expression (9) of the worker's value function to substitute into the bargaining outcome (2), we get a generic expression of the mobility wage. For any $p' \leq p$:

$$\phi(p',p) = \beta p+(1-\beta)p'-(1-\beta)^2\lambda_1\int_{p'}^{p} \frac{\overline{F}(x)-\overline{F}(t(x))}{\rho+\delta+\lambda_1\beta\left[\overline{F}(x)-\overline{F}(t(x))\right]+\lambda_1\overline{F}(t(x))}dx$$
$$= p-(1-\beta)\int_{p'}^{p} \frac{\rho+\delta+\lambda_1\overline{F}(x)}{\rho+\delta+\lambda_1\beta\left[\overline{F}(x)-\overline{F}(t(x))\right]+\lambda_1\overline{F}(t(x))}dx. \quad (14)$$

Note that the formal difference from (A17) brought about by the wage floor is the presence of $\overline{F}(t(x))$ in the integrand. In fact the model used in the main text can be seen as a special case of this Appendix where $w_{\min}$ is so low that $t(p) \geq p_{\max}$ for all p.

The expression (14) however only applies when $p \leq t(p')$, i.e. when the minimum wage constraint does not bind in the bargaining process between firms p and p'. In the converse case, i.e. when p is high enough relative to p' that workers would be willing to move from firm p' to firm p for a wage below the institutional minimum, then the mobility wage is $\phi_{\min}(p)$ defined in (3). Combining (9) evaluated at $w = p$ and $w = \phi_{\min}(p)$, (13), and the definition (3), we obtain:

$$\phi_{\min}(p) = w_{\min}+\frac{(\rho+\delta+\lambda_1)\beta}{\rho+\delta+\lambda_1\beta}$$
$$\times\left(p-w_{\min}-\lambda_1(1-\beta)\int_{t^{-1}(p)}^{p} \frac{\overline{F}(x)}{\rho+\delta+\lambda_1\beta\left[\overline{F}(x)-\overline{F}(t(x))\right]+\lambda_1\overline{F}(t(x))}dx\right). \quad (15)$$

Note that this expression is only confounded with $w_{\min}$ if $\beta = 0$.

Entry wages. We finally turn to the specific case of workers who are hired directly from the unemployment pool, and whom we left aside from the analysis up to now. Looking at the implied bargaining outcome (1) for those workers, we see that they receive a wage of $\phi_{\min}(p)$ whenever $V_0 \leq V(w_{\min},p)$, and some (greater) wage $\phi_0(p)$ in the converse case.[5] Again following the same line of arguments as in the no-minimum wage case (Appendix A.2), one can show that this latter wage is defined by $\phi_0(p) = \phi(p_{\inf},p)$, where $p_{\inf}$—the minimum viable productivity—is itself defined in a similar fashion to (A18):

$$p_{\inf} = b+(\lambda_0-\lambda_1)\int_{p_{\inf}}^{p_{\max}} \frac{\beta\overline{F}(x)+(1-\beta)\overline{F}(t(x))}{\rho+\delta+\lambda_1\beta\left[\overline{F}(x)-\overline{F}(t(x))\right]+\lambda_1\overline{F}(t(x))}dx, \quad (16)$$

[5] While this latter configuration is theoretically possible, it may be deemed unlikely that an unemployed "unskilled" worker (i.e. a worker from skill category #3 or 4 in the terminology of this paper) would turn down any job offer at the minimum wage. Our estimation procedure, however, will be valid independently of these issues (see below).

with the additional restriction that $p_{\inf}$ has to exceed $w_{\min}$. Note that $p_{\inf}$ depends on $w_{\min}$ indirectly via $t(\cdot)$ under the integral.

The wage equation: $\mathbb{E}(w|p)$**.** We now require an expression of the within-firm distribution of wages, which we shall ultimately use to derive the theoretical firm-level mean wage $\mathbb{E}(w|p)$, a moment for which we have an empirical counterpart. As a preliminary remark, we should note that the presence of a minimum wage does not affect the rules of job- or wage-mobility: a worker employed at a type-p firm and earning a wage of w would still join any firm of type $p' > p$ upon receiving an offer, and would still renegotiate his/her wage upward upon meeting a firm of type $p' \geq q(w, p)$. As a consequence, the derivation of the various distributions of interest—essentially $\ell(p)$ and $G(w|p)$—follows exactly the same steps as in Appendix A.3, and the expressions of these distributions are formally unchanged.

What is likely to change due to the introduction of $w_{\min}$, however, is the lower support of the within-firm wage distribution. As we saw in the previous sub-section, the entry wage paid by any firm type p to a worker hired from the unemployment pool is $\max\{\phi_{\min}(p), \phi(p_{\inf}, p)\}$. Moreover, a worker hired out of some competitor firm of type $p' < p$ will receive a wage equal to $\max\{\phi_{\min}(p), \phi(p', p)\}$, implying that the lower bound of wage offers made by firm p to a worker it poaches from another firm is $\max\{\phi_{\min}(p), \phi(p_{\min}, p)\}$. So depending on the particular parameter values, various mass points may appear at $\phi(p_{\inf}, p)$, $\phi(p_{\min}, p)$, or $\phi_{\min}(p)$.

Bearing all this in mind, we can now proceed exactly as in Appendix A.4 to derive the firm-level mean wage. The following first step takes account of all possible configurations for $\phi(p_{\inf}, p)$, $\phi(p_{\min}, p)$, and $\phi_{\min}(p)$:

$$\begin{aligned}\mathbb{E}(w|p) = &\int_{\max\{\phi_{\min}(p),\phi(p_{\min},p)\}}^{p} w dG(w|p) \\ &+[G(\max\{\phi_{\min}(p), \phi(p_{\min}, p)\}|p) - G(\max\{\phi_{\min}(p), \phi(p_{\inf}, p)\}|p)]\cdot\max\{\phi_{\min}(p), \phi(p_{\min}, p)\} \\ &+ G(\max\{\phi_{\min}(p), \phi(p_{\inf}, p)\}|p)\cdot\max\{\phi_{\min}(p), \phi(p_{\inf}, p)\}. \quad (17)\end{aligned}$$

Integration by parts then yields:

$$\begin{aligned}\mathbb{E}(w|p) = &\, p - \int_{\max\{\phi_{\min}(p),\phi(p_{\min},p)\}}^{p} G(w|p)\, dw \\ &+G(\max\{\phi_{\min}(p), \phi(p_{\inf}, p)\}|p)\cdot[\max\{\phi_{\min}(p), \phi(p_{\min}, p)\} - \max\{\phi_{\min}(p), \phi(p_{\inf}, p)\}|p]. \quad (18)\end{aligned}$$

Next invoking our "free-entry" assumption $p_{\min} = p_{\inf}$ (see Appendix A.4), we see that the last term in the above equation disappears. Finally using the change of variables $w = \phi(q, p)$, recalling the expression of $G(w|p)$ as a function of $F(\cdot)$ and the parameters that we established in equation (10), and recalling the fact that $q[\phi_{\min}(p), p] = t^{-1}(p)$, we obtain:[6]

$$\begin{aligned}\mathbb{E}(w|p) = &\, p - \left[1 + \kappa_1 \overline{F}(p)\right]^2 \\ &\times \int_{\max\{t^{-1}(p),p_{\min}\}}^{p} \frac{(1-\beta)\left[1 + (1-\sigma)\kappa_1 \overline{F}(q)\right]}{\left(1 + (1-\sigma)\kappa_1\left[\beta\overline{F}(q) + (1-\beta)\overline{F}(t(q))\right]\right)\left(1+\kappa_1\overline{F}(q)\right)^2} dq. \quad (19)\end{aligned}$$

[6]Besides the various additional possible mass points of $G(\cdot|p)$ that have to be taken into account, the only (technical) innovation brought about by the presence of a minimum wage is that the expression for $\frac{\partial \phi}{\partial q}$ which is needed at an intermediate stage of the derivation in (A21) is now slightly more involved and has to be taken from differentiation of (14).

Once again, this definition of $\mathbb{E}(w|p)$ only differs from the no-minimum-wage case (A21) by the presence of $\overline{F}[t(q)]$ in the integral term.[7]

4 Estimation

4.1 Determination of the threshold function $t(\cdot)$

As may be clear from the various expressions derived in the previous subsection, the main complication brought to any estimation/simulation exercise of our model by the presence of a minimum wage is that estimation or simulation involves the determination of the threshold function $t(\cdot)$, of which we only gave an implicit definition in (4) as $V(p,p) = V[w_{\min}, t(p)]$. The difficulty that we face is to come up with an "operational" characterization of $t(\cdot)$.

Combining (4) with the definition (3) of $\phi_{\min}(p)$ and the bargaining outcome (2), one sees that an equivalent characterization of $t(p)$ is

$$\phi_{\min}[t(p)] = \phi(p, t(p)). \tag{20}$$

Substituting (14) and (15) into the latter relationship, one obtains:

$$p - w_{\min} = \lambda_1 \int_p^{t(p)} \frac{\beta F(x) + (1-\beta)\left[\overline{F}(x) - \overline{F}(t(x))\right]}{\rho + \delta + \lambda_1 \beta \left[\overline{F}(x) - \overline{F}(t(x))\right] + \lambda_1 \overline{F}(t(x))} dx, \tag{21}$$

which characterizes $t(p)$ in a very implicit and directly hardly exploitable way. We can nonetheless characterize $t(p)$ more explicitly over certain subsets of the range of productivity parameters $[p_{\min}, p_{\max}]$ by taking the following steps.

First, let us introduce $p_1 = t^{-1}(p_{\max})$. Because p_1 is such that $\overline{F}[t(p)] = 0$ for all $p \geq p_1$, and because the only things we need to be able to compute $\mathbb{E}(w|p)$ from (19) are $\overline{F}[t(p)]$ and $t^{-1}(p)$ when $p \in [p_{\min}, p_{\max}]$, we can safely restrain the determination of $t(\cdot)$ to the interval $[p_{\min}, p_1]$. Moreover, (21) implies the following characterization of p_1:

$$p_1 = w_{\min} + \lambda_1 \int_{p_1}^{p_{\max}} \frac{\beta F(x) + (1-\beta)\overline{F}(x)}{\rho + \delta + \lambda_1 \beta \overline{F}(x)} dx. \tag{22}$$

So the first thing to do, given a values of β and all other parameters, is to determine p_1 by (numerically) solving (22). If it happens to be less than $p_{\min}$, then it implies that the minimum wage is too low to interfere with wage determination in the market at hand. In the converse case, we must still determine $t(\cdot)$ over $[p_{\min}, p_1]$.

To this end, we take the differential form of (21):

$$t'(p) = \frac{1 + \kappa_1 (1-\sigma) \frac{\beta F(p) + (1-\beta)\left[\overline{F}(p) - \overline{F}(t(p))\right]}{1 + \kappa_1(1-\sigma)\left[\beta \overline{F}(p) + (1-\beta)\overline{F}(t(p))\right]}}{\kappa_1 (1-\sigma) \frac{\beta F(t(p)) + (1-\beta)\left[\overline{F}(t(p)) - \overline{F}(t^{(2)}(p))\right]}{1 + \kappa_1(1-\sigma)\left[\beta \overline{F}(t(p)) + (1-\beta)\overline{F}(t^{(2)}(p))\right]}}, \tag{23}$$

where we reintroduce the notation κ_1 and σ introduced in the main text, and where $t^{(2)}(\cdot)$ denotes the composition of $t(\cdot)$ with itself. We want to solve (23) over $[p_{\min}, p_1]$ with initial condition $t(p_1) = p_{\max}$. The difficulty here lies in the presence of $t^{(2)}(\cdot)$ in the r.h.s. of (23), which because of that presence is not an ordinary differential equation. Following the same line

[7] Plus the presence of $t^{-1}(p)$ in the integral's lower bound. However, it is straightforward to see from the definition of $t(p)$ that a low enough $w_{\min}$ implies $t^{-1}(p) \leq p_{\min}$ for all $p \in [p_{\min}, p_{\max}]$.

of ideas as above, we can however define $p_2 = t^{-1}(p_1) = t^{(-2)}(p_{\max})$. This new threshold is such that $\overline{F}\left[t^{(2)}(p)\right] = 0$ for all $p \in [p_2, p_1]$. Hence over this latter interval, (23) simplifies into the following ODE:

$$t'(p) = \frac{1 + \kappa_1 (1-\sigma) \frac{\beta F(p) + (1-\beta)\left[\overline{F}(p) - \overline{F}(t(p))\right]}{1 + \kappa_1(1-\sigma)\left[\beta \overline{F}(p) + (1-\beta)\overline{F}(t(p))\right]}}{\kappa_1 (1-\sigma) \frac{\beta F(t(p)) + (1-\beta)\overline{F}(t(p))}{1+\kappa_1(1-\sigma)\beta \overline{F}(t(p))}}, \tag{24}$$

which we can solve numerically "backward", i.e. starting at p_1, still with the initial condition $t(p_1) = p_{\max}$. The point at which the thus obtained solution reaches p_1 is p_2.

We thus now have a characterization of $t(p)$ for any $p \in [p_2, p_1]$. If p_2 is found to be less than $p_{\min}$, then we have everything we need to compute $\mathbb{E}(w|p)$ from (19).[8] In practice, this will always turn out to be the case, i.e. we will always find $t^{(2)}(p_{\min}) \geq p_{\max}$.

4.2 Checking the impact of the minimum wage on our estimates of the workers' bargaining power

Procedure. We use the following 3-step procedure:

1. Construct $t(\cdot)$ recursively as exposed in the previous subsection using estimates from the no-minimum wage case as parameter values;

2. Re-estimate the various β's by NLS regression of firm-level mean wages on firm productivity using equation (19);

3. Compare the resulting new estimates of β (say $\widehat{\beta}_{w_{\min}}$) with the estimated $\widehat{\beta}$ obtained from the model without a wage floor (see Table 3).

Thus at step 1 of this procedure we construct the threshold function $t(\cdot)$ using a set of parameter estimates obtained from a different model (one without a wage floor) than the "true" model (which has a wage floor at $w_{\min}$). As can be seen from the characterization of $t(\cdot)$ in (21), the parameters involved are the transition parameters (λ_1 and δ—see Table 2 for values), the production function parameters (ξ and the α's—see Table 3 for values) through the firms' productivity types p, and most importantly the bargaining power β. Because the job mobility process implied by our model is unchanged by the introduction of a minimum wage, the transition parameters λ_1 and δ (which were estimated from job spell durations only—see Section 3.3 in the main text) are also unaffected by the minimum wage and there is no reason indeed to re-estimate them. As for the production function parameters, these were estimated jointly with the bargaining power using the wage equation (see Section 3.4) and as such are likely to be affected by the introduction of $w_{\min}$, which changes the form of the wage equation somewhat. However, we checked the consistency of our estimated of the ξ's and α's based on the wage equation with direct GMM estimates of the production function in which no wage data is used (see Appendix B). Hence we deem it legitimate to consider the values of the ξ's and α's reported in Table 3 as still valid within the model with a minimum wage.

What is a priori more problematic is to take up the estimates of β from Table 3 to construct the $t(\cdot)$ function. Because β is precisely the parameter that we end up re-estimating in the

[8] If not, then in principle we could go for more interations, i.e. define $p_3 = t^{(-3)}(p_{\max})$, solve (23) over $[p_3, p_2]$ using our knowledge of $t^{(2)}([p_3, p_2]) = t([p_2, p_1])$, and so on until we reach a number n such that $t^{(-n)}(p_{\max}) \leq p_{\min}$.

presence of a minimum wage, and because any estimation of β obviously heavily relies on the particular form of the wage equation, the $t(\cdot)$ function constructed at step 1 is merely an approximation of the "true" threshold $t(\cdot)$. Comparison of the "old" estimates $\widehat{\beta}$ with the "updated" estimates $\widehat{\beta}_{w_{\min}}$ at step 3 will thus tell us how good or bad an approximation we made at step 1.

Results. We report the two sets of estimates in Table 1.[9] The striking result is that $\widehat{\beta}$ and $\widehat{\beta}_{w_{\min}}$ are always very close,[10] meaning that (1) our approximation of the threshold function $t(\cdot)$ at step 1 of the procedure is very reasonable, and (2) that taking account of minimum wages leaves our conclusions about the distribution of bargaining power parameters across skill categories virtually unchanged. In particular, we confirm our conclusion that low-skill labor categories have no other source of rent than between-firm competition for their services.

TABLE 1: The impact of minimum wages
on bargaining power estimates

Industry	Labor category	$\widehat{\beta}$ (from Table 3)	$\widehat{\beta}_{w_{\min}}$
Manufacturing	1	0.35	0.35
	2	0.13	0.13
	3	0.00	0.00
	4	0.00	0.00
Construction	1	0.98	0.98
	2	0.26	0.26
	3	0.15	0.14
	4	0.17	0.07
Trade	1	0.38	0.37
	2	0.33	0.26
	3	0.14	0.07
	4	0.00	0.00
Services	1	0.16	0.15
	2	0.00	0.00
	3	0.08	0.00
	4	0.00	0.00

[9] In the reported exercise we set total annual labor cost at the minimum wage level to equal 95,000 FRF (14,483 Euros), which is close to the 1993 figure. It is a bit on the high side of the average figure for the whole 1993-2000 period, due to the various payroll tax cuts that were enacted in France in the second half of the 1990s.

[10] A close look reveals that the point estimates $\widehat{\beta}_{w_{\min}}$ are marginally smaller than $\widehat{\beta}$ in some cases (mostly low-skill categories), yet at our level of precision these differences can probably be considered negligible.